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PERFORMANCE ANALYSIS OF CLOSED LOOP SYSTEM WITH A TAILOR MADE PARAMETERIZATION

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Abstract. *Here using the framework of a tailor made parameterization for a closed loop system, we study the performance analysis problem where a closed loop transfer function is parameterized using the parameters of an open loop plant model, and utilizing knowledge of a feedback controller. When the plant model and feedback controller are all polynomial forms, a recursive least squares method with forgetting schemes is proposed to verify that this recursive method can be regarded as a regularization least squares problem. Based on the parameter vector, one uncertainty bound about the parameter vector is constructed to reflect the identification accuracy by using the statistical probability theory. Using a tailor made parameterization form, some results about robust control theory and related stability property are used to give a preliminary performance analysis corresponding to the closed loop transfer function. Generally this preliminary performance analysis is extended to a transfer function matrix form which is constituted by three transfer functions. The worst case performance at frequencies is analyzed by solving one standard convex optimization problem involving some linear matrix inequality constraints.*
Keywords: Tailor made parameterization, Performance analysis, Confidence internal, Linear matrix inequality

1. **Introduction.** Now many systems operate under feedback control situation, and it is due to required safety of operation or to unstable behavior of the plant, as occurs in many industrial processes such as paper production, glass production, and separation process like crystallization. In closed loop system, the feedback controller is added to return the collected output back to the collected input. Then one error signal from the input and feedback output can be imposed on the plant to generate one correction action which makes the output converge to a given value. The essences of closed loop system are to decrease the error using the negative feedback controller, and to correct the deviation from the given value automatically. As the closed loop structure can suppress the errors coming from the internal or external disturbances to achieve the achieved control goal, so the closed loop structure is most needed in all of our engineering.

There are many subjects on closed loop system, for example, closed loop system identification, closed loop controller design, and closed loop performance monitoring and diagnosis. There are three identification methods for closed loop system, i.e., direct approach, indirect approach and joint input-output approach, where the feedback is neglected in direct approach and the plant model is identified directly using only input-output data. For the indirect approach, the feedback effect is considered and the input-output data from the whole closed loop condition are used to identify the plant model. The joint input-output approach is very similar to indirect approach. The joint input-output approach requires two separate steps: (1) identification of the closed loop system, and (2) recalculation of the open loop model. For the problem of how to design feedback controller in closed loop system, generally two strategies are used to design the feedback controller in closed loop structure, i.e., model based design and direct data driven design. The primary step of model based design is to construct the plant model in closed loop system using system identification theory and apply this mathematical model in the next process of designing controller. Conversely for the direct data driven design method, the modeling process is not needed and the controller is directly designed using only the observed input-output data under closed loop condition. Through comparing these two strategies, this direct data driven method is worth studying deeply in future. However, now as the first model based design strategy is more applied widely, then we need to do much research on closed loop system identification. The performance of model based control system is often limited. This primarily arises from the quality of their underlying model that affects the closed loop performance. The plant-model mismatch always exists, because various changes occur in closed loop system over time that may cause the mismatch and invalidate the model identified at the stage of those control systems. Closed loop performance monitoring and diagnosis comprise a crucial step in maintenance of model based control system. In the event of performance degradation, diagnostic tools allow us to verify if the unsatisfactory closed loop operation results from the idea of plant-model mismatch. Some references on closed loop system are given as follows. In [1], prediction error identification of linearly parameterized models is considered in the situation where the system is in the model set. In [2], a robust stability and performance analysis is presented for an uncertainty set delivered by classical prediction error identification. In [3], the robust de-convolution filtering is addressed, when the system and noise dynamics are obtained by parametric system identification. The closed loop system identification for single input single output systems with a linear time invariant controller is extended for multivariable state space system in [4]. In [5,6], a joint robust state feedback control/input design procedure is presented to guarantee the stability and prescribed closed loop performance using models identified from experimental data. In [7], the idea of plug and play is merged into the robust distributed control and one plug and play robust distributed control algorithm is formulated to design the feedback controller in closed loop system [8]. The experiment design problem from a dual point of view and in a closed loop setting is proposed in [9], and one cheapest identification experiment will give an uncertainty set that is within the required bounds. A D-optimal input design method for finite impulse response type closed loop system is given in [10], the optimization of the determinant of the Fish information matrix is expressed as a convex optimization problem. Closed loop performance monitoring is studied to detect whether an observed deviation from nominal performance is due to a disturbance or due to a control relevant system change [11]. The closed loop performance diagnosis approach and the decision rule are presented for linear time invariant systems with disturbance [12].

Generally, here we concentrate on performance analysis of closed loop system deeply. All theories are based on a tailor made parameterization used to parameterize the closed loop system. A tailor made parameterization combines those two separate steps from indirect closed loop identification. This means that knowledge of the closed loop system and knowledge of the controller are employed into the parameterization of the closed loop system. This method applies knowledge of the controller and minimizes an error between the true closed loop transfer function and identified closed loop, using a parameterization model of the open loop model only. Here we study the tailor made parameterization method in a linear framework, where the plant model and controller are all parameterized as polynomials. To identify the closed loop parameter vector, a recursive least squares method with forgetting schemes is proposed. This recursive least squares method with forgetting schemes achieves the reformulation of the classical recursive least squares with forgetting schemes as a regularized least squares problem. In order to reflect the identification accuracy, we apply the statistical probability framework to deriving the variance matrix of the unknown parameters. This variance matrix is decomposed into one inter product form which is used to construct one uncertainty bound about the unknown parameter estimation. This uncertainty bound is called by the confidence interval and it constitutes the guaranteed confidence region test with respect to the model parameter estimation under closed loop condition. Using only one plant model, we define a Vinnicombe distance between its true and identified plant model. Then we use the results of some robust control theories such as the Vinnicombe gap between plant and its related stability property to give a preliminary performance analysis. Generally this preliminary performance analysis is extended to a transfer function matrix form which is constituted by three transfer functions. The worst case performance at frequencies is analyzed by solving one standard convex optimization problem involving some linear matrix inequality constraints.

The paper is organized as follows. In Section 2, some preliminaries and problems are formulated in the closed loop system structure with a tailor made parameterization. In Section 3, a recursive least squares method with forgetting schemes is proposed to identify the unknown parameters, and one confidence interval is constructed to include the unknown parameters with an achieved probability level. In Section 4, using only one plant model, a Vinnicombe distance or a Vinnicombe gap is studied to give a preliminary performance analysis corresponding to the closed loop system with a tailor made parameterization. In Section 5, the preliminary performance analysis is extended to a transfer function matrix form, where a convex optimization problem with linear matrix inequality constraints is solved to provide one optimal value. In Section 6, one simulation example illustrates the effectiveness of the proposed theories. Section 7 ends the paper with final conclusion.

2. Closed Loop System Description.

Consider the following closed loop system configuration in Figure 1.

In Figure 1, $G_0(q)$ is a true plant model, $H_0(q)$ is a noise filter, and they are all linear time invariant transfer functions. $C(q)$ is a stable linear time invariant feedback controller; here we assume this controller is priori known. The excited signal $r(t)$ and external disturbance $e(t)$ are uncorrelated. $e(t)$ is a white noise with zero mean value and variance σ^2. $v(t)$ is a colored noise which can be obtained by passing white noise $e(t)$ through that noise filter $H_0(q)$. $u(t)$ and $y(t)$ are the input-output signals with respect to plant model $G_0(q)$. q is the time delay operator, which means that $qu(t) = u(t+1)$.

The closed loop system configuration in Figure 1 appears in many practical engineering problems, for example, flight simulation. Flight simulation is a speed servo system with high precision position. The driven element of flight simulation is an electric motor, and the essence of the control structure in flight simulation is a closed loop system corresponding to the position or speed of that electric motor. According to the analysis of the servo control system, one negative feedback part is added to reduce the sensitivity in the closed loop system, while the cascade regulator is introduced in each feedback control structure in order to reduce the dependence on the electric motor's parameter.

Here we give an example about the pitch position tracking loop from flight simulation to verify the feasibility of our iterative correlation tuning control approach in precision servo control system. In the closed loop system of flight simulation, the photoelectric

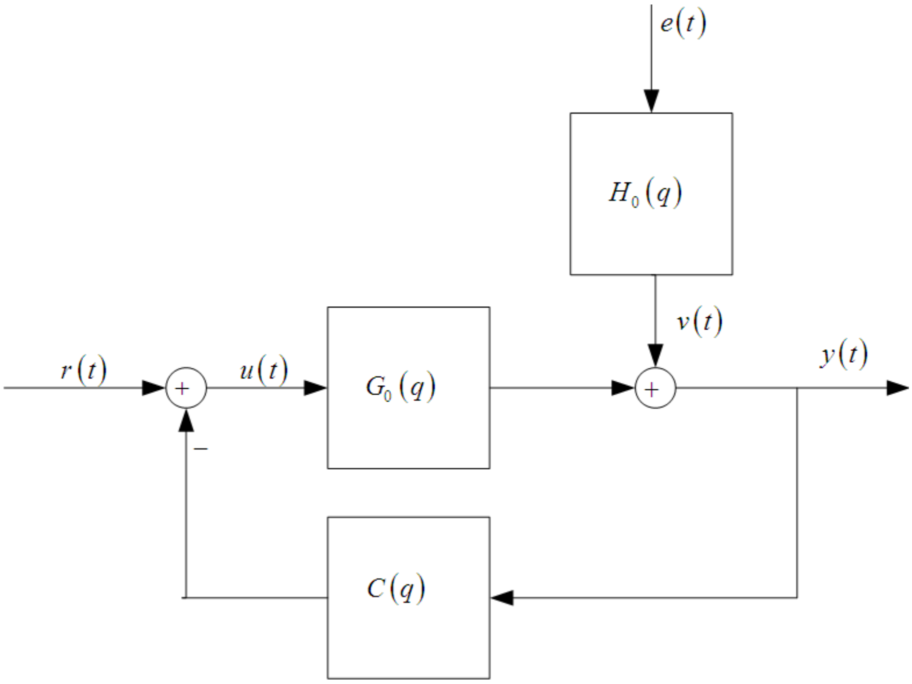

FIGURE 1. Closed loop system configuration

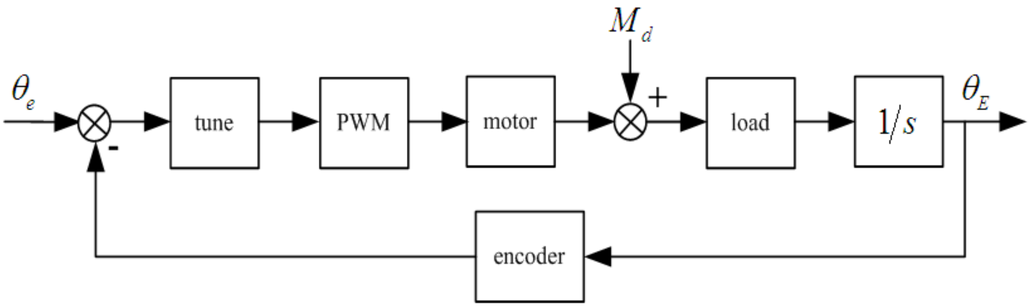

FIGURE 2. The simplified pitch position tracking loop

encoder is mounted on the outer pitch frame, and the angular position signal collected at outer pitch frame is regarded as the position feedback part. After the difference between two angular positions goes through the position correlation part and power amplifier part, then this difference will make the electric motor start to rotate. The pitch position tracking loop from flight simulation is simplified in Figure 2.

In Figure 2 the input signal is the relative angular signal of inner pitch loop, and this input signal is collected by one photoelectric encoder located in inner pitch frame. It means that one photoelectric encoder collects the angular position signal to send one position feedback part. The transfer function model of that simplified pitch position tracking loop can be seen in Figure 3.

In Figure 3 we regard the encoder as a constant and merge it in the power amplifier, and then the close loop system is a unit feedback. θ_{me} is the input signal with respect to the electric motor, and the controller in this position tracking loop is the classical PID controller.

Observing the closed loop system configuration, we obtain the following transfer function form.

$$y(t) = G_0(q)r(t) - G_0(q)C(q)y(t) + H_0(q)e(t) \tag{1}$$

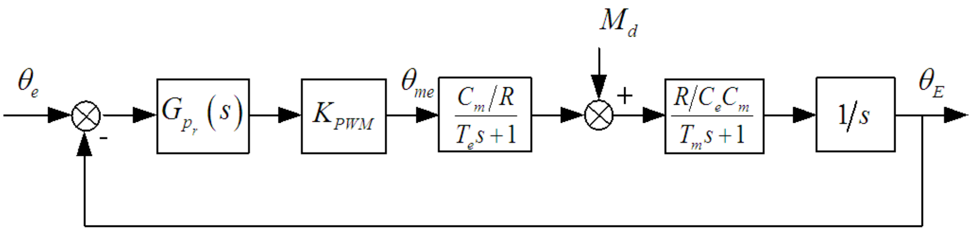

FIGURE 3. The transfer function model of that simplified pitch position tracking loop

Continuing to do some simple computations, we get

$$\begin{cases} y(t) = \dfrac{G_0(q)}{1+G_0(q)C(q)}r(t) + \dfrac{H_0(q)}{1+G_0(q)C(q)}e(t) \\ u(t) = \dfrac{1}{1+G_0(q)C(q)}r(t) - \dfrac{C(q)H_0(q)}{1+G_0(q)C(q)}e(t) \end{cases} \tag{2}$$

To simplify the analysis process, one sensitivity function is defined as

$$S_0(q) = \frac{1}{1+G_0(q)C(q)}$$

Applying the above defined sensitivity function, the output of closed loop system can be rewritten as

$$y(t,\theta) = G_0(q)S_0(q)r(t) + H_0(q)S_0(q)e(t)$$

Introducing one unknown parameter vector θ into the closed loop system, the parameterized form corresponding to Equation (2) is given by

$$y(t) = \frac{G(q,\theta)}{1+G(q,\theta)C(q)}r(t) + \frac{H(q,\theta)}{1+G(q,\theta)C(q)}e(t) \tag{3}$$

where θ denotes the unknown parameter vector, and it exists in the parameterized plant model $G(q,\theta)$ and noise filter $H(q,\theta)$ respectively. The goal of closed loop identification is to identify the unknown parameter vector from one given data set $Z^N = \{r(t), y(t)\}_{t=1}^N$ and priori known controller $C(q)$, where N denotes the total number of observed data.

According to Equation (3), the prediction of output $y(t,\theta)$ can be calculated as the one step ahead prediction.

$$\hat{y}(t,\theta) = \frac{G(q,\theta)}{H(q,\theta)}r(t) + \frac{H(q,\theta)-1-G(q,\theta)C(q)}{H(q,\theta)}y(t) \tag{4}$$

Construct one step ahead prediction error or residual as

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t,\theta) = \frac{1+G(q,\theta)C(q)}{H(q,\theta)}\left[y(t) - \frac{G(q,\theta)}{1+G(q,\theta)C(q)}r(t)\right] \tag{5}$$

In prediction error algorithm, using input-output data set $Z^N = \{r(t), y(t)\}_{t=1}^N$ with the number N, the unknown parameter vector is identified by solving an optimization problem.

$$\hat{\theta}_N = \arg\min_\theta V_N\left(\theta, Z^N\right) = \arg\min_\theta \frac{1}{N}\sum_{t=1}^N \varepsilon^2(t,\theta) \tag{6}$$

The above Equation (6) is similar to the classical prediction error algorithm and direct approach. In next section it will be made clear that a tailor made parameterization is used. The parameterized plant model $G(q,\theta)$ and feedback controller $C(q)$ are all assumed to be polynomials. Then we propose a recursive least squares method with forgetting schemes to identify the unknown parameter vector θ. Based on this identified parameter vector, one confidence interval of unknown parameter vector is constructed under closed loop condition.

3. **Confidence Interval Analysis with a Tailor Made Parameterization.** Let the plant model $G(q,\theta)$ be parameterized as one polynomial.

$$G(q,\theta)=\frac{B(q,\theta)}{A(q,\theta)}=\frac{b_1q^{-1}+\cdots+b_{n_b}q^{-n_b}}{1+a_1q^{-1}+\cdots+a_{n_a}q^{-n_a}} \tag{7}$$

where $\theta=\begin{bmatrix} a_1 & \cdots & a_{n_a} & b_1 & \cdots & b_{n_b}\end{bmatrix}^T$. Similarly the feedback controller is parameterized as

$$C(q)=\frac{N_c(q)}{D_c(q)}=\frac{n_0+n_1q^{-1}+\cdots+n_{n_N}q^{-n_N}}{1+d_1q^{-1}+\cdots+d_{n_D}q^{-n_D}} \tag{8}$$

where $N_c(q)$ and $D_c(q)$ are coprime polynomials. Based on these two polynomial forms (7) and (8), the parameterization of the output predictor is given by

$$\hat{y}(t/t-1,\theta)=\frac{D_c(q)B(q,\theta)}{D_c(q)A(q,\theta)+N_c(q)B(q,\theta)}r(t) \tag{9}$$

The denominator of the closed loop transfer function can be written as a function of the open loop unknown parameter vector θ.

$$D_c(q)A(q,\theta)+N_c(q)B(q,\theta)=1+\begin{bmatrix} q^{-1} & q^{-2} & \cdots & q^{-n}\end{bmatrix}\theta_{cl} \tag{10}$$

The order of the closed loop polynomial is given by

$$n=\max(n_a+n_D,n_b+n_N)$$

The closed loop parameter vector θ_{cl} is given as

$$\theta_{cl}=S\theta+\rho \tag{11}$$

Matrix S and vector ρ are parameterized as

$$\rho=\begin{bmatrix} d_1 & \cdots & d_{n_D} & 0 & \cdots & 0\end{bmatrix}^T\in R^n,\ S=\begin{bmatrix} P_D & P_N\\ 0 & 0\end{bmatrix}$$

$$P_D=\begin{bmatrix} 1 & 0 & \cdots & 0\\ d_1 & 1 & \cdots & \vdots\\ d_2 & d_1 & \ddots & \vdots\\ \vdots & & & 1\\ d_{n_D} & \ddots & & d_1\\ 0 & & \ddots & d_2\\ \vdots & & & \vdots\\ 0 & \cdots & 0 & d_{n_D}\end{bmatrix},\ P_N=\begin{bmatrix} n_0 & 0 & \cdots & 0\\ n_1 & n_0 & \cdots & \vdots\\ n_2 & n_1 & \cdots & \vdots\\ \vdots & \vdots & \cdots & n_0\\ n_{n_N} & & \cdots & n_1\\ \vdots & & \cdots & \vdots\\ 0 & \cdots & & n_{n_N}\end{bmatrix} \tag{12}$$

When the feedback controller $C(q)$ is priori known, then matrix S and vector ρ can be constructed by using parameters from coprime polynomials.

Rearranging Equation (9), we obtain

$$[D_c(q)A(q,\theta)+N_c(q)B(q,\theta)]\,y(t)=[D_c(q)B(q,\theta)]r(t) \tag{13}$$

Substituting (10), (11) and (12) into (13), it yields

$$\begin{aligned}&\left(1+\begin{bmatrix} q^{-1} & q^{-2} & \cdots & q^{-n}\end{bmatrix}\left[\begin{pmatrix} P_D & P_N\end{pmatrix}\theta+\rho\right]\right)y(t)\\ =&\begin{bmatrix} q^{-1} & q^{-2} & \cdots & q^{-n}\end{bmatrix}\begin{bmatrix} 0 & P_D\end{bmatrix}\theta r(t)\end{aligned} \tag{14}$$

Expanding above equation, we see that

$$\begin{aligned} y(t) = & \left(\begin{bmatrix} r(t-1) & r(t-2) & \cdots & r(t-n) \end{bmatrix} \begin{bmatrix} 0 & P_D \end{bmatrix} \right. \\ & \left. - \begin{bmatrix} y(t-1) & y(t-2) & \cdots & y(t-n) \end{bmatrix} \begin{pmatrix} P_D & P_N \end{pmatrix} \right) \theta \end{aligned}$$

Define one vector $\varphi(t)$ as

$$\begin{aligned} \varphi^T(t) = & \left(\begin{bmatrix} r(t-1) & r(t-2) & \cdots & r(t-n) \end{bmatrix} \begin{bmatrix} 0 & P_D \end{bmatrix} \right. \\ & \left. - \begin{bmatrix} y(t-1) & y(t-2) & \cdots & y(t-n) \end{bmatrix} \begin{pmatrix} P_D & P_N \end{pmatrix} \right) \end{aligned}$$

Then output of the closed loop system can be written as

$$y(t) = \varphi^T(t)\theta \tag{15}$$

Vector $\varphi(t)$ is similar to the classical regression vector. A common way to identify the unknown parameter vector θ in (15) relies on the recursive least squares with forgetting schemes, where parameter vector estimation $\hat{\theta}_t$ is given as

$$\hat{\theta}_t = \arg\min_{\theta} V_1(\theta) \tag{16}$$

where the loss function is defined as

$$V_1(\theta) = \sum_{s=1}^{t} \lambda^{t-s} \left(y(s) - \varphi^T(s)\theta \right) \tag{17}$$

The forgetting factor $\lambda \in [0, 1]$ operates as an exponential weight which decreases with more remote data. Optimization problem (16) admits the recursive solution.

$$\begin{cases} R_t = \lambda R_{t-1} + \varphi(t)\varphi^T(t) \\ \hat{\theta}_t = \hat{\theta}_{t-1} + R_t^{-1}\varphi(t) \left(y(t) - \varphi^T(t)\hat{\theta}_{t-1} \right) \end{cases} \tag{18}$$

Defining $P_t = R_t^{-1}$, then one equivalent recursion is obtained.

$$\begin{aligned} & \hat{\theta}_t = \hat{\theta}_{t-1} + K_t \left(y(t) - \varphi^T(t)\hat{\theta}_{t-1} \right) \\ & K_t = \frac{P_{t-1}\varphi(t)}{\lambda + \varphi^T(t)P_{t-1}\varphi(t)}, \; P_t = \frac{1}{\lambda} \left(I - K_t\varphi^T(t) \right) P_{t-1} \end{aligned} \tag{19}$$

Observing optimization problem (16) again, let $Q_t = diag \begin{pmatrix} 1 & \cdots & \lambda^{t-1} \end{pmatrix}$ and consider

$$\begin{aligned} \hat{\theta}_t = & \arg\min_{\theta} \left(y(t) - \varphi^T(t)\theta \right)^2 + \lambda \sum_{i=1}^{t-1} \left[\left(\varphi^T(i) \left(\theta - \hat{\theta}_{t-1} \right) \right)^2 \right. \\ & \left. - 2 \left(y(i) - \varphi^T(i)\hat{\theta}_{t-1} \right) \varphi^T(t) \left(\theta - \hat{\theta}_{t-1} \right) \right] \lambda^{t-i-1} \end{aligned} \tag{20}$$

where we use the following relation

$$\hat{\theta}_{t-1} = \arg\min_{\theta} \sum_{i=1}^{t-1} \left(y(i) - \varphi^T(i)\theta \right)^2 \lambda^{t-i-1}$$

By using optimality condition, it holds that

$$\hat{\theta}_t = \arg\min_{\theta} \left(y(t) - \varphi^T(t)\theta \right)^2 + \lambda \left(\theta - \hat{\theta}_{t-1} \right)^T R_{t-1} \left(\theta - \hat{\theta}_{t-1} \right) \tag{21}$$

where the updating law is Equation (19). Equation (21) shows that the recursive least squares with forgetting scheme can be regarded as regularization least squares problem.

Based on optimization problem (6), define the asymptotic limit parameter estimate θ^* as

$$\theta^* = \arg\min_{\theta} \lim_{N\to\infty} E\left\{V_N\left(\theta, Z^N\right)\right\}$$

where E denotes the expectation operator. In the common identification process, assume that there always exists one true parameter vector θ_0 such that

$$G(q,\theta_0) = G_0(q),\ H(q,\theta_0) = H_0(q)$$

This above assumption shows that the identified model is contained in the considered model set, and then the asymptotic covariance matrix of the parameter vector is obtained.

$$P_\theta = \operatorname{cov}\hat{\theta}_N = \sigma_0^2 \left\langle \varphi(t), \varphi(t)\right\rangle^{-1} \tag{22}$$

where $\langle \varphi(t), \varphi(t)\rangle$ denotes some inter product operator, φ is the negative gradient of the predictor error, i.e., it can be computed from Equation (15).

$$\varphi(t,\theta) = -\frac{\partial\varepsilon(t,\theta)}{\partial\theta} = \frac{\partial\hat{y}(t,\theta)}{\partial\theta}$$

On the basis of (22), the following asymptotic result can be got.

$$\hat{\theta}_N \overset{N\to\infty}{\longrightarrow} \theta_0$$

It shows that the parameter estimator $\hat{\theta}_N$ will converge to its limit θ_0, and further $\hat{\theta}_N$ will asymptotically converge ($N \to \infty$) to normally distributed random variable with mean θ_0 and variance P_θ.

$$\sqrt{N}\left(\hat{\theta}_N - \theta_0\right) \to \mathbb{N}\left(0, P_\theta\right),\ \text{as } N \to \infty$$

This asymptotic result is rewritten in a quadratic form, and then we get one λ^2 distribution.

$$N\left(\hat{\theta}_N - \theta_0\right)^T P_\theta^{-1}\left(\hat{\theta}_N - \theta_0\right) \overset{N\to\infty}{\longrightarrow} \lambda_n^2 \tag{23}$$

where n is the number of degrees of freedom in the λ^2 distribution, being equal to the dimension of the parameter vector. Equation (23) implies that the random variable $\hat{\theta}_N$ satisfies one uncertainty bound.

$$\hat{\theta}_N \in D\left(\alpha, \theta_0\right) = \left\{\theta / N\left(\theta - \theta_0\right)^T P_\theta^{-1}\left(\theta - \theta_0\right) \le \lambda_{n,\alpha}^2\right\} \tag{24}$$

with $\lambda_{n,\alpha}^2$ corresponding to a probability level α in λ_n^2 distribution. However, now in order to quantify the uncertainty on θ_0 rather than on $\hat{\theta}_N$, for every realization of $\hat{\theta}_N$, it holds that

$$\hat{\theta}_N \in D\left(\alpha, \theta_0\right) \Leftrightarrow \theta_0 \in D\left(\alpha, \hat{\theta}_N\right)$$

It signifies that

$$\theta_0 \in D\left(\alpha, \hat{\theta}_N\right) = \left\{\theta / N\left(\hat{\theta}_N - \theta\right)^T P_\theta^{-1}\left(\hat{\theta}_N - \theta\right) \le \lambda_{n,\alpha}^2\right\} \quad \text{with probability } \alpha \tag{25}$$

Equations (24) and (25) give the confidence intervals of unknown parameter vector under closed loop condition. The probability level of the event $\hat{\theta}_N \in D\left(\alpha, \theta_0\right)$ which holds is at least α. From the above statistical derivation, we obtain one kind of performance analysis corresponding to the unknown parameter vector in closed loop system. Also when considered in a tailor made parameterization, the negative gradient of the prediction (22) is exactly the regression vector (15).

4. **Performance Analysis Only on One Transfer Function.** Here we use some robust control theories to give a preliminary performance analysis. Combining Equations (15) and (10), the closed loop transfer function can be reformulated as

$$\begin{aligned}\frac{D_c(q)B(q,\theta)}{D_c(q)A(q,\theta)+N_c(q)B(q,\theta)} &= \frac{\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right]\left[\begin{array}{cc} 0 & P_D\end{array}\right]\theta}{1+\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right](S\theta+\rho)} \\ = \frac{Z_2\theta}{1+\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right]\rho+\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right]\theta} &= \frac{Z_2\theta}{a+Z_1\theta}\end{aligned} \tag{26}$$

where we apply the parameterized plant model $G(q,\theta)$, and column vectors Z_1, Z_2 are defined as follows respectively.

$$\begin{cases} Z_1=\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right],\ Z_2=\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right]\left[\begin{array}{cc} 0 & P_D\end{array}\right] \\ a=1+\left[\begin{array}{cccc} q^{-1} & q^{-2} & \cdots & q^{-n}\end{array}\right]\rho \end{cases} \tag{27}$$

To simplify the mathematical derivation, we use $T(\theta)$ to denote the above closed loop transfer function. As the closed loop system is considered here, we use the closed loop transfer function $T(\theta)$ in our performance analysis, not the former open loop transfer function $G(q,\theta)$. After substituting the true parameter vector θ_0 into the above closed loop transfer function $T(\theta)$, we obtain the true closed loop transfer function $T(\theta_0)$ as

$$T_0=T(\theta_0)=\frac{Z_2\theta_0}{a+Z_1\theta_0}$$

Remark 4.1. *In reality the true closed loop transfer function T_0 does not exist; here it is used in our performance analysis. Ideally when the number N tends to ∞, then we have the following asymptotic result.*

$$T\left(\hat{\theta}_N\right)\overset{N\to\infty}{\longrightarrow}T_0=T(\theta_0)$$

As the above asymptotic result is an ideal case, we define one measure to qualify the quantity between the parameterized closed loop transfer function and its true value. An alternative measure from robust control theory is the Vinnicombe distance as

$$\delta_v(T(\theta),T_0)=\begin{cases}\max\limits_w k\left(T(\theta),T_0\right)=\max\limits_w\dfrac{|T(\theta)-T_0|}{\sqrt{1+|T_0|^2}\sqrt{1+|T(\theta)|^2}} & \text{if (29) is satisfied} \\ 1 & \text{otherwise}\end{cases} \tag{28}$$

The condition to be satisfied in order to have $\delta_v(T(\theta),T_0)<1$ is

$$\begin{cases} 1+T_0^*T(\theta)(jw)\neq 0 \text{ for all } w \\ wno\left(1+T_0^*T(\theta)(jw)\right)+\eta(T(\theta))-\tilde{\eta}(T_0^*)=0\end{cases} \tag{29}$$

where $T^*(q)=T(-q)$, $\tilde{\eta}(T)$ denotes the number of closed right half plane of T, while $\eta(T)$ denotes the number of open right half plane poles of T, $wno(T)$ denotes the winding number about the origin of $T(q)$ as q follows the standard Nyquist D-contour. Here $\delta_v(T(\theta),T_0)$ is the Vinnicombe distance between $T(\theta)$ and T_0. From robust control theory, the worst case Vinnicombe distance is at least one optimal value $\sqrt{\gamma}$. This requirement is equivalent to the following inequality.

$$\delta_v(T(\theta),T_0)=\max_w\frac{|T(\theta)-T_0|}{\sqrt{1+|T_0|^2}\sqrt{1+|T(\theta)|^2}}\leq\sqrt{\gamma} \tag{30}$$

Taking square operation on both sides, one inequality is easily obtained.

$$\left(\frac{|T(\theta)-T_0|}{\sqrt{1+|T_0|^2}\sqrt{1+|T(\theta)|^2}}\right)^2 \leq \gamma$$

Expanding the above inequality, we obtain

$$(|T(\theta)-T_0|)^2 \leq \gamma\left(1+|T_0|^2\right)\left(1+|T(\theta)|^2\right)$$
$$\Updownarrow$$
$$T_0^*T_0 + T^*(\theta)T(\theta) - T_0^*T(\theta) - T_0T^*(\theta) - \gamma\left(1+T_0^*T_0\right) - \gamma\left(1+T_0^*T_0\right)T^*(\theta)T(\theta) \leq 0 \tag{31}$$

We regard $T^*(\theta)T(\theta)$ as a free variable and formulate a quadratic function corresponding to $T^*(\theta)T(\theta)$, and then one linear matrix inequality can be got.

$$\begin{pmatrix} T^*(\theta) \\ 1 \end{pmatrix}\begin{pmatrix} 1-\gamma\left(1+T_0^*T_0\right) & -T_0 \\ -T_0^* & T_0^*T_0-\gamma\left(1+T_0^*T_0\right) \end{pmatrix}\begin{pmatrix} T(\theta) \\ 1 \end{pmatrix} \leq 0 \tag{32}$$

Substituting $T(\theta) = \frac{Z_2\theta}{a+Z_1\theta}$ into the above linear matrix inequality, we obtain

$$\begin{pmatrix} \dfrac{Z_2\theta}{a+Z_1\theta} \\ 1 \end{pmatrix}^*\begin{pmatrix} 1-\gamma\left(1+T_0^*T_0\right) & -T_0 \\ -T_0^* & T_0^*T_0-\gamma\left(1+T_0^*T_0\right) \end{pmatrix}\begin{pmatrix} \dfrac{Z_2\theta}{a+Z_1\theta} \\ 1 \end{pmatrix} \leq 0 \tag{33}$$

By pre-multiplying (33) by $(a+Z_1\theta)^*$ and post-multiplying it by $(a+Z_1\theta)$, we have

$$\begin{pmatrix} Z_2\theta \\ a+Z_1\theta \end{pmatrix}^*\begin{pmatrix} 1-\gamma\left(1+T_0^*T_0\right) & -T_0 \\ -T_0^* & T_0^*T_0-\gamma\left(1+T_0^*T_0\right) \end{pmatrix}\begin{pmatrix} Z_2\theta \\ a+Z_1\theta \end{pmatrix} \leq 0 \tag{34}$$

which is equivalent to the following constraint after complex mathematical derivation and with $Q = (1+T_0^*T_0)$.

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^*\begin{pmatrix} (1-\gamma Q)Z_2^2 - T_0^*Z_1Z_2 - T_0Z_1Z_2 + \left(T_0^*T_0-\gamma Q\right)Z_1^2 & -T_0aZ_2 + \left(T_0^*T_0-\gamma Q\right)aZ_1 \\ -T_0^*aZ_2 + \left(T_0^*T_0-\gamma Q\right)aZ_1 & \left(T_0^*T_0-\gamma Q\right)a^2 \end{pmatrix}\begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0 \tag{35}$$

To simplify the above expression, we introduce three variables as

$$\begin{cases} a_{11} = (1-\gamma Q)Z_2^2 - T_0^*Z_1Z_2 - T_0Z_1Z_2 + \left(T_0^*T_0-\gamma Q\right)Z_1^2 \\ a_{12} = -T_0aZ_2 + \left(T_0^*T_0-\gamma Q\right)aZ_1,\ a_{22} = \left(T_0^*T_0-\gamma Q\right)a^2 \end{cases}$$

Then we obtain one Theorem 4.1.

Theorem 4.1. *In the performance analysis process of closed loop system, to qualify the distance between the parameterized closed loop transfer function and its true value, the requirement that the worst case Vinnicombe distance is equal to one optimal value $\sqrt{\gamma}$ can be reformulated as an optimization problem with linear matrix inequality constraints.*

$$\begin{aligned} &\min_{\gamma} \quad \gamma \\ &\textit{subject to} \begin{pmatrix} a_{11} & a_{12} \\ a_{12}^* & a_{22} \end{pmatrix} \leq 0 \end{aligned} \tag{36}$$

This above optimization problem can be solved directly by the Matlab Toolbox. After one optimal value γ is solved, then the worst case Vinnicombe distance is equal to this optimal value.

5. **Performance Analysis on One Transfer Function Matrix.** As the above part studies the performance analysis of closed loop system only on one closed loop transfer function, but from Equation (2), there are four closed loop transfer functions which are used to constitute one closed loop transfer function matrix form. Its parameterized form is

$$H(G,C)=\begin{pmatrix}\dfrac{G(q,\theta)}{1+G(q,\theta)C(q)} & \dfrac{H(q,\theta)}{1+G(q,\theta)C(q)}\\ \dfrac{1}{1+G(q,\theta)C(q)} & -\dfrac{C(q)H(q,\theta)}{1+G(q,\theta)C(q)}\end{pmatrix}$$

where we consider the case that $H(q,\theta)\equiv 1$. If $H(q,\theta)\neq 1$, we also assume $H(q,\theta)$ can be parameterized as a polynomial. Then the following mathematical derivation is similar. Let $H(q,\theta)\equiv 1$, and the above closed loop transfer function matrix $H(G,C)$ is reduced to

$$H(G,C)=\begin{pmatrix}\dfrac{G(q,\theta)}{1+G(q,\theta)C(q)} & \dfrac{1}{1+G(q,\theta)C(q)}\\ \dfrac{1}{1+G(q,\theta)C(q)} & -\dfrac{C(q)}{1+G(q,\theta)C(q)}\end{pmatrix} \tag{37}$$

From Equation (7), we rewrite open loop plant model $G(q,\theta)$ as

$$\begin{aligned}&G(q,\theta)=\frac{B(q,\theta)}{A(q,\theta)}=\frac{b_1q^{-1}+\cdots+b_{n_b}q^{-n_b}}{1+a_1q^{-1}+\cdots+a_{n_a}q^{-n_a}}=\frac{Z_4\theta}{1+Z_3\theta}\\&Z_3=\begin{pmatrix}q^{-1} & \cdots & q^{-n_a} & 0 & \cdots & 0\end{pmatrix},\ Z_4=\begin{pmatrix}0 & \cdots & 0 & q^{-1} & \cdots & q^{-n_b}\end{pmatrix}\end{aligned} \tag{38}$$

Substituting (8) and (38) into each element of matrix $H(G,C)$, we obtain respectively

$$\begin{cases}\dfrac{G(q,\theta)}{1+G(q,\theta)C(q)}=\dfrac{\frac{Z_4\theta}{1+Z_3\theta}}{1+\frac{Z_4\theta}{1+Z_3\theta}\times\frac{N_c}{D_c}}=\dfrac{Z_4\theta D_c}{(1+Z_3\theta)D_c+Z_4\theta N_c}\\ \dfrac{1}{1+G(q,\theta)C(q)}=\dfrac{1}{1+\frac{Z_4\theta}{1+Z_3\theta}\times\frac{N_c}{D_c}}=\dfrac{(1+Z_3\theta)\,D_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c}\\ -\dfrac{C(q)}{1+G(q,\theta)C(q)}=-\dfrac{N_c}{D_c}\times\dfrac{(1+Z_3\theta)D_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c}=-\dfrac{(1+Z_3\theta)\,N_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c}\end{cases} \tag{39}$$

In a tailor made parameterization case, after substituting (39) into matrix $H(G,C)$, then its parameterized form is got.

$$H(\theta)=\begin{pmatrix}\dfrac{Z_4\theta D_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c} & \dfrac{(1+Z_3\theta)\,D_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c}\\ \dfrac{(1+Z_3\theta)\,D_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c} & -\dfrac{(1+Z_3\theta)\,N_c}{(1+Z_3\theta)\,D_c+Z_4\theta N_c}\end{pmatrix} \tag{40}$$

Based on some above definitions, given a plant model $G(q,\theta)$ and a stabilizing controller $C(q)$, the performance of a closed loop system $\begin{bmatrix} C & G\end{bmatrix}$ is defined as the following frequency function.

$$J\left(G,W_l,W_r,\Omega\right)=\sigma_1\left(W_lH(\theta)W_r\right) \tag{41}$$

where Ω denotes the frequency point, and W_l, W_r are diagonal weights.

$$W_l=\begin{pmatrix}W_{l1} & 0\\ 0 & W_{l2}\end{pmatrix},\ W_r=\begin{pmatrix}W_{r1} & 0\\ 0 & W_{r2}\end{pmatrix}$$

$\sigma_1(A)$ denotes the largest singular value of matrix A. This matrix A can be computed as

$$A = W_l H(\theta) W_r = \begin{pmatrix} \dfrac{W_{l1} Z_4 \theta D_c W_{r1}}{(1+Z_3\theta) D_c + Z_4\theta N_c} & \dfrac{W_{l1}(1+Z_3\theta) D_c W_{r2}}{(1+Z_3\theta) D_c + Z_4\theta N_c} \\ \dfrac{W_{l2}(1+Z_3\theta) D_c W_{r1}}{(1+Z_3\theta) D_c + Z_4\theta N_c} & -\dfrac{W_{l2}(1+Z_3\theta) N_c W_{r2}}{(1+Z_3\theta) D_c + Z_4\theta N_c} \end{pmatrix} \tag{42}$$

From knowledge of matrix theory, we see that the worst case performance at frequency point Ω is equal to one optimal value $\sqrt{\gamma}$ which is equivalent to the following inequality.

$$\max_{\Omega} \sigma_1(A_r) = \sqrt{\gamma} \tag{43}$$

Equation (43) is equivalent to

$$\lambda_1(A^*A) = \gamma \tag{44}$$

where $\lambda_1(A^*A)$ denotes the largest eigenvalue of A^*A. It means that we need to solve one largest eigenvalue problem by using linear matrix inequality condition.

$$\lambda_1(A^*A) \le \gamma \tag{45}$$

where a new matrix A^*A can be computed through complex matrix product operation. As matrix A is a rank one matrix, then the problem $\lambda_1(A^*A) \le \gamma$ is equivalent to

$$\begin{pmatrix} \begin{array}{c} \dfrac{[W_{l1} Z_4\theta D_c W_{r1}]^2}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \\ +\dfrac{[W_{l2}(1+Z_3\theta) D_c W_{r1}]^2}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \end{array} & \begin{array}{c} \dfrac{W_{l1} Z_4\theta D_c W_{r1} W_{l1}(1+Z_3\theta) D_c W_{r2}}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \\ -\dfrac{W_{l2}(1+Z_3\theta) D_c W_{r1} W_{l2}(1+Z_3\theta) N_c W_{r2}}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \end{array} \\ \begin{array}{c} \dfrac{W_{l1}(1+Z_3\theta) D_c W_{r2} W_{l1} Z_4\theta D_c W_{r1}}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \\ -\dfrac{W_{l2}(1+Z_3\theta) N_c W_{r2} W_{l2}(1+Z_3\theta) D_c W_{r1}}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \end{array} & \begin{array}{c} \dfrac{[W_{l1}(1+Z_3\theta) D_c W_{r2}]^2}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \\ +\dfrac{[W_{l2}(1+Z_3\theta) N_c W_{r2}]^2}{[(1+Z_3\theta) D_c + Z_4\theta N_c]^2} \end{array} \end{pmatrix} \le \gamma I_2 \tag{46}$$

By pre-multiplying (46) by $[(1+Z_3\theta) D_c + Z_4\theta N_c]^2$ and post-multiplying it by $[(1+Z_3\theta)D_c + Z_4\theta N_c]^2$, we have

$$\begin{pmatrix} [W_{l1} Z_4\theta D_c W_{r1}]^2 + [W_{l2}(1+Z_3\theta) D_c W_{r1}]^2 & \begin{array}{c} W_{l1} Z_4\theta D_c W_{r1} W_{l1}(1+Z_3\theta) D_c W_{r2} \\ -W_{l2}(1+Z_3\theta) D_c W_{r1} W_{l2}(1+Z_3\theta) N_c W_{r2} \end{array} \\ \begin{array}{c} W_{l1}(1+Z_3\theta) D_c W_{r2} W_{l1} Z_4\theta D_c W_{r1} \\ -W_{l2}(1+Z_3\theta) N_c W_{r2} W_{l2}(1+Z_3\theta) D_c W_{r1} \end{array} & \begin{array}{c} [W_{l1}(1+Z_3\theta) D_c W_{r2}]^2 \\ +[W_{l2}(1+Z_3\theta) N_c W_{r2}]^2 \end{array} \end{pmatrix}$$
$$\le \gamma [(1+Z_3\theta) D_c + Z_4\theta N_c]^2 I_2$$

We regard $\theta^*\theta$ as a free variable and formulate a quadratic function corresponding to $\theta^*\theta$. As here the closed loop transfer function matrix is considered, four elements exist in this matrix. Then four linear matrix inequalities can be easily be got.

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* \begin{pmatrix} \begin{array}{c} W_{l1}^2 Z_3^2 D_c^2 W_{r2}^2 + W_{l2}^2 Z_3^2 N_c^2 W_{r2}^2 \\ -\gamma (Z_3^2 D_c^2 + Z_4^2 N_c^2 + 2Z_3 D_c Z_4 N_c) \end{array} & \begin{array}{c} W_{l1}^2 Z_3 D_c^2 W_{r2}^2 + W_{l1}^2 Z_3 N_c^2 W_{r2}^2 \\ -\gamma (Z_3 D_c^2 + Z_4 D_c N_c) \end{array} \\ * & W_{l1}^2 D_c^2 W_{r2}^2 + W_{l2}^2 N_c^2 W_{r2}^2 - \gamma \end{pmatrix} \begin{pmatrix} \theta \\ 1 \end{pmatrix} \le 0$$
$$\Leftrightarrow \begin{pmatrix} \theta \\ 1 \end{pmatrix}^* M_1 \begin{pmatrix} \theta \\ 1 \end{pmatrix} \le 0$$

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* \begin{pmatrix} [W_{l1}Z_4D_cW_{r1}]^2 + [W_{l2}Z_3D_cW_{r1}]^2 & W_{l2}^2Z_3D_c^2W_{r1}^2 \\ -\gamma\left(Z_3^2D_c^2 + Z_4^2N_c^2 + 2Z_3D_cZ_4N_c\right) & -\gamma\left(Z_3D_c^2 + Z_4D_cN_c\right) \\ * & W_{l2}^2D_c^2W_{r1}^2 - \gamma \end{pmatrix} \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0$$

$$\Leftrightarrow \begin{pmatrix} \theta \\ 1 \end{pmatrix}^* M_2 \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0$$

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* \begin{pmatrix} W_{l1}Z_3D_cW_{r2}W_{l2}Z_4D_cW_{r1} & \frac{1}{2}W_{l1}D_cW_{r2}W_{l1}Z_4D_cW_{r1} \\ -W_{r2}Z_3^2N_cW_{r2}W_{l2}D_cW_{r1} & -W_{l2}Z_3N_cW_{r2}D_cW_{r1} \\ * & -W_{r2}N_cW_{r2}W_{l1}D_cW_{r1} \end{pmatrix} \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0$$

$$\Updownarrow$$

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* M_3 \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0$$

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* \begin{pmatrix} W_{l1}Z_4D_cW_{r1}W_{l1}Z_3D_cW_{r2} & \frac{1}{2}W_{l1}Z_4D_cW_{r1}W_{l1}D_cW_{r2} \\ -W_{l2}Z_3W_{r1}W_{l2}Z_3N_cD_cW_{r2} & -W_{l2}Z_3W_{r1}W_{l2}N_cD_cW_{r2} \\ * & -W_{r2}N_cW_{r2}W_{l1}D_cW_{r1} \end{pmatrix} \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0$$

$$\Updownarrow$$

$$\begin{pmatrix} \theta \\ 1 \end{pmatrix}^* M_4 \begin{pmatrix} \theta \\ 1 \end{pmatrix} \leq 0 \tag{47}$$

To simplify the above four linear matrix inequalities, we introduce four matrices M_1, M_2, M_3, M_4 to denote them. The above mathematical derivation is very difficult, if reader or reviewer wants to check them, please ask for the first author. Then we obtain another Theorem 5.1.

Theorem 5.1. *Consider a closed loop system plotted in Figure 1 and the plant model $G(q,\theta)$, a stabilizing controller $C(q)$ are all parameterized as their tailor made parameterization form. The worst case performance at frequency point Ω is equal to one optimal value $\sqrt{\gamma}$. This requirement can be formulated as the following standard convex optimization problem involving linear matrix inequality constraints evaluated at the frequency point.*

$$\begin{aligned} &\min_{\gamma,\tau_1,\tau_2,\tau_3} \quad \gamma \\ &\textit{subject to } \tau_1 \geq 0, \tau_2 \geq 0, \tau_3 \geq 0, M_1 - \tau_1 M_2 - \tau_2 M_3 - \tau_3 M_4 \leq 0 \end{aligned} \tag{48}$$

Comparing Equation (48) with the result in [2], we conclude that our results are a generation of [2]. These two optimization problems (36) and (48) can be solved by many convex optimization algorithms such as fast gradient projection algorithm, active set algorithm, ellipsoidal algorithm, and trust region algorithm.

6. **Simulation Example.** To prove the confidence interval analysis under closed loop condition with a tailor made parameterization, we consider one simulation system.

$$y(t) = G_0(q)r(t) - G_0(q)C(q)y(t) + H_0(q)e(t)$$

where $G_0(q)$, $H_0(q)$ and $C(q)$ are assumed to be as follows respectively.

$$G_0(q) = \frac{0.01293q^{-1} + 0.1062q^{-2} + 0.1058q^{-3} + 0.01279q^{-4}}{1 - 0.2482q^{-1} + 1.091q^{-2} - 0.2441q^{-3} + 0.9822q^{-4}}, \quad H_0(q) = 1,\ C(q) = 1$$

Here the noise model is 1 and it shows the external disturbance acting on closed loop reduces to the white noise disturbance. The feedback is the common positive feedback. The noise $e(t)$ is a white noise with zero mean and unit variance, and input $r(t)$ is similar to noise $e(t)$. In order to analyze the confidence region of the model parameter and cross correlation function, we choose the number of observed data set $\{y(t), w(t)\}_{t=1}^{N}$ as $N =$

500, and apply direct approach to identifying the unknown parameters in parameterized plant model $G_0(q)$. The qualities of nine model parameters affect the output response directly. So the identification accuracy or credibility of model parameters can be all measured by the output response of closed loop.

The whole output frequency response curves are shown in Figure 4, based on estimated model parameters. The middle curve is the actual true amplitude curve from Bode plot tool. When the estimated model parameters are contained in the uncertainty bound with probability level 0.99, the amplitude curves lie above or low the middle curve. From Figure 4, we see these three curves are very close and the middle amplitude curve lies between two confidence amplitude curves with probability level 0.99.

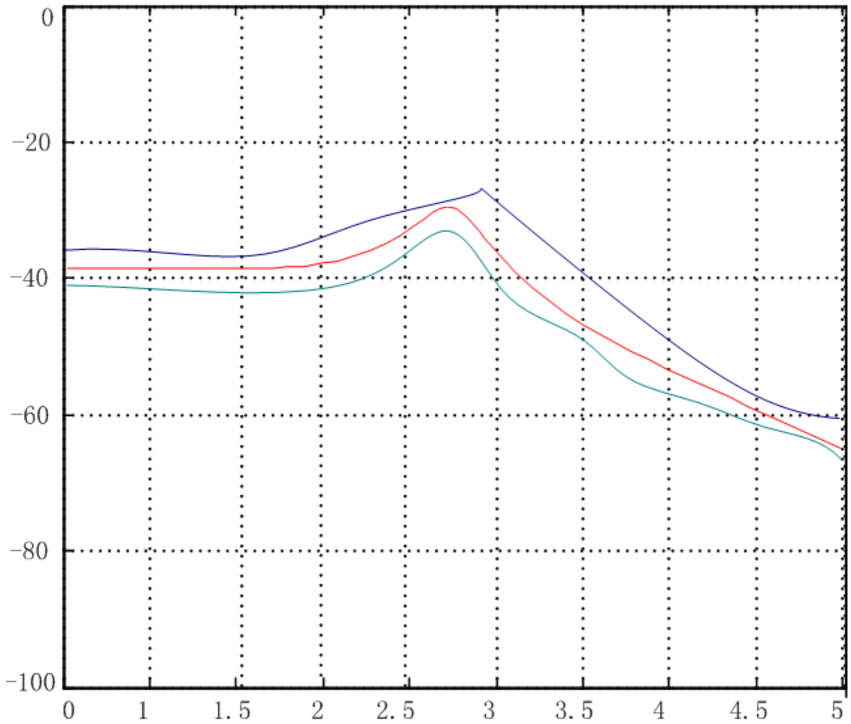

FIGURE 4. Confidence region of amplitude in Bode plot

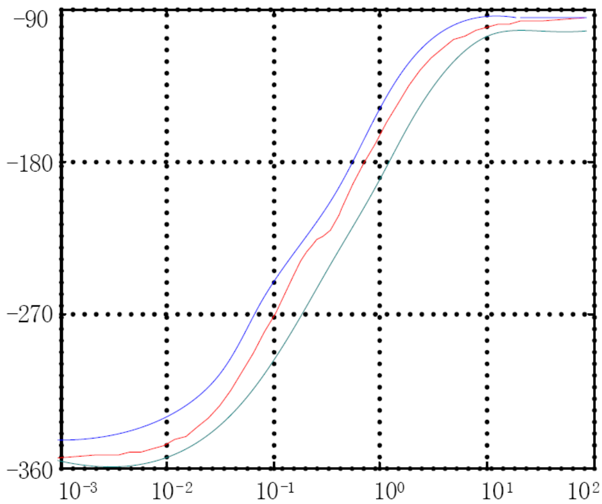

FIGURE 5. Confidence region of phase in Bode plot

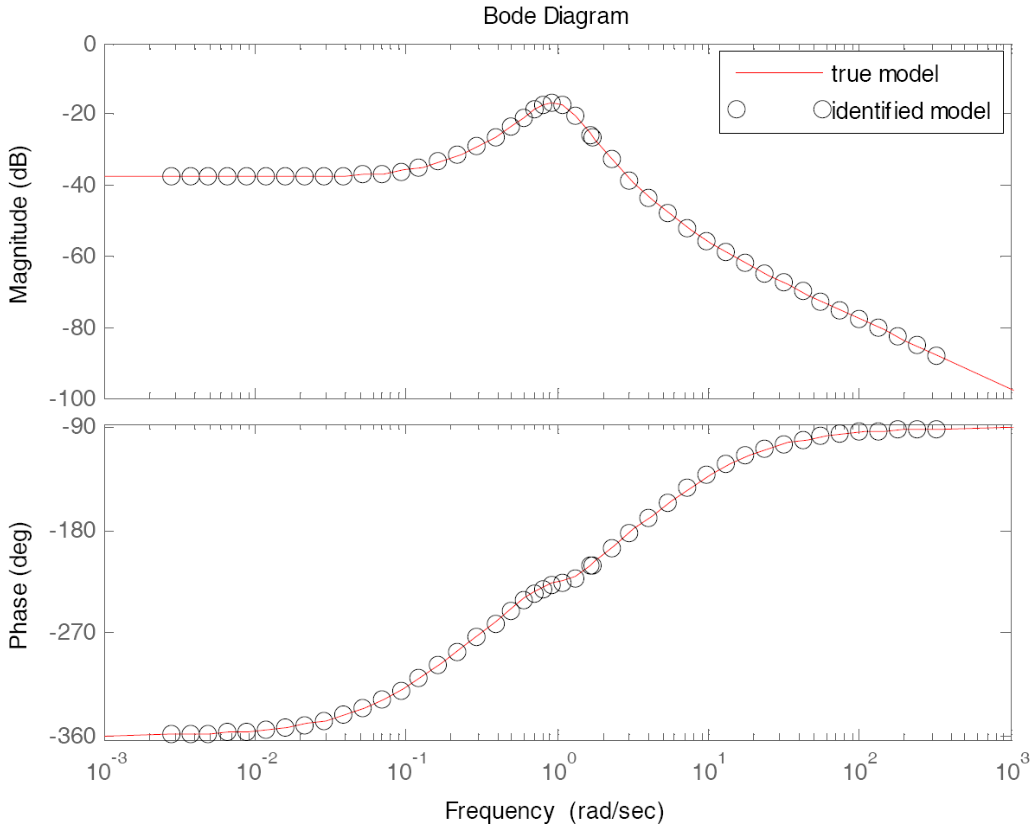

FIGURE 6. Comparison of the true model and its identified model

As using Matlab simulation tool to simulate the output response of Bode plot in closed loop, the phase plot is got with amplitude plot simultaneously. The confidence region phase plot is given in Figure 5, and the middle phase curve lies also between two confidence phase curves with the probability level 0.99. This is similar to the derivation of Figure 4.

To verify the efficiency of the identified model $G\left(\hat{\theta}_N\right)$ and make sure that this identified model can be used to replace the true model, we compare the Bode responses through true model $G_0(q)$ and its identified model $G\left(\hat{\theta}_N\right)$ respectively in Figure 6. From Figure 6, we see that these two Bode response curves coincide with each other, and it means that the model error $\tilde{G}(q)$ will converge to zero with time increase.

7. **Conclusions.**

In this paper, we consider the problem of performance analysis in closed loop system where the plant model and controller are all parameterized as their tailor made parameterization forms. Under this framework of a tailor made parameterization, we study the confidence internal analysis corresponding to the parameter vector and the performance analysis only on one transfer function. Then we extend the result to performance analysis on one transfer function matrix. However, here we do not study the closed loop performance monitoring and experiment design, and these two subjects are our next goals.

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