

ROBUST EXPONENTIAL STABILITY OF UNCERTAIN FUZZY STOCHASTIC NEUTRAL NEURAL NETWORKS WITH MIXED TIME-VARYING DELAYS

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ABSTRACT. *The robust exponential stability problem for a class of uncertain fuzzy stochastic neutral neural networks systems with mixed delays is concerned about. Based on the Lyapunov functional and the stochastic stability theory, the sufficient conditions are developed in terms of linear matrix inequalities (LMIs). Examples and simulations are provided to illustrate the effectiveness and the less conservatism of the proposed method.*

Keywords: Robust exponential stability, Stochastic neutral neural networks, Linear matrix inequality (LMI), Mixed time-varying delays

1. **Introduction.** During the past several decades, neural networks have gained great attention because of their massively potential applications in pattern classification, reconstruction of moving image and combinatorial optimization. In such applications, to ensure the equilibrium point of the designed networks stable is a very important issue. In addition, time delay, uncertainties and stochastic disturbance are considered as three main factors to affect the stability performance of various types of neural networks, so the stability analysis of stochastic uncertain neural networks with various time delays has been widely investigated by many researchers and many novel results have been reported, see, e.g., [1-10] and the references therein.

On the other hand, many practical systems such as chemical engineering system, heat exchanges and lossless transmission system are described by neutral functional differential equation that involves the delays in both its state and the derivative of the state, this kind of system is called neutral-type system or neutral system, which is another kind of time-delay system, it contains the information of past state, and time delay occurs in the derivatives of system state. Because of the existence of neutral term, the stability analysis of neutral delay systems seems to be a more attractive and complicated issue, and during the past decades, considerable meaningful results about stability analysis, H_∞ and L_2 - L_∞ filters design of stochastic systems with neutral term have been obtained and reported, see [11-20] and the references therein. Especially in [19], the stability result for neutral stochastic neural networks with time-varying delays is obtained and in [20], L_2 - L_∞ filter problem is studied, and the introduction of Lemma 2.2 has obtained better results than the use of Finsler Lemma, free-weighting matrix and model-transformation.

In recent many decades, fuzzy systems have attracted rapidly growing interest. Among them, Takagi-Sugeno (T-S) model, which is described by a set of IF-THEN rule, has been considered as a very powerful and convenient tool to approximate complex nonlinear systems, and many significant research achievements about stability analysis, state estimation, filter design, dissipativity analysis of the T-S model of fuzzy systems have been obtained, see, for example, [21-26] and the reference therein. To the best of authors' knowledge, there are not many results about the robust exponential stability on fuzzy stochastic neutral neural networks system with mixed delay published in recent years, which motivates our idea.

In this study, the robust exponential stability results are obtained for a class of uncertain fuzzy stochastic neutral neural networks by constructing proper Lyapunov functional, stochastic stability theory and linear matrix inequality approach. The mixed delays comprise discrete and distributed time-delays, and the parameter uncertainties are time-varying and norm-bounded. A new method is introduced to deal with neutral terms in the studied systems, and more effective results are obtained by comparing with the existing results.

By comparing with the results in [19,20], the contributions of this paper exist in the following aspects.

- A new approach is adopted in uncertain fuzzy stochastic neutral neural networks system to deal with neutral terms, and the new results have been obtained.
- The neuron activation function is assumed to satisfy sector-bounded condition, which is more general and less restrictive than Lipschitz condition, so less conservatism results can be expected.

This paper is organized as follows. In Section 2, system description is formulated and definition, lemmas are introduced. The main results are given and proved in Section 3. In Section 4, numerical example and simulation are given to verify the proposed results. Conclusion and future research direction have been made in Section 5.

Notation: Throughout this paper, if not explicit, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq) 0$ means that the symmetric matrix M is positive-definite (positive-semidefinite, negative, negative-semidefinite). $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalues of the corresponding matrix. The superscript “ T ” stands for the transpose of a matrix; the shorthand $diag\{\dots\}$ denotes the block diagonal matrix; $\|\cdot\|$ represents the Euclidean norm for vector or the spectral norm of matrices. I refers to an identity matrix of appropriate dimensions. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation, and $*$ means the symmetric terms. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Problem Statement and Preliminaries. Consider the Takagi-Sugeno uncertain fuzzy stochastic neutral neural networks system with mixed time-delays described by the following IF-THEN rule:

Rule i : IF $\theta_1(t)$ is η_1^i , $\theta_2(t)$ is $\eta_2^i, \dots, \theta_p(t)$ is η_p^i , THEN

$$\begin{aligned} & d[x(t) - G_i(x(t - \tau(t)))] \\ &= \left[-A_i(t)x(t) + A_{1i}(t)f(x(t)) + A_{2i}(t)f(x(t - \tau(t))) + A_{3i}(t) \int_{t-\delta(t)}^t f(x(s))ds \right] dt \\ &+ \left[D_i(t)x(t) + D_{1i}(t)x(t - \tau(t)) + D_{2i}(t) \int_{t-\delta(t)}^t f(x(s))ds \right] d\omega(t), \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, r$, r is the number of IF-THEN rules, $x(t) \in \mathbb{R}^n$ is the state vector, the premise variables vector $\theta(t) = [\theta_1(t), \theta_2(t), \dots, \theta_p(t)]^T$ is the function of state variables, η_j^i are the fuzzy set, $f(x(t)) = [f_1(x(t)), f_2(x(t)), \dots, f_n(x(t))]^T \in \mathbb{R}^n$ is the neuron activation

function with $f(0) = 0$, and n denotes the number of neurons in a neural network; $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_m(t)]^T \in \mathbb{R}^m$ is an m -dimension Brownian motion defined on a complete probability space (Ω, \mathcal{F}, P) , which satisfies

$$\mathbb{E}\{d\omega(t)\} = 0, \quad \mathbb{E}\{d\omega^2(t)\} = dt.$$

$\tau(t)$ is the discrete time delay and $\delta(t)$ is the distributed delay, which are assumed to satisfy

$$0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) = \mu, \quad 0 \leq \delta(t) \leq \delta,$$

where τ , μ and δ are some positive scalar constants, we denote $\rho(G_i)$ is the spectral radius of G_i , which satisfies $\rho(G_i) < 1$, $A_i(t) = A_i + \Delta A_i(t)$, $A_{1i}(t) = A_{1i} + \Delta A_{1i}(t)$, $A_{2i}(t) = A_{2i} + \Delta A_{2i}(t)$, $A_{3i}(t) = A_{2i} + \Delta A_{3i}(t)$, $D_i(t) = D_i + \Delta D_i(t)$, $D_{1i}(t) = D_{1i} + \Delta D_{1i}(t)$, $D_{2i}(t) = D_{2i} + \Delta D_{2i}(t)$, A_i , A_{1i} , A_{2i} , A_{3i} , D_i , D_{1i} and D_{2i} are known real constant matrices with appropriate dimensions, and $A_i = \text{diag}\{a_1, a_2, \dots, a_n\}$ is a diagonal matrix with positive entries $a_i > 0$, ($i = 1, 2, \dots, n$), $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, $\Delta A_{3i}(t)$, $\Delta D_i(t)$, $\Delta D_{1i}(t)$, $\Delta D_{2i}(t)$ are unknown matrices representing norm-bounded parameter uncertainties, which are assumed to satisfy

$$\begin{aligned} & \left[\begin{array}{ccccccc} \Delta A_i(t) & \Delta A_{1i}(t) & \Delta A_{2i}(t) & \Delta A_{3i}(t) & \Delta D_i(t) & \Delta D_{1i}(t) & \Delta D_{2i}(t) \end{array} \right] \\ & = M_i H_i(t) \left[\begin{array}{ccccccc} N_{1i} & N_{2i} & N_{3i} & N_{4i} & N_{5i} & N_{6i} & N_{7i} \end{array} \right], \end{aligned} \tag{2}$$

where M_i , N_{1i} , N_{2i} , N_{3i} , N_{4i} , N_{5i} , N_{6i} and N_{7i} are known real constant matrices and $H_i(t)$ is an unknown time-varying matrix function satisfying

$$H_i^T(t) H_i(t) \leq I, \quad \forall i \in S. \tag{3}$$

The parameter uncertainties $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, $\Delta A_{3i}(t)$, $\Delta D_i(t)$, $\Delta D_{1i}(t)$, $\Delta D_{2i}(t)$ and $\Delta D_{3i}(t)$ are said to be admissible if both (2) and (3) hold.

By using a standard fuzzy inference method, the defuzzified output of system (1) can be expressed by the following global model:

$$\begin{aligned} & d[x(t) - G_i(x(t - \tau(t)))] \\ & = \sum_{i=1}^r h_i(\theta(t)) \left\{ \left[-A_i(t)x(t) + A_{1i}(t)f(x(t)) + A_{2i}(t)f(x(t - \tau(t))) \right. \right. \\ & \quad \left. \left. + A_{3i}(t) \int_{t-\delta(t)}^t f(x(s)) ds \right] dt + \left[D_i(t)x(t) + D_{1i}(t)x(t - \tau(t)) \right. \right. \\ & \quad \left. \left. + D_{2i}(t) \int_{t-\delta(t)}^t f(x(s)) ds \right] d\omega(t) \right\}, \end{aligned} \tag{4}$$

where

$$h_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{j=1}^r h_j(\theta(t))}, \quad \mu_i(\theta(t)) = \prod_{j=1}^p \eta_j^i(\theta_j(t)),$$

and $\eta_j^i(\theta_j(t))$ is the grade of membership of $\eta_j(t)$ in η_j^i , then by the theory of fuzzy sets, we have

$$\mu_i(\theta(t)) \geq 0, \quad (i = 1, 2, \dots, r), \quad \sum_{i=1}^r h_i(\theta(t)) > 0.$$

Therefore, it is implied that

$$h_i(\theta(t)) \geq 0, \quad (i = 1, 2, \dots, r), \quad \sum_{i=1}^r h_i(\theta(t)) = 1. \tag{5}$$

The following lemmas, assumption and definition are introduced for the convenient proof of our main results.

Definition 2.1. *The neutral stochastic delay system (1) is said to be mean-square exponentially stable if there exist a pair of scalars α, β such that*

$$\mathbb{E} \{ \|x(t)\|^2 \} \leq \alpha e^{-\beta t} \sup_{\theta \in [-\tau, 0]} \mathbb{E} \{ \|\phi(0)\|^2 \}.$$

Assumption 2.1. *For $i \in \{1, 2, \dots, n\}$, $\forall x, y \in \mathbb{R}, x \neq y$, the neuron activation function $f(\cdot)$ is continuous, bounded and satisfies:*

$$[f(x) - f(y) - \Lambda_1(x - y)]^T [f(x) - f(y) - \Lambda_2(x - y)] < 0, \tag{6}$$

where Λ_1 and Λ_2 are some constant known matrices.

Remark 2.1. *In this paper, the above assumption is made on neuron activation function, which is called sector-bounded neuron activation function. When $\Lambda_1 = \Lambda_2 = -\Lambda$, the condition (6) becomes*

$$[f(x) - f(y)]^T [f(x) - f(y)] \leq (x - y)^T \Lambda^T \Lambda (x - y).$$

So it is less restrictive than the descriptions on both the sigmoid activation functions and the Lipschitz-type activation functions.

Lemma 2.1. [20] *For $\sigma > 0$ and two scalars: a, b with $0 < a < b < \sigma$, let n -dimensional vector functions $x(t), \hat{f}(t)$, and a matrix $D \in \mathbb{R}^{n \times n}$ satisfy the neutral differential equations:*

$$\frac{d[x(t) - Dx(t - \sigma)]}{dt} = \hat{f}(t), \quad t \geq 0, \tag{7}$$

where the initial condition $x(\theta) = \psi(\theta)$ ($\theta \in [-\sigma, 0]$). For any constant matrix $W > 0, W = W^T \in \mathbb{R}^{n \times n}$, if the following integrals are well defined, then

$$-(b - a) \int_{t-b}^{t-a} \hat{f}^T(s) W \hat{f}(s) \leq \eta(t) \Omega \eta^T(t), \tag{8}$$

where $\eta(t) = [x^T(t - a) \ x^T(t - b) \ x^T(t - a - \sigma) \ x^T(t - b - \sigma)]$, and

$$\Omega = \begin{bmatrix} -W & W & WD & -WD \\ * & -W & -WD & WD \\ * & * & -D^T W D & D^T W D \\ * & * & * & -D^T W D \end{bmatrix} \tag{9}$$

Lemma 2.2. [7] *For any positive matrix $M = M^T \in \mathbb{R}^{n \times n}$, scalar $\varrho > 0$, vector function $\omega : [0, \varrho] \rightarrow \mathbb{R}^n$ such that the integrations are well defined, the following inequality holds:*

$$\left[\int_0^\varrho \omega(s) ds \right]^T M \left[\int_0^\varrho \omega(s) ds \right] \leq \varrho \int_0^\varrho \omega^T(s) M \omega(s) ds.$$

Lemma 2.3. [7] *For given proper matrices D, E and F with $F^T F \leq I$ and scalar $\epsilon > 0$, the following inequality holds*

$$DFE + E^T F^T D^T \leq \epsilon DD^T Q + \epsilon^{-1} EE^T.$$

3. Main Results. In this section, firstly, when the parameter uncertainties are not taken into account, the system (4) becomes the following one:

$$\begin{aligned}
 & d[x(t) - G_i(x(t - \tau(t)))] \\
 = & \sum_{i=1}^r h_i(\theta(t)) \left\{ \left[-A_i x(t) + A_{1i} f(x(t)) + A_{2i} f(x(t - \tau(t))) + A_{3i} \int_{t-\delta(t)}^t f(x(s)) ds \right] dt \right. \\
 & \left. + \left[D_i x(t) + D_{1i} x(t - \tau(t)) + D_{2i} \int_{t-\delta(t)}^t f(x(s)) ds \right] d\omega(t) \right\}, \tag{10}
 \end{aligned}$$

then we have the following Theorem 3.1.

Theorem 3.1. *For given $\tau > 0$, $\mu > 0$, $\delta > 0$, the system described by (10) is mean square exponentially stable in the sense of Definition 2.1, if there exist positive-definiton matrices P , Q_1 , Q_2 , Q_3 , Q_4 , Q_5 and real matrix of appropriate dimensions matrices W_i such that the following LMIs hold:*

$$\Theta_i = \begin{bmatrix} \Theta_{11i} & \Theta_{12i} & -Q_3 G_i & P_i A_{3i} & \Theta_{15i} & P A_{2i} & -A_i^T W_i^T & D_i^T P_i \\ * & \Theta_{22i} & \Theta_{23i} & -G_i^T P A_{3i} & -G_i^T P A_{1i} & \Theta_{26i} & 0 & D_{1i}^T P_i \\ * & * & \Theta_{33i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_5 & 0 & 0 & A_{3i}^T W_i^T & D_{2i}^T P_i \\ * & * & * & * & \Theta_{55i} & 0 & A_{1i}^T W_i^T & 0 \\ * & * & * & * & * & \Theta_{66i} & A_{2i}^T W_i^T & 0 \\ * & * & * & * & * & * & \Theta_{77i} & 0 \\ * & * & * & * & * & * & * & -P_i \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
 \Theta_{11i} &= Q_1 + Q_2 - Q_3 - P_i A_i - A_i^T P - \lambda_{1i} F_1, \quad \Theta_{12i} = A_i^T P_i G_i + Q_3(G_i + I), \\
 \Theta_{15i} &= P A_{1i} - \lambda_{1i} F_2, \quad \Theta_{22i} = -(1 - \mu)Q_1 - (G_i + I)^T Q_3(G_i + I) - \lambda_{2i} F_1, \\
 \Theta_{23i} &= (G_i + I)^T Q_3 G_i, \quad \Theta_{26i} = -G_i^T P A_{2i} - \lambda_{2i} F_2, \\
 \Theta_{33i} &= -(1 - 2\mu)Q_2 - G_i^T Q_3 G_i, \quad \Theta_{55i} = \delta^2 Q_5 + Q_4 - \lambda_{1i} I, \\
 \Theta_{66i} &= -(1 - \mu)Q_4 - \lambda_{2i} I, \quad \Theta_{77i} = -W_i - W_i^T + \tau^2 Q_3.
 \end{aligned}$$

Proof: For the convenience of proof, we denote

$$g_i(t) = -A_i x(t) + A_{1i} f(x(t)) + A_{2i} f(x(t - \tau(t))) + A_{3i} \int_{t-\delta(t)}^t f(x(s)) ds,$$

$$\sigma_i(t) = D_i x(t) + D_{1i} x(t - \tau(t)) + D_{2i} \int_{t-\delta(t)}^t f(x(s)) ds,$$

and

$$\frac{d[x(t) - G_i x(t - \tau(t))]}{dt} = \eta(t). \tag{12}$$

Then system (10) can be rewritten as

$$d[x(t) - G_i x(t - \tau(t))] = \sum_{i=1}^r h_i(\theta(t)) \{g_i(t) dt + \sigma_i(t) d\omega(t)\}. \tag{13}$$

By Lemma 2.1 and (12), the following equation can be obtained (see Lemma 2.2 in [20])

$$\begin{aligned} \int_{t-\tau(t)}^t \eta(s) ds &= x(t) - G_i x(t - \tau(t)) - x(t - \tau(t)) + G_i x(t - 2\tau(t)) \\ &= x(t) - (G_i + I)x(t - \tau(t)) + G_i x(t - 2\tau(t)). \end{aligned} \quad (14)$$

Choose a Lyapunov-Krasovskii functional candidate as

$$V(x(t), t) = \sum_{n=1}^5 V_n(x(t), t), \quad (15)$$

where

$$V_1(x(t), t) = [x(t) - G_i(x(t - \tau(t)))]^T P[x(t) - G_i(x(t - \tau(t)))],$$

$$V_2(x(t), t) = \int_{t-\tau(t)}^t x^T(s) Q_1 x(s) ds + \int_{t-2\tau(t)}^t x^T(s) Q_2 x(s) ds,$$

$$V_3(x(t), t) = \tau \int_{-\tau(t)}^0 \int_{t+\alpha}^t \eta^T(s) Q_3 \eta(s) ds d\alpha,$$

$$V_4(x(t), t) = \int_{t-\tau(t)}^t f^T(x(s)) Q_4 f(x(s)) ds,$$

$$V_5(x(t), t) = \delta \int_{t-\delta(t)}^t \int_{t+\theta}^t f^T(x(s)) Q_5 f(x(s)) ds d\theta,$$

Then, the stochastic differential of $V(x(t), t)$ along system (10) can be obtained as follows:

$$dV(x(t), t) = \mathcal{L}V(x(t), t) dt + 2x(t)^T P \sigma_i(t) d\omega(t), \quad (16)$$

where

$$\mathcal{L}V(x(t), t) = \mathcal{L} \sum_{n=1}^5 V_n(x(t), t), \quad (17)$$

and

$$\mathcal{L}V_1(x(t), t) = \sum_{i=1}^r h_i(\theta(t)) \left\{ 2[x(t) - G_i x(t - \tau(t))]^T P g_i(t) + \sigma_i^T(t) P \sigma_i(t) \right\}, \quad (18)$$

$$\begin{aligned} \mathcal{L}V_2(t, x(t)) &= x^T(t) (Q_1 + Q_2) x(t) - (1 - \mu) x^T(t - \tau(t)) Q_1 x(t - \tau(t)) \\ &\quad - (1 - 2\mu) x^T(t - 2\tau(t)) Q_2 x(t - 2\tau(t)), \end{aligned} \quad (19)$$

$$\mathcal{L}V_3(t, x(t)) \leq \tau^2 \eta^T(t) Q_3 \eta(t) - \tau(t) \int_{t-\tau(t)}^t \eta^T(s) Q_3 \eta(s) ds, \quad (20)$$

$$\mathcal{L}V_4(x(t), t) = f^T(x(t)) Q_4 f(x(t)) - (1 - \mu) f^T(x(t - \tau(t))) Q_4 f(x(t - \tau(t))), \quad (21)$$

$$\mathcal{L}V_5(x(t), t) = \delta^2 f^T(x(s)) Q_5 f(x(s)) - \delta \int_{t-\delta(t)}^t f^T(x(s)) Q_5 f(x(s)) ds, \quad (22)$$

By Lemma 2.2, we have

$$\begin{aligned}
 -\delta \int_{t-\delta(t)}^t f^T(x(s))Q_5f(x(s))ds &\leq -\delta(t) \int_{t-\delta(t)}^t f^T(x(s))Q_5f(x(s))ds \\
 &\leq - \left[\int_{t-\delta(t)}^t f(x(s))ds \right]^T Q_5 \left[\int_{t-\delta(t)}^t f(x(s))ds \right].
 \end{aligned}
 \tag{23}$$

By Lemma 2.1,

$$-\tau(t) \int_{t-\tau(t)}^t \eta^T(s)Q_3\eta(s)ds \leq \zeta(t)\Omega\zeta^T(t),
 \tag{24}$$

where $\zeta(t) = [x^T(t) \ x^T(t - \tau(t)) \ x^T(t - 2\tau(t))]$ and

$$\Omega = \begin{bmatrix} -Q_3 & Q_3(G_i + I) & -Q_3G_i \\ * & -(G_i + I)^TQ_3(G_i + I) & (G_i + I)^TQ_3G_i \\ * & * & -G_i^TQ_3G_i \end{bmatrix},$$

then by (5) and (13), for any compatible dimensions matrix W_i , we can get

$$2\eta^T(t) \sum_{i=1}^r h_i(\theta(t)) \{W_i[(g_i(t) - \eta(t))dt + \sigma_i(t)d\omega(t)]\} = 0.
 \tag{25}$$

From (6), for $i = 1, 2, 3, \dots, n$, we have

$$\begin{aligned}
 [f_i(x_i(t)) - l_i^-x(t)]^T [f_i(x_i(t)) - l_i^+x(t)] &\leq 0, \\
 [f_i(x_i(t - \tau(t))) - l_i^-x(t - \tau(t))]^T [f_i(x_i(t - \tau(t))) - l_i^+x(t - \tau(t))] &\leq 0.
 \end{aligned}
 \tag{26}$$

Then there exist scalars $\lambda_{1i} > 0$ and $\lambda_{2i} > 0$ such that

$$-\lambda_{1i} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix}^T \begin{bmatrix} F_1 & F_2 \\ * & I \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix} \geq 0,
 \tag{27}$$

$$-\lambda_{2i} \begin{bmatrix} x_i(t - \tau(t)) \\ f_i(x_i(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} F_1 & F_2 \\ * & I \end{bmatrix} \begin{bmatrix} x_i(t - \tau(t)) \\ f_i(x_i(t - \tau(t))) \end{bmatrix} \geq 0,
 \tag{28}$$

where $F_1 = \frac{l_i^-l_i^+ + l_i^+l_i^-}{2}$, $F_2 = -\frac{l_i^- + l_i^+}{2}$.

By adding (19)-(25) and (27) and (28) to the right side of (16), we have

$$\begin{aligned}
 dV(t, x(t)) &= \mathcal{L}V(x(t), t)dt + 2x(t)^TP\sigma_i(t)d\omega(t) \\
 &\leq \xi^T(t)\bar{\Theta}_i\xi(t)dt + 2\eta^T(t)W_i\sigma_i(t)d\omega(t) + 2x(t)^TP\sigma_i(t)d\omega(t),
 \end{aligned}
 \tag{29}$$

where

$$\begin{aligned}
 \xi^T(t) &= \left[x^T(t)x^T(t - \tau(t))x^T(t - 2\tau(t)) \left[\int_{t-\delta(t)}^t f(x(s))ds \right]^T f^T(x(t))f^T(x(t - \tau(t)))\eta^T(t) \right], \\
 \bar{\Theta}_i &= \begin{bmatrix} \Theta_{11i} & \Theta_{12i} & -Q_3G_i & PA_{3i} & \Theta_{15i} & PA_{2i} & A_i^TW_i^T \\ * & \Theta_{22i} & \Theta_{23i} & -G_i^TPA_{3i} & -G_i^TPA_{1i} & \Theta_{26i} & 0 \\ * & * & \Theta_{33i} & 0 & 0 & 0 & 0 \\ * & * & * & -Q_5 & 0 & 0 & A_{3i}^TW_i^T \\ * & * & * & * & \Theta_{55i} & 0 & A_{1i}^TW_i^T \\ * & * & * & * & * & \Theta_{66i} & A_{2i}^TW_i^T \\ * & * & * & * & * & * & \Theta_{77i} \end{bmatrix} + \begin{bmatrix} D_i^T \\ D_{1i}^T \\ 0 \\ D_{2i}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} P \begin{bmatrix} D_i^T \\ D_{1i}^T \\ 0 \\ D_{2i}^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
 \end{aligned}
 \tag{30}$$

By Schur complement, $\bar{\Theta}_i < 0$ is equal to $\Theta_i < 0$, and then integrating from 0 to t on both sides of (29) and taking the mathematical expectation, we have

$$\begin{aligned} \mathbb{E}\{V(x(t), t)\} &\leq \mathbb{E}\{V(x(0), t)\} + \int_0^t \mathbb{E} \{ \xi^T(t) \Theta_i \xi(t) dt \} \\ &\leq \mathbb{E}\{V(x(0), t)\} - \lambda \int_0^t \mathbb{E} \{ |x(t)|^2 dt \}, \end{aligned} \tag{31}$$

where $\lambda = \min \{ \lambda_{\min}(-\bar{\Theta}_i) \} > 0$. From (15), it is easy to know that there exists a scalar $a = \min\{\lambda_{\min}(P_i)\}$ such that the following inequality holds:

$$V(x(t), t) \geq a |x(t)|^2. \tag{32}$$

From (30), we can have

$$\mathbb{E} \{ |x(t)|^2 \} \leq a^{-1} \mathbb{E}\{V(x(0), t)\} - a^{-1} \lambda \int_0^t \mathbb{E} \{ |x(t)|^2 dt \}. \tag{33}$$

By Gronwall's inequality and (33) we can obtain

$$\mathbb{E} \{ |x(t)|^2 \} \leq a^{-1} V(x(0), t) e^{-a^{-1} \lambda t}. \tag{34}$$

Note that there exists a scalar $c > 0$ such that

$$a^{-1} V(x(0), t) \leq c \sup_{\theta \in [-\tau, 0]} |\phi(\theta)|^2, \tag{35}$$

and then we can obtain

$$\mathbb{E} \{ |x(t)|^2 \} \leq \alpha \sup_{\theta \in [-r, 0]} \mathbb{E} \{ |\phi(0)|^2 \} e^{-\tilde{\lambda} t}, \tag{36}$$

where $\alpha = a^{-1}c$, $\tilde{\lambda} = a^{-1}\lambda$. By Definition 2.1, we can obtain system (10) is exponentially stable in mean square sense. This completes the proof.

Remark 3.1. Lemma 2.1 is a good way to reducing conservatism when studying the stability of neutral stochastic time-delay systems, so it has been utilized widely in neutral time-delay systems (see Example 5.1 in [20] and the references therein).

When uncertainties exist, for system (1), the following Theorem 3.2 can be easily obtained.

Theorem 3.2. For given scalars $\tau > 0$, $\mu > 0$, $\delta > 0$, the system described by (1) is robust exponentially stable for all admissible uncertainties satisfying (2) and (3), if there exist positive-definiton matrices P , Q_1 , Q_2 , Q_3 , Q_4 and Q_5 , positive scalars λ_{1i} , λ_{2i} , β_{1i} and β_{2i} such that the following LMIs hold:

$$\Delta_i = \begin{bmatrix} \Delta_{11i} & \Delta_{12i} & -Q_3 G_i & \Delta_{14i} & \Delta_{15i} & \Delta_{16i} & -A_i^T W_i^T & D_i^T P & P M_i & 0 \\ * & \Delta_{22i} & \Delta_{23i} & \Delta_{24i} & -G_i^T P A_{1i} & \Theta_{26i} & 0 & D_{1i}^T P & -G_i^T P M_i & 0 \\ * & * & \Delta_{33i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Delta_{44i} & \Delta_{45i} & \Delta_{46i} & A_{3i}^T W_i^T & D_{2i}^T P & 0 & 0 \\ * & * & * & * & \Delta_{55i} & \Delta_{56i} & A_{1i}^T W_i^T & 0 & 0 & 0 \\ * & * & * & * & * & \Delta_{66i} & A_{2i}^T W_i^T & 0 & 0 & 0 \\ * & * & * & * & * & * & \Delta_{77i} & 0 & W_i M_i & 0 \\ * & * & * & * & * & * & * & -P & 0 & P M_i \\ * & * & * & * & * & * & * & * & -\beta_{1i} & 0 \\ * & * & * & * & * & * & * & * & * & -\beta_{2i} \end{bmatrix} < 0, \tag{37}$$

where

$$\begin{aligned} \Delta_{11i} &= \Theta_{11i} + \beta_{1i}N_{1i}^T N_{1i} + \beta_{2i}N_{5i}^T N_{5i}, \quad \Delta_{12i} = A_i^T P_i G_i + Q_3(G_i + I) + \beta_{2i}N_{5i}^T N_{6i}, \\ \Delta_{14i} &= P A_{3i} - \beta_{1i}N_{1i}^T N_{4i} + \beta_{2i}N_{5i}^T N_{7i}, \quad \Delta_{15i} = P A_{1i} - \lambda_{1i}F_2 - \beta_{1i}N_{1i}^T N_{2i}, \\ \Delta_{16i} &= P A_{2i} - \beta_{1i}N_{1i}^T N_{3i}, \quad \Delta_{22i} = -(1 - \mu)Q_1 - (G_i + I)^T Q_3(G_i + I) - \epsilon_{2i}F_1 + \beta_{2i}N_{6i}^T N_{6i}, \\ \Delta_{23i} &= (G_i + I)^T Q_3 G_i, \quad \Delta_{24i} = -G_i^T P A_{3i} + \beta_{2i}N_{6i}^T N_{7i}, \\ \Delta_{26i} &= -G_i^T P A_{2i} - \lambda_{2i}F_2, \quad \Delta_{33i} = -(1 - 2\mu)Q_2 - G_i^T Q_3 G_i, \\ \Delta_{44i} &= -Q_5 + \beta_{1i}N_{4i}^T N_{4i} + \beta_{2i}N_{7i}^T N_{7i}, \quad \Delta_{45i} = \beta_{1i}N_{4i}^T N_{2i}, \\ \Delta_{46i} &= \beta_{1i}N_{4i}^T N_{3i}, \quad \Delta_{55i} = \delta^2 Q_5 + Q_4 - U_1 + \beta_{1i}N_{2i}^T N_{2i}, \quad \Delta_{56i} = \beta_{1i}N_{2i}^T N_{3i}, \\ \Delta_{66i} &= -(1 - \mu)Q_4 - \lambda_2 I + \beta_{1i}N_{3i}^T N_{3i}, \quad \Delta_{77i} = -W_i - W_i^T + \tau^2 Q_3. \end{aligned}$$

Proof: Substituting (3) into (9), according to Schur complement, (9) is equal to

$$\Theta_i + \phi_1^T H_i(t) \phi_2 + \phi_2^T H_i^T(t) \phi_1 + \phi_3^T H_i(t) \phi_4 + \phi_4^T H_i^T(t) \phi_3 < 0, \quad (38)$$

where

$$\begin{aligned} \phi_1 &= [M_i^T P \quad -M_i^T P G_i \quad 0 \quad 0 \quad 0 \quad 0 \quad M_i^T W_i^T \quad 0], \quad \phi_2 = [-N_{1i} \quad 0 \quad 0 \quad N_{4i} \quad N_{2i} \quad N_{3i} \quad 0 \quad 0], \\ \phi_3 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad M_i^T P], \quad \phi_4 = [N_{5i} \quad N_{6i} \quad 0 \quad N_{7i} \quad 0 \quad 0 \quad 0 \quad 0]. \end{aligned}$$

By Lemma 2.3, there exist positive scalars β_{1i} and β_{2i} such that the following inequality holds:

$$\Theta_i + \frac{1}{\beta_{1i}} \phi_1^T \phi_1 + \beta_{1i} \phi_2^T \phi_2 + \frac{1}{\beta_{2i}} \phi_3^T \phi_3 + \beta_{2i} \phi_4^T \phi_4 < 0. \quad (39)$$

By Schur complement, (39) is equal to (37). So the proof is completed.

When there are no fuzzy rule and distribution delay, system (1) will reduce to the following one, which has been researched and discussed by [19,20]

$$\begin{aligned} d[x(t) - G(x(t - \tau(t)))] &= [-A(t)x(t) + A_1(t)f(x(t)) + A_2(t)f(x(t - \tau(t)))]dt \\ &+ [D(t)x(t) + D_1(t)x(t - \tau(t))]d\omega(t), \end{aligned} \quad (40)$$

where

$$\begin{aligned} A(t) &= A + \Delta A(t), \quad A_1(t) = A_1 + \Delta A_1(t), \quad A_2(t) = A_2 + \Delta A_2(t), \\ D(t) &= D + \Delta D(t), \quad D_1(t) = D_1 + \Delta D_1(t), \end{aligned}$$

and

$$[\Delta A(t) \quad \Delta A_1(t) \quad \Delta A_2(t) \quad \Delta D(t) \quad \Delta D_1(t)] = MH(t) [N_1 \quad N_2 \quad N_3 \quad N_5 \quad N_6], \quad (41)$$

then for system (40), we have the following Corollary 3.1.

Corollary 3.1. For given scalars $\tau > 0$, $\mu > 0$, the system (40) is robustly exponentially stable for all admissible uncertainties satisfying (2) and (3), if there exist positive-definiton matrices P , Q_1 , Q_2 , Q_3 and Q_4 , matrix W , positive scalars λ_1 , λ_2 , β_1 and β_2 such that the following LMI holds:

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & -Q_3 G & \Delta_{15} & \Delta_{16} & -A^T W^T & D^T P & P M & 0 \\ * & \Delta_{22} & \Delta_{23} & -G^T P A_1 & \Delta_{26} & 0 & D_1^T P & -G^T P M & 0 \\ * & * & \Delta_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Delta_{55} & \Delta_{56} & A_1^T W^T & 0 & 0 & 0 \\ * & * & * & * & \Delta_{66} & A_2^T W^T & 0 & 0 & 0 \\ * & * & * & * & * & \Delta_{77} & 0 & W M & 0 \\ * & * & * & * & * & * & -P & 0 & P M \\ * & * & * & * & * & * & * & -\beta_1 & 0 \\ * & * & * & * & * & * & * & * & -\beta_2 \end{bmatrix} < 0, \quad (42)$$

where

$$\begin{aligned}\Delta_{11} &= Q_1 + Q_2 - Q_3 - PA - A^T P - \lambda_1 F_1 + \beta_1 N_1^T N_1 + \beta_2 N_5^T N_5, \\ \Delta_{12} &= A^T P G + Q_3(G + I) + \beta_2 N_5^T N_6, \quad \Delta_{15} = PA_1 - \lambda_1 F_2 - \beta_1 N_1^T N_3, \\ \Delta_{16} &= PA_2 - \beta_1 N_1^T N_2, \quad \Delta_{22} = -(1 - \mu)Q_1 - (G + I)^T Q_3(G + I) - \lambda_2 F_1 + \beta_2 N_6^T N_6, \\ \Delta_{23} &= (G + I)^T Q_3 G, \quad \Delta_{26} = -G^T P A_2 - \lambda_2 F_2, \\ \Delta_{33} &= -(1 - 2\mu)Q_2 - G^T Q_3 G, \quad \Delta_{55} = Q_4 - \lambda_1 I + \beta_1 N_2^T N_2, \\ \Delta_{56} &= \beta_1 N_2^T N_3, \quad \Delta_{66} = -(1 - \mu)Q_4 - \lambda_2 I + \beta_1 N_3^T N_3, \quad \Delta_{77} = -W - W^T + \tau^2 Q_3.\end{aligned}$$

The proof of Corollary 3.1 is similar to that of Theorem 3.1, so it is omitted here.

4. Numerical Example. In this section, two numerical examples are presented to demonstrate the effectiveness of the developed method on the obtained results.

Example 4.1. Consider system (4) with the following defuzzified output ($i = 1, 2$):

$$\begin{aligned}& d[x(t) - G_i(x(t - \tau(t)))] \\ &= \sum_{i=1}^2 h_i(\theta(t)) \left\{ \left[-A_i(t)x(t) + A_{1i}(t)f(x(t)) + A_{2i}(t)f(x(t - \tau(t))) \right. \right. \\ &\quad \left. \left. + A_{3i}(t) \int_{t-\delta(t)}^t f(x(s))ds \right] dt + \left[D_i(t)x(t) + D_{1i}(t)x(t - \tau(t)) \right. \right. \\ &\quad \left. \left. + D_{2i}(t) \int_{t-\delta(t)}^t x(s)ds \right] d\omega(t) \right\},\end{aligned}\tag{43}$$

where

$$\begin{aligned}G_1 &= \begin{bmatrix} -0.2 & 0 \\ 0.06 & 0.15 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.3 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} -5 & 0.1 \\ 1 & -3 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0.5 & -0.3 \\ 0.5 & 2 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0.5 & -0.3 \\ 0.5 & 2.0 \end{bmatrix}, \quad A_{31} = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & -0.3 \end{bmatrix}, \\ G_2 &= \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.3 & 1.2 \\ 2 & -0.5 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 0.2 & 1 \\ 2.5 & -0.6 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1.2 & 0.2 \\ -0.5 & 1.0 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 1.0 & 0.3 \\ -0.4 & 0.8 \end{bmatrix}, \quad A_{32} = \begin{bmatrix} -0.2 & 0.1 \\ -0.3 & 0.25 \end{bmatrix},\end{aligned}$$

and $M_{11} = 0.2I$, $M_{12} = 0.3I$, $M_{21} = 0.2I$, $M_{22} = 0.2I$, $N_{11} = 0.2I$, $N_{12} = 0.1I$, $N_{21} = 0.2I$, $N_{22} = 0.2I$, $N_{31} = 0.1I$, $N_{32} = 0.2I$, $\tau = 0.4$, $\mu = 0.5$, $\delta = 0.1$. Substituting the parameters above into LMI (37), by Matlab LMI toolbox, one feasible solution can be obtained as follows:

$$\begin{aligned}P &= \begin{bmatrix} 0.1536 & -0.0046 \\ -0.0046 & 0.1497 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.0812 & -0.0415 \\ -0.0415 & 0.0864 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1.4662 & -0.4043 \\ -0.4043 & 1.2144 \end{bmatrix}, \\ R_3 &= \begin{bmatrix} 0.0088 & -0.0058 \\ -0.0058 & 0.0122 \end{bmatrix}, \quad R_4 = \begin{bmatrix} 0.0520 & 0.0051 \\ 0.0051 & 0.0565 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.0881 & -0.0174 \\ -0.0174 & 0.0902 \end{bmatrix}, \\ Q_2 &= \begin{bmatrix} 0.0704 & -0.0139 \\ -0.0139 & 0.0717 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 0.0529 & -0.0102 \\ -0.0102 & 0.0537 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 0.0318 & -0.0060 \\ -0.0060 & 0.0322 \end{bmatrix}, \\ Q &= \begin{bmatrix} 0.0318 & -0.0060 \\ -0.0060 & 0.0322 \end{bmatrix}, \quad \gamma = 297.6609.\end{aligned}$$

The fuzzy membership functions are taken as $h_1(\theta(t)) = \sin^2(x(t))$, $h_2(\theta(t)) = 1 - \sin^2(x(t))$, the four random initial values are chosen for $x_1(t)$ and $x_2(t)$ respectively, and

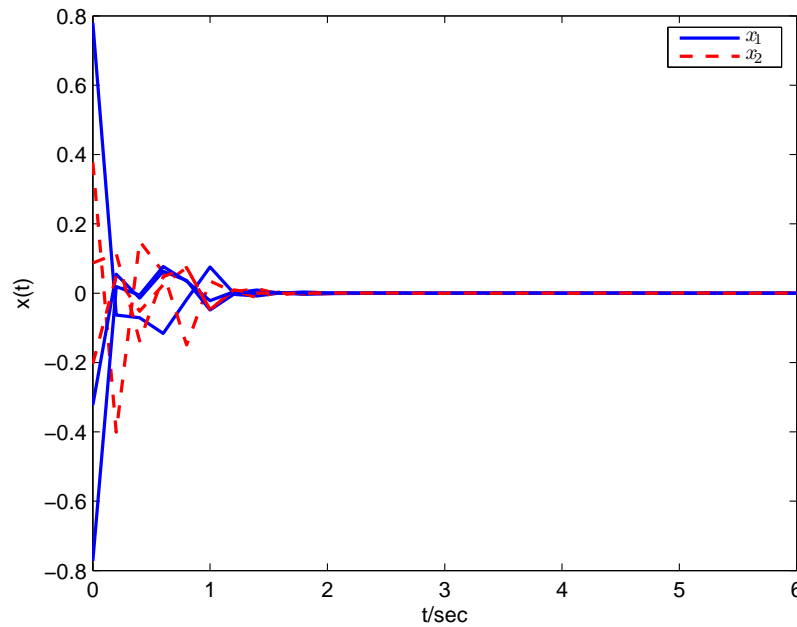


FIGURE 1. State curves of $x(t)$

the simulation results of the state response are plotted in Figure 1. From the simulation results we can see that for different initial values of state, the considered system is robustly stable.

Example 4.2. Consider the neutral stochastic neural networks (40) with the following parameters (Example 1 in [20]):

$$G = \begin{bmatrix} 0.35 & 0 \\ 0.2 & 0.6 \end{bmatrix}, A = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.15 \end{bmatrix}, A_1 = \begin{bmatrix} -0.1 & 0.4 \\ 0.2 & -0.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0.1 & -1 \\ -1.4 & 0.4 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.23 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.3 \end{bmatrix}, M = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, N_1 = [0.1 \ 0.2],$$

$$N_2 = [0.5 \ 0.1], N_3 = [-0.2 \ 0.2], N_5 = [-0.1 \ 0.2], N_6 = [0.3 \ 0.1].$$

Take the activation function as $f_1(x(t)) = f_2(x(t)) = \tanh(x(t))$, so it can be verified from Assumption 2.1 that $F_1 = \text{diag}\{0, 0\}$, $F_2 = \text{diag}\{-0.5, -0.5\}$.

When the activation function is chosen as $f_1(x(t)) = f_2(x(t)) = \tanh(x(t))$, l_i^- and l_i^+ in (26) are 0 and 1 respectively, so F_1 and F_2 can be obtained easily.

In order to testify the effectiveness of our proposed method, theory analysis has been done and the results about the upper bounds of delays τ for different μ are listed in Table 1, where “–” means that the LMI has no feasible solution. From Table 1 we can see that our approach is effective.

TABLE 1. Maximum allowable bounds of τ for different μ

μ	0.003	0.005	0.05	0.1	0.15
Corollary 3.1	1.1194	1.0918	0.6119	0.2110	0.0691
[19]	–	–	–	–	–
[20]	–	–	–	–	–

Remark 4.1. *It must be pointed out that when μ is set by 0.0035, the maximum allowable value of τ in Corollary 3.1 is 1.1124, but in [19,20], the upper bounds τ_{\max} of time delay value are more than 5×10^9 and 2.8263×10^{18} , respectively.*

5. Conclusions. In this paper, we have investigated the robust stability for a class of uncertain fuzzy stochastic neutral neural networks with mixed delays. By constructing a proper Lyapunov functional, employing stochastic stability analysis theory, the delay-dependent conditions have been proposed such that the studied systems are robust exponentially stable. Finally, examples and simulations are provided to illustrate the less conservatism and effectiveness of the developed approach. Our future research direction can be extended to the l_2 - l_∞ and l_∞ filtering design of stochastic neutral neural networks and Markovian jump systems.

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