

## ANALYSIS AND IMPROVEMENT OF THE INFLUENCE OF MEASUREMENT NOISE ON MVC BASED CONTROLLER PERFORMANCE ASSESSMENT

SHENG ZHAO<sup>1,2</sup>, GUANGHUI YANG<sup>1</sup>, ZHENGJIANG ZHANG<sup>1,\*</sup> AND CHONG CHEN<sup>2</sup>

<sup>1</sup>College of Physics and Electronic Information Engineering

<sup>2</sup>The Key Laboratory of Low-Voltage Apparatus Intellectual Technology of Zhejiang  
Wenzhou University

Chashan Gaojiaoyuan Dist., Wenzhou 325000, P. R. China

\*Corresponding author: zzjiang108@gmail.com

Received September 2017; revised December 2017

**ABSTRACT.** *Industrial systems require healthy controllers with high level of control performance. Controller performance assessment (CPA) is very crucial to recognize and correct the process control with malfunctions. Minimum variance control (MVC) introduces a method to evaluate the performance of controller, and is widely used in CPA because of the attractive theoretical and computational properties associated with it. The measurement noise is not explicitly considered in the MVC based CPA. However, the evaluated control system cannot work under the ideal condition in the actual process. It is usually influenced by the measurement noise. This paper analyzes the influence of the measurement noise on the results of MVC based CPA. The digital filter combined with MVC based CPA is proposed to reduce the influence of measurement noise and improve the results of MVC based CPA. The effectiveness of the proposed method is demonstrated by the theoretical derivation, simulations of both univariate and multivariate control systems and experiments of a constant-current constant-frequency DC/AC converter.*

**Keywords:** Minimum variance control, Controller performance assessment, Measurement noise, Digital filter

**1. Introduction.** There are a large number of control loops operating under different conditions in industrial processes. The performance of these control loops may be influenced by many factors such as the change of feedstock, malfunction of the equipment and different types of disturbance and will gradually deteriorate [1]. Controller performance assessment (CPA), which indicates the condition of the current control performance compared with the ideal control performance, is very important to recognize and correct the process control with malfunctions. Therefore, CPA has attracted growing research interests in both academic and industrial societies.

As pointed out by Hoo et al. (2003), about 60% of all the industrial controllers have performance problems [2]. Thus, the practical application of CPA is very important and a rapid development of research in CPA in the last two decades. Moving CPA from an academic research paper to an industrial application introduces new and interesting engineering problems. Paulonis and Cox (2003) reported a CPA application for 40 plants located at nine international sites and involving some 14,000 control loops [3]. These different industrial company situations will influence how the implementation should be achieved. Some companies may construct a global solution, and other companies may prefer a local plant solution. Jelali (2006) provided a list of research articles and their industrial applications in the chemical, petrochemical and other industries [4]. Bauer et al. (2016) presented results from a survey on industrial application of CPA in the process

industries [5]. The practical application of CPA has been crucial in the past and will remain to be important in the future.

As a classical method for CPA, minimum variance control (MVC) plays an important role in the controller benchmarking and design. The basic idea of MVC is to express the minimum variance prediction of the plant output in terms of the control input such that the predicted output can be driven to follow a desired output by solving a simple linear equation for appropriate control action [6]. Since Åström (1970) firstly developed the theory for MVC of stochastic control systems [7], MVC has become an active research area and many techniques for MVC have been proposed in the past decade. Harris (1989) used the MVC theory and proposed an MVC based index for CPA [8]. Accordingly, this index is defined as the ratio of the minimum achievable variance and the actual output variance, whose value varies from 0 to 1. The MVC benchmark has been widely accepted in the research field of CPA. Many researchers develop the extended use of the MVC based benchmark. Chen and Kong (2009) developed a new method to estimate the minimum variance bounds and the achievable variance bounds for the assessment of the batch control system where the iterative learning control was applied [9]. Ko and Edgar (2001) extended the use of MVC to the CPA of cascade control system [10]. Within recent years, MVC is extended to be used in time-variant process systems [11-13], model predictive controller [14-17], decentralized controller [18], adaptive optics systems [19], nonlinear multivariate systems [20,21], data mining [22,23] and batch processes [1,24]. Other types of CPA benchmarks have also been developed based on the MVC theory, such as generalized minimum variance (GMV) benchmark [20,25,26], and the linear quadratic Gaussian (LQG) benchmark [27].

The theory for MVC of stochastic control systems does not explicitly consider the influence of measurement noise. In fact, it is impractical that the evaluated control system can work under the ideal condition. Measurements from the physical sensors always contain unwanted noise. Most process measurements are generally corrupted by measurement noise [28]. With the influence of measurement noise, the feedback signal is no longer the actual output signal, but the measured signal. The results of CPA will be gradually deteriorated. However, the influence of the measurement noise on the results of MVC based CPA is rarely analyzed in theory.

Shown in this paper are several developments on MVC based CPA as follows.

- a) Considering measurement noise in the sensor device, the influence of the measurement noise on the results of MVC based CPA is analyzed.
- b) The digital filter combined with MVC based CPA is proposed to reduce the influence of measurement noise and improve the results of MVC based CPA.
- c) The effectiveness of the proposed method is demonstrated by the theoretical derivation, the simulations of both univariate and multivariate control systems and the experiments of a constant-current constant-frequency DC/AC converter.

The rest of the paper is organized as follows. The MVC based CPA is briefly reviewed in Section 2. The influence of the measurement noise on the results of MVC based CPA is analyzed through mathematical derivation in Section 3. The digital filter combined with MVC based CPA is proposed to be used to reduce the influence of measurement noise on the results of CPA in Section 4. Three case studies, including simulations and experiments, are used to illustrate the effectiveness of the digital filter combined with MVC based CPA in Section 5. Finally, conclusions are discussed in Section 6.

2. MVC Based CPA.

2.1. MVC based CPA in ideal stochastic univariate system. Considering an ideal single input single output (SISO) control system, the structure is shown in Figure 1, where the set point is  $r(t)$ , and the output signal is  $y(t)$ . In MVC based CPA,  $r(t)$  is generally assumed to be zero. The output signal is measured by an ideal sensor and compared with the set point to generate the error  $e(t)$ . And then the controller  $C_0(z^{-1})$  derives the control signal  $u(t)$ . A disturbance  $d(t)$ , which is assumed to be zero mean and Gaussian white noise of variance  $\sigma^2$ , is introduced into the control system. The output signal of the SISO control system can be described by:

$$y(t) = z^{-k} \frac{B(z^{-1})}{A(z^{-1})} u(t) + \frac{D(z^{-1})}{A(z^{-1})} d(t) \tag{1}$$

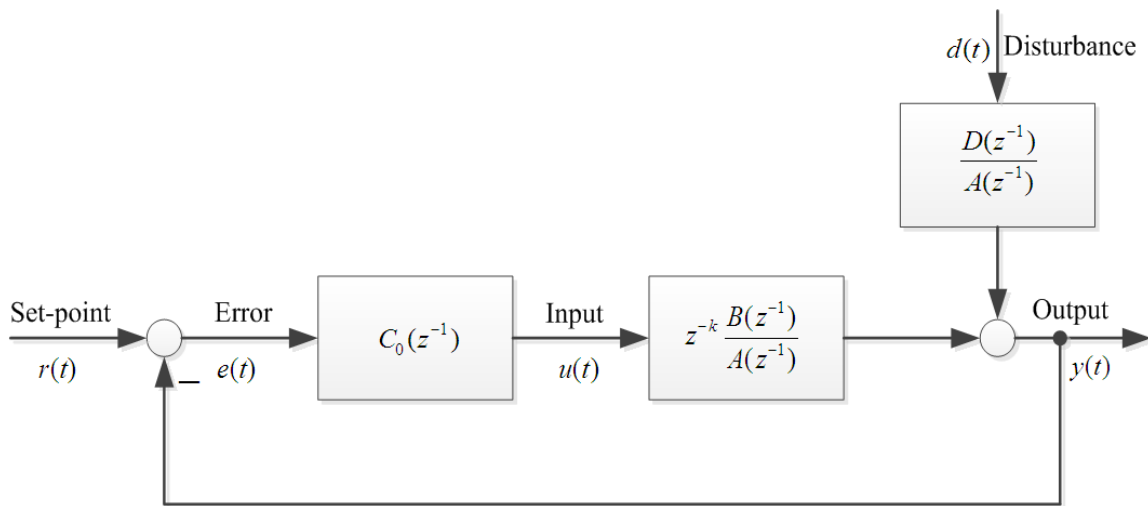


FIGURE 1. The ideal SISO control system structure

Since  $z^{-k}$  in Equation (1) represents a  $k$ -step delay in the control signal. The output signal can be rewritten as:

$$y(t+k) = \frac{B(z^{-1})}{A(z^{-1})} u(t) + \frac{D(z^{-1})}{A(z^{-1})} d(t+k) \tag{2}$$

The following Diophantine equation can be defined to effectively split  $D(z^{-1})$  into two parts, one related to the past time and the other related to the future time.

$$D(z^{-1}) = A(z^{-1})F(z^{-1}) + z^{-k}G(z^{-1}) \tag{3}$$

Substituting Equation (3) into Equation (2), after some manipulations, it yields:

$$y(t+k) = \frac{B(z^{-1})F(z^{-1})}{D(z^{-1})} u(t) + \frac{G(z^{-1})}{D(z^{-1})} y(t) + Fd(t+k) \tag{4}$$

Note that the first two terms on the right-hand side of Equation (4) are dependent on the process control law, and the third term is independent of the control action. Therefore, the variance of the output can be derived as

$$\begin{aligned} J(t) &= E [y(t+k)^2] \\ &= E \left[ \left( \frac{B(z^{-1})F(z^{-1})}{D(z^{-1})} u(t) + \frac{G(z^{-1})}{D(z^{-1})} y(t) \right)^2 \right] + E [(Fd(t+k))^2] \end{aligned} \tag{5}$$

To achieve MVC, the following control law can be generated:

$$\frac{B(z^{-1})F(z^{-1})}{D(z^{-1})}u(t) + \frac{G(z^{-1})}{D(z^{-1})}y(t) = 0 \quad (6)$$

$$u(t) = -\frac{G(z^{-1})}{B(z^{-1})F(z^{-1})}y(t) \quad (7)$$

The output variance under MVC is calculated by

$$J_{MVC}(t) = E[(Fd(t+k))^2] \quad (8)$$

However, most control systems do not satisfy MVC control law. Therefore, in the expression defining the future output, there is an extra term whose variance is nonzero. This term is a result of the controller not being a minimum variance controller

$$y(t+k) = \hat{y}(t) + Fd(t+k) \quad (9)$$

The variance of the output may be obtained as:

$$J(t) = E[y(t+k)^2] = E[\hat{y}(t)^2] + E[(Fd(t+k))^2] = J_0(t) + J_{MVC}(t) \quad (10)$$

In order to obtain a universal tool for comparing different systems, the following MVC based controller performance index is defined

$$\eta = \frac{J_{MVC}(t)}{J_0(t) + J_{MVC}(t)} \in [0, 1] \quad (11)$$

The above performance index provides a performance indicator that is normalized and bounded. From the index, the engineers can see how close to minimum variance the system is controlled. When  $\eta = 1$ , the control system is controlled under ideal minimum variance.  $\eta = 0$  indicates the case of the worst control.

**2.2. MVC based CPA in ideal stochastic multivariate system.** The ideal stochastic feedback control system with multiple-input multiple-output (MIMO) is shown in Figure 2. The MIMO linear time-invariant stationary stochastic process model can be described by

$$\mathbf{y}(t) = \mathbf{A}(z^{-1})\mathbf{u}(t) + \mathbf{B}(z^{-1})\mathbf{d}(t) \quad (12)$$

where  $\mathbf{A}(z^{-1})$  and  $\mathbf{B}(z^{-1})$  are proper, rational transfer function matrices for the process plant and disturbances respectively;  $\mathbf{y}(t)$  and  $\mathbf{u}(t)$  are the output vector and input vector

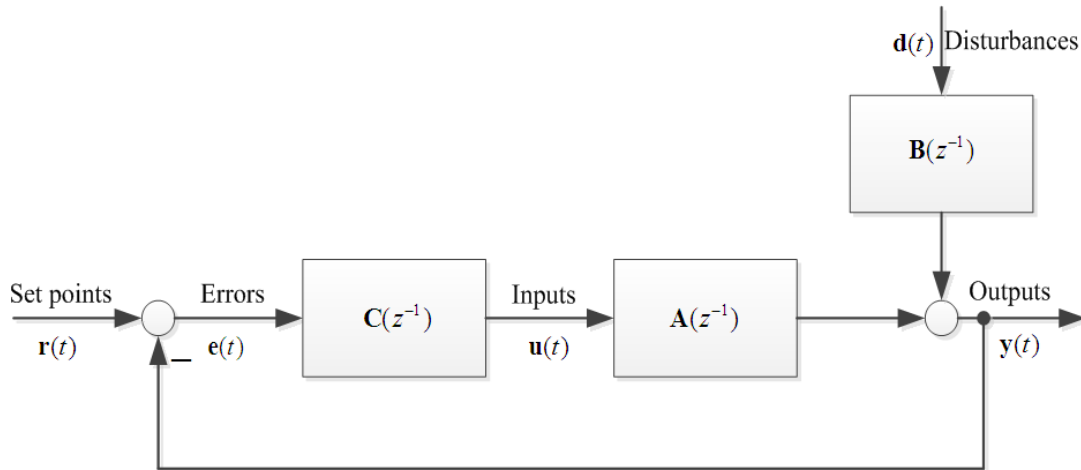


FIGURE 2. The ideal stochastic multivariate feedback control system

separately, and  $\mathbf{d}(t)$  represents a white noise vector of disturbances with zero mean and covariance matrix  $\Sigma_{\mathbf{d}}$ .

Assumed  $\mathbf{A}(z^{-1})$  is a full rank transfer function matrix and  $\mathbf{D}(z^{-1})$  is the corresponding unitary interactor matrix. They can be evaluated that

$$\lim_{z^{-1} \rightarrow 0} \mathbf{D}(z^{-1})\mathbf{A}(z^{-1}) = \lim_{z^{-1} \rightarrow 0} \mathbf{A}'(z^{-1}) = \mathbf{K} \tag{13}$$

where  $\mathbf{A}'(z^{-1})$  is the delay-free transfer function matrix of  $\mathbf{A}(z^{-1})$ , and  $\mathbf{K}$  is a full rank constant matrix. Therefore, the process model can be rewritten as

$$\mathbf{y}(t) = \mathbf{D}(z^{-1})^{-1}\mathbf{A}'(z^{-1})\mathbf{u}(t) + \mathbf{B}(z^{-1})\mathbf{d}(t) \tag{14}$$

Pre-multiplying both sides of Equation (14) by  $z^{-d}\mathbf{D}(z^{-1})$ , where  $d$  is the maximum delay order of interactor matrix  $\mathbf{D}(z^{-1})$ , yields

$$z^{-d}\mathbf{D}(z^{-1})\mathbf{y}(t) = z^{-d}\mathbf{A}'(z^{-1})\mathbf{u}(t) + z^{-d}\mathbf{D}(z^{-1})\mathbf{B}(z^{-1})\mathbf{d}(t) \tag{15}$$

Letting  $\mathbf{y}'(t) = z^{-d}\mathbf{D}(z^{-1})\mathbf{y}(t)$  and  $\mathbf{N}(z^{-1}) = z^{-d}\mathbf{D}(z^{-1})\mathbf{B}(z^{-1})$ , Equation (15) becomes

$$\mathbf{y}'(t) = z^{-d}\mathbf{A}'(z^{-1})\mathbf{u}(t) + \mathbf{N}(z^{-1})\mathbf{d}(t) \tag{16}$$

The following Diophantine equation can be defined to effectively split  $\mathbf{N}(z^{-1})$  into two parts, one related to the past time and the other related to the future time.

$$\mathbf{N}(z^{-1}) = \mathbf{F}(z^{-1}) + z^{-d}\mathbf{G}(z^{-1}) \tag{17}$$

Substituting Equation (17) into Equation (16) yields:

$$\mathbf{y}'(t) = z^{-d}(\mathbf{A}'(z^{-1})\mathbf{u}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)) + \mathbf{F}(z^{-1})\mathbf{d}(t) \tag{18}$$

Since the last term in Equation (18) cannot be influenced by the control action, the multivariate MVC can be achieved by setting the first term on the right-hand side of Equation (18) to zero,

$$z^{-d}(\mathbf{A}'(z^{-1})\mathbf{u}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)) = 0 \tag{19}$$

For an arbitrary linear multivariate controller, the following inequality yields

$$\begin{aligned} &tr[Cov(\mathbf{y}(t))] = tr[Cov(\mathbf{y}'(t))] \\ &= tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})\mathbf{y}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)))] + tr[Cov(\mathbf{F}(z^{-1})\mathbf{d}(t))] \\ &\geq tr[Cov(\mathbf{F}(z^{-1})\mathbf{d}(t))] \end{aligned} \tag{20}$$

Therefore, the multivariate MVC benchmark can be defined as

$$J_{MMVC} = tr[Cov(\mathbf{F}(z^{-1})\mathbf{d}(t))] \tag{21}$$

Defining  $J_{M0}(t) = tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})\mathbf{y}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)))]$ , the trace of the covariance of the outputs  $tr[Cov(\mathbf{y}(t))]$  may be obtained as:

$$J_M(t) = tr[Cov(\mathbf{y}(t))] = J_{M0}(t) + J_{MMVC}(t) \tag{22}$$

In order to obtain a universal tool for comparing different systems, the following multivariate MVC based controller performance index for multivariate feedback control system is defined

$$\eta = \frac{J_{MMVC}(t)}{J_{M0}(t) + J_{MMVC}(t)} \in [0, 1] \tag{23}$$

When  $\eta = 1$ , the multivariate control system is controlled under ideal minimum variance.  $\eta = 0$  indicates the case of the worst control.

The theory for MVC of stochastic control systems does not explicitly consider the influence of measurement noise. The practical application of MVC based CPA introduces

a new and interesting problem that most industrial process measurements are generally corrupted by measurement noise. With the influence of measurement noise, the results of CPA will be gradually deteriorated. Therefore, the next section will analyze the influence of the measurement noise on the results of MVC based CPA through mathematical derivation.

**3. Influence of Measurement Noise on MVC Based CPA.** The measurement noise is not explicitly considered in the above ideal SISO control system. However, the evaluated control system cannot work under the ideal condition in the actual process. Measured process data inevitably contain some inaccurate information because measurements are obtained by imperfect instruments. Therefore, the feedback signal of the output will be influenced by the measurement noise. The measurement noise would affect the results of CPA. Figure 3 shows the SISO control system structure under the consideration of the measurement noise. The raw process measurement ( $y_m(t)$ ) can be expressed as

$$y_m(t) = y(t) + \varepsilon(t) \quad (24)$$

where  $\varepsilon(t)$  is the measurement noise assumed to be normally distributed  $\varepsilon(t) \sim N(0, \rho^2)$ .  $\rho^2$  is the variance of measurement noise.

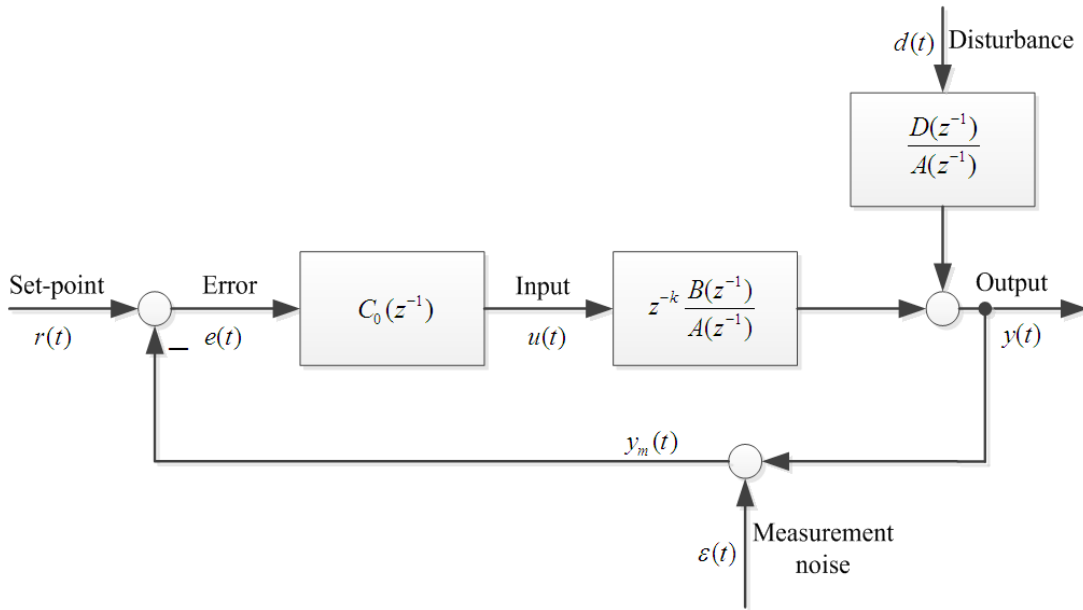


FIGURE 3. The SISO control system structure with measurement noise

Under the consideration of the measurement noise, the feedback signal of the output can be rewritten as

$$\begin{aligned} y_m(t+k) &= \frac{BF}{D}u(t) + \frac{G}{D}(y_m(t)) + Fd(t+k) \\ &= \frac{BF}{D}u(t) + \frac{G}{D}(y(t) + \varepsilon(t)) + Fd(t+k) \\ &= \left[ \frac{BF}{D}u(t) + \frac{G}{D}y(t) \right] + Fd(t+k) + \frac{G}{D}\varepsilon(t) \end{aligned} \quad (25)$$

The variance of feedback signal can be obtained as

$$\sigma_{y_m}^2 = E \left[ \left( \frac{BF}{D}u(t) + \frac{G}{D}y(t) \right)^2 \right] + E [(Fd(t+k))^2] + E \left[ \left( \frac{G}{D}\varepsilon(t) \right)^2 \right] \quad (26)$$

The feedback signal of the controller not being a minimum variance controller can be shown as this term

$$y_m(t+k) = \hat{y}(t) + Fd(t+k) + \frac{G}{D}\varepsilon(t) \tag{27}$$

The variance of feedback signal can be rewritten as

$$\sigma_{y_m}^2 = E [(\hat{y}(t))^2] + E [(Fd(t+k))^2] + E \left[ \left( \frac{G}{D}\varepsilon(t) \right)^2 \right] = \sigma_{soc}^2 + J_{MVC}(t) \tag{28}$$

where  $\sigma_{soc}^2 = E [(\hat{y}(t))^2] + E \left[ \left( \frac{G}{D}\varepsilon(t) \right)^2 \right]$ .

The controller performance index can be calculated as

$$\eta_m = \frac{J_{MVC}(t)}{\sigma_{soc}^2 + J_{MVC}(t)} \tag{29}$$

Note that the first term of  $\sigma_{soc}^2$  is generated by the controller not being a minimum variance controller, and the second term is generated by the measurement noise. Compared with Equation (11), Equation (29) indicates that the measurement noise would decrease the value of controller performance index ( $\eta_m < \eta$ ).

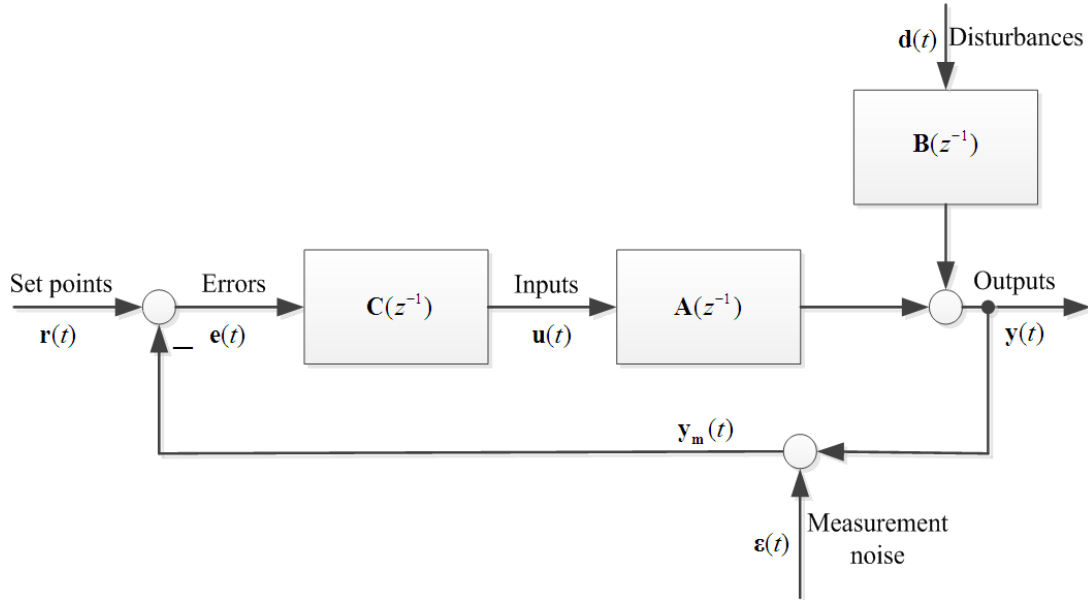


FIGURE 4. The stochastic multivariate system under the consideration of measurement noise

Likewise, the stochastic multivariate system with measurement noise can be described by Figure 4. The MIMO stochastic process model can be described by

$$\mathbf{y}(t) = \mathbf{A}(z^{-1})\mathbf{u}(t) + \mathbf{B}(z^{-1})\mathbf{d}(t) \tag{30}$$

where  $\mathbf{A}(z^{-1})$  and  $\mathbf{B}(z^{-1})$  are proper, rational transfer function matrices for the process plant and disturbances respectively;  $\mathbf{C}(z^{-1})$  is transfer function matrices for the controllers in Figure 4;  $\mathbf{y}(t)$  and  $\mathbf{u}(t)$  are the output vector and the input vector separately;  $\mathbf{d}(t)$  represents a white noise vector of disturbances with zero mean and covariance matrix  $\Sigma_d$ .

The vector of measured signals  $\mathbf{y}_m(t)$  can also be assumed to be adequately described by the additive noise model

$$\mathbf{y}_m(t) = \mathbf{y}(t) + \boldsymbol{\varepsilon}(t) \tag{31}$$

where  $\boldsymbol{\varepsilon}(t)$  is vector of the measurement noise, which is assumed to be normally distributed ( $\boldsymbol{\varepsilon}(t) \sim N(0, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}})$ ), where  $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$  is the covariance matrix of  $\boldsymbol{\varepsilon}(t)$ .

The controller uses the vector of measured signals  $\mathbf{y}_m(t)$  to tune the control law as follows

$$\mathbf{u}(t) = -\mathbf{C}(z^{-1})\mathbf{y}_m(t) = -\mathbf{C}(z^{-1})(\mathbf{y}(t) + \boldsymbol{\varepsilon}(t)) \quad (32)$$

The generalized outputs can be derived as

$$\begin{aligned} \mathbf{y}'(t) &= z^{-d}(\mathbf{A}'(z^{-1})\mathbf{u}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)) + \mathbf{F}(z^{-1})\mathbf{d}(t) \\ &= z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})(\mathbf{y}(t) + \boldsymbol{\varepsilon}(t)) + \mathbf{G}(z^{-1})\mathbf{d}(t)) + \mathbf{F}(z^{-1})\mathbf{d}(t) \end{aligned} \quad (33)$$

For an arbitrary linear multivariate controller, the following inequality yields

$$\begin{aligned} &tr[Cov(\mathbf{y}(t))] = tr[Cov(\mathbf{y}'(t))] \\ &= tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})(\mathbf{y}(t) + \boldsymbol{\varepsilon}(t)) + \mathbf{G}(z^{-1})\mathbf{d}(t)))] \\ &\quad + tr[Cov(\mathbf{F}(z^{-1})\mathbf{d}(t))] \\ &= tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})\mathbf{y}(t) + \mathbf{G}(z^{-1})\mathbf{d}(t)))] + tr[Cov(\mathbf{F}(z^{-1})\mathbf{d}(t))] \\ &\quad + tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})\boldsymbol{\varepsilon}(t)))] \end{aligned} \quad (34)$$

Therefore, the trace of the covariance of the outputs  $tr[Cov(\mathbf{y}(t))]$  considering measurement noise may be obtained as:

$$J_{Mm}(t) = tr[Cov(\mathbf{y}(t))] = J_m(t) + J_M(t) \quad (35)$$

where  $J_m(t) = tr[Cov(z^{-d}(-\mathbf{A}'(z^{-1})\mathbf{C}(z^{-1})\boldsymbol{\varepsilon}(t)))]$ .

The controller performance index can be calculated as

$$\eta_m = \frac{J_{MMVC}(t)}{J_m(t) + J_M(t)} = \frac{J_{MMVC}(t)}{J_m(t) + J_{M0}(t) + J_{MMVC}(t)} \quad (36)$$

Note that the first term on the right-hand side of Equation (35) is generated by the measurement noise, and the second term is generated by the process disturbances. Compared with Equation (23), Equation (36) indicates that the measurement noise would decrease the value of controller performance index ( $\eta_m < \eta$ ).

Considering measurement noise in the sensor device, the results of MVC based CPA will be gradually deteriorated from the above analysis. In order to reduce the influence of measurement noise and improve the results of MVC based CPA, the digital filter combined with MVC based CPA is proposed in the next section.

**4. Digital Filter Combined with MVC Based CPA.** In order to reduce the measurement noise, various classical digital filters have been designed. They have their own advantages as well as shortcomings. The digital filter provides a running average and absorbs some of the short term variations caused by noise. A trade-off between the amount of noise attenuation and the time delay after filtering is required to improve the performance of the digital filters. This can be accomplished by tuning the filter parameters [29]. Considering the complexity of implementation and the effectiveness of the filter, exponential filter is a relatively good filter, which can effectively decrease the measurement noise and make the output signal more stable. Besides, its implementation is simple. Therefore, exponential filter is used as the digital filter in this paper to reduce the influence of measurement noise.

The univariate system uses the digital filter in the control system to decrease the measurement noise, so the block diagram of the system can be shown by Figure 5. In Figure 5, the exponential filter for univariate system is added within the feedback loop. The



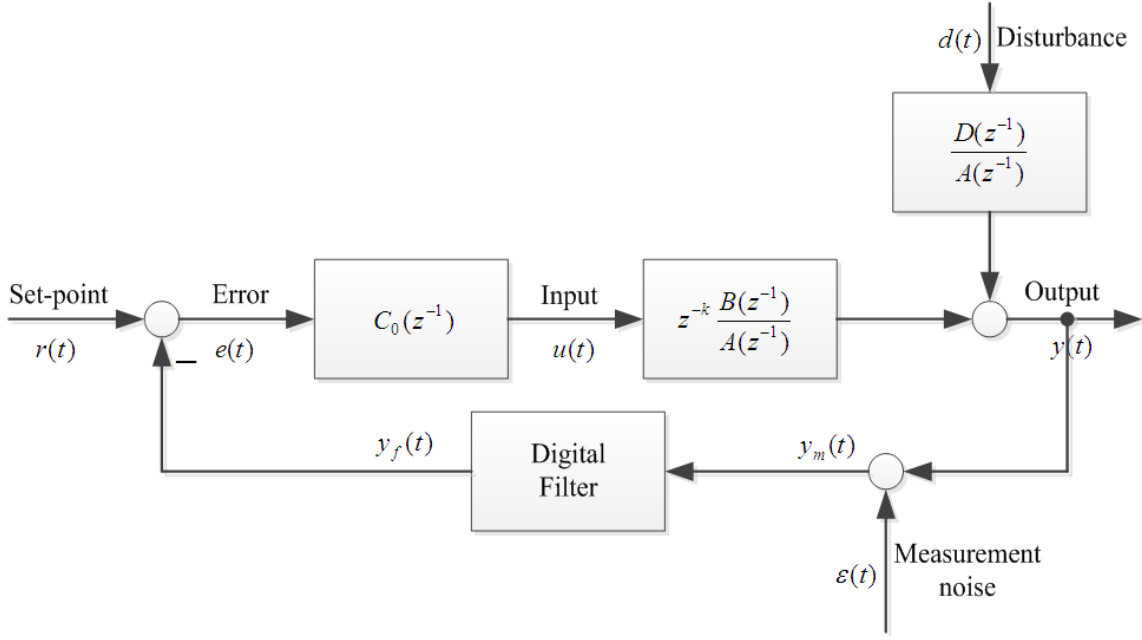


FIGURE 5. The SISO control system structure with digital filter

feedback signal is no longer the measured signal, but the filtered signal. The exponential filter can be described by the following equation:

$$y_f(t) = \theta y_m(t) + (1 - \theta)y_f(t - 1) \tag{37}$$

where  $y_f(t)$  is the filtered signal with initialization  $y_f(0) = y_m(0)$ .  $y_f(t - 1)$  is the previous value of the filtered signal.  $\theta$  is the filter parameter, which is limited to  $0 < \theta < 1$ .

By using digital filter, the variance of feedback signal can be obtained as

$$\sigma_{y_f}^2 = \theta^2 \sigma_{y_m}^2 + (1 - \theta)^2 \sigma_{y_f}^2 \tag{38}$$

Therefore,

$$\sigma_{y_f}^2 = \frac{\theta^2}{1 - (1 - \theta)^2} \sigma_{y_m}^2 = \frac{\theta}{2 - \theta} \sigma_{y_m}^2 \tag{39}$$

The filter parameter  $\theta$  is limited to  $0 < \theta < 1$ ; therefore,  $\sigma_{y_f}^2 < \sigma_{y_m}^2$ .

The filtered signal  $y_f(t)$  is assumed to be adequately described by the additive noise model,

$$y_f(t) = y(t) + \gamma(t) \tag{40}$$

where  $\gamma(t)$  is normally distributed ( $\gamma(t) \sim N(0, \sigma_{y_f}^2)$ ).

Under the consideration of digital filter, the feedback signal of the output can be rewritten as

$$\begin{aligned} y_f(t + k) &= \frac{BF}{D}u(t) + \frac{G}{D}(y(t) + \gamma(t)) + Fd(t + k) \\ &= \left[ \frac{BF}{D}u(t) + \frac{G}{D}y(t) \right] + Fd(t + k) + \frac{G}{D}\gamma(t) \end{aligned} \tag{41}$$

The variance of feedback signal can be obtained as

$$\sigma_{y_f}^2 = E \left[ \left( \frac{BF}{D}u(t) + \frac{G}{D}y(t) \right)^2 \right] + E [(Fd(t + k))^2] + E \left[ \left( \frac{G}{D}\gamma(t) \right)^2 \right] \tag{42}$$

Under the consideration of digital filter, the feedback signal of the controller not being a minimum variance controller can be shown as this term

$$y_f(t+k) = \hat{y}(t) + Fd(t+k) + \frac{G}{D}\gamma(t) \quad (43)$$

The variance of feedback signal can be rewritten as

$$\sigma_{y_f}^2 = E[(\hat{y}(t))^2] + E[(Fd(t+k))^2] + E\left[\left(\frac{G}{D}\gamma(t)\right)^2\right] = \sigma_{soc-f}^2 + J_{MVC}(t) \quad (44)$$

where  $\sigma_{soc-f}^2 = E[(\hat{y}(t))^2] + E\left[\left(\frac{G}{D}\gamma(t)\right)^2\right]$ .

The controller performance index can be calculated as

$$\eta_f = \frac{J_{MVC}(t)}{\sigma_{soc-f}^2 + J_{MVC}(t)} \quad (45)$$

Note that the first term of  $\sigma_{soc-f}^2$  is generated by the controller not being a minimum variance controller, and the second term is generated by the additive noise of the filtered signal.

Since  $\sigma_{y_f}^2 < \sigma_{y_m}^2$ , the following equation is satisfied comparing Equation (45) with Equation (29) and Equation (11).

$$\eta_m < \eta_f < \eta \quad (46)$$

Equation (46) indicates that the controller performance index by using digital filter can be more accurate than the controller performance index by using the measured signal. Under the consideration of digital filter, the influence of measurement noise on the MVC based CPA is effectively decreased.

Likewise, for the multivariate system, the block diagram of the system can be shown in Figure 6. In Figure 6, the exponential filter for multivariate system is added within the feedback loop. The vector of feedback signals is no longer the vector of measured

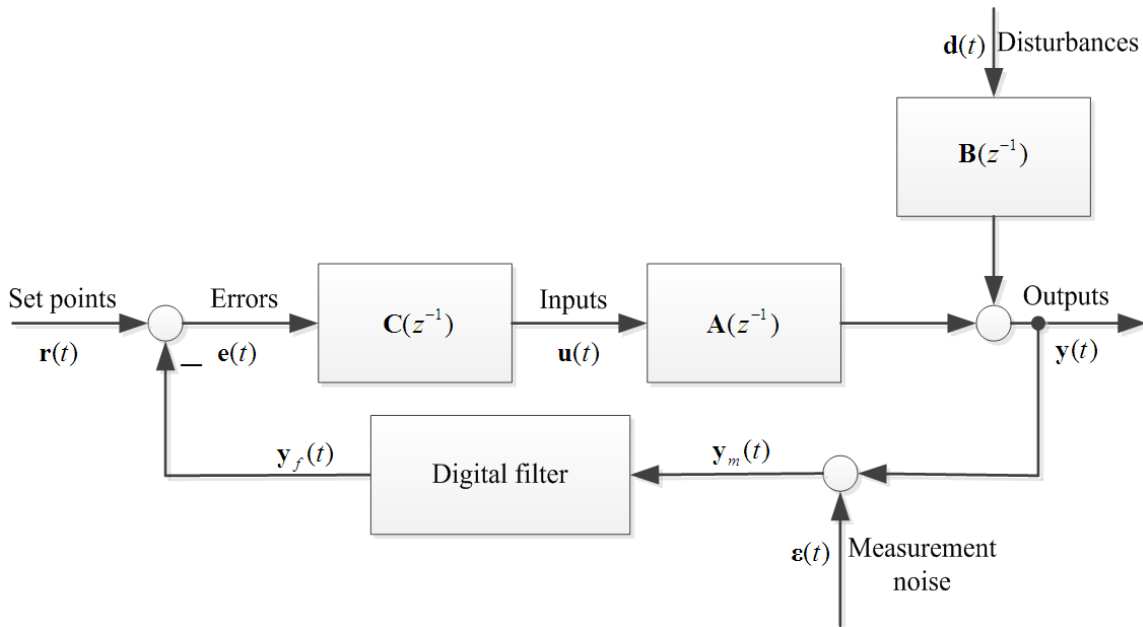


FIGURE 6. The MIMO control system structure with digital filter

signals, but the vector of the filtered signals. The vector of filtered signals  $\mathbf{y}_f(t)$  can also be described by

$$\mathbf{y}_f(t) = \theta \mathbf{y}_f(t) + (1 - \theta) \mathbf{y}_f(t - 1) \quad (47)$$

The vector of filtered signals  $\mathbf{y}_f(t)$  is still assumed to be adequately described by the additive noise model

$$\mathbf{y}_f(t) = \mathbf{y}(t) + \boldsymbol{\varepsilon}_f(t) \quad (48)$$

where  $\boldsymbol{\varepsilon}_f(t)$  is normally distributed ( $\boldsymbol{\varepsilon}_f(t) \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}_f)$ ) and  $\boldsymbol{\Sigma}_f$  is the corresponding covariance matrix derived from Equation (48),

$$\boldsymbol{\Sigma}_f = \frac{\theta^2}{1 - (1 - \theta)^2} \boldsymbol{\Sigma}_\varepsilon = \frac{\theta}{2 - \theta} \boldsymbol{\Sigma}_\varepsilon \quad (49)$$

As the parameter  $\theta$  has a range of  $0 < \theta \leq 1$ , the trace of the covariance matrix  $\boldsymbol{\Sigma}_f$  is smaller than the trace of the covariance matrix  $\boldsymbol{\Sigma}_\varepsilon$  of the measured signals. With the consideration of the exponential filter, the controller uses the vector of feedback signals  $\mathbf{y}_f(t)$  to tune the control law as follows.

$$\mathbf{u}(t) = -\mathbf{C}(z^{-1}) \mathbf{y}_f(t) = -\mathbf{C}(z^{-1}) (\mathbf{y}(t) + \boldsymbol{\varepsilon}_f(t)) \quad (50)$$

The vector of generalized outputs can be derived as

$$\begin{aligned} \mathbf{y}'(t) &= z^{-d} (\mathbf{A}'(z^{-1}) \mathbf{u}(t) + \mathbf{G}(z^{-1}) \mathbf{d}(t)) + \mathbf{F}(z^{-1}) \mathbf{d}(t) \\ &= z^{-d} (-\mathbf{A}'(z^{-1}) \mathbf{C}(z^{-1}) (\mathbf{y}(t) + \boldsymbol{\varepsilon}_f(t)) + \mathbf{G}(z^{-1}) \mathbf{d}(t)) + \mathbf{F}(z^{-1}) \mathbf{d}(t) \end{aligned} \quad (51)$$

For an arbitrary linear multivariate controller, the following inequality yields

$$\begin{aligned} &tr[Cov(\mathbf{y}(t))] = tr[Cov(\mathbf{y}'(t))] \\ &= tr [Cov (z^{-d} (-\mathbf{A}'(z^{-1}) \mathbf{C}(z^{-1}) (\mathbf{y}(t) + \boldsymbol{\varepsilon}_f(t)) + \mathbf{G}(z^{-1}) \mathbf{d}(t)))] \\ &\quad + tr [Cov (\mathbf{F}(z^{-1}) \mathbf{d}(t))] \\ &= tr [Cov (z^{-d} (-\mathbf{A}'(z^{-1}) \mathbf{C}(z^{-1}) \mathbf{y}(t) + \mathbf{G}(z^{-1}) \mathbf{d}(t)))] + tr [Cov (\mathbf{F}(z^{-1}) \mathbf{d}(t))] \\ &\quad + tr [Cov (z^{-d} (-\mathbf{A}'(z^{-1}) \mathbf{C}(z^{-1}) \boldsymbol{\varepsilon}_f(t)))] \end{aligned} \quad (52)$$

Therefore, the trace of the covariance of the outputs  $tr[Cov(\mathbf{y}(t))]$  with digital filter may be obtained as:

$$J_{Mm}(t) = tr[Cov(\mathbf{y}(t))] = J_f(t) + J_M(t) \quad (53)$$

where  $J_m(t) = tr [Cov (z^{-d} (-\mathbf{A}'(z^{-1}) \mathbf{C}(z^{-1}) \boldsymbol{\varepsilon}_f(t)))]$ .

The controller performance index can be calculated as

$$\eta_f = \frac{J_{MMVC}(t)}{J_f(t) + J_M(t)} = \frac{J_{MMVC}(t)}{J_f(t) + J_{M0}(t) + J_{MMVC}(t)} \quad (54)$$

The following equation is satisfied comparing Equation (54) with Equation (36) and Equation (23).

$$\eta_m < \eta_f < \eta \quad (55)$$

Equation (55) also indicates that the controller performance index by using digital filter can be more accurate than the controller performance index by using the measured signal.

In the next section, the effectiveness of the proposed method is illustrated by the simulations of both univariate and multivariate control systems and the experiments of a constant-current constant-frequency DC/AC converter.

**5. Case Studies.** In order to illustrate the effectiveness of the digital filter combined with MVC based CPA, three case studies are presented in this section. The classical univariate and multivariate control systems in Case Study 1 and Case Study 2 are widely used in the literature for CPA. Therefore, they are simulated to investigate the performance of the digital filter combined with MVC based CPA. In Case Study 3, the constant-current constant-frequency DC/AC converter is widely used in power electronics. The CPA for the DC-AC converter is very important because a high precise feedback control scheme is required. Therefore, the digital filter combined with MVC based CPA is applied in a constant-current constant-frequency DC/AC converter. Experimental results can illustrate the effectiveness of the digital filter combined with MVC based CPA.

The extant techniques in [8,30] do not explicitly consider the influence of measurement noise and use the measurement data for CPA. In the case studies, the CPA with MVC based on the measured signal (the traditional method) is also conducted for comparison. The comparison of the results can illustrate the effectiveness of the proposed method.

**5.1. Case Study 1: SISO control system.** The example is taken from [30,31] and selected for evaluation and comparison of the proposed strategies. The SISO control system of this illustrative example is shown in Figure 7. The plant considered in this example has been discretized and it includes a time delay of two sampling intervals. The process ARMAX model can be described as follows.

$$y(t) = z^{-3} \frac{0.08}{1 - 0.92z^{-1}} u(t) + \frac{1}{1 - z^{-1}} d(t) \quad (56)$$

where  $d(t)$  is the zero mean Gaussian random variable, which has a standard deviation of  $\sigma = 0.1$ . The actual MVC controller can be calculated analytically and it is selected to be a PI controller [30,31],  $C_0(z^{-1}) = \frac{k_1 + k_2 z^{-1}}{1 - z^{-1}}$ , where  $k_1 = \frac{1}{3*0.08}$ ,  $k_2 = \frac{-0.92}{3*0.08}$ . The magnitude of measurement noise is generally smaller than that of process disturbance. Therefore, the standard deviation  $\rho$  of the measurement noise is firstly assumed to be 0.03. And then, the standard deviation  $\rho$  is increased to be 0.1 slowly in order to investigate the influence of the different magnitudes of measurement noise on the results of CPA. Table

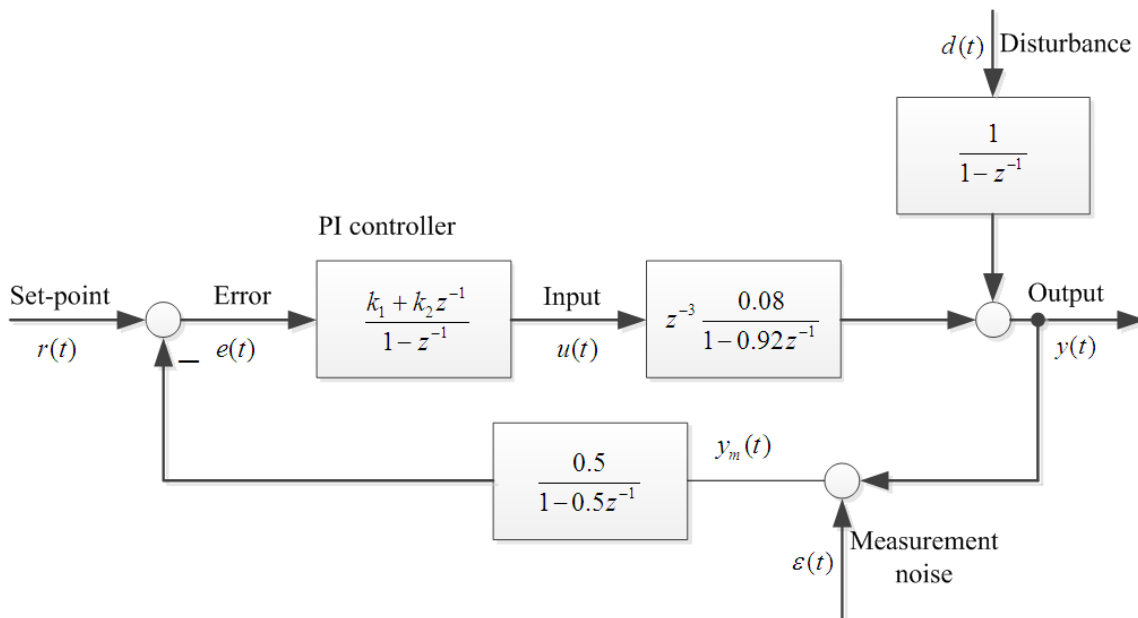


FIGURE 7. The SISO control system structure in Case Study 1

TABLE 1. Comparison results of CPA in Case Study 1

Standard deviation of measurement noise	CPA based on ideal signal: $\eta$	CPA based on measured signal: $\eta_m$	CPA based on filtered signal: $\eta_f$
0.03	1	0.7395	0.9642
0.04	1	0.6998	0.9365
0.05	1	0.666	0.9118
0.06	1	0.6365	0.8893
0.07	1	0.6103	0.8685
0.08	1	0.5869	0.8493
0.09	1	0.5656	0.8312
0.1	1	0.5416	0.8142

1 summarizes the comparison of the results of CPA with MVC based on the measured signal and the filtered signal in this case study.

Since the parameters of the PI controller are selected to be the optimal parameters of the MVC controller and do not change in the simulations, the value of controller performance index of CPA based on ideal signal (feedback signal without measurement noise) is 1.0. Table 1 shows that the results of CPA based on the measured signal are severely affected by the measurement noise. When the standard deviation of the measurement noise increases larger, the value of controller performance index of CPA becomes worse. The results of CPA based on the measured signal cannot actually indicate the real performance of the controller. However, the results of CPA based on filtered signal are more accurate compared with that based on measured signal. As the standard deviation of the measurement noise increases larger, the value of controller performance index of CPA based on filtered signal changes a little. Obviously, the use of digital filter can improve the results of CPA.

**5.2. Case Study 2: MIMO control system.** The MIMO control system of this illustrative example is taken from [31] and is shown in Figure 8. The process ARMAX model can be described as follows.

$$\begin{cases} y_1(t) = P_1 u_1(t) + N_1 d_1(t) + P_2 u_2(t) + N_2 d_2(t) \\ y_2(t) = P_3 u_1(t) + N_3 d_1(t) + P_4 u_2(t) + N_4 d_2(t) \end{cases} \quad (57)$$

where  $d_1(t)$  and  $d_2(t)$  are the zero mean Gaussian random variables, which have a standard deviation of  $\sigma = 0.1$ . There are two outputs  $y_1(t)$ ,  $y_2(t)$  and control signals  $u_1(t)$ ,  $u_2(t)$ . Other information of the control system is shown as:

$$P(z^{-1}) = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} = \begin{bmatrix} \frac{z^{-1}}{1 - 0.4z^{-1}} & \frac{4z^{-2}}{1 - 0.1z^{-1}} \\ \frac{0.3z^{-1}}{1 - 0.1z^{-1}} & \frac{z^{-2}}{1 - 0.8z^{-1}} \end{bmatrix} \quad (58)$$

$$N(z^{-1}) = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{1 - 0.5z^{-1}} & \frac{-0.6z^{-2}}{1 - 0.5z^{-1}} \\ \frac{0.5z^{-1}}{1 - 0.5z^{-1}} & \frac{1}{1 - 0.5z^{-1}} \end{bmatrix} \quad (59)$$

$$C(z^{-1}) = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} = \begin{bmatrix} \frac{0.5 - 0.2z^{-1}}{1 - 0.5z^{-1}} & 0 \\ 0 & \frac{0.5 - 0.2z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} \end{bmatrix} \quad (60)$$

$$F(z^{-1}) = \begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix} = \begin{bmatrix} \frac{0.85}{1 - 0.15z^{-1}} & 0 \\ 0 & \frac{0.85}{1 - 0.15z^{-1}} \end{bmatrix} \quad (61)$$

The standard deviations of the measurement noise are assumed to be 0.03. And then, both of the standard deviations are increased to be 0.1 slowly. Table 2 summarizes the comparison of the results of CPA with MVC based on the measured signals and the filtered signals in this case study.

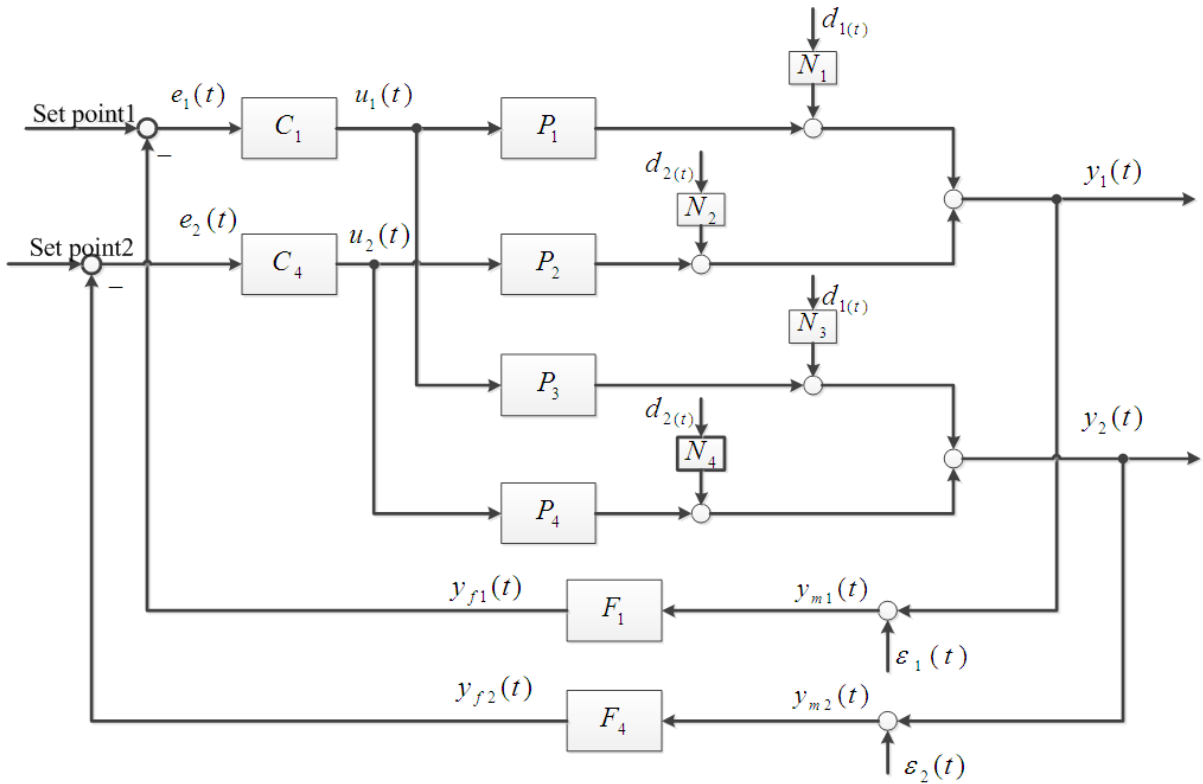


FIGURE 8. The MIMO control system structure in Case Study 2

As can be seen from Table 2 that the results of CPA based on the measured signals are also severely affected by the measurement noise. When the standard deviations of the measurement noise increase larger, the values of controller performance index of CPA will become worse. The results of CPA based on filtered signals are more accurate compared with that based on measured signals. Compared with the results of CPA in SISO control system in Case Study 1, it is illustrated that the measurement noise can deteriorate the results of CPA more severely in MIMO control system. The use of digital filter can also improve the results of CPA in MIMO control system. However, when the standard deviations of the measurement noise increase too large, the results of CPA based on filtered signals are also deteriorated severely. The results of CPA in MIMO control system are more sensitive with the measurement noise.

TABLE 2. Comparison results of CPA in Case Study 2

Standard deviation of measurement noise	CPA based on ideal signal: $\eta$	CPA based on measured signal: $\eta_m$	CPA based on filtered signal: $\eta_f$
0.03	1	0.4728	0.9189
0.04	1	0.4283	0.8379
0.05	1	0.3938	0.7745
0.06	1	0.3660	0.7229
0.07	1	0.3428	0.6796
0.08	1	0.3231	0.6425
0.09	1	0.3060	0.6102
0.1	1	0.2910	0.5818

**5.3. Case Study 3: Constant-current constant-frequency DC-AC converter.**

Constant-current constant-frequency DC-AC converters are widely used in AC power-conditioning systems such as uninterruptible power supplies, grid connected photovoltaic systems and other industrial facilities [32]. In order to improve the performance of the DC-AC converter, high precise feedback control schemes are proposed. In this section, a fast sampling rate feedback control scheme is used to improve transient response and tracking accuracy. Figure 9 shows the feedback control scheme for the DC-AC converter, where the set point is  $r_{\sin}(t)$ , which is an ideal constant-current constant-frequency sine signal; the output signal of the DC-AC converter is  $y_{\sin}(t)$ .  $C_0(z^{-1})$  is the feedback controller, which is designed to be a PID controller in the experiments.  $G_p(z^{-1})$  is the transfer function of the plant. A general output signal is defined as

$$y(t) = y_{\sin}(t) - r_{\sin}(t) \tag{62}$$

The general set point is transformed to be zero. The experiments of CPA based on the measured signal and the filtered signal are implemented in the device, which is shown in Figure 10.

When the feedback signal is based on the measured signal, the measured output signal of the DC-AC converter is shown in Figure 11, and the measured general output signal is shown in Figure 12. As can be seen from Figure 11 and Figure 12 that the measured signal contains measurement noise, which severely influences the results of CPA. The

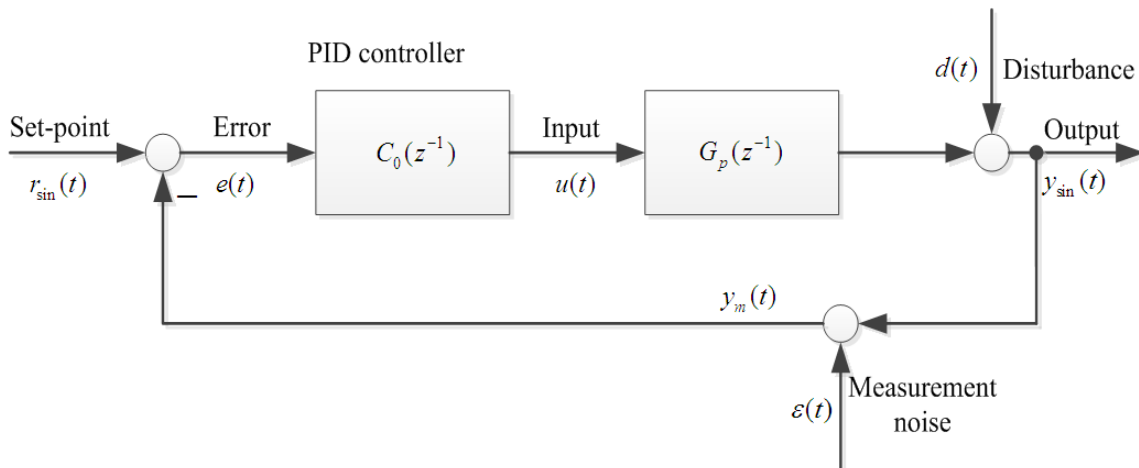


FIGURE 9. The feedback control scheme for the DC-AC converter



FIGURE 10. The device for the experiments of CPA based on the measured signal and the filtered signal

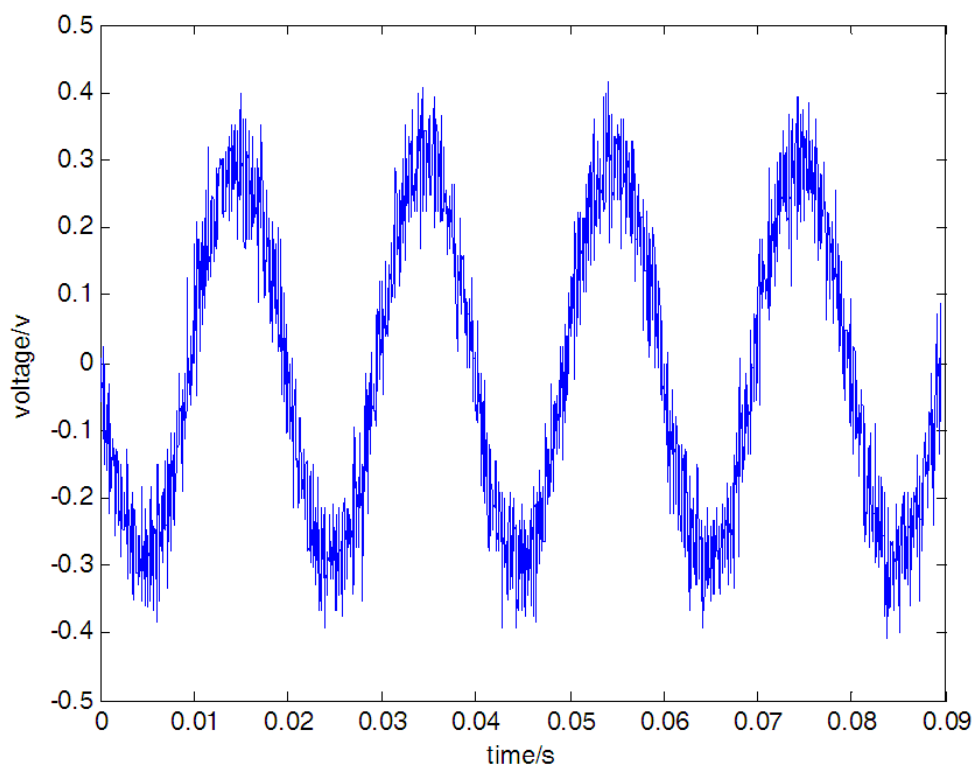


FIGURE 11. The measured output signal of the DC-AC converter

value of controller performance index of CPA based on measured general output signal is only 0.3196, which does not actually indicate the real performance of the controller. When the feedback signal is based on the filtered signal, the filtered output signal of the DC-AC converter is shown in Figure 13, and the filtered general output signal is shown in Figure 14. Based on the results of filtered general output signal, the value of controller performance index of CPA is 0.5341. The results of experiments show that the measurement noise would blemish the performance of controller and result in inaccurate CPA. With the use of digital filter, the influence of measurement noise on the performance



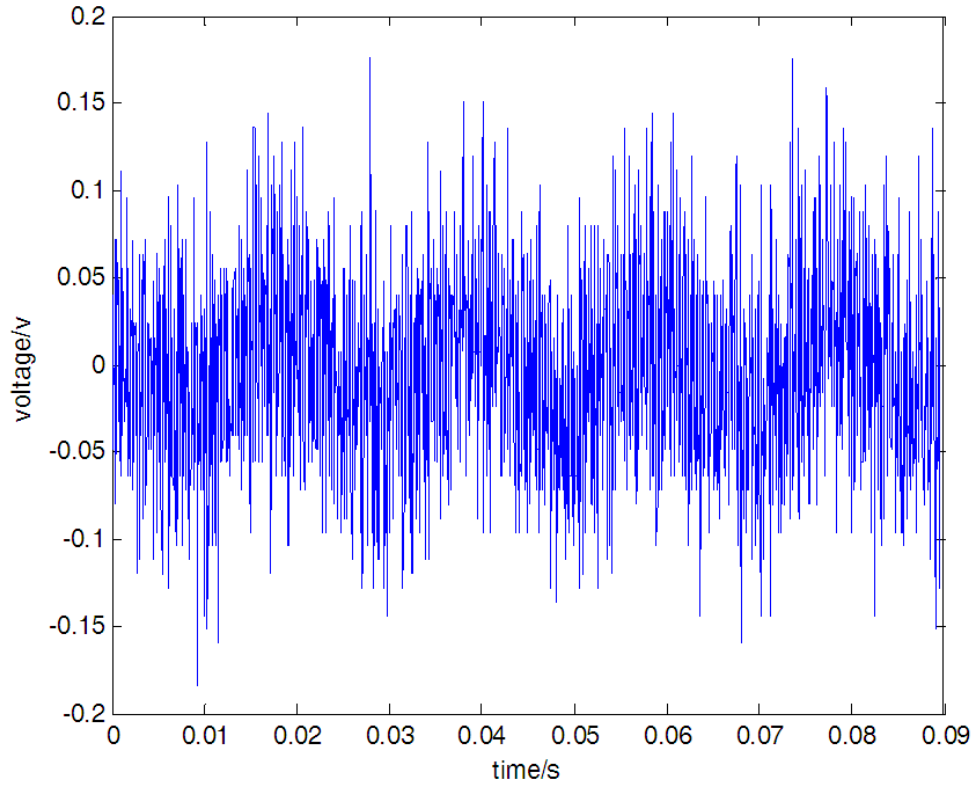


FIGURE 12. The measured general output signal of the DC-AC converter

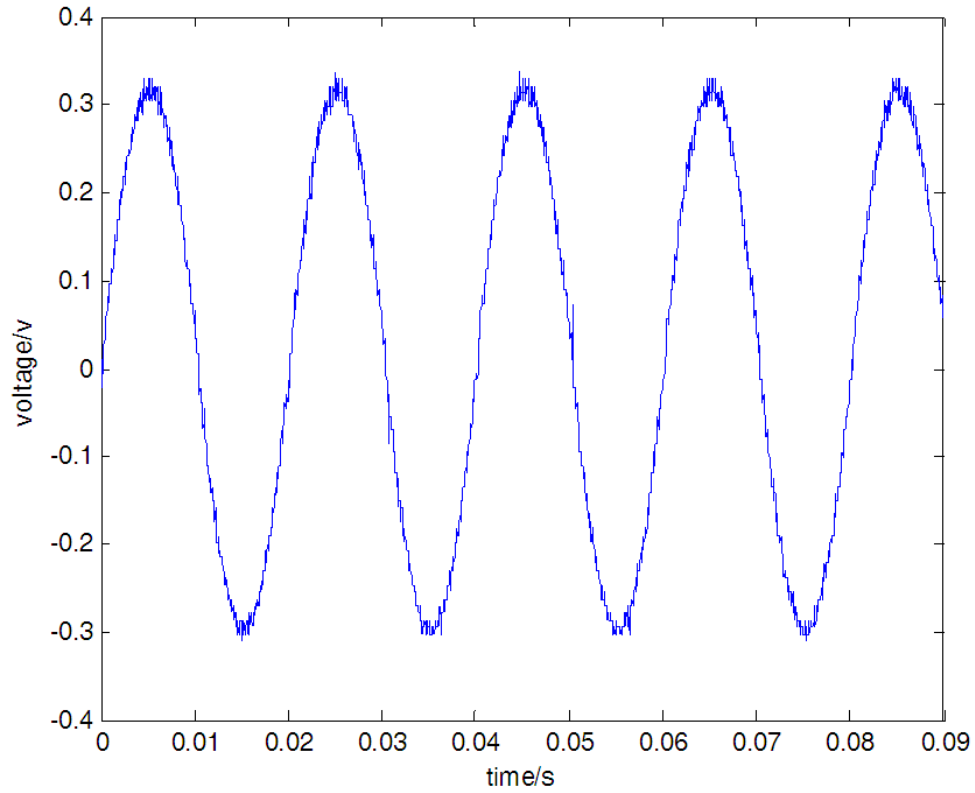


FIGURE 13. The filtered output signal of the DC-AC converter

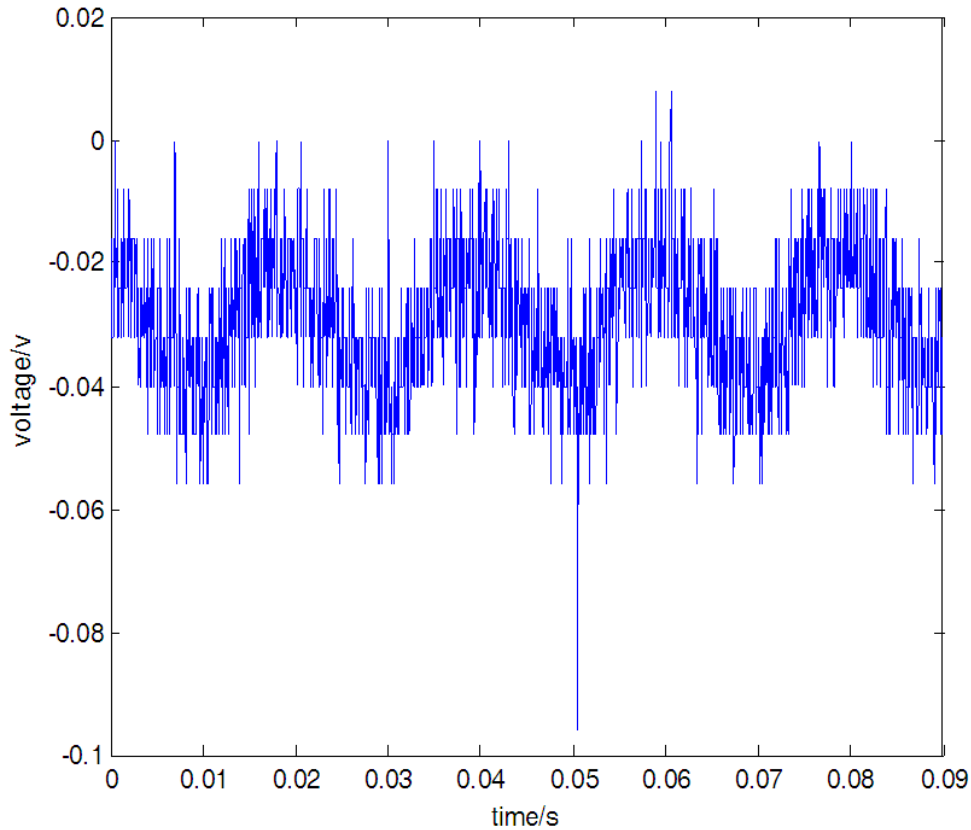


FIGURE 14. The filtered general output signal of the DC-AC converter

of controller would be decreased. The accuracy of CPA based on filtered signal would be improved.

**6. Conclusions.** This paper firstly introduces the method of CPA based on MVC. The influence of the measurement noise on the results of MVC based CPA is analyzed through derivation. The digital filter combined with MVC based CPA is proposed to be used to reduce the influence of measurement noise on the results of CPA. In order to illustrate the effectiveness of the digital filter combined with MVC based CPA, three case studies are presented in Case Study 1 and Case Study 2, and both univariate and multivariate control systems are simulated. Results of simulations show that the use of digital filter can improve the results of CPA in Case Study 3, and the digital filter combined with MVC based CPA is applied in a constant-current constant-frequency DC/AC converter. Experimental results show the measurement noise would blemish the performance of controller and result in inaccurate CPA. With the use of digital filter, the accuracy of CPA can be improved. Since measurement information may also be corrupted by gross errors, usually caused by malfunctioning instruments, measurement device biases or process deficiencies. The gross errors introduce inaccurate information and also deteriorate the results of CPA. Further work will be concentrated on how to improve the results of CPA considering the influence of gross errors.

**Acknowledgment.** This work is partially supported by the National Natural Science Foundation of China (No. 61703309), the Science and Technology Planning Project of Zhejiang Province (No. 2015C311157; 2014NM005), and the Program of “Xinmiao” Talents in Zhejiang Province (No. 2017R426019). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

## REFERENCES

- [1] Z. Yan, C. L. Chan and Y. Yao, Multivariate control performance assessment and control system monitoring via hypothesis test on output covariance matrices, *Industrial & Engineering Chemistry Research*, vol.54, no.19, pp.5261-5272, 2015.
- [2] K. A. Hoo, M. J. Piovoso, P. D. Schnelle and D. A. Rowan, Process and controller performance monitoring: Overview with industrial applications, *International Journal of Adaptive Control & Signal Processing*, vol.17, nos.7-9, pp.635-662, 2003.
- [3] M. A. Paulonis and J. W. Cox, A practical approach for large-scale controller performance assessment, diagnosis, and improvement, *Journal of Process Control*, vol.13, no.2, pp.155-168, 2003.
- [4] M. Jelali, An overview of control performance assessment technology and industrial applications, *Control Engineering Practice*, vol.14, no.5, pp.441-466, 2006.
- [5] M. Bauer, A. Horch, L. Xie, M. Jelali and N. Thornhill, The current state of control loop performance monitoring – A survey of application in industry, *Journal of Process Control*, vol.38, pp.1-10, 2016.
- [6] Z. Li and R. J. Evans, Minimum-variance control of linear time-varying systems, *Automatica*, vol.33, no.8, pp.1531-1537, 1997.
- [7] K. J. Åström, *Introduction to Stochastic Control Theory*, Academic Press, New York, 1970.
- [8] T. J. Harris, Assessment of control loop performance, *The Canadian Journal of Chemical Engineering*, vol.67, no.5, pp.856-861, 1989.
- [9] J. Chen and C. K. Kong, Performance assessment for iterative learning control of batch units, *Journal of Process Control*, vol.19, no.6, pp.1043-1053, 2009.
- [10] B. S. Ko and T. F. Edgar, Performance assessment of multivariable feedback control systems, *Automatica*, vol.37, no.6, pp.899-905, 2001.
- [11] B. Huang, Minimum variance control and performance assessment of time-variant processes, *Journal of Process Control*, vol.12, no.6, pp.707-719, 2002.
- [12] F. B. Olaleye, B. Huang and E. Tamayo, Feedforward and feedback controller performance assessment of linear time-variant processes, *Industrial & Engineering Chemistry Research*, vol.43, no.2, pp.589-596, 2004.
- [13] W. Zhang, X. Wang and Z. L. Wang, Performance assessment of control loop with time-variant disturbance dynamics based on multi-model mixing minimum variance control, *Acta Automatica Sinica*, vol.9, pp.2037-2044, 2014.
- [14] T. Sato and A. Inoue, Improvement of tracking performance in self-tuning PID controller based on generalized predictive control, *International Journal of Innovative Computing, Information and Control*, vol.2, no.3, pp.491-503, 2006.
- [15] R. S. Patwardhan, S. L. Shah and K. Z. Qi, Assessing the performance of model predictive controllers, *The Canadian Journal of Chemical Engineering*, vol.80, no.5, pp.954-966, 2002.
- [16] C. A. Harrison and S. J. Qin, Minimum variance performance map for constrained model predictive control, *Journal of Process Control*, vol.19, no.7, pp.1199-1204, 2009.
- [17] P. D. Domański and M. Ławryńczuk, Assessment of predictive control performance using fractal measures, *Nonlinear Dynamics*, vol.89, no.2, pp.773-790, 2017.
- [18] A. Y. Sendjaja and V. Kariwala, Minimum variance benchmark for performance assessment of decentralized controllers, *Industrial & Engineering Chemistry Research*, vol.51, no.11, pp.4288-4298, 2012.
- [19] C. Kulcsár, H. F. Raynaud, C. Petit and J. M. Conan, Minimum variance prediction and control for adaptive optics, *Automatica*, vol.48, no.9, pp.1939-1954, 2012.
- [20] Y. Alipouri and J. Poshtan, A linear approach to generalized minimum variance controller design for MIMO nonlinear systems, *Nonlinear Dynamics*, vol.77, no.3, pp.935-949, 2014.
- [21] M. J. Grimble and P. Majecki, Non-linear generalised minimum variance control using unstable state-dependent multivariable models, *IET Control Theory & Applications*, vol.7, no.4, pp.551-564, 2013.
- [22] L. Das, B. Srinivasan and R. Rengaswamy, A novel framework for integrating data mining with control loop performance assessment, *AIChE Journal*, vol.62, no.1, pp.146-165, 2016.
- [23] S. A. Khamseh, A. K. Sedigh, B. Moshiri and A. Fatehi, Control performance assessment based on sensor fusion techniques, *Control Engineering Practice*, vol.49, pp.14-28, 2016.
- [24] Y. Wang, H. Zhang, S. Wei, D. Zhou and B. Huang, Control performance assessment for ILC-controlled batch processes in a 2-D system framework, *IEEE Trans. Systems, Man, and Cybernetics: Systems*, 2017.

- [25] M. J. Grimble, Controller performance benchmarking and tuning using generalised minimum variance control, *Automatica*, vol.38, no.12, pp.2111-2119, 2002.
- [26] Y. Zhao, H. Su, J. Chu and Y. Gu, Multivariable control performance assessment based on generalized minimum variance benchmark, *Chinese Journal of Chemical Engineering*, vol.18, no.1, pp.86-94, 2010.
- [27] B. Huang and S. L. Shah, *Performance Assessment of Control Loops: Theory and Applications*, Springer, Berlin, 1999.
- [28] Z. Zhu, Z. Meng, T. Cao, Z. Zhang and Y. Dai, Particle filter-based robust state and parameter estimation for nonlinear process systems with variable parameters, *Measurement Science and Technology*, vol.28, no.6, 2017.
- [29] S. Narasimhan and C. Jordache, *Data Reconciliation and Gross Error Detection: An Intelligent Use of Process Data*, Gulf Professional Publishing, Houston, 2000.
- [30] A. J. Hugo, Performance assessment of single-loop industrial controllers, *Journal of Process Control*, vol.16, no.8, pp.785-794, 2006.
- [31] D. Uduehi, A. Ordys, M. Grimble, P. Majecki and H. Xia, Controller benchmarking procedures – Data-driven methods, *Process Control Performance Assessment*, Springer, London, 2007.
- [32] Y. Ye, Y. Wu, G. Xu and B. Zhang, Cyclic repetitive control of CVCF PWM DC-AC converters, *IEEE Trans. Industrial Electronics*, vol.64, no.12, pp.9399-9409, 2017.