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THE PARAMETERIZATION OF ALL ROBUST STABILIZING MULTI-PERIOD REPETITIVE CONTROLLERS FOR MIMO TD PLANTS WITH THE SPECIFIED INPUT-OUTPUT CHARACTERISTIC

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Abstract. *In this paper, we investigate the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic. The multi-period repetitive control system is a type of servomechanism for a periodic reference input. When multi-period repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the multi-period repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem. Recently, the parameterization of all robust stabilizing multi-period repetitive controllers for time-delay plants was obtained by Chen et al. In addition, Sakanushi et al. proposed that for multiple-input/multiple-output time-delay plants. However, using their method, it is difficult to specify the low-pass filter in the internal model for the periodic reference input of which the role is to specify the input-output characteristic, because the low-pass filter is related to four free parameters in the parameterization. To specify the input-output characteristic easily, this paper proposes the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that the input-output characteristic can be specified beforehand.*

Keywords: Repetitive control, Multi-period repetitive controller, Parameterization, Robust stability, Multiple-input/multiple-output systems

1. **Introduction.** A modified repetitive control system is a type of servomechanism for a periodic reference input, i.e., it follows a periodic reference input without steady state error, even when there exists a periodic disturbance or an uncertainty of a plant [1, 2, 3, 4, 5]. However, the modified repetitive control system has a bad effect on the disturbance attenuation characteristic [6], in that at certain frequencies, the sensitivity to disturbances of a control system with a conventional repetitive controller becomes twice as worse as that of a control system without a repetitive controller. Gotou et al. overcame this problem by proposing a multi-period repetitive control system [6]. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristic [7, 8]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [9, 10] using the time advance compensation described in [7, 8, 11]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [12].

As we know, the multi-period repetitive controller design procedure is usually complicated by the fact that only those controllers are allowed for which the closed-loop system is stable. References [13, 14, 15, 16, 17, 18, 19, 20] showed that it was possible to parameterize all stabilizing controllers for a particular system in a very effective manner. When designing a controller with specific properties, one can simply and without loss of generality search over the space of all stable transfer functions. The parameterization, which is based on the coprime factorization of the plant, has been widely used for designing control systems, and provides an elegant and efficient way towards solving the stabilizing and design problem, with which all stabilizing controllers are characterized and thus a constrained design procedure can be replaced by an unconstrained optimization. The parameterization of all stabilizing multi-period repetitive controllers was solved in [21, 22].

When multi-period repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the multi-period repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem [23]. However, multi-period repetitive controllers in [21, 22] cannot guarantee the stability of control system for plants with uncertainties. Satoh et al. proposed the parameterization of all robust stabilizing multi-period repetitive controllers for plants with uncertainties [24]. However, the method in [24] cannot guarantee the stability of control system for time-delay plants with uncertainties. To solve this problem, Chen et al. proposed the parameterization of all robust stabilizing multi-period repetitive controllers for time-delay plants [25]. Sakanushi et al. expanded the result in [25] and proposed the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants [26].

However, using the method in [26], it is complex to specify the low-pass filter in the internal model for the periodic reference input of which the role is to specify the input-output characteristic, because the low-pass filter is related to four kinds of free parameters in the parameterization proposed by Sakanushi et al. When we design a robust stabilizing multi-period repetitive controller, if the low-pass filter is set beforehand, we can specify the input-output characteristic more easily than in the method employed in [26]. This is achieved by parameterizing all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic, which is the parameterization when the low-pass filter is set beforehand. However, no paper has considered the problem of obtaining the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic. In addition, the parameterization is useful to design stabilizing controllers [13, 14, 15, 16, 17]. Therefore, the problem of obtaining the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic is important to solve.

In this paper, we propose the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that the low-pass filter in the internal model for the periodic reference input is set beforehand. A parameterization is derived on the basis of the definition of the internal stability. This paper is organized as follows. In Section 2, the plant under consideration is formally defined and some important background is introduced. In Section 3, the parameterization is derived for MIMO plants with time-delays. In Section 4, some control characteristics are explained. In Section 5 some applications of the parameterization are discussed. Finally, conclusions are given in Section 6.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	the largest singular value of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI - A)^{-1}B + D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.
$\mathcal{L}^{-1}\{\cdot\}$	the inverse Laplace transformation of $\{\cdot\}$.

2. **Problem Formulation.** Consider the unity feedback control system in

$$\begin{cases} y = G(s)e^{-sL}u + d \\ u = C(s)(r - y) \end{cases}, \tag{1}$$

where $G(s)e^{-sL}$ is the multiple-input/multiple-output time-delay plant, $L > 0$ is the time-delay, and $G(s) \in R^{m\times p}(s)$ is assumed to be stabilizable and detectable. $C(s)$ is the multi-period repetitive controller with the m-th input and p-th output defined later, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance, $y \in R^m$ is the output and $r \in R^m$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \ \ (\forall t \geq 0). \tag{2}$$

It is assumed that $m \leq p$ and $\operatorname{rank} G(s) = m$. The nominal plant of $G(s)e^{-sL}$ is denoted by $G_m(s)e^{-sL_m}$, where $G_m(s) \in R^{m\times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)e^{-sL}$ and the nominal plant $G_m(s)e^{-sL_m}$ is written as

$$G(s)e^{-sL} = \left(e^{-sL_m}I + \Delta(s)\right)G_m(s), \tag{3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all functions satisfying

$$\bar{\sigma}\{\Delta(j\omega)\} < |W_T(j\omega)| \quad (\forall\omega \in R_+), \tag{4}$$

where $W_T(s) \in R(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{5}$$

where $T(s)$ is given by

$$T(s) = \left(I + G_m(s)e^{-sL_m}C(s)\right)^{-1} G_m(s)C(s). \tag{6}$$

According to [6, 9, 10, 21, 22], the general form of multi-period repetitive controller $C(s)$ which makes the output y follow the periodic reference input r with period T in (1) with small steady state error, is written by

$$C(s) = C_0(s) + \sum_{i=1}^{N} C_i(s)C_{ri}(s), \tag{7}$$

where N is an arbitrary positive integer, $T_i > 0 \in R$ $(i = 1, \ldots, N)$, $C_0(s) \in R^{p\times m}(s)$, $C_i(s) \in R^{p\times m}(s)$ $(i = 1, \ldots, N)$ satisfying rank $C_i(s) = m$ $(i = 1, \ldots, N)$, and $C_{ri}(s)$ $(i = 1, \ldots, N)$ is the internal model for the periodic signal with period T written as

$$C_{ri}(s) = q_i(s)e^{-sT_i}\left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1}, \tag{8}$$

where $q_i(s) \in R^{m\times m}(s)$ $(i = 1, \ldots, N)$ is low-pass filter satisfying $\sum_{i=1}^{N} q_i(0) = I$ and rank $q_i(s) = m$ $(i = 1, \ldots, N)$.

In order to compare the difference between the characteristic of the internal model of modified repetitive controller and that of the internal model of multi-period repetitive controller, we show Bode plots of $\left(1 - q(s)e^{-Ts}\right)^{-1}$ and $\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1}$ in Figure 1 and Figure 2, respectively, where $q(s) = 1/(0.001s + 1)$, $T = 1$, $N = 5$, $q_1(s) = q(s)$, $q_i(s) = q_1(s)\cdot\{0.001s/(0.001s+1)\}^{i-1}$ $(i = 2, \ldots, N)$ and $T_i = T\cdot i$ $(i = 1, \ldots, N)$. Figures 1 and 2 show that, the gains of multi-period structure are higher than those of single-period structure. Therefore, in order to obtain good tracking precision or disturbance attenuation performance, the multi-period structure is employed. In addition, from the same reason, we find that even if the period T has some uncertainty, using the multi-period repetitive controller, we have good tracking precision or disturbance attenuation performance.

The general form of the multi-period repetitive controller $C(s)$ is shown in Figure 3. Gotou et al. [6] proposed the design method for multi-period repetitive controller as

$$T_i = T \cdot i \ \ (i = 1, \ldots, N). \tag{9}$$

On the other hand, Yamada et al. [10] proposed the design method for multi-period repetitive controller such that T_i $(i = 1, \ldots, N)$ do not necessarily satisfy (9). Therefore,

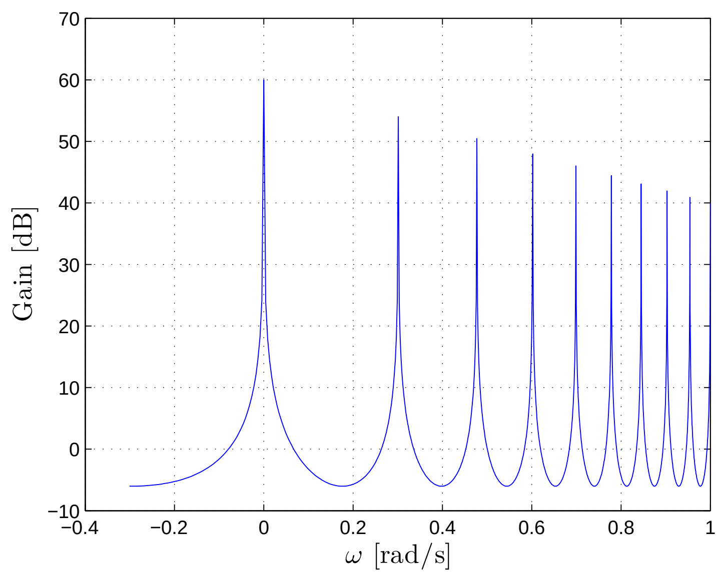

FIGURE 1. Bode plots of $\left(1 - q(s)e^{-Ts}\right)^{-1}$

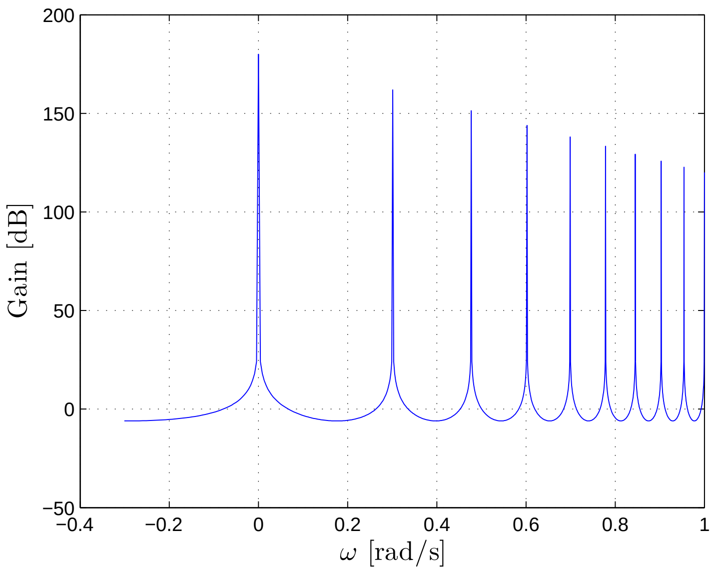

FIGURE 2. Bode plots of $\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1}$

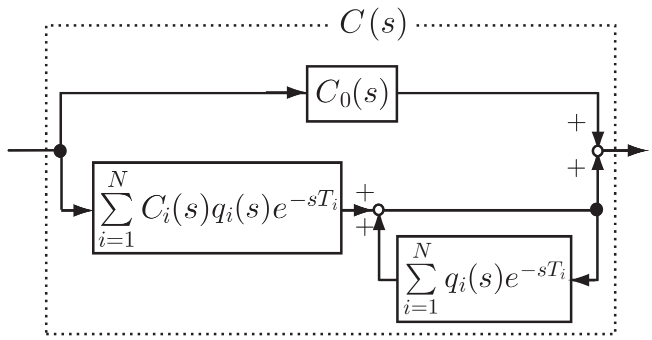

FIGURE 3. Structure of a multi-period repetitive controller

in this paper, we attach importance to the generality and assume that T_i $(i = 1, \ldots, N)$ do not necessarily satisfy (9).

According to [6, 9, 10, 21, 22], if the low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ satisfy

$$\bar{\sigma}\left\{I - \sum_{i=1}^{N} q_i(j\omega_k)\right\} \simeq 0 \quad (\forall k = 0, 1, \ldots, n), \tag{10}$$

where ω_k $(i = 0, 1, \ldots, n)$ is the frequency component of the periodic reference input r written by

$$\omega_k = \frac{2\pi}{T}k \quad (k = 0, 1, \ldots, n) \tag{11}$$

and ω_n is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error. Using the result in [26], in order for $q_i(s)$ $(i = 1, \ldots, N)$ to satisfy (10) in wide frequency range, we must design $q_i(s)$ $(i = 1, \ldots, N)$ to be stable and of minimum phase. If we obtain the parameterization of all robust stabilizing multi-period repetitive controllers such that

$q_i(s)$ $(i = 1, \ldots, N)$ in (7) is settled beforehand, we can design the robust stabilizing multi-period repetitive controller satisfying (10) more easily than the method in [26].

The problem considered in this paper is to propose the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic. That is, when $q_i(s) \in RH_\infty^{m\times m}$ $(i = 1, \ldots, N)$ are set beforehand, we obtain the parameterization of all controllers $C(s)$ in (7) satisfying (5) for multiple-input/multiple-output time-delay plants $G(s)e^{-sL}$ in (3) with any uncertainty $\Delta(s)$ satisfying (4).

3. **The Parameterization.** In this section, we clarify the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic.

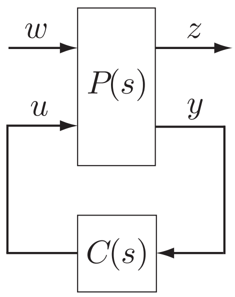

FIGURE 4. Block diagram of H_∞ control problem

In order to obtain the parameterization of all robust stabilizing multi-period repetitive controllers for time-delay plants with specified input-output characteristic, we must see that controllers $C(s)$ satisfy (5). The problem of obtaining the controller $C(s)$, which is not necessarily a multi-period repetitive controller, satisfying (5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Figure 4. $P(s)$ is selected such that the transfer function from w to z in Figure 4 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{l} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t - L_m) \\ z(t) = C_1 x(t) + D_{12} u(t) \\ y(t) = C_2 x(t) + D_{21} w(t) \end{array}\right. , \tag{12}$$

where $A \in R^{n\times n}$, $B_1 \in R^{n\times m}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{m\times n}$, $C_2 \in R^{m\times n}$, $D_{12} \in R^{m\times p}$, $D_{21} \in R^{m\times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy following assumptions:

1) (C_2, A) is detectable, and (A, B_2) is stabilizable.
2) D_{12} has full column rank, and D_{21} has full row rank.
3) $\text{rank}\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \quad (\forall \omega \in R_+)$,
   $\text{rank}\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \quad (\forall \omega \in R_+)$.
4) $C_1 A^i B_2 = 0 \quad (i = 0, 1, 2, \ldots)$.

Under these assumptions, from [27], the following lemma holds true.

Lemma 3.1. *There exists an H_∞ controller $C(s)$ for the generalized plant $P(s)$ in (12) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ written by*

$$\left\{\begin{array}{l} \dot{q}(t) = Aq(t) + B_1 w(t) + \tilde{B}_2 u(t) \\ \tilde{z}(t) = C_1 q(t) + D_{12} u(t) \\ \tilde{y}(t) = C_2 q(t) + D_{21} w(t) \end{array}\right. , \tag{13}$$

where $\tilde{B}_2 = e^{-AL_m} B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ control input for the generalized plant $\tilde{P}(s)$ in (13),

$$u(t) = \mathcal{L}^{-1}\{C(s)\tilde{y}(s)\} \tag{14}$$

is an H_∞ control input for the generalized plant $P(s)$ in (12), where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2 \int_{-L_m}^{0} e^{-A(\tau+L_m)} B_2 u(t+\tau) d\tau\right\}. \tag{15}$$

From Lemma 3.1 and [23], the following lemma holds true.

Lemma 3.2. *If controllers satisfying (5) exist, both*

$$\begin{aligned} &X\left(A - \tilde{B}_2 D_{12}^{\dagger} C_1\right) + \left(A - \tilde{B}_2 D_{12}^{\dagger} C_1\right)^T X + X\Big\{B_1 B_1^T \\ &- \tilde{B}_2 \left(D_{12}^T D_{12}\right)^{-1} \tilde{B}_2^T\Big\} X + \left(D_{12}^{\perp} C_1\right)^T D_{12}^{\perp} C_1 = 0 \end{aligned} \tag{16}$$

and

$$\begin{aligned} &Y\left(A - B_1 D_{21}^{\dagger} C_2\right)^T + \left(A - B_1 D_{21}^{\dagger} C_2\right) + Y\Big\{C_1^T C_1 \\ &- C_2^T \left(D_{21} D_{21}^T\right)^{-1} C_2\Big\} + B_1 D_{21}^{\perp} \left(B_1 D_{21}^{\perp}\right)^T = 0 \end{aligned} \tag{17}$$

have solutions $X \geq 0$ and $Y \geq 0$ such that

$$\rho(XY) < 1 \tag{18}$$

and both

$$A - \tilde{B}_2 D_{12}^{\dagger} C_1 + \left\{B_1 B_1^T - \tilde{B}_2 \left(D_{12}^T D_{12}\right)^{-1} \tilde{B}_2^T\right\} X \tag{19}$$

and

$$A - B_1 D_{21}^{\dagger} C_2 + Y\left\{C_1^T C_1 - C_2^T \left(D_{21} D_{21}^T\right)^{-1} C_2\right\} \tag{20}$$

have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (5) is given by

$$C(s) = C_{11}(s) + C_{12}(s)Q(s)(I - C_{22}(s)Q(s))^{-1} C_{21}(s), \tag{21}$$

where

$$\left[\begin{array}{cc} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{array}\right] = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array}\right], \tag{22}$$

$$\begin{aligned} A_c = &\ A + B_1 B_1^T X - \tilde{B}_2\left(D_{12}^{\dagger} C_1 + E_{12}^{-1} \tilde{B}_2^T X\right) \\ &- (I - YX)^{-1}\left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1}\right)\left(C_2 + D_{21} B_1^T X\right), \end{aligned}$$

$$B_{c1} = (I - YX)^{-1}\left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1}\right),$$

$$B_{c2} = (I - YX)^{-1}\left(\tilde{B}_2 + YC_1^T D_{12}\right) E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^{\dagger} C_1 - E_{12}^{-1}\tilde{B}_2^T X,$$

$$C_{c2} = -E_{21}^{-1/2}\left(C_2 + D_{21}B_1^T X\right),$$

$$D_{c11} = 0, \quad D_{c12} = E_{12}^{-1/2}, \quad D_{c21} = E_{21}^{-1/2}, \quad D_{c22} = 0, \quad E_{12} = D_{12}^T D_{12}, \quad E_{21} = D_{21}D_{21}^T$$

and $Q(s) \in H_\infty^{p\times m}$ *is any function satisfying* $\|Q(s)\|_\infty < 1$.

Remark 3.1. *$C(s)$ in (21) is written using Linear Fractional Transformation (LFT). Using homogeneous transformation, (21) is rewritten by*

$$\begin{aligned} C(s) &= \left(Z_{11}(s)Q(s) + Z_{12}(s)\right)\left(Z_{21}(s)Q(s) + Z_{22}(s)\right)^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \end{aligned} \tag{23}$$

where $Z_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ and $\tilde{Z}_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ are defined by

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{12}(s) - C_{11}(s)C_{21}^{-1}(s)C_{22}(s) & C_{11}(s)C_{21}^{-1}(s) \\ -C_{21}^{-1}(s)C_{22}(s) & C_{21}^{-1}(s) \end{bmatrix} \tag{24}$$

and

$$\begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{21}(s) - C_{22}(s)C_{12}^{-1}(s)C_{11}(s) & C_{12}^{-1}(s)C_{11}(s) \\ -C_{22}(s)C_{12}^{-1}(s) & C_{12}^{-1}(s) \end{bmatrix} \tag{25}$$

and satisfying

$$\begin{aligned} &\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix}\begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} \\ &= I = \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix}\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix}. \end{aligned} \tag{26}$$

Using Lemma 3.1, Lemma 3.2 and Remark 3.1, the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic is given by following theorem.

Theorem 3.1. *If multi-period repetitive controllers satisfying (5) exist, both (16) and (17) have solutions $X \geq 0$ and $Y \geq 0$ such that (18) and both (19) and (20) have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all robust stabilizing multi-period repetitive control laws with specified input-output characteristic satisfying (5) is given by*

$$u(t) = \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\}, \tag{27}$$

where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2\int_{-L_m}^{0} e^{-A(\tau+L_m)}B_2 u(t+\tau)d\tau\right\} \tag{28}$$

and

$$\begin{aligned} C(s) &= \left(Z_{11}(s)Q(s) + Z_{12}(s)\right)\left(Z_{21}(s)Q(s) + Z_{22}(s)\right)^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \end{aligned} \tag{29}$$

where $Z_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ and $\tilde{Z}_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ are defined by (24) and (25) and satisfying (26), $C_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ are given by (22) and $Q(s) \in H_\infty^{p\times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$Q(s) = \left(Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s)q_i(s)e^{-sT_i}\right)\left(Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}\right)^{-1}, \quad (30)$$

$Q_{n0}(s) \in RH_\infty^{p\times m}$, $Q_{d0}(s) \in RH_\infty^{m\times m}$, $Q_{ni}(s) \in RH_\infty^{p\times m}$ $(i = 1, \ldots, N)$ and $Q_{di}(s) \in RH_\infty^{m\times m}$ $(i = 1, \ldots, N)$ are any functions satisfying

$$\text{rank } \{Z_{11}(s)(Q_{n0}(s) + Q_{ni}(s)) + Z_{12}(s)(Q_{d0}(s) + Q_{di}(s))\} = m \ \ (i = 1, \ldots, N) \quad (31)$$

Proof: First, the necessity is shown. That is, we show that if the multi-period repetitive controller $C(s)$ in (7) stabilizes the control system in (1) robustly and $q_i(s)$ $(i = 1, \ldots, N)$ are set beforehand, then $C(s)$ is written by (29) and (30), respectively. From Lemma 3.2 and Remark 3.1, the parameterization of all robust stabilizing controllers $C(s)$ for $G(s)e^{-sL}$ is written by (29), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (7) stabilizes the control system in (1) robustly and $q_i(s)$ $(i = 1, \ldots, N)$ are set beforehand, then $Q(s)$ in (29) is written by (30). Substituting $C(s)$ in (7) for (29), we have (30), where

$$Q_{n0}(s) = -N_{0n}(s)N_d(s), \quad (32)$$

$$Q_{ni}(s) = -N_{in}(s) + N_{0n}(s)N_d(s) \ \ (i = 1, \ldots, N), \quad (33)$$

$$Q_{d0}(s) = D_{0n}(s)D_d(s)N_{0d}(s)N_d(s) \quad (34)$$

and

$$Q_{di}(s) = (D_{in}(s) - D_{0n}(s)D_d(s))\, N_{0d}(s)N_d(s) \quad (i = 1, \ldots, N). \quad (35)$$

Here, $N_{0n}(s) \in RH_\infty^{p\times m}$, $N_{in}(s) \in RH_\infty^{p\times m}$ $(i = 1, \ldots, N)$, $N_{0d}(s) \in RH_\infty^{m\times m}$, $N_d(s) \in RH_\infty^{m\times m}$, $D_{0n}(s) \in RH_\infty^{m\times m}$, $D_{in}(s) \in RH_\infty^{m\times m}$ $(i = 1, \ldots, N)$, $D_{0d}(s) \in RH_\infty^{m\times m}$ and $D_d(s) \in RH_\infty^{m\times m}$ are coprime factors satisfying

$$\tilde{Z}_{21}(s)C_0(s) - \tilde{Z}_{11}(s) = D_{0n}(s)D_{0d}^{-1}(s), \quad (36)$$

$$\tilde{Z}_{21}(s)C_i(s)D_{0d}(s) = D_{in}(s)D_d^{-1}(s) \ \ (i = 1, \ldots, N), \quad (37)$$

$$\left(\tilde{Z}_{22}(s)C_0(s) - \tilde{Z}_{12}(s)\right) D_{0d}(s)D_d(s) = N_{0n}(s)N_{0d}^{-1}(s) \quad (38)$$

and

$$\tilde{Z}_{22}(s)C_i(s)D_{0d}(s)D_d(s)N_{0d}(s) = N_{in}(s)N_d^{-1}(s) \quad (i = 1, \ldots, N). \quad (39)$$

From (32)-(35), all of $Q_{n0}(s)$, $Q_{ni}(s)$ $(i = 1, \ldots, N)$, $Q_{d0}(s)$ and $Q_{di}(s)$ $(i = 1, \ldots, N)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (7) stabilizes the control system in (1) robustly and $q_i(s)$ $(i = 1, \ldots, N)$ are set beforehand, $Q(s)$ in (29) is written by (30). From the assumption of rank $C_i(s) = m$ $(i = 1, \ldots, N)$ and from (37) and (39),

$$\text{rank } D_{in}(s) = m \ \ (i = 1, \ldots, N) \quad (40)$$

and

$$\text{rank } N_{in}(s) = m \ \ (i = 1, \ldots, N) \quad (41)$$

hold true. From (40), (41), (33) and (35), (31) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty^{p\times m}$ are settled by (29) and (30), respectively, then the controller $C(s)$ is written by the form in

(7) and rank $C_i(s) = m$ $(i = 1, \dots, N)$ hold true. Substituting (30) into (29), we have (7), where $C_0(s)$ and $C_i(s)$ $(i = 1, \dots, N)$ are denoted by

$$C_0(s) = (Z_{11}(s)Q_{n0}(s) + Z_{12}(s)Q_{d0}(s))\,(Z_{21}(s)Q_{n0}(s) + Z_{22}(s)Q_{d0}(s))^{-1}, \tag{42}$$

and

$$\begin{aligned} C_i(s) = &\{Z_{11}(s)\,(Q_{n0}(s) + Q_{ni}(s)) + Z_{12}(s)\,(Q_{d0}(s) + Q_{di}(s))\} \\ &(Z_{21}(s)Q_{n0}(s) + Z_{22}(s)Q_{d0}(s))^{-1} \quad (i = 1, \dots, N). \end{aligned} \tag{43}$$

We find that if $C(s)$ and $Q(s)$ are settled by (29) and (30), respectively, then the controller $C(s)$ is written by the form in (7). From (31) and (43),

$$\text{rank } C_i(s) = m \ \ (i = 1, \dots, N) \tag{44}$$

holds true. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

4. **Control Characteristics.** In this section, we explain control characteristics of the control system in (1) using the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants. In addition, roles of $Q_{n0}(s)$, $Q_{ni}(s)$, $Q_{d0}(s)$ and $Q_{di}(s)$ in (30) are clarified.

From Theorem 3.1, $Q(s)$ in (30) must be included in H_∞. Since $Q_{ni}(s) \in RH_\infty$ in (30), if $\left(Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}\right)^{-1} \in H_\infty$, then $Q(s)$ satisfies $Q(s) \in H_\infty$. That is, the role of $Q_{d0}(s)$ and $Q_{di}(s)$ is to assure $Q(s) \in H_\infty$, and the role of $Q_{n1}(s)$ and $Q_{n2}(s)$ is to guarantee $\|Q(s)\|_\infty < 1$.

Next, the input-output characteristic of the control system in (1) is shown. The transfer function $S(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ of the control system in (1) is written by

$$S(s) = S_n(s)S_d^{-1}(s), \tag{45}$$

where

$$Sn(s) = \left\{I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right\} C_{21}^{-1}(s)\,(-C_{22}Q_{n0}(s) + Q_{d0}(s)) \tag{46}$$

and

$$\begin{aligned} S_d(s) = &Z_{21}(s)Q_{n0}(s) + \sum_{i=1}^{N} (Z_{21}(s)Q_{ni}(s) + Z_{22}(s)Q_{di}(s))\,q_i(s)e^{-sT_i} \\ &+ Z_{22}(s)Q_{d0}(s) + G(s)\Bigg\{ Z_{11}(s)Q_{n0}(s) + Z_{12}(s)Q_{d0}(s) \\ &+ \sum_{i=1}^{N} (Z_{11}(s)Q_{n2}(s) + Z_{12}(s)Q_{d2}(s))\,q_i(s)e^{-sT_i} \Bigg\}. \end{aligned} \tag{47}$$

According to (46), if $Q_{n0}(s)$, $Q_{d0}(s)$, $Q_{ni}(s)$ and $Q_{di}(s)$ are selected satisfying (10), then

$$\bar{\sigma}\big\{S_n(j\omega_i)\big\} \le \bar{\sigma}\left\{I - q(j\omega_i)\right\} \bar{\sigma}\big\{C_{21}^{-1}(j\omega_i)\big\}\bar{\sigma}\big\{ (-C_{22}Q_{n1}(j\omega_i) + Q_{d1}(j\omega_i)) \big\} \simeq 0, \tag{48}$$

the output $y(s)$ follows the periodic reference input $r(s)$ with frequency components

$$\omega_i = \frac{2\pi}{T} i \ \ (i = 0, 1, \dots, \hbar) \tag{49}$$

without steady state error.

Next, the disturbance attenuation characteristic of the control system in (1) is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ of the control system in (1) is written by (45). From (45), for ω_i $(i = 0, 1, \ldots, \hbar)$ in (10) of the frequency component of the disturbance $d_1(s)$ that is the same as that of the periodic reference input $r(s)$, if (48) holds, then the disturbance $d(s)$ is attenuated effectively. This implies that the disturbance with the same frequency component ω_i $(i = 0, 1, \ldots, \hbar)$ of the periodic reference input $r(s)$ is attenuated effectively. That is, the role of $Q_{n2}(s)$ and $Q_{d2}(s)$ is to specify the disturbance attenuation characteristic for the disturbance with the same frequency component ω_i $(i = 0, 1, \ldots, \hbar)$ of the periodic reference input $r(s)$. When the frequency components of disturbance $d(s)$, $\bar{\omega}_k$ $(k = 0, 1, \ldots, h)$, are not equal to ω_i $(i = 0, 1, \ldots, \hbar)$, even if

$$\bar{\sigma}\left\{I - q(j\omega_k)\right\} \simeq 0, \tag{50}$$

the disturbance $d(s)$ cannot be attenuated, because

$$e^{-j\bar{\omega}_k T} \neq 1 \tag{51}$$

and

$$\bar{\sigma}\left\{I - q\left(j\bar{\omega}_k\right) e^{-j\bar{\omega}_k T}\right\} \neq 0. \tag{52}$$

In order to attenuate the frequency components $\bar{\omega}_k$ $(k = 0, 1, \ldots, h)$ of the disturbance $d(s)$, we need to satisfy

$$\bar{\sigma}\left\{-C_{22}\left(j\bar{\omega}_k\right) Q_{n0}\left(j\bar{\omega}_k\right) + Q_{d0}\left(j\bar{\omega}_k\right)\right\} \simeq 0. \tag{53}$$

This implies that the disturbance $d(s)$ with frequency components $\bar{\omega}_k \neq \omega_i$ $(i = 0, 1, \ldots, \hbar,$ $k = 0, 1, \ldots, h)$ can be attenuated effectively. That is, the role of $Q_{n1}(s)$ and $Q_{d1}(s)$ is to specify disturbance attenuation characteristics for disturbance of frequency $\omega_d \neq \omega_i$ $(i = 0, 1, \ldots, \hbar)$.

From above discussion, the role of $Q_{ni}(s)$ and $Q_{di}(s)$ is to specify the input-output characteristic for the periodic reference input $r(s)$ and to specify for the disturbance $d(s)$ of which the frequency components are equivalent to that of the periodic reference input $r(s)$. The role of $Q_{n0}(s)$ and $Q_{d0}(s)$ is to specify for the disturbance $d(s)$ of which the frequency components are different from that of the periodic reference input $r(s)$.

5. **Numerical Example.** In this section, numerical examples are made to illustrate the validity of the proposed approach. Consider the problem to obtain the parameterization of all robust stabilizing multi-period repetitive controllers for the set of plants $G(s)$ written by (3), where

$$G_m(s) = \begin{bmatrix} \dfrac{s+3}{(s-2)(s+9)} & \dfrac{2}{(s-2)(s+9)} \\ \dfrac{s+3}{(s-2)(s+9)} & \dfrac{s+4}{(s-2)(s+9)} \end{bmatrix} \tag{54}$$

and

$$W_T(s) = \frac{s+425}{520}. \tag{55}$$

The period T of the periodic reference input r is given by $T = 10$ [sec], and the time-delay L_m is given by $L_m = 5$ [sec]. We settle $N = 3$ and the low-pass filters of multiple-period repetitive controller in (8) as

$$q_1(s) = \frac{1}{0.002s+1} I, \tag{56}$$

$$q_2(s) = q_1(s)\left(\frac{0.002s}{0.002s+1}\right)I \tag{57}$$

and

$$q_3(s) = q_2(s)\left(\frac{0.002s}{0.002s+1}\right)I. \tag{58}$$

Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 3.1, the parameterization of all robust stabilizing controllers $C(s)$ is obtained. In addition, we find that $C_{22}(s)$ is of minimum phase as

$$C_{22}(s) = \begin{bmatrix} \dfrac{-450.9}{s+437.3} & \dfrac{278.7}{s+437.3} \\ \dfrac{278.7}{s+437.3} & \dfrac{450.9}{s+437.3} \end{bmatrix}. \tag{59}$$

Since $C_{22}(s)$ is of minimum phase, we set $Q_{n0}(s)$, $Q_{ni}(s)$, $Q_{d0}(s)$ and $Q_{di}(s)$ in (30) as

$$Q_{d0}(s) = I \in RH_\infty, \tag{60}$$

$$Q_{n0}(s) = C_{22}^{-1}(s)\bar{q}_d(s) \in RH_\infty, \tag{61}$$

$$Q_{ni}(s) = 0 \in RH_\infty \tag{62}$$

and

$$Q_{di}(s) = -\left(I - \bar{q}_d(s)\right) \in RH_\infty, \tag{63}$$

where $\bar{q}_d(s)$ is written by

$$\bar{q}_d(s) = \frac{1}{0.002\,s+1}I. \tag{64}$$

In order to verify that $Q(s)$ in (30) belongs to H_∞ and satisfy $\|Q(s)\|_\infty < 1$, we show the Nyquist plot of det $\left(Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}\right)$ and the largest singular value plot of $Q(s)$ are shown in Figure 5 and Figure 6, respectively. Since the Nyquist plot of

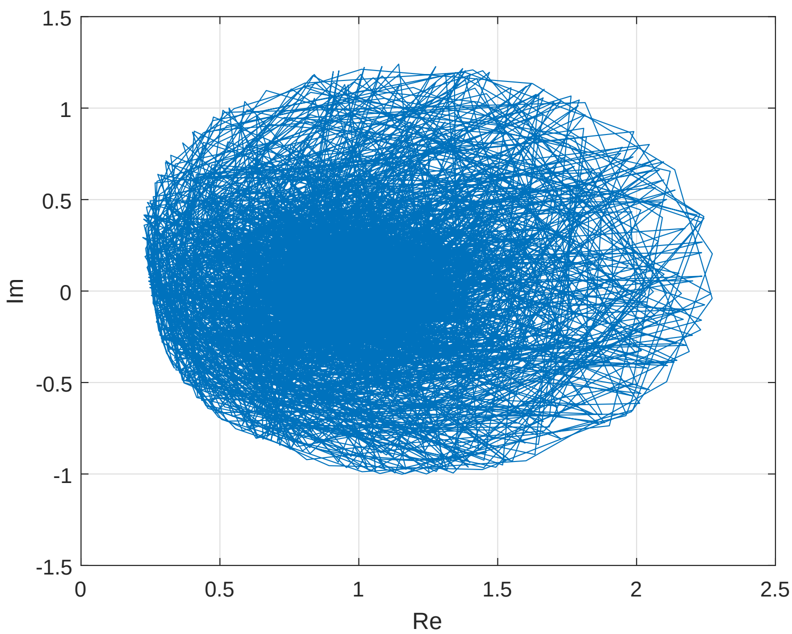

FIGURE 5. The Nyquist plot of det $\left(Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}\right)$

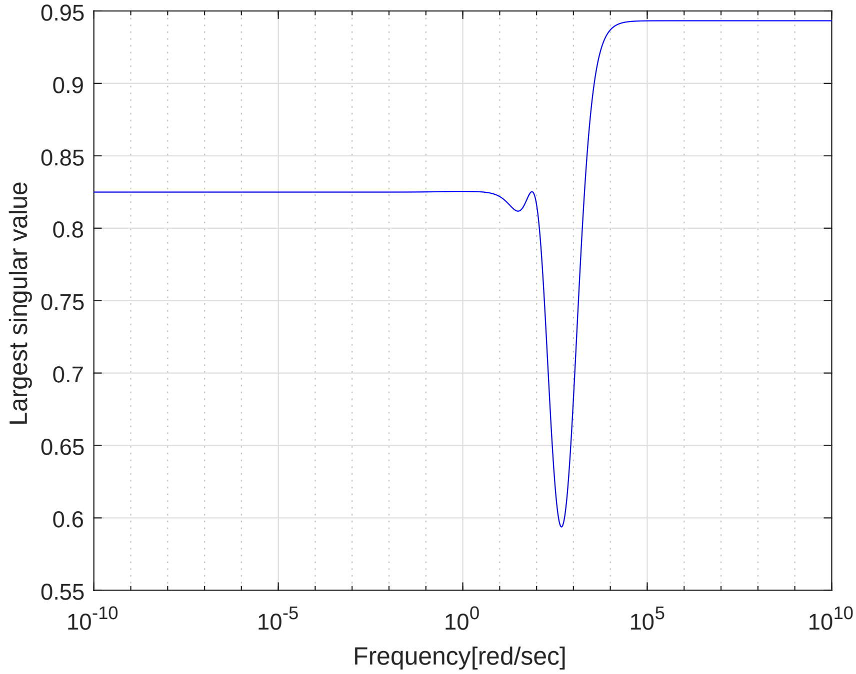

FIGURE 6. The largest singular value plot of $Q(s)$

$\det(Q_{d1}(s)+Q_{d2}(s)e^{-sT})$ does not encircle the origin, we find that $Q(s)$ in (30) is included in H_∞. Figure 6 illustrates $\bar{\sigma}\{Q(j\omega)\} \simeq 0.9432 < 1$ $(\forall\omega \in R)$, i.e., $\|Q(s)\|_\infty \simeq 0.9432 < 1$. Using above-mentioned parameters, we have a robust stabilizing multi-period repetitive controller.

When $\Delta(s)$ is given by

$$\Delta(s) = \begin{bmatrix} \dfrac{s-100}{s+500} & \dfrac{-100}{s+500} \\ \dfrac{-200}{s+500} & \dfrac{s-100}{s+500} \end{bmatrix} \tag{65}$$

and the designed robust stabilizing multi-period repetitive controller $C(s)$ is used, the tracking error $e(t) = r(t) - y(t)$ in (1) for the periodic reference inputs

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\dfrac{2\pi}{T}t\right) \\ 2\sin\left(\dfrac{2\pi}{T}t\right) \end{bmatrix} \tag{66}$$

is shown in Figure 7. Here, the largest steady state peak to peak relative error is 0.002%. These tracking errors are small and can be further reduced by modifying the cutoff frequency of the low-pass filters $q(s)$ and $\bar{q}_d(s)$.

Next, the disturbance attenuation characteristic is shown. The response of the output $y(t)$ for the disturbance $d(t)$ of which the frequency components are equivalent to that of the periodic reference input $r(t)$

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\dfrac{2\pi}{T}t\right) \\ 2\sin\left(\dfrac{2\pi}{T}t\right) \end{bmatrix} \tag{67}$$

is shown in Figure 8. Figure 8 shows that the disturbance $d(t)$ is attenuated effectively. Next we show that the response of the output $y(t)$ for the disturbance $d(t)$ of which the frequency components are different from that of the periodic reference input $r(t)$. The response of the output $y(t)$ for the disturbance $d(t)$ of which the frequency components

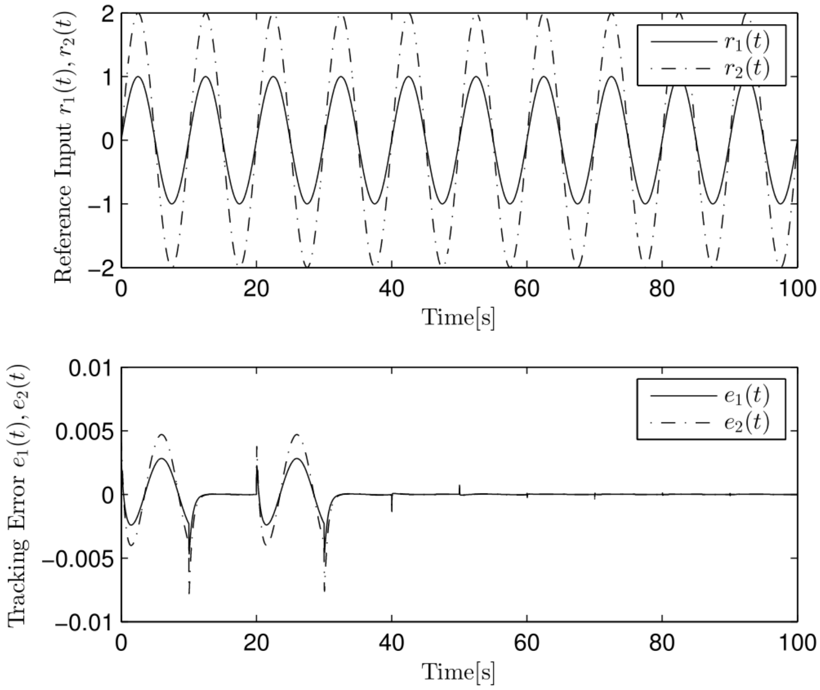

FIGURE 7. Response of the error $e(t)$ for the reference input $r(t)$ in (66)

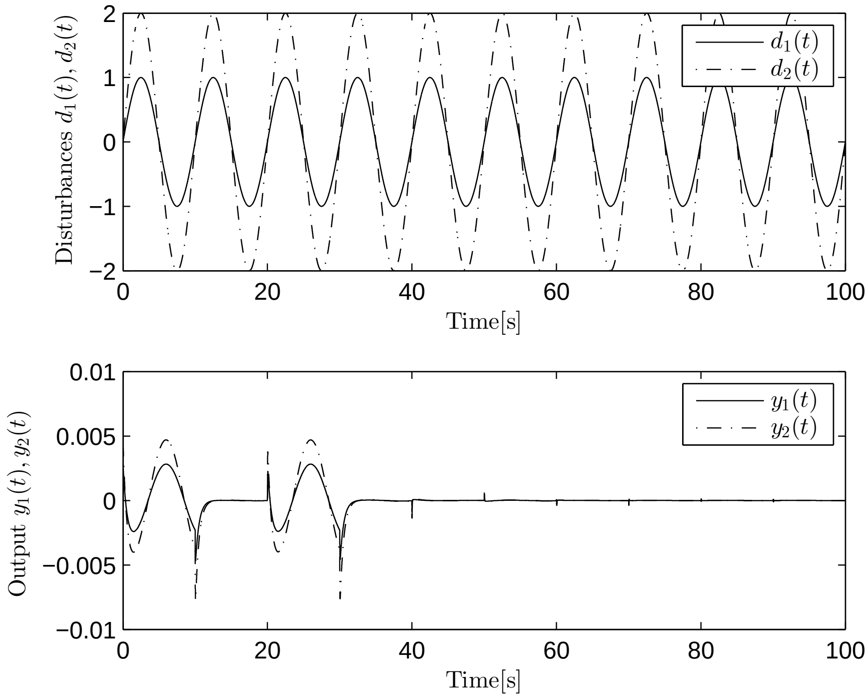

FIGURE 8. Response of the output $y(t)$ for the disturbance $d(t)$ in (67)

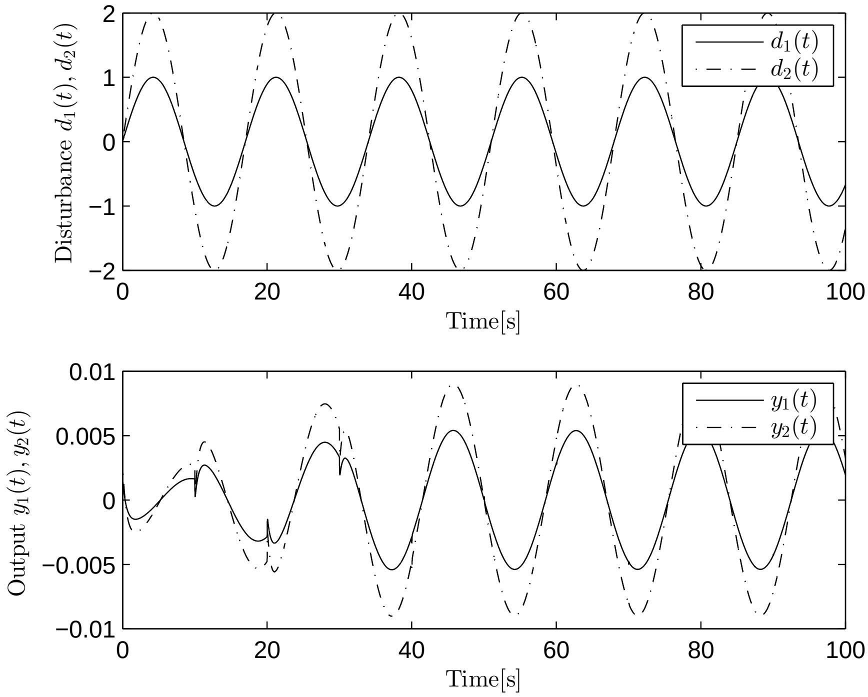

FIGURE 9. Response of the output $y(t)$ for the disturbance $d(t)$ in (68)

are different from that of the periodic reference input $r(t)$

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\dfrac{2\pi}{1.7T}t\right) \\ 2\sin\left(\dfrac{2\pi}{1.7T}t\right) \end{bmatrix} \tag{68}$$

is shown in Figure 9. Figure 9 shows that even if the frequency components of the disturbance $d(s)$ are different from that of the periodic reference input, the disturbance $d(t)$ is attenuated effectively. The results shown in Figure 7, Figure 8 and Figure 9 demonstrate that the design method presented here provided not only good robustness, but also satisfactory tracking performance for the reference inputs and attenuation performance for the disturbance.

In some cases, the period T of the reference input has uncertainty. Next, we show the responce of the tracking error when the period T of the reference input has uncertainty. The tracking error $e(t) = r(t) - y(t)$ in (1) for the periodic reference inputs r

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\dfrac{2\pi}{T+\Delta T}t\right) \\ 2\sin\left(\dfrac{2\pi}{T+\Delta T}t\right) \end{bmatrix} \tag{69}$$

is shown in Figure 10, where ΔT is an uncertainty of the period of the reference input. Here, the largest steady state peak to peak relative errors are 0.01%, 0.002%, 0.1% and 0.5% for $\Delta T = -0.5T$, $\Delta T = 0$, $\Delta T = 0.3T$ and $\Delta T = 0.7T$, respectively. These tracking errors are small and can be further reduced by modifying the cutoff frequency of the low-pass filters $q(s)$ and $\bar{q}_d(s)$. The results shown in Figure 10 demonstrate that the design

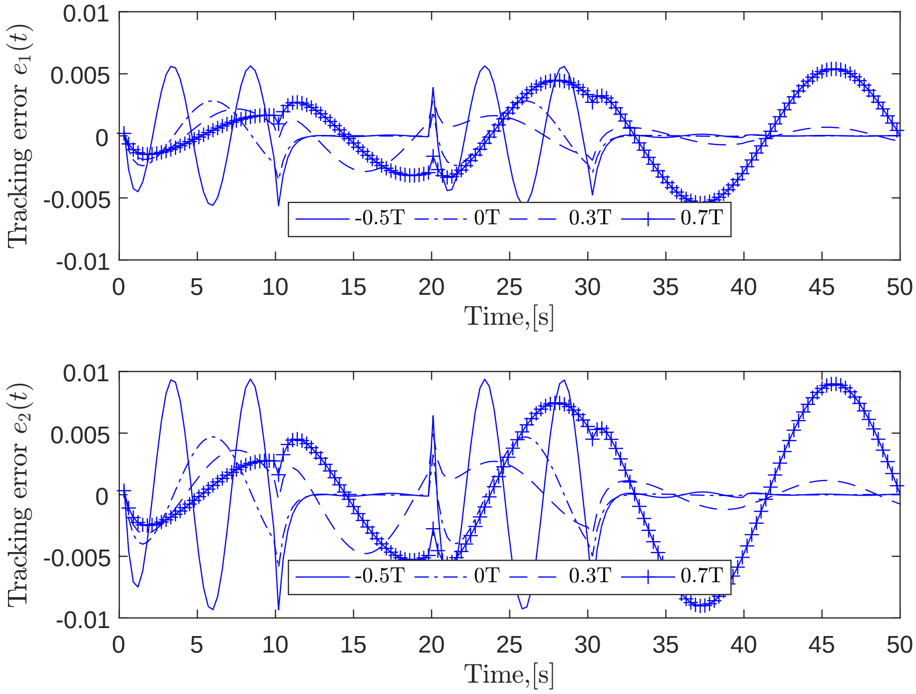

FIGURE 10. Response of the error $e(t)$ for the reference input $r(t)$ in (69)

method presented here provided satisfactory tracking performance for the reference inputs with uncertain period.

6. **Conclusions.** In this paper, we proposed the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand. Advantages of the multi-period repetitive control system using the proposed parameterization are that its input-output characteristic is easily specified and the system to guarantee the robust stability is easy to design. This control system is expected to have practical applications in, for example, engines, electrical motors and generators, converters, and other machines that perform cyclic tasks.

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REFERENCES

[1] S. Hara, T. Omata and M. Nakano, Stability condition and synthesis methods for repetitive control systems, *Trans. of Society of Instrument and Control Engineers*, vol.22, no.1, pp.36-42, 1986.

[2] Y. Yamamoto and S. Hara, The internal model principle and stabilizability of repetitive control systems, *Trans. of Society of Instrument and Control Engineers*, vol.22, no.8, pp.830-834, 1986.

[3] S. Hara and Y. Yamamoto, Stability of multivariable repetitive control systems – Stability condition and class of stabilizing controllers, *Trans. of Society of Instrument and Control Engineers*, vol.22, no.12, pp.1256-1261, 1986.

[4] S. Hara, Y. Yamamoto, T. Omata and M. Nakano, Repetitive control system: A new type of servo system for periodic exogenous signals, *IEEE Trans. Automatic Control*, vol.AC-33, no.7, pp.659-668, 1988.

[5] G. Weiss, Repetitive control systems: Old and new ideas, *Systems and Control in the Twenty-First Century*, pp.389-404, 1997.
[6] M. Gotou, S. Matsubayashi, F. Miyazaki, S. Kawamura and S. Arimoto, A robust system with an iterative learning compensator and a proposal of multi-period learning compensator, *J. Soc. Instrument and Control Engineers*, vol.31, no.5, pp.367-374, 1987.
[7] H. Sugimoto and K. Washida, A proposition of modified repetitive control with corrected dead time, *Trans. of Society of Instrument and Control Engineers*, vol.34, no.6, pp.645-647, 1998.
[8] H. Sugimoto and K. Washida, A design method for modified repetitive control with corrected dead time, *Trans. of Society of Instrument and Control Engineers*, vol.34, no.7, pp.761-768, 1998.
[9] T. Okuyama, K. Yamada and K. Satoh, A design method for repetitive control systems with a multi-period repetitive compensator, *Theoretical and Applied Mechanics Japan*, vol.51, pp.161-167, 2002.
[10] K. Yamada, K. Satoh, T. Arakawa and T. Okuyama, A design method for repetitive control systems with multi-period repetitive compensator, *Transactions of the Japan Society of Mechanical Engineers Series C*, vol.69, no.686, pp.2691-2699, 2003 (in Japanese).
[11] H. L. Broberg and R. G. Molyet, A new approach to phase cancellation in repetitive control, *Proc. of the 29th IEEE IAS*, pp.1766-1770, 1994.
[12] M. Steinbuch, Repetitive control for systems with uncertain period-time, *Automatica*, vol.38, pp.2103-2109, 2002.
[13] D. C. Youla, J. J. Bongiorno and H. Jabr, Modern Wiener-Hopf design of optimal controllers, Part I: The single-input-output case, *IEEE Trans. Automatic Control*, vol.AC-21, pp.3-13, 1976.
[14] V. Kucera, *Discrete Linear System: The Polynomial Equation Approach*, Wiley, 1979.
[15] C. A. Desoer, R. W. Liu, J. Murray and R. Saeks, Feedback system design: The fractional representation approach to analysis and synthesis, *IEEE Trans. Automatic Control*, vol.AC-25, pp.399-412, 1980.
[16] J. J. Glaria and G. C. Goodwin, A parameterization for the class of all stabilizing controllers for linear minimum phase system, *IEEE Trans. Automatic Control*, vol.AC-39, pp.433-434, 1994.
[17] M. Vidyasagar, *Control System Synthesis – A Factorization Approach*, MIT Press, 1985.
[18] K. Yamada and T. Moki, The parameterization of all internally stabilizing controllers for linear minimum phase systems and their characteristic, *Transactions of the Japan Society of Mechanical Engineers Series C*, vol.68, pp.3628-3635, 2002 (in Japanese).
[19] K. Yamada, K. Satoh, Y. Mei, T. Hagiwara, I. Murakami and Y. Ando, The parametrization for the class of all proper internally stabilizing controllers for multiple-input/multiple-output minimum phase systems, *ICIC Express Letters*, vol.3, no.1, pp.67-72, 2009.
[20] Y. Zhao, T. Sakanushi, K. Yamada, N. Smitthimedhin and H. Yamaguchi, The parameterization of all stabilizing two-degree-of-freedom multi-period repetitive controllers for multiple-input/multiple-output plants with specified frequency characteristic, *Proc. of the 54th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE)*, pp.418-421, 2015.
[21] K. Yamada, K. Satoh and T. Arakawa, The parameterization of all stabilizing multiperiod repetitive controllers, *Int. Conf. Cybernetics and Information Technologies, System and Applications*, vol.2, pp.358-363, 2004.
[22] K. Yamada, K. Satoh and T. Arakawa, A design method for multiperiod repetitive controllers (design method using the parameterization of all multiperiod repetitive controllers), *Transactions of the Japan Society of Mechanical Engineers Series C*, vol.71, no.710, pp.2945-2952, 2005.
[23] J. C. Doyle, K. Glover, P. P. Khargonekar and B. A. Francis, State-space solution to standard and control problems, *IEEE Trans. Automatic Control*, vol.AC-34, pp.831-847, 1989.
[24] K. Satoh, K. Yamada and Y. Mei, The parametrization of all robust stabilizing multi-period repetitive controllers, *Theoretical and Applied Mechanics*, vol.55, pp.125-132, 2006.
[25] Z. Chen, T. Sakanushi, K. Yamada, S. Tohnai, N. L. T. Nguyen and Y. Zhao, The parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants, *ICIC Express Letters*, vol.6, no.1, pp.41-46, 2012.
[26] T. Sakanushi, Y. Zhao, K. Yamada and N. Smitthimedhin, Robust stabilizing multi-period repetitive controllersfor multiple-input/multiple-output time-delay plants, *Proc. of the 11th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology*, 2014.
[27] N. Abe and A. Kojima, *Control in Time-Delay and Distributed Parameter Systems*, Corona Publishing, 2007.