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MARGIN SAVING MODEL PREDICTIVE CONTROL WITH OPERATING POINT BACK-OFF

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Abstract. *In chemical processes, design margins must be added on the nominal values of design variables during process design period, so the operating point will move into the feasible domain and there are sufficient operating margins between current operating point and process constraints. When plants running, process operators and engineers usually hope the operating margin consumption to be as little as possible in dynamic control process in order to keep more operating space for operation optimization. Model predictive control can deal with the problems of constrained control and multivariable control, but the optimal control sequence may reach the boundary of the feasible domain due to constraints. Therefore, a margin saving model predictive control algorithm is proposed through adding a margin loss function indicating operating margin reduction amount to the optimal control performance index function of conventional model predictive control, making the operating point back-off from the boundary of the feasible domain. Through the steady-state analysis, the margin saving effects for manipulated variables and the tracking effects for controlled variables can be found. Finally, a simulation case is given to illustrate the effectiveness of the algorithm.*

Keywords: Process systems, Process control, Margin saving, Model predictive control, Margin loss

1. **Introduction.** There always are all kinds of uncertainties in chemical processes, such as slow time-varying parameters, long time lag, and disturbances. So design margins must be added on the nominal values of design variables during process design period, then the operating point will move into the feasible domain and there are sufficient operating margins between current operating point and process constraints [1]. When plants running, operation optimization and dynamic control will consume the operating margin for manipulated variables [2,3], but process operators and engineers usually hope the less operating margin consumption for dynamic control and more operating space for operation optimization. Model predictive control (MPC) can deal with the problems of constrained control and multivariable control [4,5], so it is possible to coordinate the operating margins for manipulated variables in dynamic control and the control errors for controlled variables. In conventional MPC, the precision of controlled variables tracking the expectations is taken as the control performance standard, and the consumed margins for manipulated variables are never taken into consideration. However, the control performance cannot be judged absolutely good or absolutely bad. When control engineers only pursue that the controlled variables track the expectations accurately, the manipulated variables may swing seriously, the wave crest or wave trough will touch the boundaries of process constraints, causing the potential safety problem. So now the optimal control of current MPC algorithm may reach the boundary of the feasible domain due to constraints, the operating

margins are completely consumed and process operators and engineers always think that there are problems in MPC algorithm. To solve the above problems, adding a margin loss function which indicates the situation of manipulated variables consuming margin into the optimal control performance index function of conventional MPC, we build a new framework of MPC which is called as margin saving model predictive control to trade off the precision of controlled variables tracking the expectations and the consumed margins for manipulated variables comprehensively in the dynamic control process.

To illustrate the correctness and scientificity of margin saving model predictive control, the philosophy principles are used to deeply analyze the existence significance and inevitability of margin loss and margin saving problem in MPC [6]. From the perspective of practical application, this paper discusses the effects of saving margin in margin saving model predictive control and analyzes the influence on control effect of saving margin for manipulated variables.

Nowadays, many scholars have done a lot of research on MPC and improved the control performance index function in MPC. Clarke et al. [7] proposed the generalized predictive control (GPC) strategy. Sentoni et al. [8] proposed the theory of model predictive control based on state space. Bemporad and Morari [9] gave an overview of robustness in MPC. Faced with the impact of system constraints, scholars proposed the constrained model predictive control (CMPC) [10-12]. Garcia and Morshedi [13] proposed a new algorithm which utilizes a quadratic program to compute moves on process manipulated variables to keep controlled variables close to their targets while preventing violations of process constraints. Kerrigan and Maciejowski [14] described a method for computing a lower bound for the constraint violation penalty weight of the exact penalty function to guarantee that the soft-constrained MPC solution will be equal to the hard-constrained MPC solution. An infinite horizon controller that allows incorporation of input and state constraints in a receding horizon feedback strategy is developed by Rawlings and Muske [15]. Kwon and Pearson [16,17], Michalska and Mayne [18] proposed adding terminal constraints to control performance index function in order to ensure the stability of MPC. Xiao and Qian [19] added static target to control performance index function in MPC, realizing the economic indicator on the basis of the basic control requirements satisfied. However, the above studies did not consider the consumed margins for manipulated variables, this paper improves the basic control performance index function of MPC, adding a margin loss function indicating operating margin reduction amount to the optimal control performance index function for margin saving.

To study the operating margin in MPC, Sanchez-Sanchez and Ricardez-Sandoval [20] considered MPC in dynamic optimization of the long period of chemical process to calculate the design margin. Sun et al. [21] defined the operating margin and design margin in heat exchanger. Xu and Luo [22] studied the relationship between the operating margin and the control system performance in chemical process under the conditions of MPC. However, the above studies all discussed how to determine the design margins through dynamic optimization in process design period, and this paper puts forward a method of considering margin loss in MPC in process control.

The rest of this paper is organized as follows. In Section 2, the description of margin saving problem and margin loss function in MPC is discussed. In Section 3, the structure of margin saving model predictive control is presented, based on which the optimization solution of margin saving model predictive control is deduced, and the conditions for convex optimization in margin saving model predictive control are also given. In Section 4, we find the margin saving effects for manipulated variables of margin saving model predictive control through the steady-state optimal value analysis. In Section 5, we analyze the effects on controlled variables tracking the expectations through the steady-state closed-loop final value derivation of margin saving model predictive control. In Section 6, a simulation case is given to illustrate the effectiveness of the algorithm. Finally, Section 7 concludes the paper.

2. Description of Margin Saving Problem in MPC.

2.1. **Margin saving problem.** In order to avoid the influences of all kinds of uncertainties like slow time-varying parameters, long time lag, and disturbances in dynamic control, we need to increase the operating space for manipulated variables and reduce the consumption of operating margin in dynamic control. So to add a margin loss function into MPC is necessary. The new algorithm of MPC will save the operating margin for manipulated variables in dynamic control as shown in Figure 1, so this algorithm is called margin saving model predictive control.

2.2. **Margin loss function.** To consider the changes of operating margin in dynamic control quantitatively, we need to build a function representing the distance between the operating point of manipulated variables and the boundary of the constraints, which is defined as margin loss function. 2-norm is a function of the concept of "length", and is

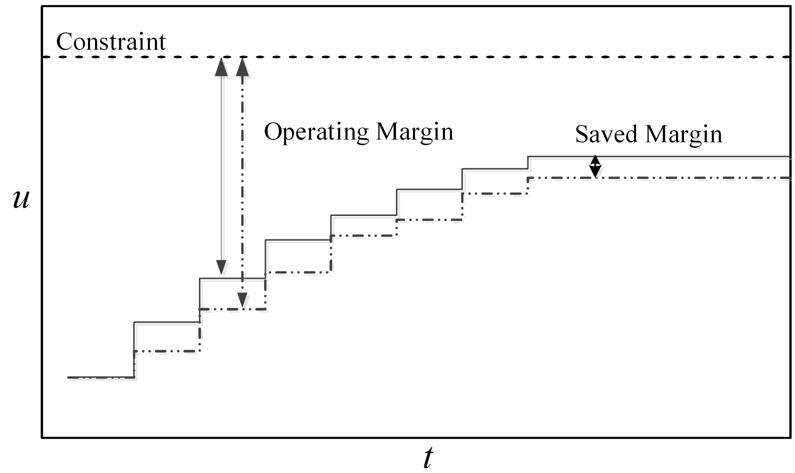


FIGURE 1. Figure of margin saving model predictive control

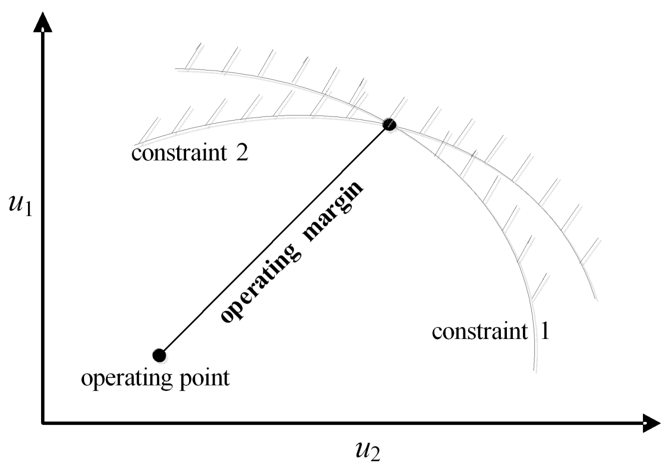


FIGURE 2. Figures of operating margin

usually used to represent the linear distance between two points or vectors matrix, so the mathematics expression of the margin loss function can be written as $f_{\mathrm{ML}} = \|u - u_{\mathrm{H}}\|^2$. u is the operating point of manipulated variables, and u_{H} is the upper constraint of manipulated variables.

3. Margin Saving Model Predictive Control.

3.1. **The structure of margin saving model predictive control.** Conventional MPC is a control algorithm based on optimization, and it determines the future control through a certain optimal performance index [24]. In order to achieve the effect of increasing operating margin in the dynamic control process and reducing the margin consumption in control, we need to improve the structure of conventional MPC by adding margin loss function into the performance index function. Eventually, we solve the optimization problem by turning into a quadratic programming.

Conventional model predictive control can be written into the form of 2-norm.

$$\min J(k) = \sum_{i=1}^{P} \|\hat{y}(k+i\,|k) - y_{\mathrm{d}}\|_Q^2 + \sum_{i=1}^{M} \|\Delta\hat{u}(k+i-1\,|k)\|_R^2$$

$\hat{y}(k+i|k)$ means the output prediction of system state from k moment to $(k+i)$ moment, y_{d} means the expectations or expected trajectory of output, and $\Delta\hat{u}(k+i-1|k)$ means the increment of input in present moment. P and M are prediction horizon and control horizon. Q is the weight matrix of tracking control item, and R is the weight matrix of changes of control energy item.

The performance index function of MPC should be the minimum of the objective function, so the margin loss function which is added into performance index function of MPC must have the form of negative. Therefore, the optimization performance index of margin saving model predictive control can be shown as

$$\begin{aligned}\min J(k) = {} & \sum_{i=1}^{P} \|\hat{y}(k+i|k) - y_{\mathrm{d}}\|_Q^2 + \sum_{i=1}^{M} \|\Delta\hat{u}(k+i-1\,|k)\|_R^2 \\ & - \sum_{i=1}^{M} \|\hat{u}(k+i-1\,|k) - u_{\mathrm{H}}\|_T^2\end{aligned} \tag{1}$$

T is the weight matrix of margin loss function. Since then, the new structure of margin saving model predictive control is proposed.

3.2. **Optimization solution of margin saving model predictive control.** It is assumed that the linear time-invariant discrete state space model in model predictive control has the following form.

$$\begin{cases} \hat{x}(k+1\,|k) = A\hat{x}(k\,|k) + B\hat{u}(k\,|k) \\ \hat{y}(k\,|k) = C\hat{x}(k\,|k) \end{cases}$$

$y(k) \in \mathrm{R}^r$ means controlled variables, $\hat{y}(k\,|k) = y(k)$ means outputs at the current moment, $\hat{x}(k\,|k) = x(k)$ means states at the current moment, $\hat{x}(k+i\,|k) \in \mathrm{R}^n$ means the estimations of system states from k moment to $k+i$ moment, $u(k) \in \mathrm{R}^m$ means control variables, and $\hat{u}(k\,|k) = u(k)$ means inputs at the current moment. $A \in \mathrm{R}^{n\times n}$ is state matrix, $B \in \mathrm{R}^{n\times m}$ is input matrix, and $C \in \mathrm{R}^{r\times n}$ is output matrix. P and M are prediction horizon and control horizon, and it is assumed that $M \le P$. When $i \ge M$, there exists $\hat{u}(k+i\,|k) = 0$. Define $\Delta\hat{u}(k+i\,|k) = \hat{u}(k+i\,|k) - \hat{u}\ (k+i-1\,|k)$ as the increment of control variables. Therefore, the P-step prediction of state can be expressed as matrix and vector form.

$$\begin{bmatrix} \hat{y}(k+1\,|k) \\ \hat{y}(k+2\,|k) \\ \vdots \\ \hat{y}(k+M\,|k) \\ \vdots \\ \hat{y}(k+P\,|k) \end{bmatrix} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^M \\ \vdots \\ CA^P \end{bmatrix} \hat{x}(k\,|k) + \begin{bmatrix} CB \\ CAB+CB \\ \vdots \\ \sum_{i=0}^{M-1} CA^iB \\ \vdots \\ \sum_{i=P-M}^{P-1} CA^iB \end{bmatrix} u(k-1)$$

$$+ \begin{bmatrix} CB & & & \\ CAB+CB & CB & & \\ \vdots & \vdots & \ddots & \\ \sum_{i=0}^{M-1} CA^iB & \sum_{i=0}^{M-2} CA^iB & \cdots & CB \\ \sum_{i=1}^{M} CA^iB & \sum_{i=1}^{M-1} CA^iB & \cdots & CAB \\ \vdots & \vdots & & \vdots \\ \sum_{i=P-M}^{P-1} CA^iB & \sum_{i=P-M}^{P-2} CA^iB & & CA^{P-M}B \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k|k) \\ \Delta\hat{u}(k+1|k) \\ \vdots \\ \Delta\hat{u}(k+M-1|k) \end{bmatrix}$$

It is assumed that system states can be measured, so we can express the prediction of output as matrix and vector form.

$$\hat{Y}(k) = S_X x(k) + S_U u(k-1) + S_{\Delta U}\Delta\hat{U}(k) = Y_0(k) + S_{\Delta U}\Delta\hat{U}(k) \tag{2}$$

And there exist

$$Y_0(k) = \begin{bmatrix} y_0(k+1) \\ y_0(k+2) \\ \vdots \\ y_0(k+P) \end{bmatrix} = S_X x(k) + S_U u(k-1),$$

$$\Delta\hat{U}(k) = \begin{bmatrix} \Delta\hat{u}(k\,|k) \\ \Delta\hat{u}(k+1\,|k) \\ \vdots \\ \Delta\hat{u}(k+M-1\,|k) \end{bmatrix}, \quad \hat{Y}(k) = \begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \vdots \\ \hat{y}(k+P|k) \end{bmatrix},$$

$$S_X = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^M \\ \vdots \\ CA^P \end{bmatrix}, \quad S_U = \begin{bmatrix} CB \\ CAB+CB \\ \vdots \\ \sum_{i=0}^{M-1} CA^iB \\ \vdots \\ \sum_{i=P-M}^{P-1} CA^iB \end{bmatrix},$$

$$S_{\Delta U} = \begin{bmatrix} CB & & & \\ CAB + CB & CB & & \\ \vdots & \vdots & \ddots & \\ \sum_{i=0}^{M-1} CA^iB & \sum_{i=0}^{M-2} CA^iB & \cdots & CB \\ \sum_{i=1}^{M} CA^iB & \sum_{i=1}^{M-1} CA^iB & \cdots & CAB \\ \vdots & \vdots & & \vdots \\ \sum_{i=P-M}^{P-1} CA^iB & \sum_{i=P-M}^{P-2} CA^iB & & CA^{P-M}B \end{bmatrix}.$$

Select Equation (1) as performance index function, and define matrixes.

$$\mathrm{E} = \begin{bmatrix} \mathrm{I}_m & & & & \\ \mathrm{I}_m & \mathrm{I}_m & & & \\ \mathrm{I}_m & \mathrm{I}_m & \mathrm{I}_m & & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathrm{I}_m & \mathrm{I}_m & \mathrm{I}_m & \mathrm{I}_m & \mathrm{I}_m \end{bmatrix}, \quad \mathrm{I}_{mM} = \begin{bmatrix} \mathrm{I}_m \\ \mathrm{I}_m \\ \mathrm{I}_m \\ \vdots \\ \mathrm{I}_m \end{bmatrix}^{\mathrm{T}}, \quad U_0 = \begin{bmatrix} u(k-1) \\ u(k-1) \\ \vdots \\ u(k-1) \\ u(k-1) \end{bmatrix},$$

$$U_{\mathrm{H}} = \begin{bmatrix} u_{\mathrm{H}} \\ u_{\mathrm{H}} \\ \vdots \\ u_{\mathrm{H}} \\ u_{\mathrm{H}} \end{bmatrix}, \quad Y_{\mathrm{d}} = \begin{bmatrix} y_{\mathrm{d}}(k+1|k) \\ y_{\mathrm{d}}(k+2|k) \\ \vdots \\ y_{\mathrm{d}}(k+P|k) \end{bmatrix},$$

$$\hat{U}(k) = \mathrm{E}\Delta\hat{U}(k) + U_0, \quad U_1 = U_0 - U_{\mathrm{H}}.$$

In addition, the weight matrices are $Q = \mathrm{diag}\{Q(1), Q(2), \ldots, Q(P)\}$, $T = \mathrm{diag}\{T(1), T(2), \ldots, T(P)\}$, $R = \mathrm{diag}\{R(1), R(2), \ldots, R(P)\}$. It is known as $\|\alpha\|_Q^2 = \alpha^{\mathrm{T}} Q\alpha$, Equation (1) can be written as Equation (3), and according to Equation (2) the above types are defined.

$$\begin{aligned} \min J(k) &= \left\|\hat{Y}(k) - Y_{\mathrm{d}}\right\|_Q^2 + \left\|\Delta\hat{U}(k)\right\|_R^2 - \|U(k) - U_{\mathrm{H}}\|_T^2 \\ &= \left\|S_{\Delta U}\Delta\hat{U}(k) + (Y_0(k) - Y_{\mathrm{d}})\right\|_Q^2 + \left\|\Delta\hat{U}(k)\right\|_R^2 - \left\|\mathrm{E}\Delta\hat{U}(k) + U_1\right\|_T^2 \\ &= \left(S_{\Delta U}\Delta\hat{U}(k) + (Y_0(k) - Y_{\mathrm{d}})\right)^{\mathrm{T}} Q \left(S_{\Delta U}\Delta\hat{U}(k) + (Y_0(k) - Y_{\mathrm{d}})\right) \\ &\quad + \left(\Delta\hat{U}(k)\right)^{\mathrm{T}} R \left(\Delta\hat{U}(k)\right) - \left(\mathrm{E}\Delta\hat{U}(k) + U_1\right)^{\mathrm{T}} T \left(\mathrm{E}\Delta\hat{U}(k) + U_1\right) \\ &= \mathrm{const} + \frac{1}{2}\Delta\hat{U}^{\mathrm{T}}(k)\Psi\Delta\hat{U}(k) + \Theta\Delta\hat{U}(k) \end{aligned} \tag{3}$$

Consider the constraints,

$$u_{\min} \le u(k) \le u_{\max}$$

$$\Delta u_{\min} \le \Delta u(k) \le \Delta u_{\max}$$

$$y_{\min} \le y(k) \le y_{\max}$$

And the above optimization problem can be expressed as the following standard secondary planning form.

$$\begin{aligned}
\min_{\Delta\hat{U}(k)} \quad & J(k) = \frac{1}{2}\Delta\hat{U}(k)^{\mathrm{T}}\Psi\Delta\hat{U}(k) + \Theta\Delta\hat{U}(k) \\
\text{s.t.} \quad & \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \Delta\hat{U}(k) \le \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}
\end{aligned}$$

where,

$$\begin{aligned}
& D_1 = \begin{bmatrix} \mathrm{E} \\ -\mathrm{E} \end{bmatrix}, \quad D_2 = \begin{bmatrix} \mathrm{I}_{mM} \\ -\mathrm{I}_{mM} \end{bmatrix}, \quad D_3 = \begin{bmatrix} S_{\Delta U} \\ -S_{\Delta U} \end{bmatrix}, \\
& \Psi = 2\left(S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}\right), \quad \Theta = 2\Big(-\left(Y_{\mathrm{d}} - Y_0(k)\right)^{\mathrm{T}} Q S_{\Delta U} - U_1^{\mathrm{T}} T \mathrm{E}\Big), \\
& d_1 = \left[u_{\max} - u(k-1), \dots, u_{\max} - u(k-1), -u_{\min} + u(k-1), \dots, -u_{\min} + u(k-1)\right]^{\mathrm{T}}, \\
& d_2 = \left[\Delta u_{\max}, \dots, \Delta u_{\max}, -\Delta u_{\min}, \dots, -\Delta u_{\min}\right]^{\mathrm{T}}, \\
& d_3 = \left[y_{\max} - y_0(k+1), \dots, y_{\max} - y_0(k+P),\ -y_{\min} + y_0(k+1), \dots, -y_{\min} + y_0(k+P)\right]^{\mathrm{T}}.
\end{aligned}$$

If no process constraints are considered, if $L = \begin{bmatrix} \mathrm{I}_m & 0_{m\times(mM-m)} \end{bmatrix}$, we can get the optimal control rate of unconstrained model predictive control.

$$\Delta U^*(k) = -L\Psi^{-1}\Theta^{\mathrm{T}} = L\left(S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}\right)^{-1}\Big(\left(Y_{\mathrm{d}} - Y_0(k)\right)^{\mathrm{T}} Q S_{\Delta U} + U_1^{\mathrm{T}} T \mathrm{E}\Big)^{\mathrm{T}}$$

3.3. **Convex optimization conditions in margin saving model predictive control.** In performance index function of margin saving model predictive control, because that the margin loss function is negative, the optimization problem in model predictive control may become a non-convex optimization problem [25], which results in no global optimal solution. In order to solve the above problems and get the global optimal solution in optimization, we should deduce the condition of convex optimization for the weight matrix T of margin loss function.

According to the definition of convex function, the general method for discriminating the convex function on the real number set is to solve its second derivative. If the function's second derivative is non-negative, it is called a convex function [26]. Therefore, if the weight matrix T of margin loss function meets the condition, $\dfrac{\partial^2 J(k)}{\partial^2 \Delta U(k)} \ge 0$, the optimization in margin saving model predictive control can be considered as convex optimization, and the global optimal solution can be obtained.

We can get the following equation from the optimal control in margin saving model predictive control.

$$\frac{\partial^2 J(k)}{\partial^2 \Delta U(k)} = S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}$$

Therefore, the conditions for convex optimization in margin saving model predictive control are

$$\mathrm{E}^{\mathrm{T}} T \mathrm{E} \le S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R$$

If the matrix $S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}$ is positive, the optimization in margin saving model predictive control is convex optimization.

4. **Margin Saving Effects for Manipulated Variables of Margin Saving Model Predictive Control.** Assumed that the initial steady-state point is 0, we introduce the optimization Equation (4) of conventional MPC. We can get the optimal steady-state solution of conventional MPC [27].

$$\begin{aligned} &\min_u \ J_1(k) = \|u - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u\|^2_R \\ &\text{s.t.} \ \ u = G^{-1}y \end{aligned} \tag{4}$$

In Equation (4), G is steady-state gain matrix. In the same way, the solution (5) can obtain the optimal solution of margin saving model predictive control which has the same parameter with conventional MPC.

$$\begin{aligned} &\min_u \ J_2(k) = \|u - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u\|^2_R - \|u - u_\mathrm{H}\|^2_T \\ &\text{s.t.} \ \ u = G^{-1}y \end{aligned} \tag{5}$$

So we have Theorem 4.1.

Theorem 4.1. *If u_1^* is the steady-state optimal value of conventional MPC, and u_2^* is the steady-state optimal value of margin saving model predictive control which has the same parameter with conventional MPC, then there exists $\|u_2^* - u_\mathrm{H}\|^2_T \geq \|u_1^* - u_\mathrm{H}\|^2_T$.*

Proof: In performance index function of conventional MPC, there exists $J_1(u_1^*) \leq J_1(u_2^*)$, which can be written as

$$\|u_1^* - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u_1^*\|^2_R \leq \|u_2^* - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u_2^*\|^2_R \tag{6}$$

In performance index function of margin saving model predictive control, there exists $J_2(u_2^*) \leq J_2(u_1^*)$, which can be written as

$$\|u_2^* - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u_2^*\|^2_R - \|u_2^* - u_\mathrm{H}\|^2_T \leq \|u_1^* - u_\mathrm{d}\|^2_{G^\mathrm{T}QG} + \|\Delta u_1^*\|^2_R - \|u_1^* - u_\mathrm{H}\|^2_T \tag{7}$$

Equation (6) plus Equation (7), we can get Equation (8)

$$\|u_2^* - u_\mathrm{H}\|^2_T \geq \|u_1^* - u_\mathrm{H}\|^2_T \tag{8}$$

This procedure completes the proof.

From Theorem 4.1, we know that u_1^* is closer to the upper constraints of the manipulated variables than u_2^*. Compared with conventional MPC, the margin saving model predictive control can increase operating margin in the current dynamic control process and make manipulated variables reduce the margin consumption in control. At the same time, it can make manipulated variables far away from constraint boundary on-line.

Now we deduce the steady-state effect of margin saving model predictive control. It is assumed that the initial steady-state point is 0, and we can get

$$\begin{aligned} &\min \ J(k) = \|u(k) - u_\mathrm{d}\|^2_{Q'} + \|\Delta u(k)\|^2_R - \|u(k) - u_\mathrm{H}\|^2_T \\ &\text{s.t.} \ \ u = G^{-1}y \end{aligned}$$

G is steady-state gain matrix, $Q' = Q^\mathrm{T}GQ$, and u_d is input variables after transformation by the output expectations. Because of $\Delta u(k) = u(k) - u(k-1)$, then

$$\begin{aligned} \min \ J(k) = {} & \|u(k) - u_\mathrm{d}\|^2_{Q'} + \|\Delta u(k)\|^2_R - \|u(k) - u_\mathrm{H}\|^2_T \\ = {} & \text{const} + u^\mathrm{T}(k)Q'u(k) - 2u^\mathrm{T}(k)Q'u_\mathrm{d} + u^\mathrm{T}(k)Ru(k) \\ & - 2u^\mathrm{T}(k)Ru(k-1) - u^\mathrm{T}(k)Tu(k) + 2u^\mathrm{T}(k)Tu_\mathrm{H} \end{aligned}$$

Then

$$\frac{\partial J(k)}{\partial u(k)} = 2\left(Q' + R - T\right)u(k) - 2\left(Q'u_\mathrm{d} + Ru(k-1) - Tu_\mathrm{H}\right) = 0$$

We can get the optimal value of control,

$$u^*(k) = \left(Q' + R - T\right)^{-1}\left(Q'u_{\mathrm{d}} + Ru(k-1) - Tu_{\mathrm{H}}\right) \tag{9}$$

Through the z transform of Equation (9), we can get Equation (10)

$$u^*(z) = \left(Q' + R - T - Rz^{-1}\right)^{-1}\left(Q'u_{\mathrm{d}}\frac{z}{z-1} - Tu_{\mathrm{H}}\frac{z}{z-1}\right) \tag{10}$$

Therefore, the final value theorem is used to deduce the steady-state value of manipulated variables.

$$\begin{aligned} u(\infty) &= \lim_{z\to 1}\frac{z-1}{z}u(z) \\ &= \lim_{z\to 1}\frac{z-1}{z}\left(Q' + R - T - Rz^{-1}\right)^{-1}\left(Q'u_{\mathrm{d}}\frac{z}{z-1} - Tu_{\mathrm{H}}\frac{z}{z-1}\right) \\ &= \left(Q' - T\right)^{-1}\left(Q'u_{\mathrm{d}} - Tu_{\mathrm{H}}\right) \end{aligned}$$

The necessary condition for control to achieve steady state is the definite matrix $\left(Q' - T\right)$ to be positive, and Theorem 4.2 is proposed.

Theorem 4.2. *When $\left(Q' - T\right)$ is a positive matrix and the constraints for manipulated variables are hard constraints $u_{\mathrm{H}} \geq u_{\mathrm{d}}$, there must be steady-state values of the manipulated variables that meet $u(\infty) \leq u_{\mathrm{d}}$.*

Proof: As it is known $u_{\mathrm{H}} \geq u_{\mathrm{d}}$, we multiply both sides by T and a minus sign. As T is positive, we can get Equation (11).

$$-Tu_{\mathrm{H}} \leq -Tu_{\mathrm{d}} \tag{11}$$

Adding $Q'u_{\mathrm{d}}$ to both sides of Equation (11), we can get Equation (12)

$$Q'u_{\mathrm{d}} - Tu_{\mathrm{H}} \leq Q'u_{\mathrm{d}} - Tu_{\mathrm{d}} \tag{12}$$

Because $\left(Q' - T\right)$ is positive, we can get $\left(Q' - T\right)^{-1}\left(Q'u_{\mathrm{d}} - Tu_{\mathrm{H}}\right) \leq u_{\mathrm{d}}$, which is the same as $u(\infty) \leq u_{\mathrm{d}}$.

This procedure completes the proof.

By Theorem 4.2, the steady-state value of margin saving model predictive control must be less than the expectations of manipulated variables, which realize the effect of increasing the operating margin.

5. **Tracking Effect for Controlled Variables of Margin Saving Model Predictive Control.** Increasing the operating margin in the dynamic control process can save margin for control, but at the same time it has a certain effect on the condition of the controlled variable tracking the given values. In order to discuss the control effect quantitatively, it is necessary to analyze the steady-state tracking effect for controlled variables of margin saving model predictive control. However, it is difficult for constrained control system to solve the analytical solution and analyze the steady-state control effect of control system. To the same control system, the stability under no constraints is the necessary condition for the stability of the constrained control system under the same control parameter [28]. At the same time, considering margin in constrained MPC, we usually make the operating point run within the constrained feasible region. Therefore, assuming that the initial steady-state points are 0, if we want to analyze the steady-state conditions of margin saving model predictive control, we must analyze the closed-loop of control system under no constraints firstly. We can know the condition of steady-state solution by putting optimal control action into the closed-loop of control system.

The discrete state space model of the controlled process is

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{13}$$

$y(k) \in \mathrm{R}^r$ is output variables, $u(k) \in R^m$ is input variables, and $x(k) \in \mathrm{R}^n$ is state variables. $A \in \mathrm{R}^{n\times n}$ is state matrix, $B \in R^{n\times m}$ is output matrix, and $C \in \mathrm{R}^{r\times n}$ is input matrix.

To simplify the analysis process, we will use single value MPC algorithm [29] to analyze the steady-state solutions, only taking the P-step predictive value of the output variables in the future.

$$\hat{y}(k+P) = CA^P x(k) + \sum_{i=1}^{P} CA^{i-1}Bu(k+P-i)$$

P is the prediction horizon. In addition, for single-valued MPC, the control horizon $M = 1$. When $i > 0$, there exits $u(k+i) = u(k)$. At the same time, if $K = CA^P$, $S(P) = \sum_{i=1}^{P} CA^{i-1}B$, the predictive value of the output variables in the future can be written as

$$\hat{y}(k+P) = Kx(k) + S(P)u(k) \tag{14}$$

In order to facilitate the analysis of tracking effect when the final output achieves the steady state, we express the margin loss function as the form of output.

$$\min J(k) = \|\hat{y}(k+P) - y_{\mathrm{d}}\|_Q^2 + \|\Delta u(k)\|_R^2 - \|\hat{y}(k+P) - y_{\mathrm{H}}\|_{S'}^2 \tag{15}$$

y_{H} is the value of output variables which correspond with the upper constraints of manipulated variables, and T' is the weight matrix of margin loss function which corresponds with output variables. As $\Delta u(k) = u(k) - u(k-1)$, Equation (14) is brought in Equation (15), and there exits

$$\begin{aligned} &\min\ J(k) \\ &= \|\hat{y}(k+P) - y_{\mathrm{d}}\|_Q^2 + \|\Delta u(k)\|_R^2 - \|\hat{y}(k+P) - y_{\mathrm{H}}\|_{T'}^2 \\ &= \|S(P)u(k) + Kx(k) - y_{\mathrm{d}}\|_Q^2 + \|u(k) - u(k-1)\|_R^2 - \|S(P)u(k) + Kx(k) - y_{\mathrm{H}}\|_{T'}^2 \\ &= \mathrm{const} + u^{\mathrm{T}}(k)S^{\mathrm{T}}(P)QS(P)u(k) + 2u^{\mathrm{T}}(k)S^{\mathrm{T}}(P)Q(Kx(k) - y_{\mathrm{d}}) + u^{\mathrm{T}}(k)Ru(k) \\ &\quad - 2u^{\mathrm{T}}(k)Ru(k-1) - u^{\mathrm{T}}(k)S^{\mathrm{T}}(P)T'S(P)u(k) - 2u^{\mathrm{T}}(k)S^{\mathrm{T}}(P)T'(Kx(k) - y_{\mathrm{H}}) \end{aligned}$$

And

$$\begin{aligned} \frac{\partial J(k)}{\partial u(k)} &= 2S^{\mathrm{T}}(P)QS(P)u(k) + 2S^{\mathrm{T}}(P)Q(Kx(k) - y_{\mathrm{d}}) + 2Ru(k) - 2Ru(k-1) \\ &\quad - 2S^{\mathrm{T}}(P)T'S(P)u(k) - 2S^{\mathrm{T}}(P)T'(Kx(k) - y_{\mathrm{H}}) = 0 \end{aligned}$$

The optimal value of control is

$$\begin{aligned} u^*(k) = &\left(S^{\mathrm{T}}(P)QS(P) + R - S^{\mathrm{T}}(P)T'S(P)\right)^{-1}\Big(S^{\mathrm{T}}(P)Q(y_{\mathrm{d}} - Kx(k)) \\ &+ Ru(k-1) - S^{\mathrm{T}}(P)T'(y_{\mathrm{H}} - Kx(k))\Big) \end{aligned} \tag{16}$$

The z transformation for Equation (16) is available, and we can get

$$\begin{aligned} u^*(z) = & \left(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1} \\ & \left(S^{\mathrm{T}}(P)Q\left(y_{\mathrm{d}}\frac{z}{z-1} - Kx(k)\right) - S^{\mathrm{T}}(P)T^{'}\left(y_{\mathrm{H}}\frac{z}{z-1} - Kx(k)\right)\right) \\ = & \left(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1} \\ & \left(-S^{\mathrm{T}}(P)\left(Q - T^{'}\right)Kx(k) + S^{\mathrm{T}}(P)Qy_{\mathrm{d}}\frac{z}{z-1} - S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\frac{z}{z-1}\right) \end{aligned} \tag{17}$$

It is assumed that the actual process also follows Equation (13), and the z transformation for Equation (13) is available,

$$\begin{aligned} zx(z) &= Ax(z) + Bu(z) \\ y(z) &= Cx(z) \end{aligned} \tag{18}$$

Equation (17) is brought in Equation (18), and we can get

$$\begin{aligned} y(z) = C\Bigg(& z\mathrm{I} - A + B\left(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1}S^{\mathrm{T}}(P) \\ & \left(Q - T^{'}\right)K\Bigg)^{-1} B\left(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1} \\ & \left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}}\frac{z}{z-1} - S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\frac{z}{z-1}\right) \end{aligned} \tag{19}$$

In order to determine the steady-state situation, the final value theorem is used for Equation (19).

$$\begin{aligned} y(\infty) = & \lim_{z\to 1}\frac{z-1}{z}y(z) \\ = & \lim_{z\to 1}\frac{z-1}{z}C\Bigg(z\mathrm{I} - A + B\Big(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} \\ & - S^{\mathrm{T}}(P)T^{'}S(P)\Big)^{-1}S^{\mathrm{T}}(P)\left(Q - T^{'}\right)K\Bigg)^{-1} \\ & B\left(S^{\mathrm{T}}(P)QS(P) + R - Rz^{-1} - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1} \\ & \left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}}\frac{z}{z-1} - S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\frac{z}{z-1}\right) \\ = & C\left(\mathrm{I} - A + B\left(S^{\mathrm{T}}(P)QS(P) - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1}S^{\mathrm{T}}(P)\left(Q - T^{'}\right)K\right)^{-1} \\ & B\left(S^{\mathrm{T}}(P)QS(P) - S^{\mathrm{T}}(P)T^{'}S(P)\right)^{-1}\left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}} - S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\right) \\ = & C(\mathrm{I} - A)^{-1}B\Big(S^{\mathrm{T}}(P)\left(Q - T^{'}\right)K(\mathrm{I} - A)^{-1}B + S^{\mathrm{T}}(P)QS(P) \\ & - S^{\mathrm{T}}(P)T^{'}S(P)\Big)^{-1}\left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}} - S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\right) \end{aligned}$$

$$
\begin{aligned}
&= C(\mathrm{I}-A)^{-1}B\left(S^{\mathrm{T}}(P)\left(Q-T^{'}\right)\left(K\left(\mathrm{I}-A\right)^{-1}B+S(P)\right)\right)^{-1} \\
&\quad \left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}}-S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\right) \qquad (20) \\
&= G_{\mathrm{m}}(1)\left(S^{\mathrm{T}}(P)\left(Q-T^{'}\right)G_{\mathrm{m}}(1)\right)^{-1}\left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}}-S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\right)
\end{aligned}
$$

$G_{\mathrm{m}}(z)$ represents the transfer function of the model, and there exists $C(\mathrm{I}-A)^{-1}B = G_{\mathrm{m}}(1)$. From Equation (20) we can get

$$
\begin{aligned}
y(\infty) &= G_{\mathrm{m}}(1)S^{\mathrm{T}}(P)\left(Q-T^{'}\right)G_{\mathrm{m}}(1)^{-1}\left(S^{\mathrm{T}}(P)Qy_{\mathrm{d}}-S^{\mathrm{T}}(P)T^{'}y_{\mathrm{H}}\right) \\
&= \left(Q-T^{'}\right)^{-1}\left(Qy_{\mathrm{d}}-T^{'}y_{\mathrm{H}}\right)
\end{aligned} \tag{21}
$$

That the matrix $\left(Q-T^{'}\right)$ is non-singular is the necessary condition to determine whether the output variables have steady-state solution. From Equation (21), the steady-state values of the output variables are functions between the output expectations y_{d} and the process upper constraints y_{H}. Therefore, in margin saving model predictive control, when the weight Q, $T^{'}$ do not tend to zero, the steady-state values of outputs cannot track the expectations accurately, and there will be some loss. However, the thought of constrained control can allow the controlled variables to follow the expectations within a certain zone. Therefore, when using margin saving model predictive control, we need to choose reasonable weight matrix of margin loss function. In order to achieve reducing the margin consumption in control, we must ensure that the controlled variables are within the given zones.

6. **Simulation and Example.** In order to verify the correctness of this method, the isothermal continuous stirred tank reactor (CSTR) is used as a simulation example [20]. The present analysis assumes that an exothermic, irreversible, first order reaction that transforms reactant A into product B takes place in the CSTR, i.e., A → B. In the CSTR, the density and the heat capacity of the mixture are assumed to be constant, and the volume hold-up keeps constant because it is perfectly controlled by the outlet flow rate of the CSTR. Figure 3 shows the CSTR system.

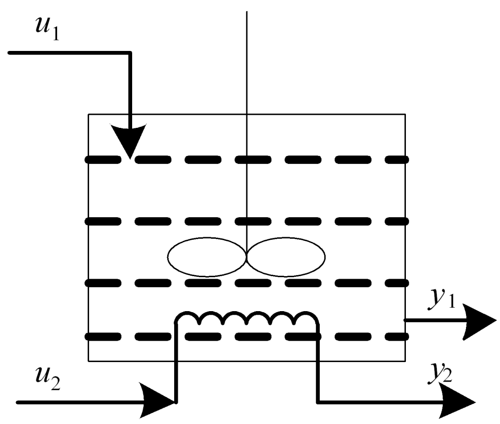


FIGURE 3. Model of CSTR system

The model of the CSTR is

$$\begin{aligned} V\frac{\mathrm{d}C}{\mathrm{d}t} &= -k_0 e^{-\frac{E}{\mathrm{R}T}}CV + Q_0(C_0 - C) \\ V\rho C_\mathrm{p}\frac{\mathrm{d}T}{\mathrm{d}t} &= D_\mathrm{h} k_0 e^{-\frac{E}{\mathrm{R}T}}CV + Q_0\rho C_\mathrm{p}(T_0 - T) - q_\mathrm{c} \\ q_\mathrm{c} &= \frac{U_\mathrm{a} Q_\mathrm{cw} C_\mathrm{pc}}{U_\mathrm{a} + Q_\mathrm{cw} C_\mathrm{pc}}(T - T_\mathrm{c}) \end{aligned} \tag{22}$$

where k_0 is the reaction rate constant; E is the reaction activation energy; R is the ideal gas constant; V is the reactor volume; C is the concentration of A in the reactor; T is the temperature of the reactor; C_p is the heat capacity of the mixture; ρ is the density of the mixture; D_h is the reaction heat; Q_0 is the inlet flow rate; C_0 and T_0 are the inlet concentration of A and the inlet temperature; q_c is the heat transfer rate of cooling water; Q_cw is the cooling water flow rate; T_c is the temperature of the cooling water; C_pc is the heat capacity of the cooling water; U_a is the heat transfer coefficient.

The inlet flow rate Q_0 and the cooling water flow rate Q_cw represent the process manipulated variables $u(t)$ whereas the temperature and the concentration of reactant A in the reactor are variables that can be measured online and represent the controlled (output) variables $y(t)$. That is $u = \begin{bmatrix} Q_0 & Q_\mathrm{cw} \end{bmatrix}^\mathrm{T}$, $y = \begin{bmatrix} C & T \end{bmatrix}^\mathrm{T}$.

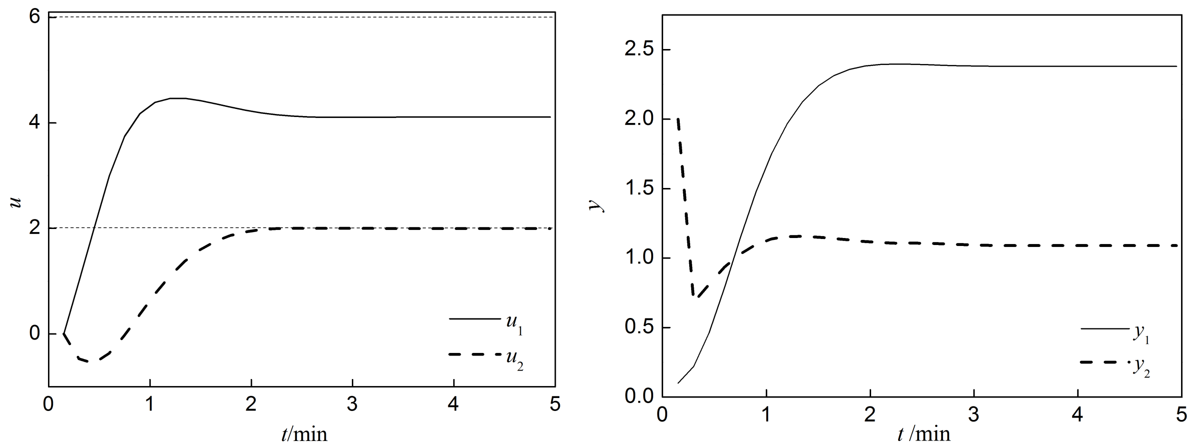
Its linear discrete state space model is shown as Equation (23)

$$x(k+1) = \begin{bmatrix} 0.74154 & -0.00154 \\ 1.19354 & 0.06567 \end{bmatrix} x(k) + \begin{bmatrix} 0.15 & 0 \\ 0 & -0.912 \end{bmatrix} u(k) \tag{23}$$

$$\text{s.t.} \quad \begin{array}{c} -1.5 \le u_1 \le 6, \quad -1.5 \le u_2 \le 2 \\ -1 \le y_1 \le 3, \quad -1 \le y_2 \le 2 \end{array}$$

When the output expectations are $y_\mathrm{d} = \begin{bmatrix} 2.38 & 1.09 \end{bmatrix}^\mathrm{T}$, we use conventional MPC to control the system. The simulation gives the curves of the controlled variables and the manipulated variables as shown in Figure 4.

From Figure 4, we can see that, the manipulated variable u_2 reaches the boundary of the feasible domain, the operating margins are completely consumed and process operators and engineers have no space for operation optimization, which is not hoped in process control.



(a) The response curves of manipulated variables (b) The response curves of controlled variables

FIGURE 4. The simulation of conventional MPC

Using margin saving model predictive control, the simulation is carried out and used to analyze the simulation results.

(1) Convex optimization analysis in margin saving model predictive control.

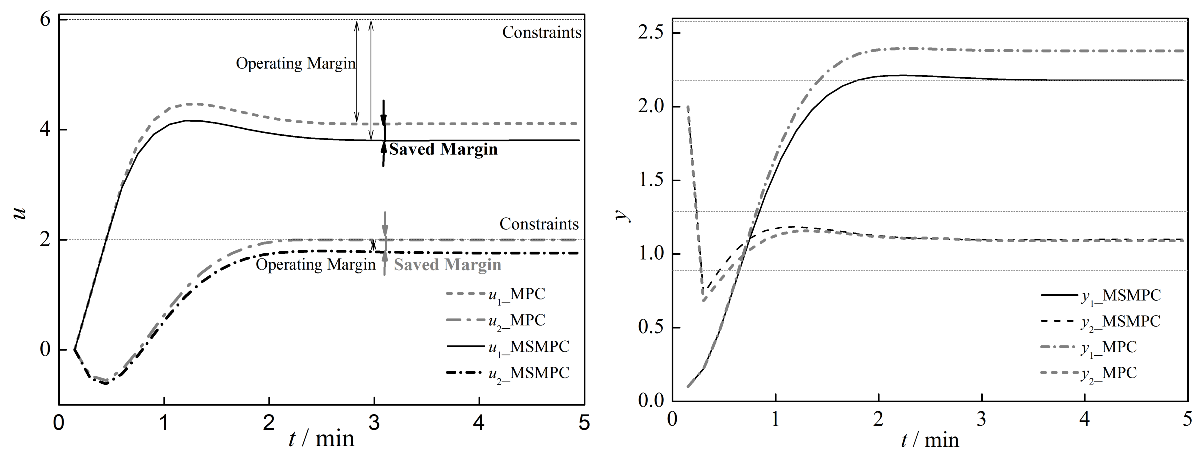
When the weight matrix Q, R are unit matrixes, the coefficient of T is $t = 0.03$. Through calculation the matrix $S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}$ is the symmetric matrix, and the eigenvalue of this matrix is

$$\det\left(S_{\Delta U}^{\mathrm{T}} Q S_{\Delta U} + R - \mathrm{E}^{\mathrm{T}} T \mathrm{E}\right) = 116.54 > 0$$

So the optimization is convex optimization, and the global optimal solution can be obtained.

(2) Dynamic response comparison between margin saving model predictive control and conventional MPC.

When the output expectations are $y_{\mathrm{d}} = \left[\begin{array}{cc} 2.38 & 1.09 \end{array}\right]^{\mathrm{T}}$, the controlled variables are allowed to be controlled in the range of $y_1 = \left[\begin{array}{cc} 2.18 & 2.58 \end{array}\right]$ and $y_2 = \left[\begin{array}{cc} 0.89 & 1.29 \end{array}\right]$. We implement the simulation of margin saving model predictive control and conventional MPC. We can get the comparison figures as shown in Figure 5.



(a) The response curves of manipulated variables (b) The response curves of controlled variables

FIGURE 5. The comparison simulation of conventional MPC and margin saving model predictive control

In Figure 5, the dotted lines are the response curves of conventional MPC, the solid lines are the response curves of margin saving model predictive control, bold lines are the response curves of u_1 and y_1, and thin lines are the response curves of u_2 and y_2. Figure 5(a) is the response curve of manipulated variables, where the dotted line is the constraints of manipulated variables. Figure 5(b) is the response curve of controlled variables, and the dotted lines are the expected zones of the controlled variables. As you can see from Figure 5, when the system expectation is $y_{\mathrm{d}} = \left[\begin{array}{cc} 2.38 & 1.09 \end{array}\right]^{\mathrm{T}}$, the operating variable u_2 reaches the boundary. By using margin saving model predictive control, the manipulated variable will back off within the constraint boundary. It not only saves the margins of manipulated variables, but also can keep more space for the future process control and optimization effectively. At the same time, the final steady-state values of the controlled variable are $y = \left[\begin{array}{cc} 2.18 & 1.10 \end{array}\right]^{\mathrm{T}}$, which are in the expect zones of the controlled variables, and satisfy the requirement for the control effect.

(3) The relationship between the performance index functions in margin saving model predictive control.

To analyze the control influence quantitatively when adding margin loss function into performance index function of MPC, we need to analyze the relationship between the performance index functions in margin saving model predictive control. J_0 is the sum function value of tracking control item and control energy item. J is the sum function value of tracking control item, control energy item and margin loss item. J_{Margin} is the function value of margin loss item. We draw the curves of the three function values in the dynamic process, as shown in Figure 6.

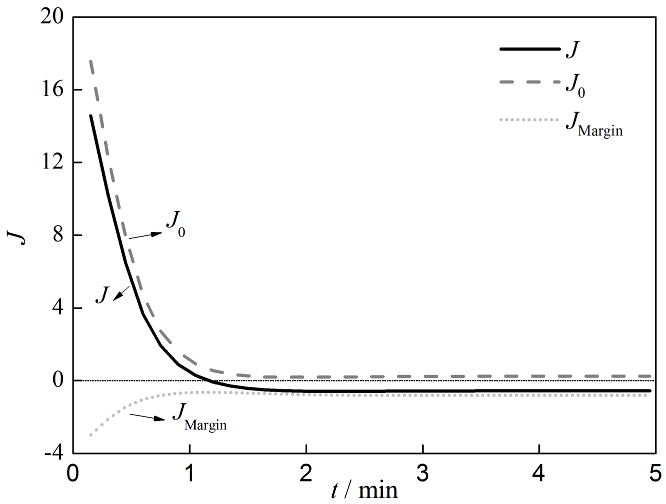


Figure 6. Dynamic curves of performance index function values

In Figure 6, the solid line is the curve of J, the short dotted line is the curve of J_0, the dotted line is the curve of J_{Margin}, as you can see from Figure 6.

a) Because of the existence of margin loss function, J_0 has been lost, and the whole function value J is reduced.

b) In general, the bigger the function value of margin loss item is, the more difficult the optimization becomes convex optimization. From Figure 6, the margin loss function value J_{Margin} accounts for the proportion of J about 1/3, which is large, and MPC still belongs to the convex optimization. Thus, we can draw the conclusion that margin saving model predictive control has good effect, and it can generally guarantee the optimization of convex optimization, which can get the global optimal solution.

7. **Conclusion.** In order to save more space for operation optimization, this paper proposed an algorithm of margin saving model predictive control to back off the operating point. Through adding a margin loss function into the performance index function of conventional MPC, we established a new framework of margin saving model predictive control. We deduced the optimization solution and analyzed the margin saving effect for manipulated variables and the tracking effects for controlled variables after adding the margin loss function.

Some future research directions of our work can be given as follows.

(1) In the long running period of chemical processes, the constraints of control variables are getting narrow with the process margin consumption. Therefore, we must analyze the situations of saving margin when the constraints get narrow.

(2) Based on multi-layer hierarchical structure in process control, we must consider the online optimization implementation of margin saving model predictive control. We can design the objective function of steady-state optimization of conventional MPC considering not only the optimal expectations but also margin loss function.

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