# THE PLASTICITY AND ELASTICITY OF STOCK PRICE VARIATIONS – PART 1: THEORY AND TECHNIQUES

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ABSTRACT. This paper proposes the conceptions of stock price plasticity and stock price elasticity. Based on the similarity between the motion law of the spring with plasticity under the action of external force and the variation features of the stock price under the drive of trading volumes, with stock prices, trading volumes, the volume of tradable stocks and the balanced price of stock included in some specially designed econometric models, the basic stock price plasticity model, the power-exponential model, the auto-regression model and the distributed-lag model and some stock price elasticity models are built. All these models can pass the economic test and statistical test. The power-exponential model and the first-order auto-regression model have ideal goodness of fit.

**Keywords:** Stock market, Stock price plasticity, Stock price elasticity, Balanced price of stock

1. Introduction. In the stock market, two indexes which are the stock price and the trading volume and their variations are the basis of the technical analysis. Meanwhile, the price-volume analysis of stock is the core content of the technical analysis of stock [1,2]. The theories about the price-volume analysis of stock place emphasis on the intrinsic laws shown by the price and trading volume sequences of stocks. Although the price-volume relation of stock has been studied for a long time, it is still the hot topic in the field of microfinance. The research about stock price mainly focuses on such aspects as the relation between trading volumes and stock price variations, and the casual relation between stock prices and trading volumes [3]. Research findings show that the relation between stock price variations and trading volumes is synchronous, and may not be monotonous; the relation between trading volumes and the absolute value of stock price variations may not be linear [4-6]. According to the research on the relation between trading volumes and stock market information disclosure, trading volume can weigh the amount of the information disclosed during the process of stock trading. Therefore, the stock price generated by the large volume is considered as the better estimation of the stock value. Scholars have more concerned about the combinations between technical analysis and Portfolio strategy in recent years [7-9].

The variation of the trading volume directly reflects whether the trading on the market is active, the continuous rising or falling of the stock price requires the corresponding trading volume. In general, when the stock price continues to rise, the stock price will continue to create new highs recently, and on the other hand, the trading volume will remain relatively high level; When the stock price continues to fall, the stock price will continue to create new lows recently, and on the other hand, the trading volume will remain relatively high level. When the trading volume shrinks, the continuously rising stock price will fall back properly and the continuously falling stock price will rally slightly.

The rise and fall of the stock price under the action of trading volumes is similar to the motion process of the stretched or compressed spring with plasticity: for a spring with plasticity, the larger the external force is, the farther from the balanced position it is, and when the external force is decreased or removed, the spring will move from the current position to the balanced position as a result of its elasticity; however, because of the existence of plasticity, the balanced position of the spring changes with its load and deformation, and the spring cannot move back to its original balanced position after the external force is removed. The larger the external force is (the spring will be farther from the balanced position) and the longer the external force lasts, the larger its plastic deformation is and the farther away from the original balanced position is, and the spring with plasticity will form a new balanced position in every moment. If a small and quick force is exerted on the spring, the spring mainly shows its elasticity, that is, it oscillates around its balanced position; if a large and slow force whose direction is constant is exerted on the spring, the spring will stay in the position where it is stretched or compressed for a long time, and the spring will mainly show its plasticity.

As for the stock price, corresponding to the balanced position of the spring, it means that there is a balanced price of stock in every moment. Driven by the ever-increasing trading volume, the stock price rises and remains high for some time, the stock price often falls back slightly when trading volume shrinks, and this pullback to the balanced price is the embodiment of the elasticity of the stock price. However, the stock price can seldom fall back to the original balanced price, and instead, to a price which is higher than the original balanced price, that is, the balanced price rises, which shows the plasticity of the stock price. Similarly, driven by the ever-increasing trading volume, the stock price falls and remains low for some time, the stock price often rallies when the trading volume shrinks, and this rally to the balanced price is the embodiment of the elasticity of the stock price. However, the stock price can seldom rise to the original balanced price, and instead, to a price which is lower than the original balanced price, that is, the balanced price decreases, which shows the plasticity of the stock price.

Based on the plasticity and elasticity embodied in the stock price variation, this paper builds several mathematical models which can reflect the plasticity and elasticity of the stock price and conducts tests on these models to study their applicability. It is not yet found in the exiting research results that use the concepts of plasticity and elasticity of stock price to study the law of change for stock price.

In Section 2, the concepts of the plasticity and elasticity of the stock price are introduced. In Section 3, the basic stock price plasticity model is constructed and the econometric tests are carried out on it. In Section 4, the basic stock price plasticity model is extended to obtain several different models, including the distributed-lag model, the power-exponential model and the model with the autoregression term. In Section 5, the stock price elasticity model and the stock price elastoplasticity model are built. Section 6 studies the change of the stock price plasticity coefficient with time and explains its meaning of investment. The last section is the conclusion.

2. The Concept of Plasticity and Elasticity of the Stock Price. Through analogical reasoning, the motion of the spring with plasticity under the action of external force and the variation law of the balanced position can be used to describe, explain and quantitatively analyze the stock price variations under the action of trading volumes. In order to introduce the concepts of the plasticity and elasticity of the stock price, this section firstly introduces the balanced price of stock which corresponds to the balanced position of the spring.

**Definition 2.1.** The balanced price of a stock is the price determined and accepted by both sellers and buyers of this stock when they are in a balanced situation, and the balanced price corresponds to the balanced position of a spring without the action of external force.

The stock price should fluctuate around its balanced price all the time. Stock price variations should follow two patterns: the first pattern is the variation of the balanced price (which exists in a hidden form because people can only see the stock price); the second pattern is the deviation of the stock price from the balanced price, which is embodied in the fluctuation of the stock price. The variation of the balanced price of stock is a substantive variation of the stock price and the deviation of the stock price from the balanced price from the balanced price is a transitory variation of the stock price which has the tendency to regress. Generally speaking, only when the deviation of the stock price from the balanced price is larger and the trading volume is higher and the situation continues for a long time will the balanced price change obviously. However, when there is the unexpected news which is unfavorable to some stock, the balanced price will fall instantaneously. This instantaneous variation of the balanced price need not the corresponding trading volume. Additionally, seasoned equity offerings, ex-right, ex-dividend and split of the stock can also cause the instantaneous variation of the balanced price.

Different from the spring, the balanced price of stock must be related to the length of the studied time interval. For example, many investors firmly believe that the current stock price remains abnormal high level in some times, but the stock price is stable, which means that investors can accept the relatively high short-term balanced price. However, they must think that the long-term balanced price of the stock should stay low. The balanced position of the spring is determined by the properties of materials, which is objective, while the balanced price of stock is determined by investors, that is, whether investors can accept it or not, which is subjective. In this paper, short-term and long-term balanced prices are applied to representing the balanced price of stocks of different time intervals.

According to the analysis of the similarity between the motion of the spring with plasticity and the variations of the stock price, the definitions of stock price plasticity and elasticity are proposed below.

**Definition 2.2.** Stock price plasticity refers to the fact that the balanced price of stock will deviate from its original position and toward to the deviation direction when the stock price deviates from its original balanced price for a period of time and a certain trading volume.

**Definition 2.3.** Stock price elasticity refers to the fact that the stock price will automatically approach to its balanced price when the stock price deviates from its original balanced price for a short time and the trading volume shrinks.

For a spring, to determine its balanced position, what people need do is to remove the external force and make it static. For the stock price, it is not easy to determine its balanced price, because the stock is either in the state of continuous trading (market opening) or in frozen state (market closing), which is similar to a spring with plasticity under the reciprocating action of the irregular external force. The balanced price of the stock should possess the following characteristics: (1) the stock price fluctuates around its balanced price; (2) the balanced price is the one accepted by most investors; (3) the X. WANG

stock price approximately equals its balanced price when the trading volume shrinks and the stock has kept backing and filling in a tight range for a long time.

For the spring with plasticity, there exists a balanced position at any moment. Similarly, for the stock in the state of trading, there should be a theoretical balanced price at any moment. When the stock price increases or decreases dramatically and the corresponding trading volume is high, the stock price will deviate a lot from its balanced price temporarily and meanwhile its balanced price will also show an obvious movement. It is not easy to determine the theoretical balanced price of the stock.

During the dramatic rise of the stock price, because the stock price remains higher than its balanced price for a long time and the high trading volume is accumulated, the balanced price of the stock will increase continuously, which should be the true rise of the stock price. At this moment, the stock price plasticity plays a leading role in this process. After the trading volume shrinks, the stock price will fluctuate around the newestablished higher balanced price. Similarly, during the dramatic fall of the stock price, because the stock price remains lower than its balanced price for a long time and the high trading volume is accumulated, the balanced price of the stock will decrease continuously, which should be the true fall of the stock price. At this moment, the stock price plasticity plays a leading role in this process. After the trading volume shrinks, the stock price will fluctuate around the new-established lower balanced price. When the trading volume is relatively low, the temporary rise and fall of the stock price have little impact on its balanced price, stock price variations cannot be accepted by the market and the stock price will approach to its balanced price in a short time. At this moment, the stock price elasticity plays a leading role in this process.

For investors who have some experience of stock investment, most of them will certainly agree with and accept the proposed idea that the concepts of the balanced price of stock, stock plasticity and stock elasticity, and these concepts are introduced to understand the variation features of the stock price. Certainly, it is not enough to only put forward these concepts, that is, they cannot be considered as the theories about the intrinsic laws of the stock price. Applicable mathematical models should be applied to describing the quantitative relations among the balanced price of stock, the stock price plasticity index and the stock price elasticity index; these models need to meet the requirement of the statistical accuracy. Meanwhile, a lot of empirical research should be made to demonstrate the rationality of the models proposed in this paper.

# 3. The Establishment and Test of the Basic Stock Price Plasticity Model.

3.1. The establishment of the basic stock price plasticity model. Econometric methods are used to build the basic stock price plasticity model containing high-frequency transaction data and its mathematical expression is as follows:

$$B_{K+1} = B_K + \alpha \frac{\sum_{i=1}^{N} Q_K^i \left( P_K^i - B_K \right)}{Q_K^F} + \varepsilon_K \tag{1}$$

where  $B_K$  represents the balanced price of the Kth day,  $B_{K+1}$  the balanced price of the K+1th day, that is the new-established balanced price;  $Q_K^i$  represents the trading volume of the *i*th transaction in the Kth day,  $P_K^i$  the price of the *i*th deal in the Kth day;  $Q_K^F$  represents the quantity of the total tradable stocks of the stock in the Kth day,  $Q_K^F$  often takes a fixed number, and N is the number of transactions in the Kth day;  $\alpha$  is the regression parameter, and  $\varepsilon_K$  is the random error term.

Here, the reason why the model (1) is built will be given. Based on experience, the higher the trading volume is and the farther the stock price deviates from its balanced

price, the larger the deviation of the balanced price will be. When  $Q_K^i$  is given, the bigger  $P_K^i - B_K$  which is the deviation of the stock price is, the larger the plastic deformation of the stock price should be; when  $P_K^i - B_K$  is given, the bigger  $Q_K^i$  is, the larger the plastic deformation of the stock price should be. For the same  $P_K^i - B_K$  and the same  $Q_K^i$ , the plastic deformation of the stock price should be inversely proportional to the quantity of tradable stocks. In the expression (1),  $\sum_{i=1}^N Q_K^i (P_K^i - B_K) / Q_K^F$  is the direct drive index that leads to the plastic deformation of the stock price. The model (1) basically conforms to the economic estimation about the variation features of the stock price under the drive of trading volumes, and  $\alpha > 0$  should be a condition of the economic test on the feasibility of the basic stock price plasticity model.

Move  $B_k$  of the expression (1) to its left side, then divide both sides by  $B_K$ , and it can be obtained that

$$\frac{B_{K+1} - B_K}{B_K} = \alpha \frac{\sum_{i=1}^N Q_K^i \left( P_K^i - B_K \right)}{Q_K^F B_K} + \varepsilon_K \tag{2}$$

When  $B_K$  has little change, model (1) and model (2) can be taken as the same model.  $(B_{K+1} - B_K)/B_K$ , the term in the left side of the expression (2), is the variation rate of the balanced price. In the right side of the expression (2),  $\sum_{i=1}^{N} Q_K^i (P_K^i - B_K)/Q_K^F B_K$  is a non-dimensional quantity and is called the stock price plasticity index which is denoted as SPPI. The variation rate of the balanced price is denoted as VRBP. The parameter  $\alpha$  can reflect the size of stock price plasticity and is called the stock price plasticity coefficient. If the value of  $\alpha$  is relatively large, it is easier for the stock price plasticity index (SPPI) to cause the balanced price variations, which means that it is easier for the balanced price to change. Model (2) can be rewritten as the following simple form:

# $VRBP = \alpha \cdot SPPI + \varepsilon$

We have explained above that the balanced price of stock must be related to the length of the time interval. When the econometric method is used to estimate the parameter  $\alpha$ , a convenient choice is that the value of  $B_K$  is replaced by some average price, such as the average stock price in 10, 20, 30 or more days. The average price in a short time represents the short-term balanced price and the average price in a long time represents the long-term balanced price. In the basic stock price plasticity model (2), there is no constant term. For the model which can reflect the variation rate of the balanced price, the value of the constant term ought to be 0, which means if the stock is bought or sold at its balanced price (SPPI = 0), its balanced price should not change. Therefore, such choice should be reasonable.

3.2. The econometric test on the basic stock price plasticity model. 22 representative stocks in different sectors are selected from Shanghai and Shenzhen stock markets in China, including large-capitalization stocks, small-capitalization stocks, blue chip stocks and average-performance stocks, as shown in Table 1. Based on the historical transaction data of the 22 stocks within 237 trading days from Jan. 2nd, 2003 to Dec. 31st, 2003, the basic stock price plasticity model is tested. The tradable stocks are selected as the  $Q_K^F$  of the 22 sample stocks, because the state-owned stocks and legal person stocks in the total capital stock cannot be traded.

The research work about the economic and statistical test for the stock price plasticity models and elasticity models proposed in this paper has been done in 2005, and these results have not been published. It is not very suitable to do some researches work using the transaction data of 2003-2005. Considering the stability of the relationship between

			Tradable	Total	
C	Ticker	Ctl			Stock
Sector	symbol	Stock name	A-Stocks	stock capital	Exchange
			(ten thousand)	(ten thousand)	
Traffic	600811	Orient Group	44197	63149	Shanghai
facilities	000011		11101	00140	Shanghai
		Shenzhen			
Finance	000001	Development	140936	194582	Shenzhen
		Bank A			
		Maanshan			
Steel	600808	Iron & Steel	60000	645530	Shanghai
	600642		20025	46069	Changhai
Real	600662	AJ Securities	16050	196769	Shanghai
estate	000005	Lujiazui Properties	10052	100700	Shanghai
	600675	China Enterprise	31856	69745	Shanghai
		North China			
	600812	Pharmaceutical	46926	116939	Shanghai
		Corporation			
		Guangzhou			
		Baivunshan			Shenzhen
Medicine	000522	Pharmaceutical	15654	37434	
		Holdings A			
	000605	Sihuan			
		Dhamma coutical	2062	2250	Charghan
		Pharmaceutical	2002	8200	Snenznen
		Corporation			
		Shenyang Jinbei			
Automobilo	600609	Automotive	36400	109266	Shanghai
Automobile		Company Limited			
maustry	600104	SAIC Motor	75599	251999	Shanghai
	000800	FAW Car	54600	162750	Shenzhen
		Jiangsu Chunlan	su Chunlan		
		Refrigerating	10004		Shanghai
Household	600854	Equipment	19824	51945	
appliances		Stock Co. Ltd			
appnances	600600	Hojor	44080	70764	Shanghai
	000030	Konka Croup A	29/10	60108	Shonzhon
	000010	Circan Charachai	22419	00190	Shenzhen
		Sinopec Snangnai			
	600688	Petrocnemical	72000	720000	Shanghai
Petrochemical		Company Limited			
industry	600633	Shanghai White	1320	15205	Shanghai
	000000	Cat Group	1020	10200	Shanghai
	000520	CSC Phoenix	29516	51916	Shenzhen
	600100	Tsinghua Tongfang	27303	57461	Shanghai
		Harbin Gong			
Electronic		Da High-tech			
	600701	Enterprise	17469	32409	Shanghai
		Development	1,100	02100	Summerical
technology					
	600817	Hongsneng	1124	8251	Shanghai
		Technology			0
	000021	Shenzhen Kaifa	20058	73293	Shenzhen
		Technology A	20000	10200	Shonzholl

TABLE 1. The related information of 22 sample stocks

Note: The data of tradable capital stock and total capital stock in Dec. 31st, 2003 are selected.

the stock price and the trading volume in stock market, we did not do these research works again using the data of recent years about the economic and statistical test.

The form of model (2) indicates that the stock price plasticity index is calculated with the high-frequency data of the stock transaction. In fact, we can use more simple data, which is the daily transaction data, rearranging expression (2) we can obtain that

$$\frac{B_{K+1} - B_K}{B_K} = \alpha \frac{W_K - B_K Q_K}{Q_K^F B_K} + \varepsilon_K$$

where  $W_K$  is the trading amount of the Kth day, and  $Q_K$  is the trading volume of the Kth day.

In the following analysis, the balanced price is replaced by the 10-day average price of the stock, which is an approximate short-term balanced price. The 22 selected sample stocks are estimated by the basic stock price plasticity model and the results are shown in Table 2. According to the results, the stock price plasticity coefficients  $\alpha$  of 22 sample stocks satisfy  $\alpha > 0$ , that is, all of them can pass the economic test and *t*-test. However, the test results of DW show that there exists obvious self-correlation of sequences and that of  $R^2$ , the goodness of fit of the model, is around 0.3 on average. Additionally, for different stocks, there are some differences in the value of the stock price plasticity coefficient.

Ticker symbol	$\begin{array}{c} \text{Regression model} \\ \text{The test values} \\ \text{of } \alpha \text{ and } t \end{array}$	$R^2$	DW	Ticker symbol	$\begin{array}{c} \text{Regression model} \\ \text{The test values} \\ \text{of } \alpha \text{ and } t \end{array}$	$R^2$	DW
600811	4.47(11.29)	0.36	0.52	000800	2.92(16.29)	0.50	0.79
000001	0.37(14.39)	0.47	0.68	600854	8.57(10.91)	0.30	0.68
600808	2.47(12.47)	0.33	0.82	600690	10.13(10.26)	0.29	0.55
600643	3.60(8.84)	0.24	0.45	000016	6.40(10.87)	0.33	0.73
600663	2.94(6.39)	0.11	0.40	600688	4.26(14.57)	0.41	0.76
600675	12.84(10.69)	0.26	0.66	600633	3.75(3.64)	0.19	0.33
600812	3.39(11.74)	0.37	0.67	000520	5.73(9.65)	0.28	0.62
000522	4.06(11.09)	0.33	0.56	600100	3.49(10.59)	0.32	0.75
000605	3.15(7.30)	0.12	0.29	600701	9.54(7.98)	0.18	0.50
600609	1.92(11.70)	0.36	0.67	600817	2.60(8.60)	0.21	0.55
600104	2.16(11.74)	0.25	0.69	000021	2.21(11.30)	0.35	0.46

TABLE 2. The calculated results of the basic stock price plasticity model (2)

The reasons why the goodness of fit of the basic stock price plasticity model is relatively low may include the following aspects: firstly, the form of the model needs to be improved; secondly, it is necessary to introduce new explanatory variables to eliminate the selfcorrelation of sequences and improve the goodness of fit of the models; thirdly, for different time windows, the model may have different stock price plasticity coefficient  $\alpha$ , and it is not very rational to use the large sample data to compute the uniform  $\alpha$ . In order to improve the goodness of fit of the basic stock price plasticity model and eliminate the selfcorrelation of sequences, this paper will extend the model (2) and build several different forms of models.

### 4. The Extension of the Basic Stock Price Plasticity Model.

4.1. The distributed-lag model of stock price plasticity. Because the variation of the balanced price of stock may depend on not only the stock price plasticity index of the current day but also the previous days, the influence of the lag-period stock price plasticity index on the current balanced price of stock mainly reflects investors' psychological recognition degree of stock price variations; this is the characteristics of the trend of stock price variations. The basic stock price plasticity model can be extended as following form:

$$VRBP_{k} = \alpha_{1}SPPI_{k} + \alpha_{2}SPPI_{k-1} + \dots + \alpha_{n+1}SPPI_{k-n} + \varepsilon_{k}$$
(3)

where  $SPPI_k$  signifies the stock price plasticity index of today and  $SPPI_{k-l}$  the stock price plasticity index of the previous day. In model (3), the stock price plasticity indexes of today and the previous n days are used to explain the variation rate of the balanced price of stock. The model (3) is called the distributed-lag model of stock price plasticity.

Take FAW Car (000800) for example and estimate the stock price plasticity model containing lagged variables to study the order of the lag period in the model (3). The results are shown in Table 3.

TABLE 3. The calculated results of the distributed-lag model of stock price plasticity of FAW Car (000800)

Sample observation interval	Lagged order	Regression model The test values of $\alpha$ and $t$	$R^2$	F-test	DW
0002.01.00	0 order	$\alpha_1 = 2.92 \ (16.29)$	0.50		0.79
2003.01.02	1st order	$\alpha_1 = 1.77, \ \alpha_2 = 1.54 \ (6.97, 6.05)$	0.56	305.10	0.53
2003.12.31	2nd order	$\alpha_1 = 1.53,  \alpha_2 = 1.04,  \alpha_3 = 0.91 (5.95, 3.64, 3.53)$	0.58	166.17	0.50
0002.01.00	0 order	$\alpha_1 = 2.72 \ (13.19)$	0.55		1.32
2003.01.22	1st order	$\alpha_1 = 1.64, \ \alpha_2 = 1.37 \ (5.45, 4.46)$	0.66	114.73	0.77
2003.04.25	2nd order	$\alpha_1 = 1.54, \ \alpha_2 = 1.21, \ \alpha_3 = 0.30 (4.72, 3.37, 0.86)$	0.66	57.47	0.71

It is easy to find that in the model (3), the goodness of fit  $R^2$  rises lightly with the increase of the lagged order. After adding lagged variables, the estimated values of the stock price plasticity coefficient  $\alpha$  become a group of estimated values and the current period estimated value is generally greater than those of the lag period, which are consistent with the experience that the influence of recent variables on the stock price is larger than that of lagged variables.

The first-order distributed-lag model of stock price plasticity is applied to testing the 22 selected stocks. According to the test results, the stock price plasticity coefficients of the 22 sample stocks satisfy  $\alpha_1 > 0$ ,  $\alpha_2 > 0$ ,  $\alpha_1 > \alpha_2$  and all variables can pass the *t*-test. Meanwhile, F-test of the model is ideal, the goodness of fit  $(R^2)$  of the model is higher than that of the basic stock price plasticity model and there is no obvious change in the values of DW test.

4.2. The stock price plasticity power-exponential model. From the analysis of the fitting residue of the basic stock price plasticity model, it can be found that the linear relation between the variation rate of the balanced price (VRBP) and the stock price plasticity index (SPPI) is not very nice. In plastic mechanics, some of the formulas describing the plasticity of solid materials are in the power-exponential form. Therefore, there may exist power function relation between VRBP and SPPI.

The expression (2) of the basic stock price plasticity model can be changed as follows:

$$VRBP = \alpha \cdot sign(SPPI) \cdot [abs(SPPI)]^m + \varepsilon$$

where *m* is an unknown power parameter, *VRBP* is the variation rate of the balanced price,  $\alpha$  is the stock price plasticity coefficient, and *SPPI* is the stock price plasticity index. In the above expression, sign(x) is a sign function: when *x* is positive, the value of sign(x) is +1; when *x* is negative, the value of sign(x) is -1; when *x* is 0, the value of sign(x) is 0, and abs(x) is the absolute value function.

Different time intervals of over 100 stocks from Shanghai and Shenzhen stock markets including the 22 sample stocks are tested by the model. According to the calculated results, when m is 0.4, all sample stocks can pass the statistical test and economic test and the goodness of fit  $(R^2)$  of the model is usually about 0.6, which is much higher than that of the basic stock price plasticity model. The stock price plasticity power-exponential model can be determined as follows:

$$VRBP = \alpha \cdot sign(SPPI) \cdot [abs(SPPI)]^{0.4} + \varepsilon$$
(4)

For the convenience of expression, let  $\omega(x) = sign(x) \cdot [abs(x)]^{0.4}$  and the model (4) can be written as follows:

$$VRBP = \alpha \cdot \omega(SPPI) + \varepsilon$$

The econometric test is made on the stock price plasticity power-exponential model (4) and the test results are shown in Table 4. From Table 4, it can be seen that all the stock price plasticity coefficients  $\alpha$  of the 22 sample stocks can pass *t*-test and satisfy  $\alpha > 0$ , that is, all of them can pass the economic test. Moreover, the goodness of fit of the model is usually about 0.6, which is much higher than that of the basic stock price plasticity model. However, the test results of DW show that the model has obvious self-correlation of sequences.

The first-order distributed-lag model of the stock price plasticity power-exponential model is:

$$VRBP = \alpha_1 \cdot \omega(SPPI) + \alpha_2 \cdot \omega(SPPI(-1)) + \varepsilon$$

TABLE 4. The calculated results of the stock price plasticity powerexponential model (4)

Ticker symbol	$\begin{array}{c} \text{Regression model} \\ \text{The test values} \\ \text{of } \alpha \text{ and } t \end{array}$	$R^2$	DW	Ticker symbol	$\begin{array}{c} \text{Regression model} \\ \text{The test values} \\ \text{of } \alpha \text{ and } t \end{array}$	$R^2$	DW
600811	2.09(18.71)	0.59	1.06	000800	1.95(22.30)	0.65	1.02
000001	2.25(19.10)	0.60	1.18	600854	2.24(17.99)	0.56	1.02
600808	1.48(20.95)	0.61	1.23	600690	2.51(17.75)	0.56	1.04
600643	2.36(19.35)	0.61	0.99	000016	2.22(18.29)	0.58	1.14
600663	1.97(17.30)	0.54	0.86	600688	2.10(24.25)	0.68	1.19
600675	2.37(18.15)	0.54	1.05	600633	1.51(12.97)	0.57	0.89
600812	1.80(17.59)	0.57	0.97	000520	1.98(17.59)	0.57	1.12
000522	2.22(19.09)	0.60	1.04	600100	1.89(18.65)	0.59	1.03
000605	2.34(13.99)	0.41	0.66	600701	3.35(18.02)	0.56	0.90
600609	1.60(18.89)	0.60	1.03	600817	1.68(15.93)	0.51	0.87
600104	1.78(20.85)	0.58	1.31	000021	2.26(24.06)	0.71	1.04

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The calculated results of the above model show that the estimated values of  $\alpha_1$  and  $\alpha_2$  are greater than 0 and  $\alpha_1$  is greater than  $\alpha_2$ . Moreover,  $\alpha_1$  and  $\alpha_2$  can pass t-test and F-test of the model. However, the goodness of fit of the above model is not much higher than that of the model (4) and DW test values still show that the model has obvious self-correlation of sequences. The corresponding estimated results will not be shown here.

4.3. The stock price plasticity model with the auto-regression term. From the analysis of the DW test values on the basic stock price plasticity model, it can be seen that the basic model has obvious self-correlation of sequences; we suggest that the lagged variable of the explained variable may be a good explanatory variable, which means that the econometric model with the auto-regression term should be selected. The basic stock price plasticity model can be extended as the form below:

$$VRBP = \alpha SPPI + \gamma_1 VRBP(-1) + \dots + \gamma_n VRBP(-n) + \varepsilon$$

The above model is called the *n*-order auto-regression model. The lag period of VRBP is used to explain VRBP of the current period, which means the investors' psychological recognition of balanced price variations is characterized by time. The variation rate of the balanced price of the previous period has an effect on that of the current period, or rather, the variation trend of the balanced price of stock is widely accepted by investors and the variation of the balanced price of stock is characterized by inertia.

According to plenty of estimated results of the above model, introducing first-order auto-regression term can improve the goodness of fit of the model, and the DW test value is approximately equal to 2.0, which indicates that the self-correlation of sequences is nearly eliminated. Introducing second-order or higher-order auto-regression term cannot significantly improve the goodness of fit. Therefore, we select the first-order auto-regression model as follows:

$$VRBP = \alpha SPPI + \gamma VRBP(-1) + \varepsilon \tag{5}$$

The first-order auto-regression model (5) is used to compute the 22 sample stocks and part of the calculated results (the estimated results of one stock in each sector are listed) are shown in Table 5. According to the table, the goodness of fit is greater than 0.76 and the highest one is 0.86, so the goodness of fit is improved a lot; all DW test values are approximately equal to 2, which means the model basically eliminates the self-correlation of sequences; the values of  $\alpha$  are greater than 0, that is, all of them can pass the economic test, and the *t*-test of  $\alpha$  and the F-test of the equation are also very good; the coefficients of the auto-regression term  $\gamma$  are relatively stable.

TABLE 5. The calculated results of the first-order auto-regression model (5)

Ticker	Regression model The test values of $\alpha$ , $\alpha$ and $t$	$\mathbb{R}^2$	F-test	DW
<u> </u>	$\frac{1}{10000000000000000000000000000000000$	0.76	797 44	1.00
600811	$\alpha = 0.86, \gamma = 0.80 \ (2.86, 20.26)$	0.70	(31.44	1.89
000001	$\alpha = 0.23, \ \gamma = 0.60 \ (12.11, 17.14)$	0.76	758.28	1.75
600808	$\alpha = 0.33, \gamma = 0.83 \ (2.08, 20.60)$	0.76	754.79	2.03
600663	$\alpha = 0.21,  \gamma = 0.90 (0.90, 29.69)$	0.81	1015.26	1.87
000605	$\alpha = 0.18,  \gamma = 0.92  (0.91, 34.40)$	0.86	1378.08	1.89
000800	$\alpha = 0.46,  \gamma = 0.85  (3.41, 25.23)$	0.86	1494.22	1.99
600690	$\alpha = 1.60, \gamma = 0.82 \ (2.30, 21.64)$	0.76	762.33	1.84
600688	$\alpha = 0.64, \ \gamma = 0.85 \ (2.78, 22.61)$	0.81	1034.74	2.04
000021	$\alpha = 0.38, \gamma = 0.85 \ (3.12, 26.37)$	0.83	1192.01	1.88

The goodness of fit of the first-order auto-regression model (5) is much better than that of the basic stock price plasticity model (2). From this, it can be seen that the balanced price variation of the previous period has a relatively strong and stable impact on that of the current period, indicating that the stock price is characterized by inertia, which is the trend in the technical analysis of the stock price.

4.4. The stock price plasticity power-exponential model with the auto-regression term. In order to be further compared with the basic stock price plasticity model, the first-order auto-regression model of the stock price plasticity power-exponential model is tested and its form is as follows:

$$VRBP = \alpha \cdot \omega(SPPI) + \gamma \cdot VRBP(-1) + \varepsilon \tag{6}$$

Part of the calculated results of the first-order auto-regression model (6) are shown in Table 6. According to the table, the goodness of fit is basically greater than 0.77 and the highest one is 0.87; DW test values are approximately equal to 2.0 and the model basically eliminates the self-correlation of sequences; all the values of  $\alpha$  are greater than 0, that is, all of them can pass the economic test, and the *t*-test of  $\alpha$  and the F-test of the equation are also very good; the coefficients of the auto-regression term  $\gamma$  are relatively stable.

Ticker	Regression model	$D^2$	E tost	DW
symbol	The test values of $\alpha$ , $\gamma$ and $t$		r-test	DW
600811	$\alpha = 0.71, \ \gamma = 0.67 \ (5.56, 14.24)$	0.77	827.40	2.03
000001	$\alpha = 0.65, \gamma = 0.70 \ (4.64, 14.50)$	0.79	866.50	2.04
600808	$\alpha = 0.48, \gamma = 0.67 (5.25, 13.58)$	0.78	850.61	2.01
600663	$\alpha = 0.43, \gamma = 0.78 \ (4.10, 19.55)$	0.82	1099.58	1.94
000605	$\alpha = 0.16, \gamma = 0.90 \ (1.39, 6.83)$	0.86	1385.69	1.91
000800	$\alpha = 0.42, \ \gamma = 0.78 \ (4.44, 19.51)$	0.87	1550.69	2.04
600690	$\alpha = 0.70, \ \gamma = 0.71 \ (4.45, 15.07)$	0.78	822.27	1.94
600688	$\alpha = 0.67, \ \gamma = 0.69 \ (5.74, 14.59)$	0.83	1166.05	2.10
000021	$\alpha = 0.70, \ \gamma = 0.69 \ (5.53, 14.68)$	0.85	1312.45	1.92

TABLE 6. The calculated results of the first-order auto-regression model (6)

4.5. The summary of all stock price plasticity models. On the basis of the basic stock price plasticity model, this paper selects several extended form of models and makes econometric tests on them. Summarizing the test results of all models plays a role in two aspects: one is to select better models to conduct applied research; the other is to analyze the better model and explain the reason why these models are better, so that we can understand the intrinsic laws of the stock price variation more deeply.

The basic stock price plasticity model, the power-exponential model, the first-order auto-regression model and the first-order distributed-lag model can all pass the economic test and statistical test, but their goodness of fit and DW test results are different. Because all these models can pass the economic test and statistical test, it can be sure that the stock price variation is characterized by plasticity and the form of the stock price plasticity model is reasonable. The first-order auto-regression model can basically eliminate the selfcorrelation of sequences and the coefficients of the corresponding auto-regression terms are relatively stable. The basic stock price plasticity model is the basis of the improved models. The stock price plasticity coefficient  $\alpha$  of the stock price plasticity power-exponential model contains sufficient information about the stock price plasticity and has theoretical value and application potential. The models with the first-order auto-regression term have higher goodness of fit and are the best stock price plasticity models.

# 5. The Establishment of the Stock Price Elasticity Model and the Stock Price Elastoplasticity Model.

5.1. The establishment of the stock price elasticity model. In physics, the Hooke's law that describes the elasticity of materials is as follows:

$$F = -k\Delta x$$

where  $\Delta x$  is the displacement, k > 0 is the elastic coefficient, F is the elastic force and the minus means that the force is in the opposite direction to the displacement. We believe that when the trading volume is low, the stock price is characterized by elasticity. The farther the stock price deviates from its balanced price, the larger the force that makes the stock price back to its balanced price is. In fact, BIAS, used in the technical analysis of the stock price, takes example by the elastic force. Imitating the form of Hooke's law, this paper builds the stock price elasticity model and its mathematical expression is as follows:

$$P_{K+1} = P_K + \beta \left( P_K - B_K \right) + \varepsilon_K \tag{7}$$

In the model (7),  $P_{K+1}$  is the average price of the K + 1th day,  $P_K$  the average price of the Kth day,  $B_K$  the balanced price of the Kth day,  $\beta$  the regression parameter and  $\varepsilon_K$ the random error term.  $W_K$  represents the trading amount of the Kth day and  $Q_K$  the trading volume of the Kth day, so  $P_K = W_K/Q_K$ .  $\beta (P_K - B_K)$  shows the size of the elastic force affecting the stock price,  $P_k - B_k$  is equivalent to the displacement and  $\beta$  is equivalent to the elastic coefficient in mechanics. According to the property of the elastic force, there should be  $\beta < 0$ . If the stock price is higher than its balanced price, that is,  $P_k - B_k > 0$ , the elastic force affecting the stock price will make the stock price decrease; if the stock price is lower than its balanced price, that is,  $P_k - B_k < 0$ , the elastic force affecting the stock price will make the stock price increase. The farther the average stock price  $P_K$  deviates from its balanced price  $B_K$  is, the larger the force that makes the stock price back to its balanced price is and the longer the deviation of the stock price  $P_{k+1} - P_k$ is.

Divide both sides of the expression (7) by  $P_K$ , and it can be rewritten as follows:

$$\frac{P_{K+1} - P_K}{P_K} = \beta \frac{P_K - B_K}{P_K} + \varepsilon_K \tag{8}$$

The left side of the model (8) is the variation rate of the average price which is denoted as VRP.  $(P_K - B_K)/P_K$  is the stock price elasticity index which is denoted as SPEI. The model (8) can be simplified as follows:

$$VRP = \beta \cdot SPEI + \varepsilon$$

The parameter  $\beta$  shows the size of the stock elasticity and is called the stock price elasticity coefficient.

The elasticity of the stock price can be embodied only when the trading volume is very low. When the trading volume is very low and the stock has consolidated for a long time, although the rise or fall of the stock price will appear occasionally, the stock price will get back around its original position very soon (in a couple of days), which is the embodiment of the stock price elasticity. After the stock price increases or decreases on higher volume for a long time, if the trading volume shrinks sharply, the rising stock price will fall back and the falling stock price will rally, which is also the embodiment of the stock price elasticity. The application range of the stock price elasticity model should be the time interval in which the trading volume is very low. The explained variable of the stock price elasticity model can determine the average price of the next day, so in fact, it is a stock price prediction model. Plenty of investment experiences indicate that the predictability of the stock price is very weak, so we cannot expect that the model (8) has a high goodness of fit.

Table 7 lists the regression results of Shenzhen Development Bank A in the observation intervals where the trading volume is relatively low; there are differences in the estimated results because of the difference of sample intervals. All the estimated values of  $\beta$  is negative, which is consistent with the economic meaning of the stock price elasticity coefficient. The *t*-test values show that the linear relation between *VRP* and *SPEI* is not very obvious and the goodness of fit index  $R^2$  is not ideal. All *DW* test values are between 1.5 and 2.0 and there is basically no self-correlation of sequences in the model. The estimated results of other sample stocks are basically in line with the results listed in Table 7.

Sample interval	Regression model The test values of $\beta$ and $t$	$R^2$	DW
2003.04.17-2003.06.02	-0.27(-3.10)	0.26	1.85
2003.11.18-2003.11.25	-0.27(-2.88)	0.09	1.89
2004.02.16-2004.02.20	-0.24(-2.12)	0.14	1.88
2004.02.20-2004.02.27	-0.24(-2.12)	0.11	1.83
2004.06.02-2004.06.14	-0.26(-2.97)	0.13	1.77
2004.09.10-2004.09.23	-0.28(-3.01)	0.14	1.87
2004.10.26-2004.11.01	-0.29(-2.12)	0.12	1.73
2005.04.01-2005.04.11	-0.27(-2.93)	0.15	1.86
2005.04.11-2005.04.22	-0.19(-2.02)	0.13	1.85

TABLE 7. The calculated results of the basic stock price elasticity model (8) with the sample stock Shenzhen Development Bank A (000001)

5.2. The stock price elastoplasticity model. Because the stock price variation is influenced not only by the elastic force but also the stock price plasticity, the stock price plasticity index and the stock price elasticity index are used as the explanatory variable to explain the variation rate of the stock price. The stock price elastoplasticity model containing the plasticity term and the elasticity term is built and its mathematical expression is as follows:

$$VRP = \alpha \cdot \omega(SPPI) + \beta \cdot SPEI + \varepsilon \tag{9}$$

where  $\omega(SPPI) = sign(SPPI) \cdot [abs(SPPI)]^{0.4}$  represents the plasticity term, VRP is the variation rate of the average stock price,  $\alpha \cdot \omega(SPPI)$  represents the influence of the stock price plasticity index on the variation rate of the price, and  $\alpha \cdot \omega(SPPI)$  embodies the stock price plasticity;  $\beta \cdot SPEI$  represents the influence of the stock price elasticity index on the variation rate of the price, and  $\beta \cdot SPEI$  embodies the stock price elasticity. There would exist a positive correlation between the SPPI and the VRP, and there would exist a negative correlation between the SPPI and the VRP, so the following conditions of the economic test should be satisfied: the stock price plasticity coefficient  $\alpha > 0$  and the stock price elasticity coefficient  $\beta < 0$ .

Take Shenzhen Development Bank A (000001) as the sample stock and take the short time interval with obvious elasticity for example to conduct empirical analyses. Table 8 lists the regression results of Shenzhen Development Bank A in the short observation intervals. According to the results, all the values of  $\alpha$  are greater than 0 and can pass the *t*-test and all the values of the stock price elasticity coefficient  $\beta$  are less than 0 and most of them can pass the *t*-test. *DW* test show that there is no self-correlation of sequences in the model. Compared with that of the basic stock price elasticity model (8), the goodness of fit  $R^2$  of this model has improved.

TABLE 8. The calculated results of the stock price elastoplasticity model (9) of Shenzhen Development Bank A (000001)

Sample interval	Regression modelThe test values of $\alpha$ , $\beta$ and $t$	$R^2$	F-test	DW
2004.01.15-2004.02.06	$\alpha = 16.68, \ \beta = -1.04 \ (2.85, -2.79)$	0.31	13.27	1.89
2004.03.09-2004.03.18	$\alpha = 23.59, \ \beta = -0.88 \ (2.53, -2.80)$	0.28	8.50	1.78
2004.03.18-2004.03.30	$\alpha = 15.08, \ \beta = -0.87 \ (1.48, -2.79)$	0.22	10.81	1.98
2004.04.05-2004.04.15	$\alpha = 18.12, \ \beta = -0.92 \ (2.43, -2.81)$	0.22	10.83	1.85
2004.06.02-2004.06.14	$\alpha = 23.18, \ \beta = -1.04 \ (2.32, -1.34)$	0.28	7.01	1.83
2004.09.10-2004.09.23	$\alpha = 25.61, \ \beta = -1.07 \ (2.43, -2.22)$	0.20	6.23	1.92
2004.09.22-2004.10.08	$\alpha = 20.43, \ \beta = -0.89 \ (2.36, -2.23)$	0.25	10.17	1.78
2005.04.01-2005.04.11	$\alpha = 19.68, \ \beta = -0.99 \ (2.18, -2.21)$	0.25	12.45	1.89

5.3. The stock price elastoplasticity model containing DUALVOL. In this section, the active buying volume and the active selling volume are introduced into the stock price elasticity model. The active buying volume reflects the demand for stocks and can make the stock price rise; the active selling volume reflects the supply of stocks and can make the stock price fall.

The mathematical expression of the stock price elastoplasticity model containing DU-ALVOL is as follows:

$$VRP = \alpha \cdot \omega(SPPI) + \beta \cdot SPEI + \gamma \cdot BSI + \varepsilon$$
(10)

where  $\gamma \cdot BSI$  represents the influence of DUALVOL on the stock price variation,  $BSI = (QB_K - QS_K)/Q_K$  is the percentage of the difference between the active buying volume and the active selling volume in the total volume,  $Q_K$  the trading volume of the Kth day,  $QB_K$  the active buying volume of the Kth day and  $QS_K$  the active selling volume of the Kth day. The percentage of the difference between them in the total volume should have a positive correlation with the average stock price, so there should be  $\gamma > 0$ .

Because the model (10) contains the plasticity term, there will be a high goodness of fit in the time interval where the trading volume is relatively high. Several tests are made on the small sample of the historical transaction data of the 22 sample stocks and the goodness of fit of the model (10) is satisfactory. A time interval with 10 trading days (from February 26, 2004 to March 10, 2004) is selected arbitrarily. Based on the historical transaction data of the 22 sample stocks in this time interval, the stock price elastoplasticity model (10) containing DUALVOL is tested. Part of the calculating results are shown in Table 9.

It can be seen that the stock price elastoplasticity model (10) containing DUALVOL can pass the economic test and the econometric test, the model (10) has relatively high goodness of fit for any small sample containing 10 trading days, so model (10) can describe the variation of the stock price under the drive of the volume very well.

Ticker	Regression model	$D^2$	F toot	DW
symbol	The test values of $\alpha$ , $\beta$ , $\gamma$ and $t$	$\mathbf{n}$	r-test	DW
600811	$\alpha = 5.47, \ \beta = -0.20, \ \gamma = 0.09 \ (2.71, 1.88, 2.47)$	0.60	10.46	1.89
000001	$\alpha = 6.74, \beta = -0.29, \gamma = 0.11 \ (2.71, 2.53, 2.76)$	0.61	10.49	1.93
600808	$\alpha = 1.95, \beta = -0.12, \gamma = 0.09 \ (2.38, 2.36, 2.08)$	0.62	12.91	1.95
600663	$\alpha = 0.54, \beta = -0.38, \gamma = 0.05 \ (2.14, 1.92, 2.60)$	0.55	12.93	1.97
000605	$\alpha = 8.08, \beta = -1.31, \gamma = 0.02 \ (2.46, 2.28, 2.61)$	0.66	10.77	1.96
000800	$\alpha = 0.71, \beta = -0.13, \gamma = 0.02 \ (2.78, 2.15, 2.29)$	0.52	10.07	1.93
600690	$\alpha = 3.54, \beta = -0.27, \gamma = 0.03 \ (2.96, 1.47, 2.77)$	0.51	9.87	1.89
600688	$\alpha = 5.27, \ \beta = -0.43, \ \gamma = 0.03 \ (2.04, 2.08, 2.67)$	0.61	10.51	1.99
000021	$\alpha = 2.87, \beta = -0.34, \gamma = 0.02 \ (1.15, 2.61, 2.51)$	0.63	11.11	1.92

TABLE 9. The calculated results of the stock price elastoplasticity model (10) containing DUALVOL (from February 26, 2004 to March 10, 2004)

6. The Change of the Stock Price Plasticity Coefficient with Time. The length of the time window for estimating the stock price plasticity coefficient can be chosen by people. In the above discussion, the data of one year is used. The longer or shorter time window can also be used to study the change of the stock price plasticity coefficient in different time intervals.

20 trading days is chosen as the time window and the stock price plasticity powerexponential model (4) is also selected. The data of the previous 20 trading days is used for regression the stock price plasticity coefficient  $\alpha$  of the day and a curve composed by the stock price plasticity coefficient can be obtained through moving time window. Figure 1 shows the curves of the stock price and the stock price plasticity coefficient of Yian Tech (000008). The fine solid line refers to the stock price, and the imaginary line represents the decuple stock price plasticity coefficient and the bold solid line is another calculated index. From Figure 1, it can be seen that with the fluctuation of the stock price, the stock price plasticity coefficient changed sharply. Before the stock price rose sharply, the stock



FIGURE 1. The variation curves of the stock price and the stock price plasticity coefficient of Yian Tech (000008)

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price plasticity coefficient had increased from 1.0-1.3 to 2.5-3.0; during the process where the stock price increased from 30 yuan per share to 120 yuan per share and decreased from 120 yuan per share to 75 yuan per share, the stock price plasticity coefficient did not change a lot; during the 5 months after May, 2000, the stock price plasticity coefficient dropped down to around 1.0, and during the steep fall of the stock price followed, the stock price plasticity coefficient increased from 1.0 to around 2.0.

According to the stock price plasticity power-exponential model (4), the stock price plasticity reflects the amount of the variation of balanced price caused by the SPPI. The larger the stock price plasticity coefficient is, the easier the balanced price changes. If the stock price plasticity coefficient is relatively large, it seems that the number of the tradable stocks becomes small, which means parts of stocks are locked and cannot be traded. Qualitatively, through the research on the variation of the stock price plasticity coefficient, the situation where tradable stocks are locked can be analyzed, which is a problem that will inevitably draw investors' attention. The related problems will be explored in subsequent research papers.

7. Conclusion. Imitating the theory about the plasticity and elasticity of materials in physics, this paper proposes the concepts of the balanced price of stock, stock price plasticity and stock price elasticity. Based on the similarity between the motion law of the spring with plasticity under the action of external force and the variation features of the stock price under the drive of trading volumes, several stock price plasticity and elasticity models are built. With trading volumes, stock prices, the volume of tradable stocks and the balanced price included in the designed econometric model, this paper tests these models, obtains several models whose imitative effects are better and explains the investment meaning of these models.

The basic stock price plasticity model, the power-exponential model, the first-order auto-regression model and the first-order distributed-lag model can all pass the economic test and statistical test, but their goodness of fit and DW test results are different. Because all these models can pass the economic test and statistical test, it can be sure that the stock price variation is characterized by plasticity and the form of the stock price plasticity model is reasonable. According to the DW test results of the basic stock price plasticity model and the stock price plasticity power-exponential model, the variation rate of the balanced price VRBP has obvious self-correlation of sequences; the first-order auto-regression model can basically eliminate the self-correlation of sequences and the coefficients of the corresponding auto-regression terms are relatively stable. The stock price plasticity coefficient of the stock price plasticity power-exponential model contains sufficient information about the stock price plasticity and is worth exploring further. This paper also builds the basic stock price elasticity model, the stock price elastoplasticity model and the stock price elastoplasticity model containing DUALVOL, and the econometric test results of these models are satisfactory. This paper also studies the investment meaning of the change of the stock price plasticity coefficient under the condition of the time window and points out the theoretical value and application potential.

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