International Journal of Innovative Computing, Information and Control
Volume **14**, Number **1**, February **2018** ISSN 1349-4198
pp. 111–126

NONLINEAR DISTURBANCE OBSERVER-BASED BACKSTEPPING CONTROL FOR A DUAL EXCITATION AND STEAM-VALVING SYSTEM OF SYNCHRONOUS GENERATORS WITH EXTERNAL DISTURBANCES

Adirak Kanchanaharuthai[1] and Ekkachai Mujjalinvimut[2]

[1]Department of Electrical Engineering
Rangsit University
Mueang Pathum Thani, Pathum Thani, Bangkok 12000, Thailand
adirak@rsu.ac.th

[2]Department of Electrical Engineering
Faculty of Engineering
King Mongkut's University of Technology Thonburi
Pracha Uthit Road, Bang Mod, Bangkok 10140, Thailand
ekkachai.muj@kmutt.ac.th

Received June 2017; revised October 2017

Abstract. *This paper presents a nonlinear stabilizing state feedback control for a dual-excited and steam-valving system of synchronous generators via a backstepping strategy combined with a nonlinear disturbance design. The disturbance observer is used to estimate unavoidably external disturbances. The resulting controller is employed to stabilize the system stability and reject undesired external disturbances. In order to demonstrate the effectiveness of the developed design, numerical simulation results are provided to illustrate that the presented control can improve dynamic performances, rapidly suppress system oscillations of the overall closed-loop dynamics, and despite having inevitably external disturbances, performs better than two conventional nonlinear control techniques: a backstepping design and an integral backstepping design.*
Keywords: Dual excitation and steam-valving system, Backstepping control, Nonlinear disturbance observer, Integral backstepping control

1. **Introduction.** It is well known that improving power system stability has attracted a lot of researcher's attention due to power systems with rapid increase of the size and complexity. Additionally, it becomes a key issue of the operation of modern power systems. Thus, there are currently a lot of effects to find high-performance stabilizing controllers which are able to mitigate the results from many severe contingencies such as voltage collapse, islanding faults, and loss of synchronism. The excitation control of synchronous generators is an effective way for transient stability enhancement of power systems [1]. Further, a variety of excitation control strategies have been proposed [1-5]. Besides from the excitation control, a steam-valving control scheme [6-8] is also a promising and effective way capable of stabilizing the overall closed-loop dynamics of synchronous generators and effectively accomplishing the control performances. However, it was found that the power system stability and operation were greatly improved with the help of a combination of generation excitation control with steam-valving control. This coordination has received a great attention in the power engineering community [9-11]. This strategy can offer an opportunity to increase a degree of freedom for achieving further desired control performances.

So far, the combination above has concentrated on the coordination of single-excited and steam-valving control. This means that the d-axis field voltage is regarded as a constant throughout consideration; nevertheless the only q-axis field voltage is utilized to achieve the desired control objectives. Thus, a coordination of d-axis and q-axis field voltages, called dual-excitation is a more effective method than the coordination of single-excited and steam-valving control. Also, this method is a promising idea of increasing greater flexibility for the system stability enhancement. Roughly speaking, this scheme can be regarded as an inclusion of the additional degree of freedoms and offers an opportunity to determine the effective control on account of both d-axis and q-axis field windings which are separately designed.

To the best knowledge of the authors, there is less attention devoted to the coordination of dual-excited and steam-valving control of synchronous generators [12-14]. With the help of a coordinated passivation scheme [12], a nonlinear control was presented to demonstrate hat dynamic performances were considerably improved superior to the feedback linearizing scheme. A design procedure based on an immersion and invariance method for a nonlinear feedback stabilizing control law was report in [13,14]. This scheme has provided an opportunity to achieve power angle stability along with frequency and voltage regulation, and to ensure that the closed-loop system dynamics are transiently and asymptotically stable. In those works, the resulting controller performed better than a coordinate passivation controller [12] in terms of the rapid damping of oscillations in all time responses following small or large disturbance. However, even if the I&I control design has presented the high-performance stabilizing controller and was applicable for many types of practical systems, it had several disadvantages such as no systematic way to select the mapping, a target dynamical system, and an energy function, respectively.

It has been found that in practice, most engineering systems have often disturbances capable of degrading inevitably the desired control performances of the closed-loop dynamics. The disturbances considered include external disturbances, parametric uncertainties and other unknown nonlinear terms. Therefore, the desired control design method needs to include the disturbance dynamics to reject the effects of the abovementioned disturbances. Recently, a disturbance observer method is an approach for compensating the result from external disturbances and mismatched disturbances/uncertainties. This method has been widely accepted in compensating the effects of disturbances. The disturbance observer is utilized to estimate disturbances appearing in the system. There are currently the development of disturbance observer design combined with most popular nonlinear control methods such as backstepping method [15] and sliding mode method [16], as presented in [16-20]. Based on the abovementioned references, disturbance observer-based control is a promising method capable of rejecting external disturbances and improving robustness against uncertainties [16] simultaneously. It also provides an effective way to handle external disturbances and system uncertainties. Additionally, disturbance observer design method can be further extended to several problems in control system societies, such as adaptive control [21], finite-time control [22], and tracking control [23]. Further, this method can be successfully applied for numerous kinds of real engineering systems such as flight control systems [16], permanent magnet synchronous motors [16], airbreathing hypersonic vehicle systems [17], power systems [18], and electrohydrolic actuator systems [23]. Those indicate important application potentials of the disturbance observer-based control method to deal with the effect of unavoidably external disturbances.

However, even though the control design methods presented in [12-14] have good control performances, external disturbances and uncertainties have not been taken into account before. Disturbances and uncertainties arising in the system inevitably may lead to undesired control performances, and eventually make the system unstable.

This paper still continues this line of investigation, but a systematic procedure to synthesize a nonlinear feedback stabilizing control law on the basis of a backstepping control [15] combined with the disturbance observer design [16] is developed to cope with the effects of external disturbances.

Therefore, the primary contributions of this work lie in that:

- The use of a nonlinear disturbance observer-based backstepping control strategy to stabilize the system in the presence of external disturbances has not been investigated before;
- The overall closed-loop system is input-to-state in spite of having external disturbances;
- In comparison with a backstepping control and an integral backstepping control, the developed control law offers better dynamic performances and a satisfactory disturbance rejection ability.

The rest of this paper is organized as follows. A dynamic model of a dual-excited and steam-valving system of synchronous generators is briefly presented, and some significant lemmas together with the problem statement are given in Section 2. Controller design and stability analysis are developed in Section 3 while simulation results are stated in Section 4. Finally, in Section 5, a conclusion is given.

2. Power System Model Description and Preliminaries.

2.1. Power system models.

In this subsection, a dynamic model of a synchronous generator with dual-excited and steam-valving controller can be obtained as follows:

$$\begin{cases} \dot{\delta} = \omega - \omega_s + \bar{d}_1(t), \\ \dot{\omega} = \dfrac{1}{M}\left(P_m - \dfrac{E'_q}{X'_{d\Sigma}}V_\infty \sin\delta - \dfrac{E'_d}{X'_{q\Sigma}}V_\infty\cos\delta - \dfrac{X'_{d\Sigma}-X'_{q\Sigma}}{2X'_{d\Sigma}X'_{q\Sigma}}V_\infty^2\sin 2\delta - D(\omega-\omega_s)\right) \\ \qquad + \bar{d}_2(t), \\ \dot{P}_m = -\dfrac{P_m - P_{me}}{T_{H\Sigma}} + \dfrac{C_H}{T_{H\Sigma}}u_G + \bar{d}_3(t), \\ \dot{E}'_q = -\dfrac{X_{d\Sigma}}{X'_{d\Sigma}T'_{d0}}E'_q + \dfrac{(X_{d\Sigma}-X'_{d\Sigma})}{X'_{d\Sigma}T'_{d0}}V_\infty\cos\delta + \dfrac{u_{fd}}{T'_{d0}} + \bar{d}_4(t), \\ \dot{E}'_d = -\dfrac{X_{q\Sigma}}{X'_{q\Sigma}T'_{q0}}E'_d - \dfrac{(X_{q\Sigma}-X'_{q\Sigma})}{X'_{q\Sigma}T'_{q0}}V_\infty\sin\delta + \dfrac{u_{fq}}{T'_{q0}} + \bar{d}_5(t), \end{cases} \tag{1}$$

where δ is the power angle of the generator, ω denotes the relative speed of the generator, $D \geq 0$ is a damping constant, and E'_q and E'_d are the q-axis and d-axis internal transient voltages, respectively. X'_d and X'_q are the d-axis and q-axis transient reactances, respectively. P_e is the electrical power delivered by the generator to the voltage at the infinite bus V_∞, ω_s is the synchronous machine speed, $\omega_s = 2\pi f$, H represents the per unit inertial constant, f is the system frequency and $M = 2H/\omega_s$. $X'_{d\Sigma} = X'_d + X_T + X_L$ is the reactance consisting of the direct axis transient reactance of SG, the reactance of the transformer, and the reactance of the transmission line X_L. Similarly, $X_{d\Sigma} = X_d + X_T + X_L$ is identical to $X'_{d\Sigma}$ except that X_d denotes the direct axis reactance of SG. $X'_{q\Sigma}$ and $X_{q\Sigma}$ denote the q-axis reactances similar to d-axis reactance. T'_{d0} and T'_{q0} are the d-axis and q-axis transient open-circuit time constants. u_{fd} and u_{fq} are the q-axis and d-axis field voltage control inputs to be designed, respectively. P_{me} is the initial value of mechanical power, and C_H is the assigned coefficient of high-pressure cylinder. $T_{H\Sigma}$ is the equivalent time constant of steam valve control systems. u_G is the steam-valving control input to be designed. $\bar{d}_j(t)$, $(j = 1, 2, 3, 4, 5)$ are external disturbances and system parameter variations.

For convenience, let us define new state variables as follows:

$$\begin{cases} x_1 = \delta - \delta_e, \\ x_2 = \omega - \omega_s, \\ x_3 = P_m - P_{me}, \\ x_4 = \dfrac{E'_q V_\infty \sin(x_1 + \delta_e) - E'_{qe} V_\infty \sin \delta_e}{X'_{d\Sigma}} + m(\sin 2(x_1 + \delta_e) - \sin 2\delta_e), \\ x_5 = \dfrac{E'_d V_\infty \cos(x_1 + \delta_e) - E'_d V_\infty \cos \delta_e}{X'_{q\Sigma}}, \end{cases} \tag{2}$$

where $m = \frac{X'_{d\Sigma} - X'_{q\Sigma}}{2X'_{d\Sigma} X'_{q\Sigma}}$.

Subsequently, after differentiating the state variables (2), we have the dynamic model of the dual excitation and steam-valving system of synchronous generators with lumped disturbances can be expressed as an affine nonlinear system as follows:

$$\dot{x} = f(x) + g(x)u(x) + d(t), \tag{3}$$

where

$$\begin{cases} f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{1}{M}(x_3 - Dx_2 - x_4 - x_5) \\ \dfrac{P_{me} - x_3}{T_{H\Sigma}} \\ \left(-a_q E'_q + b_q \cos(x_1 + \delta_e)\right) \dfrac{V_\infty \sin(x_1 + \delta_e)}{X'_{d\Sigma}} \\ \left(-a_d E'_d - b_d \sin(x_1 + \delta_e)\right) \dfrac{V_\infty \cos(x_1 + \delta_e)}{X'_{q\Sigma}} \end{bmatrix}, \\ g(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31}(x) & 0 & 0 \\ 0 & g_{42}(x) & 0 \\ 0 & 0 & g_{53}(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \dfrac{V_\infty \sin(x_1 + \delta_e)}{X'_{d\Sigma}} & 0 \\ 0 & 0 & \dfrac{V_\infty \cos(x_1 + \delta_e)}{X'_{q\Sigma}} \end{bmatrix}, \\ u(x) = \begin{bmatrix} \dfrac{C_H}{T_{H\Sigma}} u_G \\ \dfrac{u_{fd}}{T'_{d0}} \\ \dfrac{u_{fq}}{T'_{q0}} \end{bmatrix}, \; d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \\ d_4(t) \\ d_5(t) \end{bmatrix} = \begin{bmatrix} \bar{d}_1(t) \\ \bar{d}_2(t) \\ \bar{d}_3(t) \\ \dfrac{V_\infty \sin(x_1 + \delta_e)}{X'_{d\Sigma} T'_{d0}} \bar{d}_4(t) \\ \dfrac{V_\infty \cos(x_1 + \delta_e)}{X'_{q\Sigma} T'_{q0}} \bar{d}_5(t) \end{bmatrix}, \end{cases} \tag{4}$$

where $a_q = \frac{X_{d\Sigma}}{X'_{d\Sigma} T'_{d0}}$, $a_d = \frac{X_{q\Sigma}}{X'_{q\Sigma} T'_{q0}}$, $b_q = \frac{(X_{d\Sigma} - X'_{d\Sigma})}{X'_{d\Sigma} T'_{d0}} V_\infty$, $b_d = \frac{(X_{q\Sigma} - X'_{q\Sigma})}{X'_{q\Sigma} T'_{q0}} V_\infty$. The region of operation is defined in the set $\mathcal{D} = \{x \in \mathcal{S} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} |\ 0 < x_1 < \frac{\pi}{2}\}$. The open loop operating equilibrium is denoted by $x_e = [0,\ 0,\ 0,\ 0,\ 0]^T$.

For the sake of simplicity, the power system considering (3) and (4) can be expressed as follows.

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t), \\ \dot{x}_2 = \dfrac{1}{M}(x_3 - Dx_2 - x_4 - x_5) + d_2(t), \\ \dot{x}_3 = f_3(x) + g_{31}(x)\dfrac{C_H}{T_{H\Sigma}}u_G + d_3(t), \\ \dot{x}_4 = f_4(x) + g_{42}(x)\dfrac{u_{fd}}{T'_{d0}} + d_4(t), \\ \dot{x}_5 = f_5(x) + g_{53}(x)\dfrac{u_{fq}}{T'_{q0}} + d_5(t). \end{cases} \tag{5}$$

Assumption 2.1. *The external disturbances $d_j(t)$, $(j = 1, 2, 3, 4, 5)$ are bounded. Additionally, the first derivatives of the disturbances above are also bounded.*

Remark 2.1. *The assumption above is often employed in the disturbance estimator based control design technique for real engineering applications* [16]*, such as airbreathing hypersonic vehicle systems* [17]*, power systems* [18]*, and electrohydrolic actuator systems* [23]*. In particular, the derivative of the disturbance will appear in the estimation error equation (10). This assumption is necessary to be used to analyze the overall closed-loop system stability as given in Section 3.3.*

2.2. **Preliminaries.** In this subsection, some important lemmas are mentioned as follows for convenience of the reader. Consider the following system

$$\dot{x} = f(t, x, u),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m. \tag{6}$$

Definition 2.1. [24] *A continuous function $\alpha : [0, a) \to [0, +\infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$ if $a = +\infty$ and $a(r) \to +\infty$ as $r \to +\infty$.*

Lemma 2.1. [24] *Let $V : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that*

$$\alpha_1(||x||) \le V(t, x) \le \alpha_2(||x||)$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \le -W_3(x),\ \forall ||x|| \ge \rho(||u||) > 0,$$

for all $(t, x, u) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m$, where α_1 and α_2 are class $\mathcal{K}$ functions, ρ is a class $\mathcal{K}$ function, and $W_3(x)$ is a continuous positive definite function on $\mathbb{R}^n$. Then, system (6) is input-to-state stable (ISS).

Lemma 2.2. [24] *Consider the following system (6). If the following conditions are satisfied*

- *system $\dot{x} = f(t, x, u)$ is globally input-to-state stable.*
- $\lim_{t\to+\infty} u = 0$.

then the states of the system (5) will asymptotically converge to zero, that is, $\lim_{t\to+\infty} x = 0$.

Remark 2.2. *It is observed that the second subsystem of the system (5) depends upon the state variables x_3, x_4 and x_5. Therefore, the dynamic equations considered are not the strict-feedback form. The backstepping design presented in this work needs to be extended for the nonlinear control design for non-strict-feedback form.*

Problem statement: The control objective of this paper is to solve the problem of the stabilization of the system (5) with the external disturbances d, which can be formulated as follows: with the help of the nonlinear disturbance observer-based backstepping control technique [17], to design a stabilizing (state) feedback controller $u(x)$ and disturbance estimation $\hat{d}$ as follows:

$$\begin{cases} u = \phi\left(x, \hat{d}\right) \\ \dot{\hat{d}} = \varphi\left(x, u, \hat{d}\right) \end{cases} \tag{7}$$

such that the overall closed-loop systems (5) and (7) are input-to-state stable, where $\hat{d}$ is the estimate of d.

For the developed design procedure in the next section, a combination of the backstepping strategy and disturbance observe design will be presented to obtain a composite nonlinear controller (7). In comparison with the conventional backstepping method, the proposed approach will introduce the disturbance estimation terms into virtual control variables. These terms are also used for compensating the external disturbances at each step, and the estimation error dynamics are included for the closed-loop stability analysis.

3. **Controller Design and Stability Analysis.**

In this section, we aim at deriving the control laws for stabilizing the dual-excitation and steam-valving system of synchronous generators. Our proposed design process can be divided into three subsections.

- The first subsection consists of designing the nonlinear disturbance observer to online identify the unknown, but bounded, disturbances.
- The second subsection presents a design procedure combining the backstepping control law with the disturbance estimator introduced into the virtual control laws in each design step.
- Based on Lyapunov stability arguments, the overall closed-loop system in the presence of external disturbance is investigated in the final subsection. Also, the resulting controller can achieve stability and performance specifications.

3.1. **Nonlinear disturbance observer design.**

In accordance with [16-18] in order to guarantee the control performance of the system (5), the nonlinear disturbance observer can be designed as

$$\begin{cases} \hat{d}_i = \lambda_i(x_j - p_j), \ j = 1, 2, 3, 4, 5, \\ \dot{p}_1 = f_1(x) + \hat{d}_1, \\ \dot{p}_2 = f_2(x) + \hat{d}_2, \\ \dot{p}_3 = f_3(x) + g_{31}(x)\dfrac{C_H}{T_{H\Sigma}}u_G + \hat{d}_3, \\ \dot{p}_4 = f_4(x) + g_{42}(x)\dfrac{u_{fd}}{T'_{d0}} + \hat{d}_4, \\ \dot{p}_5 = f_5(x) + g_{53}(x)\dfrac{u_{fq}}{T'_{q0}} + \hat{d}_5, \end{cases} \tag{8}$$

where $\lambda_j > 0$ is a design parameter. Thus, based on (8) the disturbance estimation dynamics can be expressed in the following form:

$$\dot{\hat{d}}_j = \lambda_i\left(\dot{x}_j - \dot{p}_j\right) = \lambda\left(d_j - \hat{d}_j\right), \ j = 1, 2, 3, 4, 5. \tag{9}$$

Let us define the disturbance estimation error as $e_j = d_j - \hat{d}_j$, and we have the estimation error dynamics as follows.

$$\dot{e}_j = -\lambda_j e_j + \dot{d}_j. \tag{10}$$

3.2. **Backstepping design.** According to the concept reported in [18], the stabilization problem for the system (5) is solved by designing a backstepping control. The design procedures are developed step by step as follows.

Step 1: Considering, the first subsystem (5), a Lyapunov function is selected as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}e_1^2, \tag{11}$$

where $z_1 = x_1$. Then the time derivative of V_1 along the system trajectories becomes

$$\begin{aligned}\dot{V}_1 &= z_1(x_2 + d_1) + e_1(-\lambda_1 e_1 + \dot{d}_1)\\ &= -\lambda_1 e_1^2 + z_1 x_2 + z_1 d_1 + e_1 \dot{d}_1\\ &= -\lambda_1 e_1^2 + z_1 x_2^* + z_1(x_2 - x_2^*) + z_1 d_1 + e_1 \dot{d}_1.\end{aligned} \tag{12}$$

From (12), it is seen that x_2^* is regarded as the virtual control variable with the disturbance estimate $\hat{d}_1$ as follows.

$$x_2^* = -\left(k_1 + \frac{1}{4\epsilon_1}\right)z_1 - \hat{d}_1, \tag{13}$$

where $k_1 > 0$ and $\epsilon_1 > 0$. After substituting (13) into (12), we have

$$\begin{aligned}\dot{V}_1 &= -\left(k_1 + \frac{1}{4\epsilon_1}\right)z_1^2 - \lambda_1^2 e_1^2 + z_1 e_1 + z_1 z_2 + e_1 \dot{d}_1\\ &= -\left(k_1 + \frac{1}{4\epsilon_1}\right)z_1^2 - \lambda_1^2 e_1^2 + \frac{1}{4\epsilon_1}z_1^2 + \epsilon_1 e_1 + z_1 z_2 + e_1 \dot{d}_1\\ &= -k_1 z_1^2 - (\lambda_1 - \epsilon_1)e_1^2 + z_1 z_2 + e_1 \dot{d}_1,\end{aligned} \tag{14}$$

where $z = x_2 - x_2^*$.

Step 2: Let us define the Lyapunov function of Step 1 as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}e_2^2. \tag{15}$$

Then the time derivative of V_2 along the system trajectories is as follows:

$$\begin{aligned}\dot{V}_2 &= -k_1 z_1^2 - (\lambda_1 - \epsilon_1)e_1^2 + z_1 z_2 + e_1 \dot{d}_1 + e_2 \dot{d}_2\\ &\quad + z_2\left(\frac{1}{M}(x_3 - Dx_2 - x_4 - x_5) + d_2 - \frac{\partial x_2^*}{\partial z_1}(x_2 + d_1) - \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\right)\\ &= -k_1 z_1^2 - (\lambda_1 - \epsilon_1)e_1^2 - \lambda_2 e_2^2 + z_1 z_2 + \frac{1}{M}z_2 x_3^* - \frac{1}{M}z_2 x_4^* - \frac{1}{M}z_2 x_5^* - z_2\frac{\partial x_2^*}{\partial z_1}x_2\\ &\quad + z_2\left[\frac{1}{M}\left(x_3 - x_3^* - Dx_2 - (x_4 - x_4^*) - (x_5 - x_5^*) + d_2 - \frac{\partial x_2^*}{\partial z_1}(x_2 + d_1)\right)\right]\\ &\quad - z_2\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1 + e_1 \dot{d}_1 + e_2 \dot{d}_2.\end{aligned} \tag{16}$$

From (16), it can be observed that x_3^*, x_4^* and x_5^* are considered as the virtual control variables with the disturbance estimates $\hat{d}_1$ and $\hat{d}_2$ as follows.

$$\begin{cases} x_3^* = \dfrac{M}{3}\Bigg[-\left(k_2 + \dfrac{1}{4\epsilon_2} + \hat{c}_2\right) z_2 - z_1 + \dfrac{Dx_2}{M} - \hat{d}_2 + \dfrac{\partial x_2^*}{\partial z_1}\left(x_2 + \hat{d}_1\right)\Bigg], \\ x_4^* = -x_3^*, \\ x_5^* = -x_3^*, \end{cases} \tag{17}$$

where $k_2 > 0$, $\epsilon_2 > 0$ and $\hat{c}_2 = \frac{1}{4\epsilon_2}\left(\frac{\partial x_2^*}{\partial z_1} + \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1\right)$. After substituting (17) into (16), we obtain

$$\begin{aligned} \dot{V}_2 =& -k_1 z_1^2 - \lambda_1^2 e_1^2 - \left(k_2 + \frac{1}{4\epsilon_2} + \hat{c}_2\right) z_2^2 - \lambda_2 e_2^2 + \frac{1}{M} z_2(x_3 - x_3^*) - \frac{1}{M} z_2(x_4 - x_4^*) \\ & - \frac{1}{M} z_2(x_5 - x_5^*) + z_2 e_2 - z_2\left(\frac{\partial x_2^*}{\partial z_1} + \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1\right) e_1 + e_1 \dot{d}_1 + e_2 \dot{d}_2. \end{aligned} \tag{18}$$

Based on Young inequality, it can be straightforwardly computed of the terms in (18) as

$$e_2 z_2 \leq \frac{1}{4\epsilon_2} z_2^2 + \epsilon_2 e_2^2 \tag{19}$$

$$-z_2\left(\frac{\partial x_2^*}{\partial z_1} + \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1\right) e_1 \leq \frac{1}{4\epsilon_2}\left(\frac{\partial x_2^*}{\partial z_1} + \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1\right)^2 z_2^2 \epsilon_1^2 = \hat{c}_2 z_2^2 + \epsilon_1 e_1^2. \tag{20}$$

Substituting (19) and (20) into (18) and then defining $z_i = x_i - x_i^*$, $i = 3, 4, 5$, we get

$$\dot{V}_2 \leq -k_1 z_1^2 - k_2 z_2^2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - \epsilon_2) e_2^2 + \frac{z_2}{M}(z_3 - z_4 - z_5). \tag{21}$$

Step 3: we select a Lyapunov function as follows:

$$V_3 = V_2 + \frac{1}{2}\sum_{i=3}^{5}\left(z_i^2 + e_i^2\right). \tag{22}$$

After taking derivatives of both sides of (22), one has

$$\begin{aligned} \dot{V}_3 =& \dot{V}_2 + \sum_{i=3}^{5}(z_i \dot{z}_i + e_i \dot{e}_i) \\ =& -k_1 z_1^2 - k_2 z_2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - \epsilon_2) e_2^2 + \frac{z_2}{M}(z_3 - z_4 - z_5) \\ & + \sum_{i=3}^{5}\left[z_i\left(\dot{x}_i - \frac{\partial x_i^*}{\partial z_1}\dot{z}_1 - \frac{\partial x_i^*}{\partial z_2}\dot{z}_2 - \frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1 e_1 - \frac{\partial x_i^*}{\partial \hat{d}_2}\lambda_2 e_2\right) - \lambda_i e_i^2 + e_i \dot{d}_i \right]. \end{aligned} \tag{23}$$

Substituting $\dot{x}_i$, $(i = 3, 4, 5)$ from (5), $\dot{z}_1$, $\dot{z}_2$ and x_2^* into (23) yields

$$\begin{aligned} \dot{V}_3 \leq& -k_1 z_1^2 - k_2 z_2^2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - \epsilon_2) e_2^2 - \sum_{i=3}^{5}\left(\lambda_i e_i^2 - e_i \dot{d}_i\right) \\ & + z_3\Bigg[\frac{z_2}{M} + f_3(x) + g_{31}(x)\frac{C_H}{T_{H\Sigma}} u_G + d_3 - \frac{\partial x_3^*}{\partial z_1}\left(x_2 + \hat{d}_1\right) - \frac{\partial x_3^*}{\partial z_2}(f_2(x) + d_2) \\ & + \frac{\partial x_3^*}{\partial z_2}\frac{\partial x_2^*}{\partial x_1}(x_2 + d_1) - \frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\Bigg] - z_3\left(\frac{\partial x_3^*}{\partial \hat{d}_1}\lambda_1 e_1 + \frac{\partial x_3^*}{\partial \hat{d}_2}\lambda_2 e_2\right) \\ & + z_4\Bigg[-\frac{z_2}{M} + f_4(x) + g_{42}(x)\frac{u_{fd}}{T'_{d0}} + d_4 - \frac{\partial x_4^*}{\partial z_1}\left(x_2 + \hat{d}_1\right) - \frac{\partial x_4^*}{\partial z_2}(f_2(x) + d_2) \end{aligned}$$

$$+\frac{\partial x_4^*}{\partial z_2}\frac{\partial x_2^*}{\partial x_1}(x_2+d_1)-\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\Bigg]-z_4\left(\frac{\partial x_4^*}{\partial \hat{d}_1}\lambda_1 e_1+\frac{\partial x_4^*}{\partial \hat{d}_2}\lambda_2 e_2\right)$$
$$+z_5\Bigg[-\frac{z_2}{M}+f_5(x)+g_{53}(x)\frac{u_{fq}}{T'_{q0}}+d_5-\frac{\partial x_5^*}{\partial z_1}\left(x_2+\hat{d}_1\right)-\frac{\partial x_5^*}{\partial z_2}(f_2(x)+d_2)$$
$$+\frac{\partial x_5^*}{\partial z_2}\frac{\partial x_2^*}{\partial x_1}(x_2+d_1)-\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\Bigg]-z_5\left(\frac{\partial x_4^*}{\partial \hat{d}_1}\lambda_1 e_1+\frac{\partial x_4^*}{\partial \hat{d}_2}\lambda_2 e_2\right). \tag{24}$$

From (24), in order to achieve the desired control performance, we choose the control law as follows:

$$\begin{cases} \dfrac{C_H}{T_{H\Sigma}}u_G=\dfrac{1}{g_{31}(x)}\Bigg[-\dfrac{z_2}{M}-f_3(x)-\hat{d}_3+\dfrac{\partial x_3^*}{\partial z_1}\left(x_2+\hat{d}_1\right)+\dfrac{\partial x_3^*}{\partial z_2}\Bigg(f_2(x)+\hat{d}_2 \\ \qquad -\dfrac{\partial x_2^*}{\partial z_1}\left(x_2+\hat{d}_1\right)\Bigg)-\left(k_3+\dfrac{1}{4\epsilon_3}+\hat{c}_{31}+\hat{c}_{32}+\hat{c}_{33}+\hat{c}_{34}\right)z_3\Bigg] \\ \dfrac{u_{fd}}{T'_{d0}}=\dfrac{1}{g_{42}(x)}\Bigg[\dfrac{z_2}{M}-f_4(x)-\hat{d}_4+\dfrac{\partial x_4^*}{\partial z_1}\left(x_2+\hat{d}_1\right)+\dfrac{\partial x_4^*}{\partial z_2}\Bigg(f_2(x)+\hat{d}_2 \\ \qquad -\dfrac{\partial x_2^*}{\partial z_1}\left(x_2+\hat{d}_1\right)\Bigg)-\left(k_4+\dfrac{1}{4\epsilon_4}+\hat{c}_{41}+\hat{c}_{42}+\hat{c}_{43}+\hat{c}_{44}\right)z_4\Bigg] \\ \dfrac{u_{fq}}{T'_{q0}}=\dfrac{1}{g_{53}(x)}\Bigg[\dfrac{z_2}{M}-f_5(x)-\hat{d}_5+\dfrac{\partial x_5^*}{\partial z_1}\left(x_2+\hat{d}_1\right)+\dfrac{\partial x_5^*}{\partial z_2}\Bigg(f_2(x)+\hat{d}_2 \\ \qquad -\dfrac{\partial x_2^*}{\partial z_1}\left(x_2+\hat{d}_1\right)\Bigg)-\left(k_5+\dfrac{1}{4\epsilon_5}+\hat{c}_{51}+\hat{c}_{52}+\hat{c}_{53}+\hat{c}_{54}\right)z_5\Bigg], \end{cases} \tag{25}$$

where $\hat{c}_{i1}=\frac{1}{4\epsilon_1}\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)^2$, $\hat{c}_{i2}=\frac{1}{4\epsilon_2}\left(\frac{\partial x_i^*}{\partial z_2}+\frac{\partial x_i^*}{\partial \hat{d}_2}\lambda_2\right)^2$, $\hat{c}_{i3}=\frac{1}{4\epsilon_1}\left[\frac{\partial x_i^*}{\partial z_2}\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)\right]^2$, $\hat{c}_{i4}=\frac{1}{4\epsilon_2}\left[\frac{\partial x_i^*}{\partial x_2}\right]^2$, $i=3,4,5$.

Remark 3.1. *In accordance with the results presented in* [18], *some auxiliary terms* $\hat{c}_2$, $\hat{c}_{i1},\dots,\hat{c}_{i4}$ *are introduced into the virtual state variables (17) and the final controller (25) to deal with the crossing terms arising from the effect of disturbances, compensation errors, and system states. On the other hand, these auxiliary terms are not included in the conventional backstepping method, thereby leading to unsatisfactory control performances.*

Substituting the presented control law (25) into (24), we have

$$\begin{aligned}\dot{V}_3 =& -k_1z_1^2-k_2z_2^2-(\lambda_1-2\epsilon_1)e_1^2-(\lambda_2-\epsilon_2)e_2^2-\sum_{i=3}^{5}\left(\lambda_i e_i^2-e_i\dot{d}_i\right) \\ &+\sum_{i=3}^{5}z_i\Bigg[e_i-\frac{\partial x_i^*}{\partial z_1}e_1-\frac{\partial x_i^*}{\partial z_2}e_2+\frac{\partial x_i^*}{\partial z_2}\frac{\partial x_2^*}{\partial z_1}e_1-\frac{\partial x_i^*}{\partial x_2}e_2 \\ &+\left(k_i+\frac{1}{4\epsilon_1}+\hat{c}_{i1}+\hat{c}_{i2}+\hat{c}_{i3}+\hat{c}_{i4}\right)z_i\Bigg] \\ &-\sum_{i=3}^{5}z_i\left[\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1 e_1+\frac{\partial x_i^*}{\partial \hat{d}_2}\lambda_2 e_2+\frac{\partial x_i^*}{\partial z_2}\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\right].\end{aligned} \tag{26}$$

It is observed that some terms of the last two lines in (26) can be changed into the following inequalities:

$$-z_i\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)e_1 \le \frac{1}{4\epsilon_1}\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)^2 z_i^2+\epsilon_1 e_1^2=\hat{c}_{i1}z_i^2+\epsilon_1 e_1^2, \quad (27)$$

$$-z_i\left(\frac{\partial x_i^*}{\partial z_2}+\frac{\partial x_i^*}{\partial \hat{d}_2}\lambda_2\right)e_2 \le \frac{1}{4\epsilon_2}\left(\frac{\partial x_i^*}{\partial z_2}+\frac{\partial x_i^*}{\partial \hat{d}_2}\lambda_1\right)^2 z_i^2+\epsilon_2 e_2^2=\hat{c}_{i2}z_i^2+\epsilon_2 e_2^2, \quad (28)$$

$$\begin{aligned}-z_i\left[\frac{\partial x_i^*}{\partial z_2}\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)\right]e_1 &\le \frac{1}{4\epsilon_1}\left[\frac{\partial x_i^*}{\partial z_2}\left(\frac{\partial x_i^*}{\partial z_1}+\frac{\partial x_i^*}{\partial \hat{d}_1}\lambda_1\right)\right]^2 z_i^2+\epsilon_1 e_1^2\\ &=\hat{c}_{i3}z_i^2+\epsilon_1 e_1^2, \end{aligned}\quad (29)$$

$$-z_i\frac{\partial x_i^*}{\partial x_2}e_2 \le \frac{1}{4\epsilon_2}\left(\frac{\partial x_i^*}{\partial x_2}\right)^2 z_i^2+\epsilon_2 e_2^2=\hat{c}_{i4}z_i^2+\epsilon_2 e_2^2. \quad (30)$$

After substituting the inequalities (27)-(30) and then combining those inequalities with (26), we have

$$\dot{V}_3 \le -\sum_{j=1}^{5}\left(k_j z_j^2+e_j\dot{d}_j\right)-(\lambda_1-8\epsilon_1)e_1^2-(\lambda_2-4\epsilon_2)e_2^2-\sum_{i=3}^{5}(\lambda_i-\epsilon_i)e_i^2 \quad (31)$$

In the following subsection, we analyze the stability of the closed-loop system with the control law (25) based on the control design of this subsection.

3.3. **Stability analysis.** In this subsection, the overall closed-loop stability of the system (5) with the proposed control law (25) and the error estimation dynamics (10) are analyzed within the framework of Lyapunov theory. Before the closed-loop stability is carried out, the closed-loop system dynamics can be expressed as follows:

$$\begin{cases}\dot{z}_1 &= z_2-\left(k_1+\frac{1}{4\epsilon_1}\right)z_1+e_1\\ \dot{z}_2 &= -z_1+\frac{1}{M}(z_3-z_4-z_5)-\left(k_2+\frac{1}{4\epsilon_2}+\hat{c}_2\right)z_2+e_2-\frac{\partial x_2^*}{\partial z_1}e_1-\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1\\ \dot{z}_3 &= -\frac{1}{M}z_2-\left(k_3+\frac{1}{4\epsilon_3}+\hat{c}_{31}+\hat{c}_{32}+\hat{c}_{33}+\hat{c}_{34}\right)z_3+e_3-\frac{\partial x_3^*}{\partial z_1}e_1-\frac{\partial x_3^*}{\partial z_2}e_2\\ &\quad +\frac{\partial x_3^*}{\partial z_1}\frac{\partial x_2^*}{\partial z_1}e_1+\frac{\partial x_3^*}{\partial z_1}\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_3^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_3^*}{\partial \hat{d}_2}\lambda_2 e_2\\ \dot{z}_4 &= \frac{1}{M}z_2-\left(k_4+\frac{1}{4\epsilon_4}+\hat{c}_{41}+\hat{c}_{42}+\hat{c}_{43}+\hat{c}_{44}\right)z_4+e_4-\frac{\partial x_4^*}{\partial z_1}e_1-\frac{\partial x_4^*}{\partial z_2}e_2\\ &\quad +\frac{\partial x_4^*}{\partial z_1}\frac{\partial x_2^*}{\partial z_1}e_1+\frac{\partial x_4^*}{\partial z_1}\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_4^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_4^*}{\partial \hat{d}_2}\lambda_2 e_2\\ \dot{z}_5 &= \frac{1}{M}z_2-\left(k_5+\frac{1}{4\epsilon_5}+\hat{c}_{51}+\hat{c}_{52}+\hat{c}_{53}+\hat{c}_{54}\right)z_5+e_5-\frac{\partial x_5^*}{\partial z_1}e_1-\frac{\partial x_5^*}{\partial z_2}e_2\\ &\quad +\frac{\partial x_5^*}{\partial z_1}\frac{\partial x_2^*}{\partial z_1}e_1+\frac{\partial x_5^*}{\partial z_1}\frac{\partial x_2^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_5^*}{\partial \hat{d}_1}\lambda_1 e_1-\frac{\partial x_5^*}{\partial \hat{d}_2}\lambda_2 e_2,\\ \dot{e}_j &= -\lambda_j e_j+\dot{d}_j, \quad j=1,2,3,4,5.\end{cases} \quad (32)$$

Therefore, we can summarize the control design in the following theorem.

Theorem 3.1. *Under Assumption 2.1, the nonlinear disturbance observer-based back-stepping controller (25) can guarantee that the overall closed-loop system consisting of the* *system and the disturbance observer error dynamics (32) with the developed controller is input-to-state stable.*

Proof: To demonstrate the closed-loop stability of the presented control strategy, let us define the following Lyapunov function for the closed-loop dynamics (32).

$$V_3 = \sum_{j=1}^{5} \frac{1}{2}\left(z_j^2 + e_j^2\right). \tag{33}$$

After computing the time derivative of the Lyapunov function candidate (33), the closed-loop system can be expressed as

$$\dot{V}_3 \leq -\sum_{j=1}^{5}\left(k_j z_j^2 + e_j \dot{d}_j\right) - (\lambda_1 - 8\epsilon_1)e_1^2 - (\lambda_2 - 4\epsilon_2)e_2^2 - \sum_{i=3}^{5}(\lambda_i - \epsilon_i)e_i^2. \tag{34}$$

After selecting $\lambda_1 = a_{01} + 8\epsilon_1$, $\lambda_2 = a_{02} + 4\epsilon_2$, $\lambda_i = a_{0i} + \epsilon_i$, $(i = 3, 4, 5)$, $a_{0j} > 0$, $(j = 1, 2, 3, 4, 5)$, we obtain

$$\dot{V}_3 \leq -\sum_{j=1}^{5} k_j z_j^2 - \sum_{j=1}^{5} a_{0j} e_j^2 + \sum_{j=1}^{5} e_j \dot{d}_j \leq -\sum_{j=1}^{5} k_j z_j^2 - a_0\|e\|^2 + \|e\|\|\dot{d}\|, \tag{35}$$

where $e = [e_1, e_2, e_3, e_4, e_5]^T$, $\dot{d} = \left[\dot{d}_1, \dot{d}_2, \dot{d}_3, \dot{d}_4, \dot{d}_5\right]^T$, $a_0 = \min\{a_{01}, a_{02}, \ldots, a_{05}\}$. Besides, we rewrite the inequality (35) as

$$\dot{V}_3 \leq -\sum_{j=1}^{5} k_j z_j^2 - (1-\theta)a_0\|e\|^2 - \theta a_0\|e\|^2 + \|e\|\|\dot{d}\|, \tag{36}$$

where $0 < \theta < 1$. Provided that $\|e\| \geq \frac{\|\dot{d}\|}{a_0\theta}$, we obtain $\dot{V}_3 \leq -\sum_{j=1}^{5} k_j z_j^2 - (1-\theta)a_0\|e\|^2 \leq 0$. In accordance with Lemma 2.2, it can be concluded that the overall closed-loop dynamics (32) are input-to-state stable. This completes the proof.

Remark 3.2. *According to the result of Theorem 1 and Lemma 9.2 in* [24]*, it can be observed that under Assumption 2.1, the disturbance estimation error will approach to any specified level via a suitable selection of the observer gain* (λ_i).

Assumption 3.1. *The disturbances satisfy the condition of* $\lim_{t\to+\infty} \dot{d}_j(t) = 0$, $(j = 1, 2, 3, 4, 5)$.

Theorem 3.2. *Under Assumptions 2.1 and 3.1, the closed-loop dynamics (32) under the control law (25) and the disturbance estimation (8) will asymptotically converge to zero.*

Proof: It is seen from the closed-loop system (32) that $\dot{d}_j$ can be regarded as an input of the system. After combining Theorem 3.1 with Assumption 3.1, it follows from Lemma 2.2 that all trajectories of z_j and e_j of the closed-loop dynamics converge to zero. This means that $z_j \to 0$ and $e_j \to 0$ as $t \to +\infty$. This completes the proof.

4. **Simulation Results.** In this section, simulation results are given to indicate the effectiveness of the developed strategy. The proposed controller is evaluated via simulations of a single-machine infinite bus (SMIB) power system consisting of dual-excited and steam-valving control as shown in Figure 1 [13]. The performance of the proposed control scheme is evaluated and verified in MATLAB environment.

The physical parameters (pu.), the controller parameters, and initial conditions used for this power system model are the same as those used in [13] as follows:

- The parameters of dual-excitation and steam-valving system and transmission line: $\omega_s = 2\pi f$ rad/s, $D = 5$, $H = 5$, $f = 60$ Hz, $T'_{d0} = 10$, $T'_{q0} = 4$, $V_\infty = 1\angle 0°$, $\omega = \omega_s$, $X_q = 1.6$, $X'_q = 0.38$, $X_d = 1.6$, $X'_d = 0.23$, $T_{H\Sigma} = 0.4$, $X_T = 0.13$, $X_L = 0.17$.
- The controller parameters of the proposed controller are $\epsilon_j = 10$, $k_j = 20$, $\lambda_j = 50$, $(j = 1, 2, \ldots, 5)$.
- Initial conditions $\delta_e = 0.3445$ rad, $P_{me} = 1.2749$, $E'_{qe} = 1.0703$, $E'_{de} = 0.522$, $\hat{d}_{j0} = 0$, $(j = 1, 2, 3, 4, 5)$.

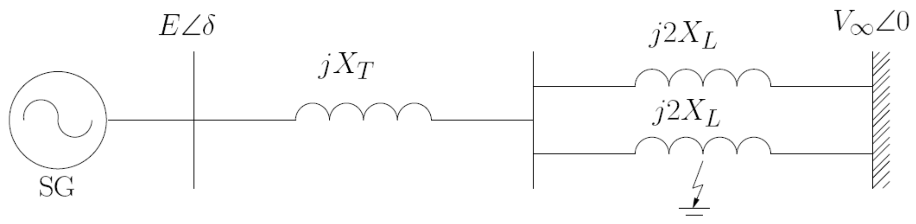

FIGURE 1. A single line diagram of SMIB model with SMES

Additionally, the external disturbances $(d_j,\ j = 1, 2, 3, 4, 5)$ acting on the underlying system are assumed to be:

$$d_1(t) = 0,\ 0 \le t \le 20,$$
$$d_2(t) = \begin{cases} 0.5\sin(2t), & 0 \le t < 5 \\ 1, & 5 \le t < 10 \\ 0.25\sin(2t)e^{-t}, & 10 \le t \le 20 \end{cases},\quad d_3(t) = \begin{cases} 0.15\cos(t), & 0 \le t < 5 \\ 2, & 5 \le t < 10 \\ 0.5\cos(t)e^{-2t}, & 10 \le t \le 20 \end{cases}$$
$$d_4(t) = \begin{cases} 0.25\sin(t), & 0 \le t < 5 \\ 2, & 5 \le t < 10 \\ 0.3\sin(t)e^{-3t}, & 10 \le t \le 20 \end{cases},\quad d_5(t) = \begin{cases} 0.2\cos(t), & 0 \le t < 5 \\ 1.5, & 5 \le t < 10 \\ 0.4\cos(t)e^{-t}, & 10 \le t \le 20 \end{cases}$$

The controller parameters are set as $\epsilon_j = 10$, $k_j = 20$, $\lambda_j = 50$, $(j = 1, 2, \ldots, 5)$. The time domain simulations are carried out to investigate the system stability enhancement and the dynamic performance of the designed controller, as given in (25), in the system in the presence of external disturbances. The control performance of the proposed controller (nonlinear disturbance observer-based backstepping controller) is compared with that of the following nonlinear controllers.

- A backstepping controller [15] is provided by

$$\begin{cases} \dfrac{C_H}{T_{H\Sigma}} u_G = \dfrac{1}{g_{31}(x)}\left[- k_3 z_3 - \dfrac{z_2}{M} - f_3(x) + \dfrac{1}{3}\left(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 + k_1\dot{x}_2)\right)\right], \\ \dfrac{u_{fd}}{T'_{d0}} = \dfrac{1}{g_{42}(x)}\left[- k_4 z_4 + \dfrac{z_2}{M} - f_4(x) - \dfrac{1}{3}\left(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 + k_1\dot{x}_2)\right)\right], \\ \dfrac{u_{fq}}{T'_{q0}} = \dfrac{1}{g_{53}(x)}\left[- k_5 z_5 + \dfrac{z_2}{M} - f_5(x) - \dfrac{1}{3}\left(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 + k_1\dot{x}_2)\right)\right], \end{cases} \tag{37}$$

with $z_j = x_j - x_j^*$, $(j = 1, 2, 3, 4, 5)$, $x_1^* = 0$, $x_2^* = -k_1 z_1$, $x_3^* = \frac{1}{3}(Dx_2 - M(k_2 z_2 + z_1 + k_1 x_2))$, $x_4^* = x_5^* = -x_3^*$, $\dot{z}_1 = -c_1 z_1 + z_2$, $\dot{z}_2 = f_2(x) + c_1 x_2$. The controller parameters of this scheme are set as $c_j = 20$, $(j = 1, 2, \ldots, 5)$.

- An integral backstepping controller is given by

$$\begin{cases} \dfrac{C_H}{T_{H\Sigma}}u_G = \dfrac{1}{g_{31}(x)}\Big[-k_3z_3 - \dfrac{z_2}{M} - f_3(x) + \dfrac{1}{3}\big(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 \\ \qquad + k_1(\dot{x}_2 + \beta\dot{x}_1) + \beta\dot{x}_2)\big)\Big], \\ \dfrac{u_{fd}}{T'_{d0}} = \dfrac{1}{g_{42}(x)}\Big[-k_4z_4 + \dfrac{z_2}{M} - f_4(x) - \dfrac{1}{3}\big(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 \\ \qquad + k_1(\dot{x}_2 + \beta\dot{x}_1) + \beta\dot{x}_2)\big)\Big], \\ \dfrac{u_{fq}}{T'_{q0}} = \dfrac{1}{g_{53}(x)}\Big[-k_5z_5 + \dfrac{z_2}{M} - f_5(x) - \dfrac{1}{3}\big(D\dot{x}_2 - M(k_2\dot{z}_2 + \dot{z}_1 \\ \qquad + k_1(\dot{x}_2 + \beta\dot{x}_1) + \beta\dot{x}_2)\big)\Big], \end{cases} \tag{38}$$

where $z_j = x_j - x_j^*$, $(j = 1,2,3,4,5)$, $x_1^* = -\beta\int_0^t x_1(\tau)d\tau$, $\beta > 0$, $x_2^* = -k_1z_1 - \beta x_1$, $x_3^* = \frac{1}{3}\left(Dx_2 - M(k_2z_2 + z_1 + k_1(x_2 + \beta x_1) + \beta x_2)\right)$, $x_4^* = x_5^* = -x_3^*$, $\dot{z}_1 = -c_1z_1 + z_2$, $\dot{z}_2 = f_2(x) + c_1(x_2 + \beta x_1) + \beta x_2$. The controller parameters of this scheme are set as $c_j = 20$, $(j = 1,2,\ldots,5)$, $\beta = 1$.

The simulation results are presented and discussed as follows. Time trajectories of the power angle, frequency, mechanical power together with d-axis and q-axis transient internal voltages under three controllers are presented in Figures 2 and 3, respectively. Also, in order to illustrate the effectiveness of disturbance observer, Figure 4 shows time histories of external disturbances and disturbance estimation. From these figures, it can be observed that the time responses can achieve the control objectives in the presence

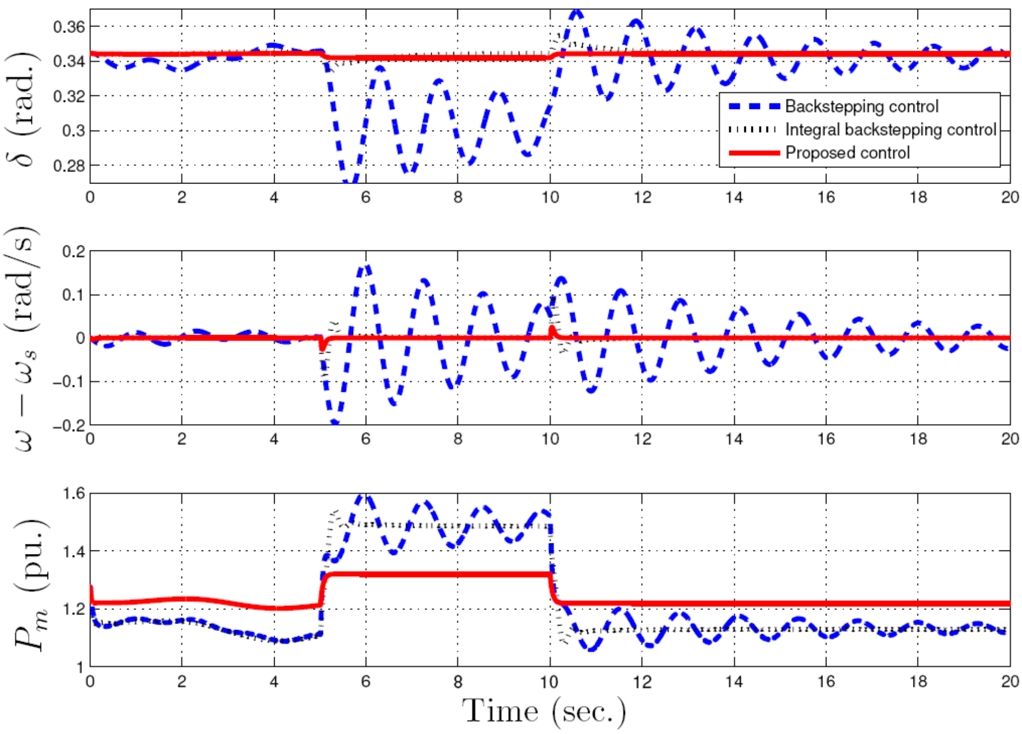

FIGURE 2. Controller performance – power angles (δ) (rad.), frequency ($\omega - \omega_s$) rad/s. and mechanical input (P_m) pu. (Solid: The proposed control, Dashed: Backstepping control, Dotted: Integral backstepping control)

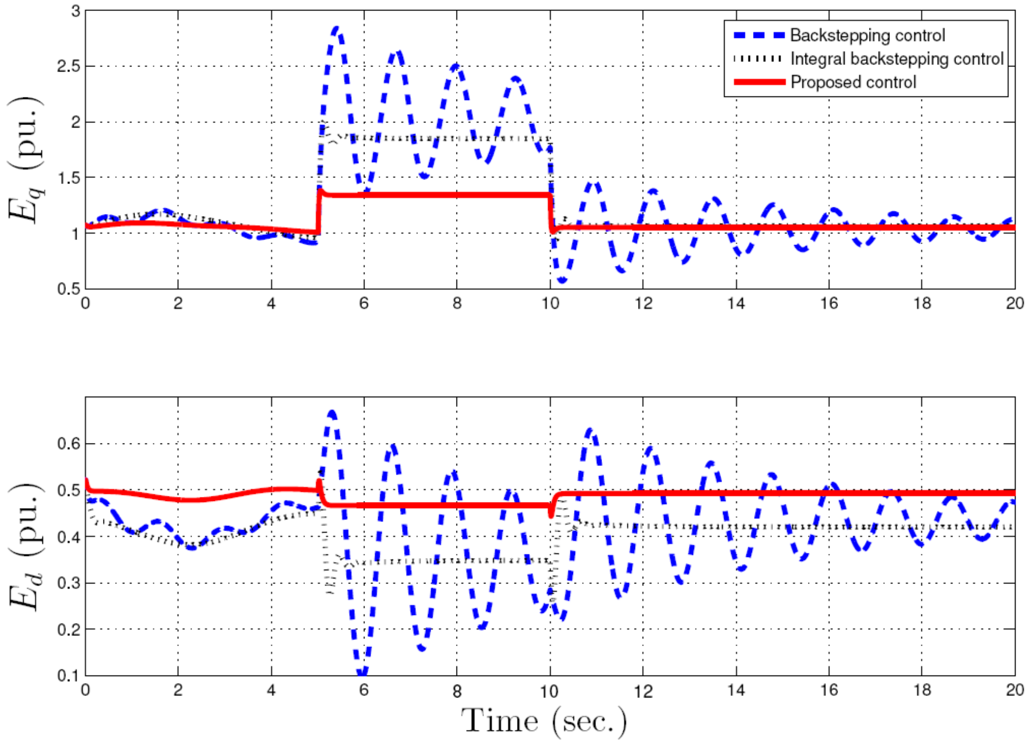

FIGURE 3. Controller performance – the q-axis transient internal voltage (E_q) (pu.) and the d-axis transient internal voltage (E_d) (pu.) (Solid: The proposed control, Dashed: Backstepping control, Dotted: Integral backstepping control)

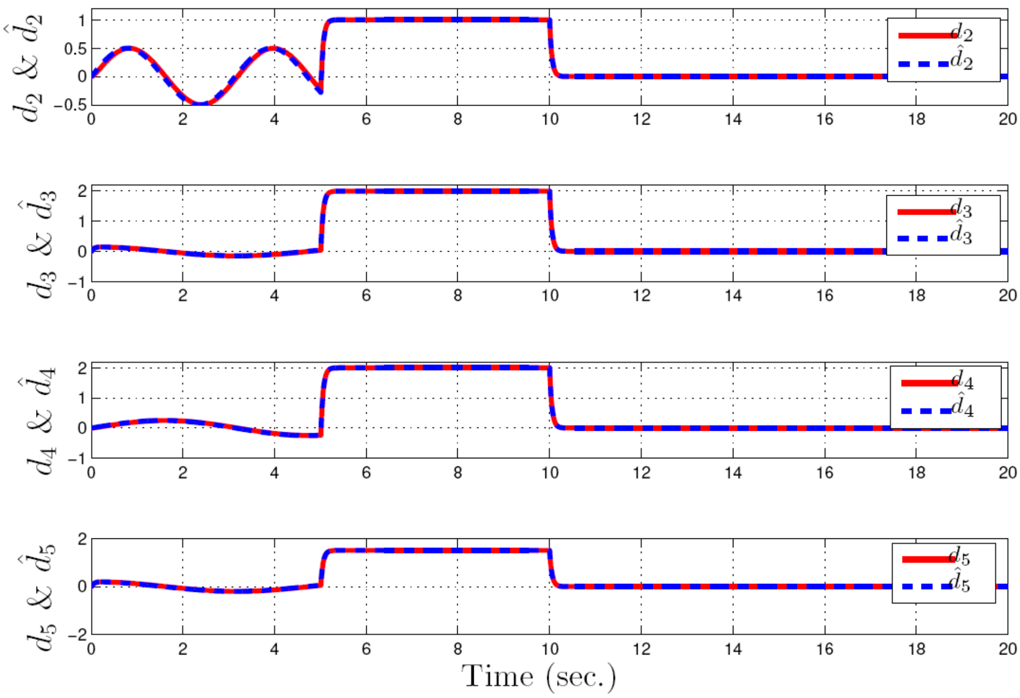

FIGURE 4. External disturbances and disturbance estimation

of external disturbances. It is obviously seen that the proposed controller offers better dynamic performances, such as a shorter settling time, a short rise time, a faster convergence rate, and no steady-state error. It is capable of accomplishing a satisfactory disturbance rejection ability despite having external disturbances. From Figures 2 and 3, one can observe that under the proposed control, the adverse effects caused by the disturbances are removed from the system after a short period. On the other hand, the backstepping control and the integral backstepping control clearly exhibit undesired control performances such as unsatisfactory overshoots, slowly suppressing system oscillations, and nonzero steady-state error. Thus, the proposed controller offers superiority over backstepping and integral backstepping controller since the disturbance observer design, employed to estimate the external disturbance, introduces the disturbance estimation to the virtual control laws in each step and the final controller (25). Figure 4 illustrates time histories of disturbances and their estimations using the proposed observer capable of effectively estimating and compensating the external disturbances successfully. Apart from this, the disturbance estimator of the developed strategy quickly approaches to the external disturbances with very fast convergence rates and no oscillations as shown in Figure 4.

From the simulation results mentioned previously, it is evident that as the presented backstepping method combined with the disturbance observer scheme is applied to the SMIB power system with external disturbances, the advantages over both conventional backstepping and integral backstepping methods are as follows.

- The proposed control law is effectively designed to stabilize the system in the presence of undesired disturbances.
- The developed control strategy can make the overall closed-loop dynamics converge more quickly to a desired equilibrium point. In particular, it obviously performs well and has considerably effective disturbance rejection ability. It offers obviously superior transient performances illustrated by the rapidly suppressing system oscillations in all time trajectories in spite of having external disturbances.
- The process of designing the desired control law includes some auxiliary terms into the virtual control laws and the final controller. These terms can counteract the crossing terms arising from disturbances, compensation errors, and system states. In contrast, these terms are not included in the conventional backstepping method and the integral backstepping method. Thus, both backstepping methods provide unsatisfactory control performances.

5. **Conclusion.** In this paper, the nonlinear disturbance observer-based backstepping control strategy has been proposed for a dual excitation and steam-valving system of synchronous generators in the presence of external disturbances. The developed approach is utilized to offer a satisfactory disturbance rejection performance and achieve improved dynamic performances. To validate the proposed scheme, a dynamic model of the dual-excited and steam-valving system of synchronous generators is used to evaluate through a simulation environment. The simulation results have demonstrated that the developed control method provides an improved transient performance and performs satisfactorily in rejecting external disturbances better than backstepping and integral backstepping methods. The comparative results with other two controllers confirm the effectiveness of the proposed controller capable of rapidly suppressing system oscillations in the closed-loop system dynamics and rejecting unavoidably external disturbances.

REFERENCES

[1] P. Kundur, *Power System Stability and Control*, McGraw-Hill, 1994.

[2] M. Galaz, R. Ortega, A. Bazanella and A. Stankovic, An energy-shaping approach to excitation control of synchronous generators, *Automatica*, vol.39, pp.111-119, 2003.

[3] R. Ortega, M. Galaz, A. Astolfi, Y. Sun and T. Shen, Transient stabilization of multi-machine power systems with nontrivial transfer conductances, *IEEE Trans. Automatic Control*, vol.50, pp.60-75, 2005.

[4] H. Liu, Z. Hu and Y. Song, Lyapunov-based decentralized excitation control for global asymptotical stability and voltage regulation of multi-machine power systems, *IEEE Trans. Power Systems*, vol.27, pp.2262-2270, 2012.

[5] Y. Liu, Q. H. Wu and X. X. Zhou, Coordinated switching controllers for transient stability of multi-machine power systems, *IEEE Trans. Power Systems*, vol.31, pp.3937-3949, 2016.

[6] L. Sun and J. Zhao, A new adaptive backstepping design of turbine main steam valve control, *Journal of Control Theory Applications*, vol.8, pp.425-428, 2010.

[7] N. Jiang, X. Chen, T. Liu, B. Liu and Y. Jing, Nonlinear steam valve adaptive controller design for the power systems, *Intelligent Control and Automation*, vol.2, pp.31-37, 2011.

[8] Q. Lu, Y. Sun and S. Wei, *Nonlinear Control Systems and Power System Dynamics*, Kluwer Academic Publishers, Boston, 2001.

[9] B. Wang and Z. Mao, Nonlinear variable structure excitation and steam valving controllers for power system stability, *Journal of Control Theory Applications*, vol.7, pp.97-102, 2009.

[10] S. Li and Y. Wang, Adaptive $\mathcal{H}_\infty$ control of synchronous generators with steam valve via Hamiltonian function method, *Journal of Control Theory Applications*, vol.2, pp.105-110, 2006.

[11] S. Xu and X. Hou, A family of robust adaptive excitation controllers for synchronous generators with steam valve via Hamiltonian function method, *Journal of Control Theory Applications*, vol.10, pp.11-18, 2012.

[12] H. Chen, H. B. Ji, B. Wang and H. S. Xi, Coordinated passivation techniques for the dual-excited and stream-valving control of synchronous generators, *IEE Proceedings – Control Theory Applications*, vol.149, pp.659-666, 2006.

[13] A. Kanchanahanathai, Immersion and invariance-based nonlinear dual-excitation and steam-valving control of synchronous generators, *International Trans. Electrical Energy Systems*, vol.24, no.12, pp.1671-1687, 2014.

[14] H. Zhang, S. Li, X. Liu, X. Ren and Z. Wang, The coordinated controller design for dual excitation and steam valve of synchronous generator, *International Journal of System Control and Information Processing*, vol.2, no.1, pp.90-98, 2017.

[15] M. Krstic, I. Kanellakopoulos and P. V. Kokotivic, *Nonlinear and Adaptive Control Design*, John Willey & Sons, 1995.

[16] S. Li, J. Yang, W.-H. Chen and X. Chen, *Disturbance Observer-Based Control: Methods and Applications*, CRC Press, 2014.

[17] H. Sun, S. Li, J. Yang and L. Guo, Non-linear disturbance observer-based back-stepping control for airbreathing hypersonic vehicle with mismathced disturbances, *IET Control Theory Applications*, vol.8, no.17, pp.1852-1865, 2014.

[18] H. Sun, S. Li, J. Yang and W. X. Zheng, Global output regulation for strict-feedback nonlinear systems with mismatched nonvanishing disturbances, *International Journal of Robust and Nonlinear Control*, vol.25, pp.2631-2645, 2015.

[19] D. Ginoya, P. D. Shendge and S. B. Phadke, Disturbance observer based sliding mode control of nonlinear mismatched uncertain systems, *Communications in Nonlinear Science and Numerical Simulation*, vol.26, pp.98-107, 2015.

[20] J. Yang, W.-H. Chen and S. Li, Non-linear disturbance observer-based robust control for systems with mismatched disturbance/uncertainties, *IET Control Theory Applications*, vol.5, pp.2053-2062, 2011.

[21] H. Sun and L. Guo, Composite adaptive disturbance observer based control and back-stepping method for nonlinear system with multiply mismatched disturbances, *Journal of the Franklin Institute*, vol.351, pp.1027-1041, 2014.

[22] X. Liu, Z. Liu, J. Shan and H. Sun, Anti-disturbance autopilot design for missile system via finite time intergral sliding mode control method and nonlinear disturbance observer technique, *Transactions of the Institute of Measurement and Control*, vol.38, pp.693-700, 2015.

[23] W. Kim, D. Shin, D. Won and C. C. Chung, Disturbance-observer-based position tracking controller in the presence of biased sinusoidal disturbance for electrohydraulic actuators, *IEEE Trans. Control Systems Technology*, vol.21, pp.2290-2298, 2013.

[24] H. K. Khalil, *Nonlinear Systems*, Prentice-Hall, 2002.