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MATHEMATICAL MODELING FOR FORECASTING THE GROSS DOMESTIC PRODUCT OF MEXICO

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Abstract. *This paper presents three mathematical models for forecasting of the Mexico's gross domestic product using regressions. Three mathematical models are: 1) linear regression; 2) exponential regression; 3) parabolic regression. The regression models have been developed through the data analysis of 82 years from 1935 until 2016. Starting from the idea that in economics, as well as in other sciences, anything has the tendency to depend on anything else and in this paper three models are developed to observe which of the three is capable of expressing the relation between the years and the gross domestic product of Mexico. According to the figures shown, it is clearly observed that the parabolic regression model is more accurate with respect to the linear and exponential regression models. Then, the parabolic regression model is the most appropriate, since it is adjusted to real conditions of the Mexico's gross domestic product, which is the main contribution of this paper.*

Keywords: Linear regression, Exponential regression, Parabolic regression

1. **Introduction.** Gross domestic product (GDP) is a monetary measure of the market value of all final goods and services produced in a period (quarterly or yearly). Nominal GDP estimates are commonly used to determine the economic performance of a whole country or region, and to make international comparisons [1].

The importance of the forecast of future economic outcomes is a vital component of the decision-making process in central banks for all countries. Monetary policy decisions affect the economy with a delay, so monetary policy authorities must be forward looking, i.e., must know what is likely to happen in the future. Gross domestic product (GDP) is one of the most important indicators of national economic activities for countries. Scientific prediction of the indicator has important theoretical and practical significance on the development of economic development goals [2].

GDP is commonly used as an indicator of the economic health of a country, as well as a gauge of a country's standard of living. Since the mode of measuring GDP is uniform from country to country, GDP can be used to compare the productivity of various countries with a high degree of accuracy. Adjusting for inflation from year to year allows for the seamless comparison of current GDP measurements with measurements from previous years or quarters. In this way, a nation's GDP from any period can be measured as a percentage relative to previous periods. As an important statistic that indicates whether an economy is expanding or contracting, GDP can be tracked over long spans of time and used in measuring a nation's economic growth or decline, as well as in determining if an economy is in recession (generally defined as two consecutive quarters of negative GDP growth) [3].

Organisation for Economic Co-operation and Development (OECD) defines GDP as "an aggregate measure of production equal to the sum of the gross values added of all resident and institutional units engaged in production (plus any taxes, and minus any subsidies, on products not included in the value of their outputs) [4].

An International Monetary Fund (IMF) publication states that "GDP measures the monetary value of final goods and services – that is, those that are bought by the final user – produced in a country in a given period of time (say a quarter or a year)" [5].

The performance of economy can be measured with the help of GDP. There are three ways in which the GDP of a country can be measured [2,6,7]:

a) GDP is the sum of gross value added of the various institutional sectors or the various industries plus taxes and less subsidies on products (which are not allocated to sectors and industries) – production approach.

b) GDP is the sum of final uses of goods and services by resident institutional units (actual final consumption and gross capital formation), plus exports and minus imports of goods and services – expenditure approach.

c) GDP is the sum of uses in the total economy generation of income account (compensation of employees, taxes on production and imports less subsidies, gross operating surplus and mixed income of the total economy) – income approach.

Figure 1 shows in detail the Mexico's gross domestic product of the last 82 years from 1935 until 2016. Also, the periods governed by each president and the economic crisis of some years are presented [8].

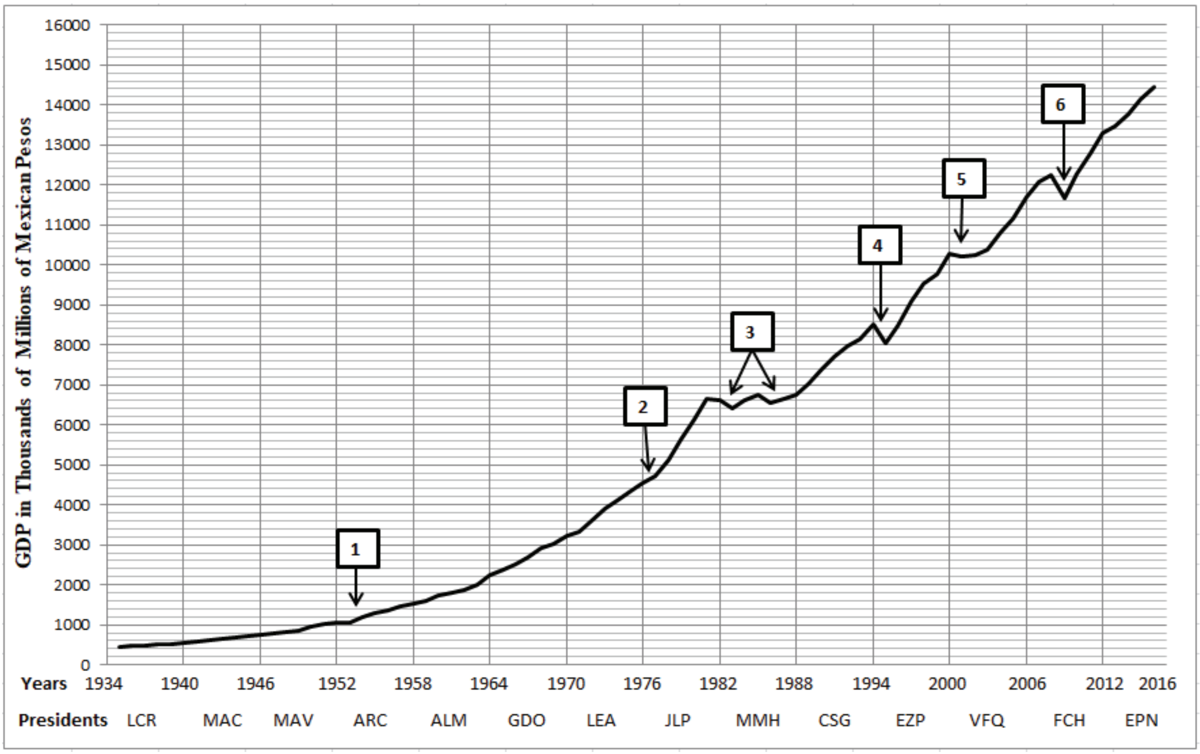

FIGURE 1. Gross domestic product of Mexico

The nomenclature used in Figure 1 is:

1. – Economic crisis of 1954.	2. – Economic crisis of 1976.
3. – Economic crisis of 1982 and 1987.	4. – Economic crisis of 1994.
5. – Economic crisis of 2001.	6. – Economic crisis of 2009.
LCR – Lázaro Cárdenas del Río.	MAC – Manuel Ávila Camacho.
MAV – Miguel Alemán Valdez.	ARC – Adolfo Ruiz Cortines.
ALM – Adolfo López Mateos.	GDO – Gustavo Díaz Ordaz.
LEA – Luis Echeverría Alvarez.	JLP – José López Portillo.
MMH – Miguel de la Madrid Hurtado.	CSG – Carlos Salinas de Gortari.
EZP – Ernesto Zedillo Ponce.	VFQ – Vicente Fox Quezada.
FCH – Felipe Calderón Hinojosa.	EPN – Enrique Peña Nieto.

The most relevant papers addressing the issue of the GDP models are as follows.

Kurita realized an econometric investigation of the interactions between Japan's housing investment and gross domestic product (GDP). A cointegrated vector autoregressive (VAR) analysis of Japan's recent time series data reveals two cointegrating relationships, which characterize the underlying long-run interactions of the variables in question. The cointegrated VAR model is then reduced to a vector equilibrium correction model, which is seen as a parsimonious representation of various dynamic interactions in the data. This paper contributes to a deeper understanding of empirical aspects of macroeconomics, and also provides useful information for the development of Japan's economic and housing policy in future, but does not present the behavior of the GDP [9].

Desai and Bhatia built a multi variable regression model to predict the GDP growth rate in India using key macroeconomic indicators such as consumer price index (CPI) inflation, manufacturing and services purchasing manager's index, interest rates and the price of crude oil. The relationships between GDP and these parameters, as well as their inter-relationships are studied in this paper using linear regression models. This paper presents a multi variable linear regression model to predict the GDP growth rate using the inflation, price of crude oil, interest rates, services and manufacturing PMI as predictors. Impact of each of these variables on GDP growth rate is studied individually to observe if it is statistically significant [10].

Banks et al. presented the recent movements in UK economic statistics in an international context to support the understanding of the UK economy by drawing international comparisons [11].

Chioma realized a study employing regression analysis to investigate the casual relationship between gross domestic product and personal consumption expenditure of Nigeria using data from 1994-2007. A non-insignificant value of 0.0514 was obtained as a slope coefficient indicating that an increase in gross domestic product has no significant effect on the personal consumption expenditure of Nigeria. This was further evidenced by the value of the coefficient of determination which was only 0.035 implying that the gross domestic product only explained about 3.5% of the personal consumption expenditure of Nigeria. This paper considers the dependent variable as the personal consumption expenditures and the independent variable as the gross domestic product taking consideration of the years of 1994 to 2007 [12].

Abbas et al. investigated the impact of foreign direct investment on GDP growth of South Asian Association for Regional Cooperation (SAARC) countries. This relationship is tested by applying multiple regression models. The change in GDP is taken as dependent variable while the changes in foreign direct investment (FDI) and the inflation are considered as independent variables. The data used for this is ranging from year 2001 to 2010 of SAARC countries. The result shows that the overall model is significant. There is a positive and significant relationship between GDP and FDI while an insignificant relationship between GDP and inflation. The countries of the SAARC are: Pakistan, India, Bangladesh, Sri Lanka, Maldives, Bhutan and Nepal [13].

Agalega and Antwi showed in the impact of macroeconomic variables on gross domestic product: Empirical Evidence from Ghana employing multiple linear regressions to establish that there exists a fairly strong positive correlation between GDP, interest rate and inflation, but inflation and interest rate could only explain movement in GDP by only 44 percent. The paper further established that there exists a positive relationship between inflation and GDP, and the relationship between interest rate and GDP is negative. It is recommended among others that the Government together with the Bank of Ghana should develop and pursue prudent monetary policies that would aim at reducing and stabilizing both the micro and macroeconomic indicators such as inflation targeting, and interest rate, so as to boast the growth of the economy. This paper considers the dependent variable as the gross domestic product and the independent variables as the inflation rate and the interest rates taking consideration of the years of 1980 to 2010 [14].

Syed and Shaikh investigated the effects of macroeconomic variables on GDP of Pakistan. Gross domestic product (GDP), a chief indicator of an economy, shows that for a long time, Pakistan economy was backward. The years after independence, the size of real GDP, per capita GDP, and their growth rates was small, but improved from 1990. In this study, the GDP of the 64 districts in Pakistan at current market prices are considered. The factors which have effects on the gross district product in the year 2010-2011 are measured here. Here principal component analysis and maximum likelihood method of factor analysis are used for the seventeen variables of the gross district products of 64 districts. This paper considers the dependent variable as the gross domestic product and the seventeen independent variables are: 1) crop and horticulture; 2) animal farming; 3) forestry and related services; 4) fishing; 5) mining and quarrying; 6) manufacturing; 7) electricity, gas and water supply; 8) construction; 9) wholesale and retail trade; 10) hotel and restaurants; 11) transport, storage and communication: 12) financial intermediation; 13) real state renting and business service; 14) public administration and defense; 15) education; 16) health and social work; 17) community social and personal services [15].

Paliu-Popa et al. gave the intensification of the world economy globalization, the production represents one of the factors characterizing the development of a country and its performance. In this context, this paper aims to establish and analyze the influence of the crude steel production (CSP) and blast furnace iron (BFI) production on the gross domestic product, using the multiple linear regression model. Data subject to the study are related to the period 2004-2014, and the research refers to the first ten countries producing crude steel and blast furnace iron, members of the European Union (EU). The paper aims to analyze the interdependence among the GDP, CSP and BFI from: Belgium, Czech Republic, Germany, Spain, France, Italy, Netherlands, Austria, Poland and United Kingdom using the linear regression model. In the analysis it comes from a set of data including, as dependent variable, the value of the GDP and as independent variables: CSP and BFI [16].

Tripathi focused on the trends of Indian growth in terms of gross domestic product and its components. And furthermore analyze if the growth accelerates or decelerates. This paper covers growth performance in India for period 1951-2012. The average annual trend growth of GDP and its components is higher during the reform phase as compared to the pre-reform phase except mining & quarrying and electricity, gas & water supply. The GDP shows an acceleration trend (positive trend) in overall and sub-periods except in the electricity, gas & water supply that is deceleration trend (negative trend) in overall and sub-periods. And in case of mining & quarrying the results are statistically insignificant.

However, the structure of GDP and its components in terms of intercepts and slopes are different in post reform phase as compared to the pre-reform phase [17].

Olej and Kfupka presented the possibility of the design and application of Takagi-Sugeno fuzzy inference system in predicting of gross domestic product development by designing a prediction models whose accuracy is superior to the models used in praxis [18].

Dritsaki shows at modeling and forecasting real GDP rate in Greece using the Box-Jenkins methodology during the period 1980-2013 with one auto-regressive-integrated-moving-average (ARIMA) model for forecasting the values of real GDP rate for 2015, 2016 and 2017. Statistical results show that Greece's real GDP rate is steadily improving [2].

Zhang studied time series data of regional GDP per capita in Sweden from 1993 to 2009, and three autoregressive models are used to model and forecast regional GDP per capita. The included models are the autoregressive integrated moving average (ARIMA) model, the vector autoregression (VAR) model and the first-order autoregression (AR(1)) model. Data from five counties were chosen for the analysis: Stockholm, Västra Götaland, Skåne, Östergötland and Jönköping, which are the top 5 ranked counties in Sweden with regard to regional GDP per capita. Data from 1993 to 2004 are used to fit the model, and then data for the last 5 years are used to evaluate the performance of the prediction. The results show that all the three models are valid in forecasting the GDP per capita in short-term. However, generally, the performance of the AR(1) model is better than that of the ARIMA model. And the predictive performance of the VAR model was shown to be the worst [19].

This paper presents three mathematical models for forecasting the gross domestic product of Mexico using regressions. The three mathematical models are: 1) linear regression; 2) exponential regression; 3) parabolic regression. The regression models have been developed through of the data analysis of 82 years from 1935 until 2016. In this paper three models are developed to observe differences and which of the three is the most appropriate, and that is adjusted to real conditions of the Mexico's gross domestic product.

The paper is organized as follows. Section 2 describes the formulas of the three mathematical models for forecasting the gross domestic product of Mexico using regressions. Subsection 2.1 shows the linear regression model. Subsection 2.2 presents the exponential regression model. Subsection 2.3 shows the parabolic regression model. Section 3 presents the three mathematical models for forecasting the Mexico's gross domestic product. Results and discussion are presented in Section 4. Conclusions (Section 5) complete the paper.

2. **Methodology.**

The models applied in this paper to the Mexico's gross domestic product are: linear regression model, exponential regression model and parabolic regression model.

2.1. **Linear regression model.**

The linear regression model that describes the relationship between two variables x and y can be expressed by the following equation [20]:

$$Y = a + bX + \varepsilon \tag{1}$$

where Y is the dependent (response) variable, a and b are parameters of the model called regression coefficients, X is the independent (explanatory) variable, and ε is the random error (residues) variable and is a normally distributed random variable with mean equal to zero.

By the above the algebraic formula with linear regression for making predictions is [20]:

$$Y = a + bX \tag{2}$$

The residual data of the linear regression model is the difference between the observed data of the dependent variable y and the fitted value or estimated $\hat{y}$.

The parameters a and b are obtained from the following equations [20]:

$$a = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2} \tag{3}$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \tag{4}$$

2.2. **Exponential regression model.** The exponential regression model that describes the relationship between two variables x and y can be expressed by the following equation [21]:

$$Y = ab^X \varepsilon \tag{5}$$

The algebraic formula with exponential regression for making predictions is [21]:

$$Y = ab^X \tag{6}$$

If we take logarithms on both sides of Equation (6), it is presented as follows:

$$\log Y = \log a + \log b^X \to \log Y = \log a + X \log b \tag{7}$$

The parameters $\log a$ and $\log b$ are obtained from the following equations [21]:

$$\log a = \frac{\sum X^2 \sum \log Y - \sum X \sum X \log Y}{n \sum X^2 - (\sum X)^2} \tag{8}$$

$$\log b = \frac{n \sum X \log Y - \sum \log Y \sum X}{n \sum X^2 - (\sum X)^2} \tag{9}$$

2.3. **Parabolic regression model.** The parabolic regression model that describes the relationship between two variables x and y can be expressed by the following equation [22]:

$$Y = a + bX + cX^2 + \varepsilon \tag{10}$$

The algebraic formula with exponential regression for making predictions is [22]:

$$Y = a + bX + cX^2 \tag{11}$$

where Y is the dependent variable, a is the constant (free term of equation), b and c are the coefficients of independent variable, and X is the independent variable.

The parameters a, b and c are obtained from the following equations [22]:

$$a = \frac{\left[(\sum x^3)^2 - \sum x^2 \sum x^4\right] \sum y + \left[\sum x \sum x^4 - \sum x^2 \sum x^3\right] \sum xy + \left[(\sum x^2)^2 - \sum x \sum x^3\right] \sum x^2 y}{(\sum x)^2 \sum x^4 - 2 \sum x \sum x^2 \sum x^3 + (\sum x^2)^3 - n \sum x^2 \sum x^4 + n(\sum x^3)^2} \tag{12}$$

$$b = \frac{\left[\sum x \sum x^4 - \sum x^2 \sum x^3\right] \sum y + \left[(\sum x^2)^2 - n \sum x^4\right] \sum xy + \left[n \sum x^3 - \sum x \sum x^2\right] \sum x^2 y}{(\sum x)^2 \sum x^4 - 2 \sum x \sum x^2 \sum x^3 + (\sum x^2)^3 - n \sum x^2 \sum x^4 + n(\sum x^3)^2} \tag{13}$$

$$c = \frac{\left[(\sum x^2)^2 - \sum x \sum x^3\right] \sum y + \left[n \sum x^3 - \sum x \sum x^2\right] \sum xy + \left[(\sum x)^2 - n \sum x^2\right] \sum x^2 y}{(\sum x)^2 \sum x^4 - 2 \sum x \sum x^2 \sum x^3 + (\sum x^2)^3 - n \sum x^2 \sum x^4 + n(\sum x^3)^2} \tag{14}$$

3. **Application of the Models.** Table 1 shows the historical data of the Mexico's gross domestic product of the last 82 years from 1935 until 2016 [8].

TABLE 1. Historical data of the Mexico's gross domestic product

GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X
440.46	1935	1066.74	1952	3019.64	1969	6762.27	1985	10226.68	2001
476.59	1936	1070.18	1953	3216.00	1970	6554.07	1986	10240.17	2002
492.08	1937	1176.85	1954	3337.00	1971	6666.96	1987	10385.86	2003
498.96	1938	1276.64	1955	3611.59	1972	6752.51	1988	10832.00	2004
526.49	1939	1364.39	1956	3895.50	1973	7029.74	1989	11160.49	2005
533.37	1940	1467.63	1957	4120.54	1974	7393.58	1990	11718.67	2006
584.99	1941	1545.06	1958	4357.24	1975	7705.20	1991	12087.60	2007
619.40	1942	1591.50	1959	4549.72	1976	7978.05	1992	12256.86	2008
641.76	1943	1720.55	1960	4703.99	1977	8132.92	1993	11680.75	2009
693.38	1944	1794.86	1961	5125.32	1978	8517.39	1994	12277.66	2010
715.75	1945	1874.89	1962	5622.38	1979	8026.90	1995	12774.24	2011
762.20	1946	2016.30	1963	6141.51	1980	8498.46	1996	13287.53	2012
789.73	1947	2238.32	1964	6665.11	1981	9090.20	1997	13468.25	2013
820.70	1948	2375.95	1965	6630.40	1982	9517.60	1998	13773.36	2014
867.16	1949	2520.79	1966	6399.24	1983	9771.44	1999	14138.96	2015
951.46	1950	2668.38	1967	6617.50	1984	10288.98	2000	14462.16	2016
1025.45	1951	2919.83	1968						

where: GDP in Thousands of Millions of Mexican Pesos

3.1. **Linear regression model.** From Table 1 the following values are obtained:

$$\sum y = 443598.98; \quad \sum x = 161991$$

$$\sum xy = 884506593.8; \quad \sum x^2 = 320059161$$

Substituting these values into Equations (3) and (4), the values of a and b are found:

$$a = -346203.4474$$

$$b = 177.9869353$$

Now substituting the values a and b into Equation (2), the linear regression model is obtained:

$$Y = -346203.4474 + 177.9869353X$$

Table 2 presents data obtained by the equation of the linear regression model.

Figure 2 shows the graphics design of the historical data and the equation of the linear regression model.

3.2. **Exponential regression model.** The $\log Y$ are found from Table 1 and are shown in Table 3.

From Table 3 the following values are obtained:

$$\sum \log Y = 289.3633; \quad \sum x = 161991$$

$$\sum X \log Y = 572541.5775; \quad \sum x^2 = 320059161$$

Substituting these values into Equations (8) and (9), the values of a and b are found:

$$\log a = -35.36059977$$

$$a = 4.359134106 \left(10^{-36}\right)$$

$$\log b = 0.01968586215$$

$$b = 1.046371403$$

Now substituting the values a and b into Equation (6), the exponential regression model is obtained:

$$Y = 4.359134106 \left(10^{-36}\right) (1.046371403)^X$$

TABLE 2. Data by equation of the linear regression model

GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X
−1798.73	1935	1227.05	1952	4252.83	1969	7100.62	1985	9948.41	2001
−1620.74	1936	1405.04	1953	4430.82	1970	7278.61	1986	10126.40	2002
−1442.75	1937	1583.02	1954	4608.80	1971	7456.59	1987	10304.38	2003
−1264.77	1938	1761.01	1955	4786.79	1972	7634.58	1988	10482.37	2004
−1086.78	1939	1939.00	1956	4964.78	1973	7812.57	1989	10660.36	2005
−908.79	1940	2116.98	1957	5142.76	1974	7990.55	1990	10838.34	2006
−730.81	1941	2294.97	1958	5320.75	1975	8168.54	1991	11016.33	2007
−552.82	1942	2472.96	1959	5498.74	1976	8346.53	1992	11194.32	2008
−374.83	1943	2650.95	1960	5676.72	1977	8524.51	1993	11372.31	2009
−196.85	1944	2828.93	1961	5854.71	1978	8702.50	1994	11550.29	2010
−18.86	1945	3006.92	1962	6032.70	1979	8880.49	1995	11728.28	2011
159.13	1946	3184.91	1963	6210.68	1980	9058.48	1996	11906.27	2012
337.12	1947	3362.89	1964	6388.67	1981	9236.46	1997	12084.25	2013
515.10	1948	3540.88	1965	6566.66	1982	9414.45	1998	12262.24	2014
693.09	1949	3718.87	1966	6744.65	1983	9592.44	1999	12440.23	2015
871.08	1950	3896.85	1967	6922.63	1984	9770.42	2000	12618.21	2016
1049.06	1951	4074.84	1968						

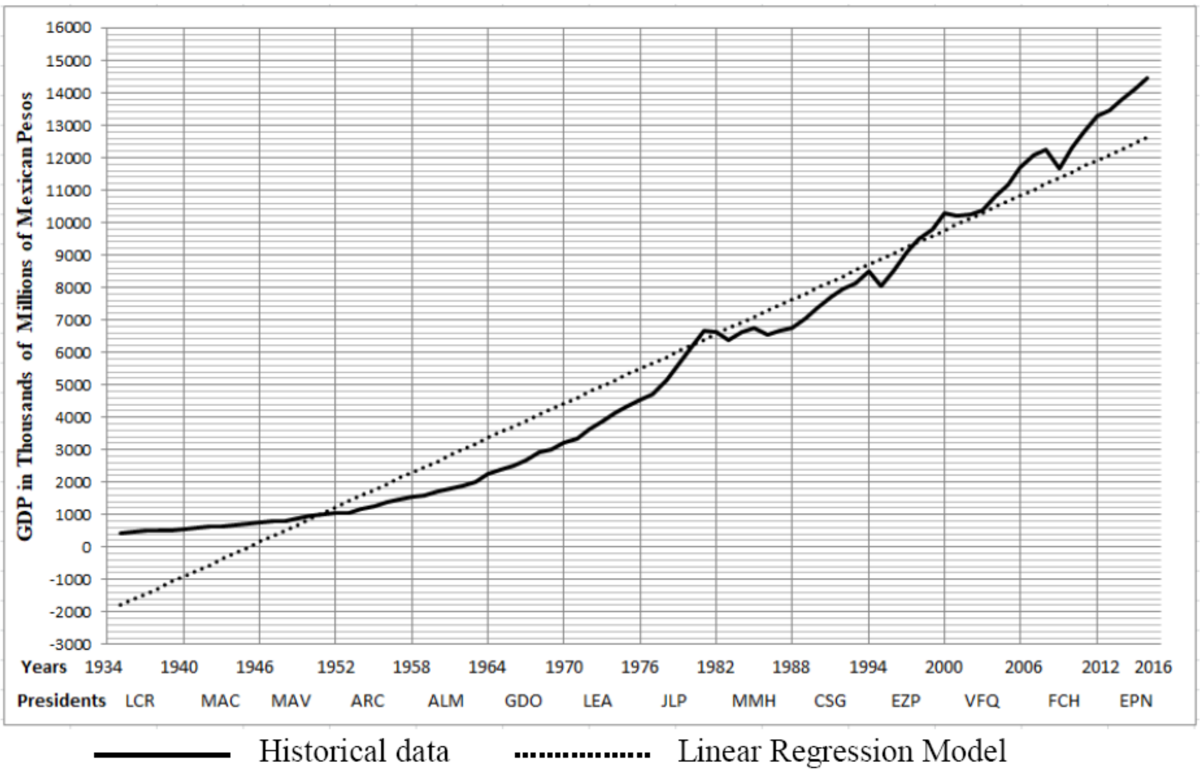

FIGURE 2. Comparison between historical data and linear regression model

TABLE 3. Values of the $\log Y$

$\log Y$	X	$\log Y$	X	$\log Y$	X	$\log Y$	X	$\log Y$	X
2.6439	1935	3.0281	1952	3.4800	1969	3.8310	1985	4.0097	2001
2.6781	1936	3.0295	1953	3.5073	1970	3.8165	1986	4.0103	2002
2.6920	1937	3.0707	1954	3.5234	1971	3.8239	1987	4.0164	2003
2.6981	1938	3.1061	1955	3.5577	1972	3.8295	1988	4.0347	2004
2.7214	1939	3.1349	1956	3.5906	1973	3.8469	1989	4.0477	2005
2.7270	1940	3.1666	1957	3.6150	1974	3.8689	1990	4.0689	2006
2.7671	1941	3.1889	1958	3.6392	1975	3.8868	1991	4.0823	2007
2.7920	1942	3.2018	1959	3.6580	1976	3.9019	1992	4.0884	2008
2.8074	1943	3.2357	1960	3.6725	1977	3.9102	1993	4.0675	2009
2.8410	1944	3.2540	1961	3.7097	1978	3.9303	1994	4.0891	2010
2.8548	1945	3.2730	1962	3.7499	1979	3.9045	1995	4.1064	2011
2.8821	1946	3.3046	1963	3.7883	1980	3.9293	1996	4.1234	2012
2.8975	1947	3.3499	1964	3.8238	1981	3.9586	1997	4.1293	2013
2.9142	1948	3.3758	1965	3.8215	1982	3.9785	1998	4.1390	2014
2.9381	1949	3.4015	1966	3.8061	1983	3.9900	1999	4.1504	2015
2.9784	1950	3.4262	1967	3.8207	1984	4.0124	2000	4.1602	2016
3.0109	1951	3.4654	1968						

TABLE 4. Data by equation of the exponential regression model

GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X
538.94	1935	1164.67	1952	2516.88	1969	5198.00	1985	10735.20	2001
563.94	1936	1218.68	1953	2633.59	1970	5439.04	1986	11233.01	2002
590.09	1937	1275.19	1954	2755.71	1971	5691.26	1987	11753.90	2003
617.45	1938	1334.32	1955	2883.50	1972	5955.17	1988	12298.94	2004
646.08	1939	1396.20	1956	3017.21	1973	6231.32	1989	12869.26	2005
676.04	1940	1460.94	1957	3157.13	1974	6520.27	1990	13466.03	2006
707.39	1941	1528.69	1958	3303.53	1975	6822.62	1991	14090.46	2007
740.19	1942	1599.57	1959	3456.71	1976	7139.00	1992	14743.86	2008
774.52	1943	1673.75	1960	3617.01	1977	7470.05	1993	15427.55	2009
810.43	1944	1751.36	1961	3784.73	1978	7816.44	1994	16142.95	2010
848.01	1945	1832.57	1962	3960.24	1979	8178.90	1995	16891.52	2011
887.34	1946	1917.55	1963	4143.88	1980	8558.17	1996	17674.80	2012
928.48	1947	2006.47	1964	4336.04	1981	8955.02	1997	18494.41	2013
971.54	1948	2099.52	1965	4537.10	1982	9370.28	1998	19352.02	2014
1016.59	1949	2196.87	1966	4747.50	1983	9804.79	1999	20249.40	2015
1063.73	1950	2298.75	1967	4967.64	1984	10259.45	2000	21188.39	2016
1113.06	1951	2405.34	1968						

Table 4 presents data obtained by the equation of the exponential regression model.

Figure 3 shows the graphics design of the historical data and the equation of the exponential regression model.

3.3. **Parabolic regression model.** From Table 1 the following values are obtained:

$$\sum y = 443598.98; \quad \sum x = 161991; \quad \sum xy = 884506593.8; \quad \sum x^2 = 320059161;$$

$$\sum x^3 = 632458383471; \quad \sum x^4 = 1249959445086825; \quad \sum x^2 y = 1763781640176$$

Substituting these values into Equations (12), (13) and (14), the values of a, b and c are found:

$$a = 6678524.992$$
$$b = -6934.882759$$
$$c = 1.800270740$$

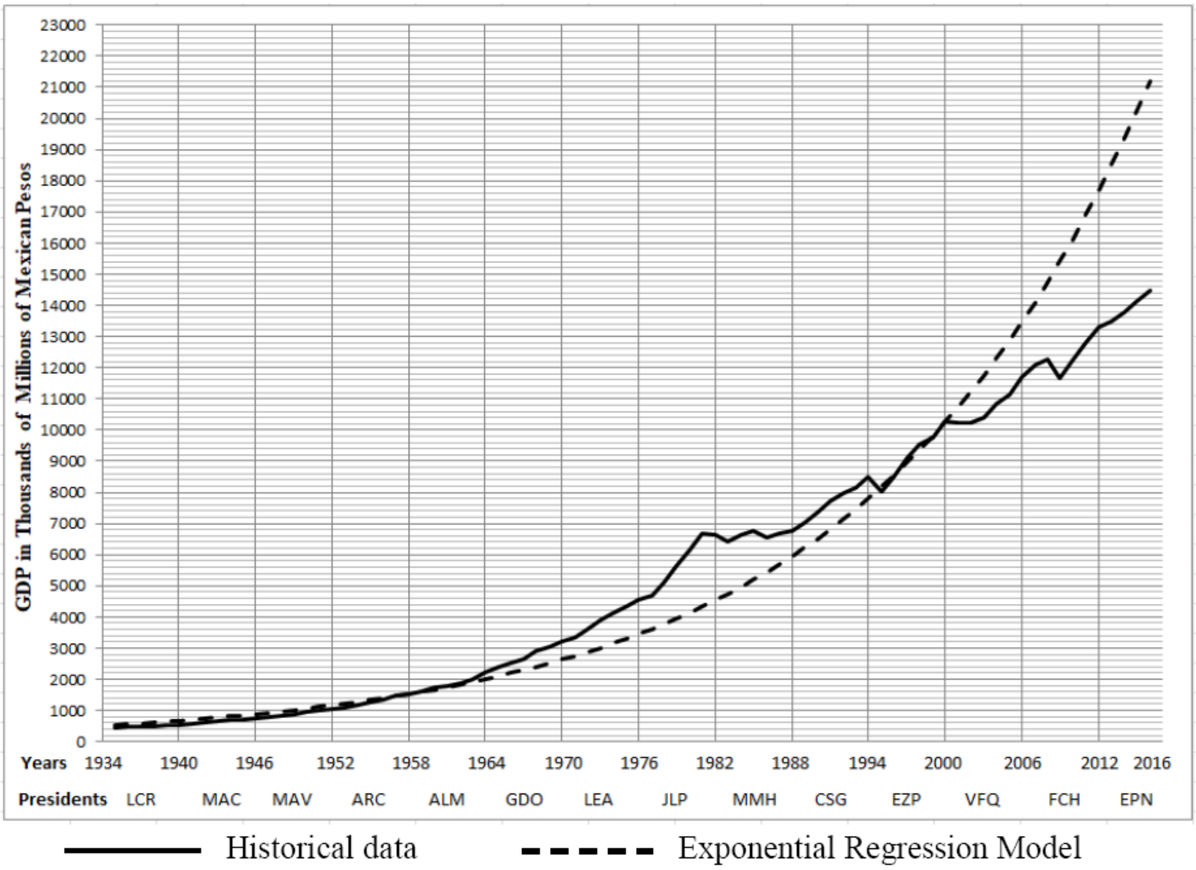

FIGURE 3. Comparison between historical data and exponential regression model

Now substituting the values a, b and c into Equation (11), the parabolic regression model is obtained:

$$Y = 6678524.992 - 6934.882759X + 1.800270740X^2$$

Table 5 presents data obtained by the equation of the parabolic regression model.

Figure 4 shows the graphics design of the historical data and the equation of the parabolic regression model.

4. **Results and Discussion.** Figure 5 shows the graphics design of the historical data, the equations of the linear regression model, exponential regression model and parabolic regression model.

Comparing each model with respect to the real historical values is presented:

Linear Regression Model:

From 1935 to 1950, the values of the linear regression model are smaller with respect to historical data, even there are negative values that are found from 1935 to 1945. From 1951 to 1997, the values of the linear regression model are larger than the historical data

TABLE 5. Data by equation of the parabolic regression model

GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X	GDP Y	Year X
145.56	1935	1212.65	1952	3320.29	1969	6254.49	1985	10110.43	2001
179.53	1936	1307.82	1953	3476.67	1970	6468.48	1986	10382.04	2002
217.10	1937	1406.60	1954	3636.66	1971	6686.08	1987	10657.24	2003
258.26	1938	1508.97	1955	3800.24	1972	6907.27	1988	10936.04	2004
303.03	1939	1614.95	1956	3967.43	1973	7132.06	1989	11218.44	2005
351.40	1940	1724.53	1957	4138.21	1974	7360.46	1990	11504.44	2006
403.36	1941	1837.70	1958	4312.60	1975	7592.45	1991	11794.05	2007
458.93	1942	1954.48	1959	4490.59	1976	7828.05	1992	12087.25	2008
518.10	1943	2074.86	1960	4672.17	1977	8067.25	1993	12384.06	2009
580.87	1944	2198.84	1961	4857.36	1978	8310.04	1994	12684.46	2010
647.24	1945	2326.42	1962	5046.15	1979	8556.44	1995	12988.47	2011
717.22	1946	2457.60	1963	5238.54	1980	8806.44	1996	13296.08	2012
790.78	1947	2592.38	1964	5434.53	1981	9060.04	1997	13607.28	2013
867.96	1948	2730.76	1965	5634.12	1982	9317.24	1998	13922.09	2014
948.73	1949	2872.74	1966	5837.31	1983	9578.03	1999	14240.50	2015
1033.10	1950	3018.32	1967	6044.10	1984	9842.43	2000	14562.51	2016
1121.07	1951	3167.50	1968						

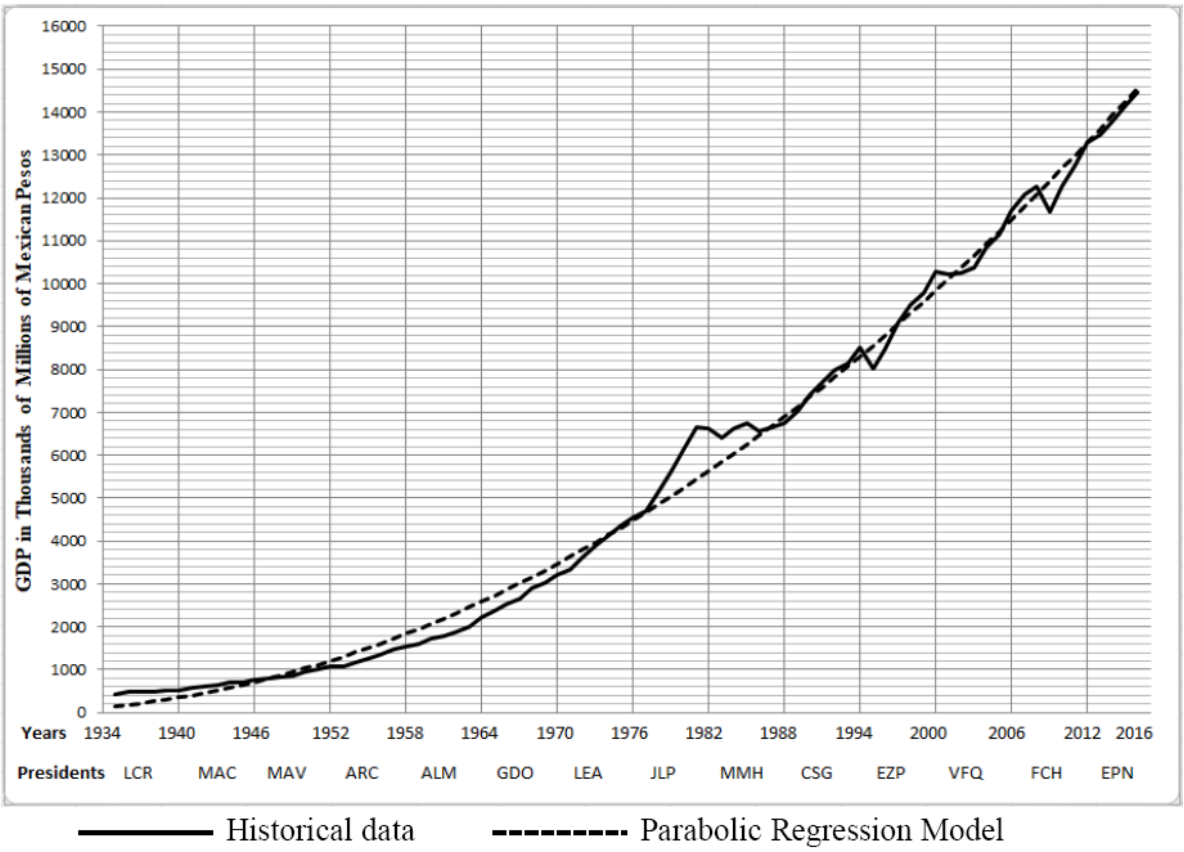

FIGURE 4. Comparison between historical data and parabolic regression model

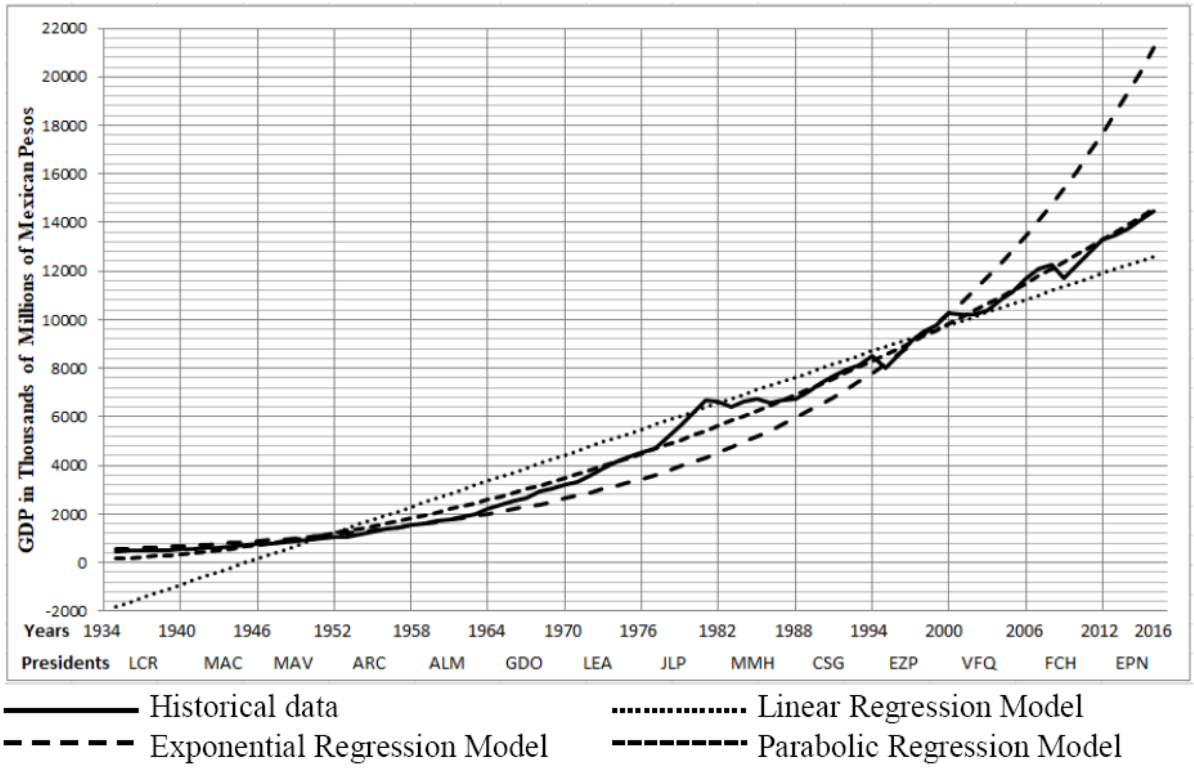

FIGURE 5. Comparison between historical data and the three models

with the exception of 1981 and 1982; these are presented to the contrary. From 1998 to 2016, the values of the linear regression model are smaller with respect to historical data.

Exponential Regression Model:

From 1935 to 1956, the values of the exponential regression model are larger than the historical data. From 1957 to 1994, the values of the historical data are larger than the exponential regression model. From 1995 to 2016, the values of the exponential regression model are larger than the historical data with the exception of 1998 and 2000; these are presented to the contrary.

Parabolic Regression Model:

From 1935 to 1946, the values of the parabolic regression model are smaller with respect to historical data. From 1947 to 1974, the values of the parabolic regression model are larger with respect to historical data. From 1975 to 2001, the values of the parabolic regression model are smaller with respect to historical data with the exception of 1987 to 1989, 1995 and 1996; these are presented to the contrary. From 2002 to 2016, the values of the parabolic regression model are smaller with respect to historical data with the exception of 2006 to 2008; these are presented to the contrary.

Figure 5 clearly shows that the parabolic regression model fits the historical data of the Mexico's gross domestic product.

Use the parabolic regression model to forecast the Mexico's gross domestic product for the years of 2017 to 2020, if there would not be an economic crisis:

$$Y_{2017} = 6678524.992 - 6934.882759(2017) + 1.800270740(2017)^2 = 14888.11564$$

$$Y_{2018} = 6678524.992 - 6934.882759(2018) + 1.800270740(2018)^2 = 15217.32533$$

$$Y_{2019} = 6678524.992 - 6934.882759(2019) + 1.800270740(2019)^2 = 15550.13554$$

$$Y_{2020} = 6678524.992 - 6934.882759(2020) + 1.800270740(2020)^2 = 15886.54630$$

5. **Conclusions.** Three models were created through the data used and estimated from 1935 to 2016.

This paper clearly demonstrates the differences among the linear regression model, exponential regression model, and parabolic regression model. The major contribution of this research is the proposal of the use of the parabolic regression model for forecasting the Mexico's gross domestic product, if there would not be an economic crisis, because the parabolic regression model is the most appropriate since it is adjusted to real conditions of the Mexico's gross domestic product.

Using the proposed model to forecast the Mexico's gross domestic product for the years of 2017, 2018, 2019 and 2020 must be 14888, 15217, 15550 and 15887. Thousands of Millions of Mexican Pesos respectively, if there would not be an economic crisis.

Results of the study will be helpful for the policy makers to formulate effective policies for attracting foreign direct investment. Furthermore, the findings of the study will also help the managerial business executives for implementing the new project ideas or taking decisions concerned with the expansion of the existing business.

The limitation of this research is that it is only applied to the Mexico's gross domestic product, because other countries can present another type of configuration.

This paper considers the dependent variable as the gross domestic product and the independent variable is the year.

Suggestions for future research are an analysis that includes other independent variables such as inflation, oil, and foreign investment.

REFERENCES

[1] D. Coyle, *GDP: A Brief but Affectionate History*, Revised and Expanded Edition, 2015.
[2] C. Dritsaki, Forecasting real GDP rate through econometric models: An empirical study from Greece, *Journal of International Business and Economics*, vol.3, no.1, pp.13-19, 2015.
[3] L. Fioramonti, *The World after GDP: Politics, Business and Society in the Post Growth Era*, 1st Edition, Kindle Edition, 2017.
[4] L. Fioramonti, *Wellbeing Economy: Success in a World without Growth*, Macmillan, 2017.
[5] T. Callen, *Gross Domestic Product: An Economy's All*, International Monetary Fund, 2016.
[6] *The European System of Accounts ESA 1995*, Eurostat, 1996.
[7] P. S. Pritzker, K. Arnold and B. C. Moyer, *Measuring the Economy – A Primer on GDP and the National Income and Product Accounts*, BEA Bureau of Economic Analysis, U.S. Department of Commerce, 2015.
[8] M. Aguirre Botello, *Termómetro de la Economía Mexicana, Indicadores Históricos 1935-2017, 80 Años de la Economía Mexicana Post Revolucionaria Resumida en 14 Sexenios Presidenciales*, http://mexicomaxico.org/Voto/PIBMex.htm.
[9] T. Kurita, Econometric modeling of the interactions between Japan's housing investment and GDP, *International Journal of Applied Economics*, vol.7, no.1, pp.56-71, 2010.
[10] N. A. Desai and M. Bhatia, A multiple variable regression model for gross domestic product growth rate prediction in India using key macroeconomic indicators, *Journal of Economics and Finance*, vol.7, no.2, pp.47-51, 2016.
[11] A. Banks, S. Hamroush, C. Taylor and M. Hardie, An international perspective on the UK – Gross domestic product, *Office for National Statistics*, vol.24, pp.1-25, 2014.
[12] N. J. Chioma, Causal relationship between gross domestic product and personal consumption expenditure of Nigeria, *African Journal of Mathematics and Computer Science Research*, vol.2, no.8, pp.179-183, 2009.
[13] Q. Abbas, S. Akbar, A. S. Nasir, H. A. Ullah and M. A. Naseem, Impact of foreign direct investment on gross domestic product, *Global Journal of Management and Business Research*, vol.11, no.8, pp.36-40, 2011.
[14] E. Agalega and S. Antwi, The impact of macroeconomic variables on gross domestic product: Empirical evidence from Ghana, *International Business Research*, vol.6, no.5, pp.108-116, 2013.
[15] A. A. S. G. Syed and M. Shaikh, Effects of macroeconomic variables on gross domestic product (GDP) in Pakistan, *Procedia Economics and Finance*, vol.5, pp.703-711, 2013.

[16] L. Paliu-Popa, I. E. Chirtoc and G. Buşan, Analysis of the interdependence between gross domestic product (GDP), steel production and iron production (10 States of European Union), *Metalurgija*, vol.55, no.2, pp.260-262, 2016.
[17] P. N. Tripathi, Gross domestic product and its components in India. Trends and issues, *Academic Journal of Economic Studies*, vol.1, no.3, pp.88-104, 2015.
[18] V. Olej and J. Kfupka, Prediction of gross domestic product development by Takagi-Sugeno fuzzy inference systems, *IEEE Proc. of International Conference on Intelligent Systems Design and Applications*, Wroclaw, pp.186-191, 2005.
[19] H. Zhang, Modeling and forecasting regional GDP in Sweden using autoregressive models, *Working Paper*, Högskolan Dalarna University, Sweden, 2013.
[20] X. Yan and X. G. Su, *Linear Regression Analysis Theory and Computing*, World Scientific Publishing Co. Pte. Ltd., 2009.
[21] R. M. Bethea, B. S. Duran and T. L. Boullion, *Statistical Methods for Engineers and Scientists*, Marcel Dekker, 1985.
[22] G. A. F. Seber and C. J. Wild, *Nonlinear Regression*, John Wiley and Sons, 1989.