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REDUCTION OF NOISE IN ON-DEMAND TYPE FEEDBACK CONTROL SYSTEMS

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Abstract. *This paper explores the ability of reducing noise in on-demand type feedback control system. A controller, i.e., generalized minimum variance control (GMVC) can be extended by using coprime factorization. The extended controller has a new design parameter, and the parameter can select the characteristic of the extended controller without changing the closed-loop characteristic. Focusing on feedback signal, the proposed controller can drive the magnitude of the feedback signal to zero if the control object was achieved. In other words, the feedback signal by the proposed method can appear on demand of achieving the control object. In order to consider the application in industry, this paper newly defines the design parameter of on-demand type feedback controller to reduce noise. A numerical example is given in order to check the characteristic of the proposed method.*

Keywords: On-demand type feedback controller, Coprime factorization, Noise reduction

1. **Introduction.** Generalized minimum variance control (GMVC) has been proposed by Clarke et al. [1]. GMVC is one of the control methods for application in industry. This control method uses generalized output which is selected to make the closed-loop system stable. The control law is derived to minimize the variance of generalized output. Once the generalized output is determined, the derived controller cannot be re-designed without changing the closed-loop system. In the case of considering the application to industry, it is desirable that both of the closed-loop system and the controller are stable in the view point of safety. Authors have proposed the extended GMVC design method [2, 3]. The extended method introduces a new design parameter into conventional GMVC by using Youla-Kucera parameterization [4]. In the method, the poles of controller can be re-designed without changing the poles of closed-loop system by its parameter. Therefore, a strong stability system, which means that both closed-loop system and controller are stable, can be obtained by re-designing stable controller. Although the authors have proposed such a design method [5] and a concept of strong stability rate [6, 7], these researches have not focused on feedback signal clearly. Under the assumption that the controlled plant is stable, the research about strong stability rate has focused on a stable open-loop output. For example, if the value of strong stability rate becomes one, the controlled output becomes equal to reference signal in the steady state whether the feedback loop is cut or not. This situation indicates that the control object is achieved and the feedback signal is not demanded (that is, the feedback signal becomes zero) in the steady state. In other words, new concept controller named as on-demand type feedback controller, whose feedback signal emerges according to the demand to make the controlled output follow the reference signal and disappears if the controlled output becomes equal to the reference

signal, can be considered [8]. In the proposed method, the role and the benefit that the feedback signal disappears contribute to designing safe systems because the output of the proposed system does not diverge even if the feedback signal becomes zero by an accident. In this paper, on-demand type feedback controller is modified for noisy environment. The control method to construct on-demand type feedback controller is GMVC in this paper. A numerical example is shown in order to verify the effectiveness of the proposed method.

This paper is organized as follows. Section 2 describes problem statement and conventional GMVC. Section 3 extends GMVC through coprime factorization and gives the proposed controller. Section 4 shows a numerical example to verify the effectiveness of on-demand type feedback controller. Section 5 summarizes the result of this paper.

Notations and Assumptions. z^{-1} means backward shift operator $z^{-1}y(t) = y(t-1)$. $A\left[z^{-1}\right]$ and $A\left(z^{-1}\right)$ mean polynomial and rational functions with z^{-1} respectively. This paper assumes that the controlled plant is stable. Steady state gain $A(1)$ of transfer function is calculated as $z^{-1} = 1$ under the assumption that input and output signals do not change with regard to time t.

2. **Problem Statement and Conventional GMVC.** A single-input single-output system is considered.

$$A\left[z^{-1}\right] y(t) = z^{-k_m} B\left[z^{-1}\right] u(t) + C\left[z^{-1}\right] \xi(t) \qquad (1)$$
$$t = 0, 1, 2, \ldots$$

$u(t)$ and $y(t)$ are input and output respectively. k_m is time delay, and $\xi(t)$ is white Gaussian noise with zero mean. $A\left[z^{-1}\right]$, $B\left[z^{-1}\right]$ and $C\left[z^{-1}\right]$ are the polynomials with degrees n, m and l.

$$\begin{aligned} A\left[z^{-1}\right] &= 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \\ B\left[z^{-1}\right] &= b_0 + b_1 z^{-1} + \cdots + b_m z^{-m} \\ C\left[z^{-1}\right] &= 1 + c_1 z^{-1} + \cdots + c_l z^{-l} \end{aligned} \qquad (2)$$

On the system (1) the following assumptions hold.

[A.1] The degrees n, m and l, and the time delay k_m are known.
[A.2] The coefficients of $A\left[z^{-1}\right]$, $B\left[z^{-1}\right]$ and $C\left[z^{-1}\right]$ are known.
[A.3] The polynomials $A\left[z^{-1}\right]$ and $B\left[z^{-1}\right]$, $A\left[z^{-1}\right]$ and $C\left[z^{-1}\right]$ are coprime.
[A.4] The polynomial $C\left[z^{-1}\right]$ is stable.

The control object is to make the output $y(t)$ follow the reference signal $w(t)$. To achieve this object, performance index J averaged over the noise is minimized.

$$\Phi(t+k_m) = P\left[z^{-1}\right] y(t+k_m) + Q\left[z^{-1}\right] u(t) - R\left[z^{-1}\right] w(t) \qquad (3)$$
$$J = \mathrm{E_X}\left[\Phi^2(t+k_m)\right] \qquad (4)$$

$\Phi(t+k_m)$ means generalized output. $P\left[z^{-1}\right]$, $Q\left[z^{-1}\right]$ and $R\left[z^{-1}\right]$ are polynomials with degrees of n_p, n_q and n_r. These polynomials are selected to obtain stable closed-loop poles. In the conventional GMVC, Diophantine equation is given for the solutions $E\left[z^{-1}\right]$ and $F\left[z^{-1}\right]$.

$$P\left[z^{-1}\right] C\left[z^{-1}\right] = A\left[z^{-1}\right] E\left[z^{-1}\right] + z^{-k_m} F\left[z^{-1}\right] \qquad (5)$$

where

$$E\left[z^{-1}\right] = 1 + e_1 z^{-1} + \cdots + e_{k_m-1} z^{-(k_m-1)} \qquad (6)$$
$$F\left[z^{-1}\right] = f_0 + f_1 z^{-1} + \cdots + f_{n_1} z^{-n_1} \qquad (7)$$
$$n_1 = \max\{n-1, n_p + l - k_m\} \qquad (8)$$

The solution $E[z^{-1}]$ of Diophantine equation is used to calculate the following polynomial $G[z^{-1}]$. $T[z^{-1}]$ gives the closed-loop characteristics.

$$G[z^{-1}] = E[z^{-1}]B[z^{-1}] + C[z^{-1}]Q[z^{-1}] \tag{9}$$
$$T[z^{-1}] = P[z^{-1}]B[z^{-1}] + Q[z^{-1}]A[z^{-1}] \tag{10}$$

From (5) and (9), the generalized output and its prediction $\hat{\Phi}(t+k_m|t)$ can be given.

$$\Phi(t+k_m) = \hat{\Phi}(t+k_m|t) + E[z^{-1}]\xi(t+k_m) \tag{11}$$
$$\hat{\Phi}(t+k_m|t) = \left(F[z^{-1}]y(t) + G[z^{-1}]u(t) - C[z^{-1}]R[z^{-1}]w(t)\right)/C[z^{-1}] \tag{12}$$

Since $\hat{\Phi}(t+k_m|t)$ and the noise term $E[z^{-1}]\xi(t+k_m)$ have no correlation each other, the control law $u(t)$ minimizing J can be obtained by the following equation.

$$\hat{\Phi}(t+k_m|t) = 0 \tag{13}$$

Then the control law is obtained as,

$$u(t) = \frac{C[z^{-1}]R[z^{-1}]}{G[z^{-1}]}w(t) - \frac{F[z^{-1}]}{G[z^{-1}]}y(t) \tag{14}$$

The closed-loop system for (14) can be given as,

$$y(t) = \frac{z^{-k_m}B[z^{-1}]R[z^{-1}]}{T[z^{-1}]}w(t) + \frac{G[z^{-1}]}{T[z^{-1}]}\xi(t) \tag{15}$$

where $T[z^{-1}]$ is defined in (10).

3. Extension of GMVC through Coprime Factorization.

3.1. Coprime factorization of controlled systems. For coprime factorization, the family of stable rational functions RH_∞ is considered,

$$RH_\infty = \left\{ G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]} \right\} \tag{16}$$

where $G_d[z^{-1}]$ is stable polynomial. Transfer function $G_p(z^{-1})$ of the system (1) from $u(t)$ to $y(t)$ is given in the form of a ratio of rational functions in RH_∞,

$$y(t) = \frac{z^{-k_m}B[z^{-1}]}{A[z^{-1}]}u(t) = G_p(z^{-1})u(t) = N(z^{-1})D^{-1}(z^{-1})u(t) \tag{17}$$

$N(z^{-1})$ and $D(z^{-1})$ are rational functions in RH_∞ and coprime each other. In the next step, the following Bezout identity is considered.

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \tag{18}$$

The solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout identity are in RH_∞. Then all the stabilizing controller is given in Youla-Kucera parameterization [4] from (17) and (18).

$$u(t) = C_1(z^{-1})w(t) - C_2(z^{-1})y(t) \tag{19}$$
$$C_1(z^{-1}) = \left(Y(z^{-1}) - U(z^{-1})N(z^{-1})\right)^{-1}K(z^{-1}) \tag{20}$$
$$C_2(z^{-1}) = \left(Y(z^{-1}) - U(z^{-1})N(z^{-1})\right)^{-1}\left(X(z^{-1}) + U(z^{-1})D(z^{-1})\right) \tag{21}$$

$U(z^{-1}), K(z^{-1}) \in RH_\infty$ are free parameters and $w(t)$ is reference signal. Then the closed-loop system is given as follows.

$$y(t) = N(z^{-1})K(z^{-1})w(t) + \frac{C[z^{-1}]}{T[z^{-1}]}\left(Y(z^{-1}) - U(z^{-1})N(z^{-1})\right)\xi(t) \tag{22}$$

If the controller is designed for settling control, the output $y(t)$ converges to $w(t)$ as time progresses. It means that the steady-state gain of closed-loop system (22) is designed to be $N(1)K(1) = 1$. Here it is noticed that the closed-loop transfer function from $w(t)$ to $y(t)$ is independent of design parameter $U(z^{-1})$.

3.2. **Concept of on-demand type feedback controller.** In the previous research [9], the authors have proposed a design method of strong stability system and defined the selection method of design parameter $U(z^{-1})$, which can equate steady state gains of the closed-loop system and the open-loop system. Through this research, it was found that the derived closed-loop system allows that the feedback signal becomes zero in the steady state because the controller is designed to make the open-loop gain equal to the closed-loop gain. It means that the feedback signal appears so as to achieve the control object, and the feedback signal becomes zero when the control object was achieved in the steady state. Therefore, this research calls such a controller as on-demand type feedback controller.

In this subsection, the concept is described briefly. It assumes that there is no noise and the feedback signal $C_2(z^{-1})y(t)$ in the stabilizing controller (19) becomes zero. Considering the open-loop system for the closed-loop system (22), the controller (19) is given as follows.

$$u(t) = \left(Y(z^{-1}) - U(z^{-1})N(z^{-1})\right)^{-1} K(z^{-1})w(t) \tag{23}$$

Because of $Y(z^{-1})D(z^{-1}) = 1 - X(z^{-1})N(z^{-1})$, the open-loop system can be obtained as the following equation.

$$\begin{aligned} y(t) &= N(z^{-1})D^{-1}(z^{-1})u(t) \\ &= \left\{1 - \left(X(z^{-1}) + U(z^{-1})D(z^{-1})\right) N(z^{-1})\right\}^{-1} N(z^{-1})K(z^{-1})w(t) \end{aligned} \tag{24}$$

The steady state output $y(t)$ of the open-loop system is given.

$$y(t) = \{1 - (X(1) + U(1)D(1))N(1)\}^{-1}N(1)K(1)w(t) \tag{25}$$

If the design parameter $U(z^{-1})$ is selected as,

$$U(z^{-1}) = -D^{-1}(1)X(1) \tag{26}$$

then the steady state output $y(t)$ in (25) can be expressed as follows.

$$y(t) = N(1)K(1)w(t) \tag{27}$$

From (27), the steady state gain of open-loop system becomes equal to the closed-loop's one, even if the feedback signal $C_2(z^{-1})y(t)$ in (19) becomes zero. In other words, the open-loop system's output becomes equal to the reference signal $w(t)$ in the steady state because $N(1)K(1)$ is designed to be 1. This means that the feedback signal of the closed-loop system becomes zero in the steady state. That is, on-demand type feedback controller can be obtained.

3.3. **Consideration of noise.** As shown in (22), when the design parameter $U(z^{-1})$ is selected as (26), the closed-loop transfer function from $\xi(t)$ to $y(t)$ is determined uniquely. In order to enable the on-demand type feedback controller to reduce the noise influence on output, the design parameter in this paper is newly given as the following rational function [2].

$$U(z^{-1}) = \frac{\beta T[z^{-1}]}{C[z^{-1}]H[z^{-1}]} \tag{28}$$

where $H[z^{-1}]$ is a stable design polynomial, and β is newly defined as the following equation.

$$\beta = -\frac{C[1]H[1]}{T[1]}D^{-1}(1)X(1) \tag{29}$$

In the steady state, $U(z^{-1})$ can be given as,

$$U(1) = -\frac{C[1]H[1]}{T[1]}D^{-1}(1)X(1)\cdot\frac{T[1]}{C[1]H[1]} = -D^{-1}(1)X(1) \tag{30}$$

This implies that the proposed controller given by (28) becomes the on-demand type feedback controller.

3.4. **Controller design for GMVC.** In the case that $P[z^{-1}]$ and $Q[z^{-1}]$ in the generalized output $\Phi(t+k_m)$ are chosen for $T[z^{-1}]$ to be stable, comparing transfer function (17) to (15), $N(z^{-1})$ and $D(z^{-1})$ can be chosen as follows.

$$N(z^{-1}) = \frac{z^{-k_m}B[z^{-1}]}{T[z^{-1}]} \tag{31}$$

$$D(z^{-1}) = \frac{A[z^{-1}]}{T[z^{-1}]} \tag{32}$$

Substituting (31) and (32) into Bezout identity (18) and comparing it to Diophantine Equation (5), the solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout identity are given.

$$X(z^{-1}) = \frac{F[z^{-1}]}{C[z^{-1}]} \tag{33}$$

$$Y(z^{-1}) = \frac{G[z^{-1}]}{C[z^{-1}]} \tag{34}$$

Then the control law (14) can be expressed as Youla-Kucera parameterization (19), (20) and (21) by selecting the following free parameters.

$$K(z^{-1}) = R[z^{-1}] \tag{35}$$

$$U(z^{-1}) = 0 \tag{36}$$

To extend the controller (14), instead of choosing $U(z^{-1})$ as 0, on-demand type feedback controller uses (28) and (29). Then the extended controller through $U(z^{-1})$ is obtained as follows.

$$\begin{aligned}&\left(H[z^{-1}]G[z^{-1}] - z^{-k_m}\beta B[z^{-1}]\right)u(t)\\ &= C[z^{-1}]H[z^{-1}]R[z^{-1}]w(t) - \left(H[z^{-1}]F[z^{-1}] + \beta A[z^{-1}]\right)y(t)\end{aligned} \tag{37}$$

To calculate this control law, the polynomial operating on $u(t)$ in the left side of (37) is divided by the leading term g_0 and the remaining term.

$$H\left[z^{-1}\right]G\left[z^{-1}\right]-z^{-k_m}\beta B\left[z^{-1}\right]=g_0+z^{-1}G'\left[z^{-1}\right] \tag{38}$$

Therefore, the control law (37) is calculated by

$$\begin{aligned}u(t)=&\frac{1}{g_0}\Big\{C\left[z^{-1}\right]H\left[z^{-1}\right]R\left[z^{-1}\right]w(t)-\left(H\left[z^{-1}\right]F\left[z^{-1}\right]+\beta A\left[z^{-1}\right]\right)y(t)\\&-G'\left[z^{-1}\right]u(t-1)\Big\}\end{aligned} \tag{39}$$

From (22), the closed-loop system can be given as the following equation.

$$y(t)=\frac{z^{-k_m}B\left[z^{-1}\right]R\left[z^{-1}\right]}{T\left[z^{-1}\right]}w(t)+\frac{H\left[z^{-1}\right]G\left[z^{-1}\right]-z^{-k_m}\beta B\left[z^{-1}\right]}{H\left[z^{-1}\right]T\left[z^{-1}\right]}\xi(t) \tag{40}$$

It is noticed that the transfer function from reference signal $w(t)$ to output $y(t)$ is independent of $U\left(z^{-1}\right)$. On the other hand, the transfer function from noise $\xi(t)$ to output $y(t)$ can be changed through $H\left[z^{-1}\right]$ in $U\left(z^{-1}\right)$. The poles of controller can be given by the following equation.

$$H\left[z^{-1}\right]G\left[z^{-1}\right]-z^{-k_m}\beta B\left[z^{-1}\right]=0 \tag{41}$$

4. **Numerical Example.** The following controlled system described in (1) is given [9].

$$\begin{aligned}A\left[z^{-1}\right]&=1-0.998775z^{-1}\\B\left[z^{-1}\right]&=14.4\\C\left[z^{-1}\right]&=1,\quad k_m=1\end{aligned}$$

Simulation steps are 4000, the initial values of output and input are assumed to be zero. The disturbance is set to be white Gaussian noise with the variance $\sigma^2=0.0005^2$. In order to design the closed-loop characteristic to be stable, the generalized output is given so as to make the controlled output $y(t)$ follow the reference signal $w(t)$.

$$\Phi(t+1)=y(t+1)+1350u(t)+1.1148w(t)$$

The amplitude of reference signal $w(t)$ is 1 from the beginning of simulation to 2000th step, and 1.5 after 2001th step. The closed-loop pole is 0.9882. Therefore, the derived closed-loop system is designed to be stable. By using $H\left[z^{-1}\right]=1-0.998z^{-1}$ and $\beta=-1.6307$, the new design parameter $U(z^{-1})$ in (28) is defined as follows.

$$U\left(z^{-1}\right)=\frac{-2224.9+2198.7z^{-1}}{1-0.998z^{-1}}$$

Then the controller's pole is 0.9808. That is, the strong stability system is obtained by the proposed controller. If the parameter is selected as $U\left(z^{-1}\right)=-D^{-1}(1)X(1)$ of the previous method [9], then the controller's pole is 0.887 and the closed-loop pole becomes equal to the proposed one. Although the proposed method can give a strong stability system, it is noticed that the new design parameter does not always provide strong stability system because it depends on the given system in (1) and the conventional controller.

Figure 1 and Figure 2 show the plant outputs by the previous method and the proposed one respectively. The dashed lines mean the reference signals $w(t)$. The solid lines show the plant outputs $y(t)$. From these figures, it can find that each output can track to reference signal. Each variance of the previous method's output and the proposed one is 0.0122 and 0.012 respectively. That is, the proposed controller can reduce the noise influence on output, compared to the previous method.

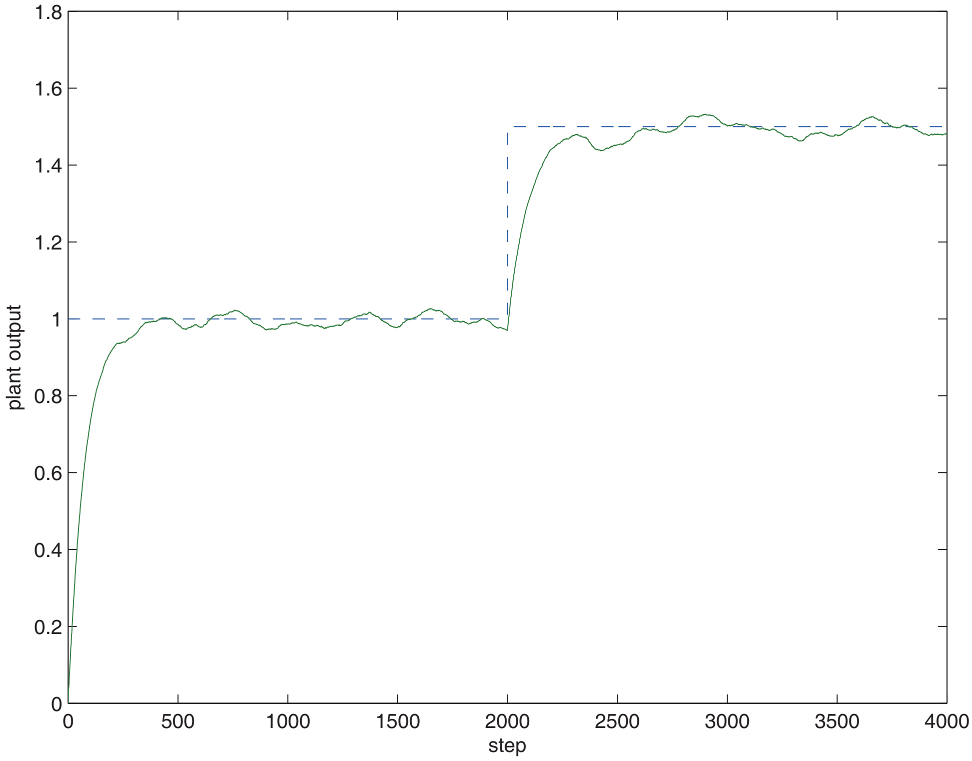

Figure 1. Previous method [9] (output)

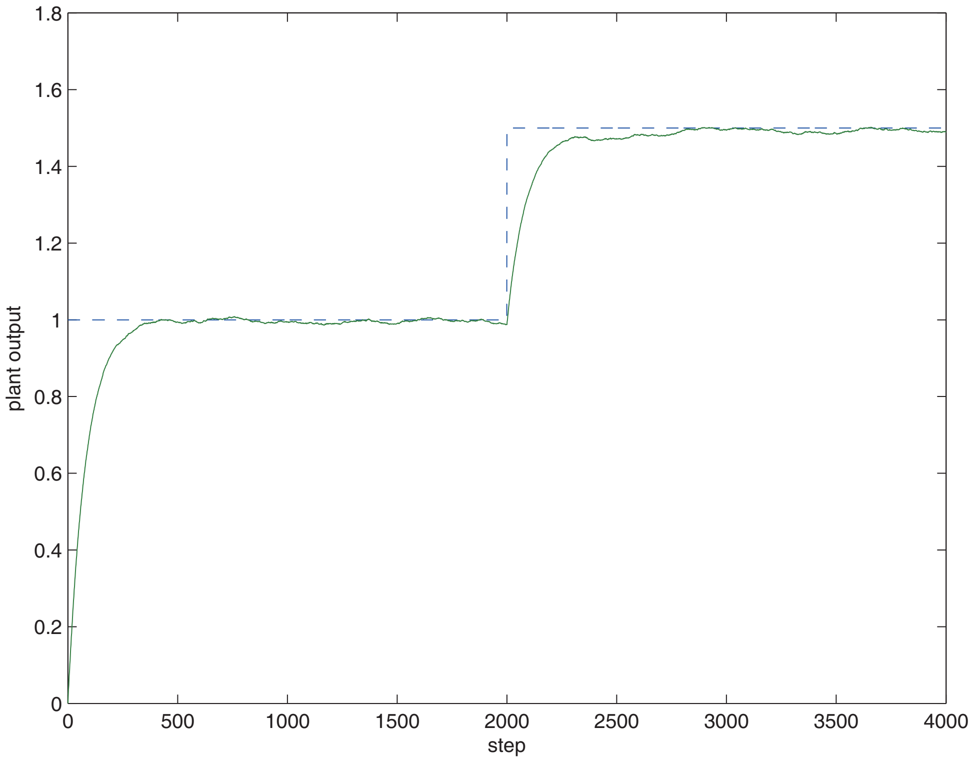

Figure 2. Proposed method (output)

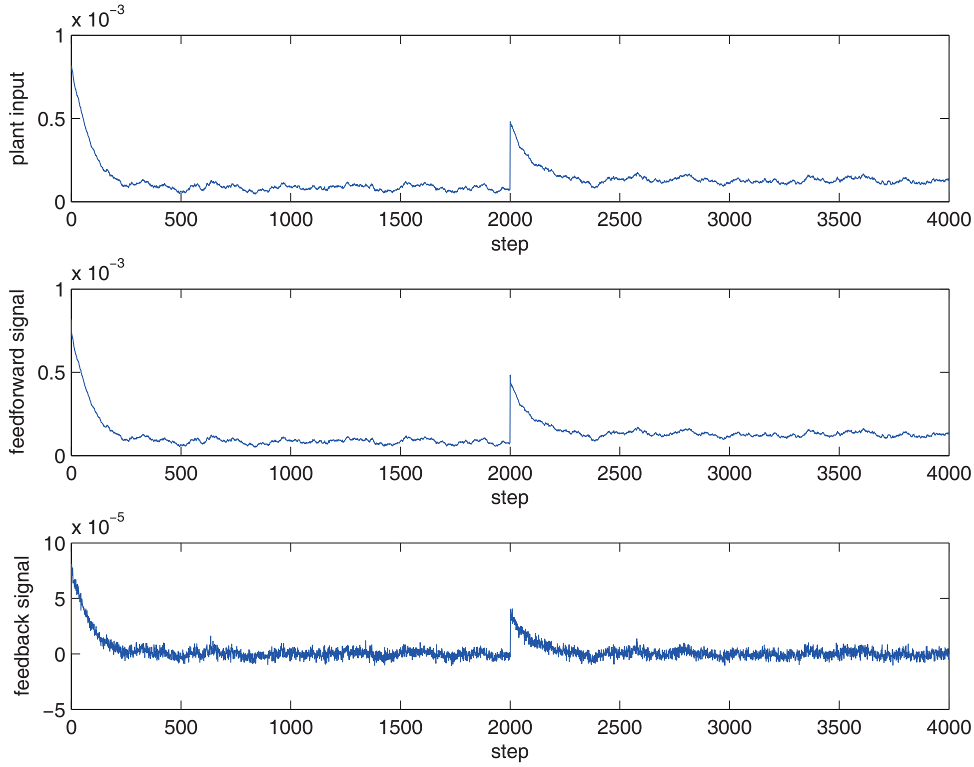

FIGURE 3. Previous method [9] (input)

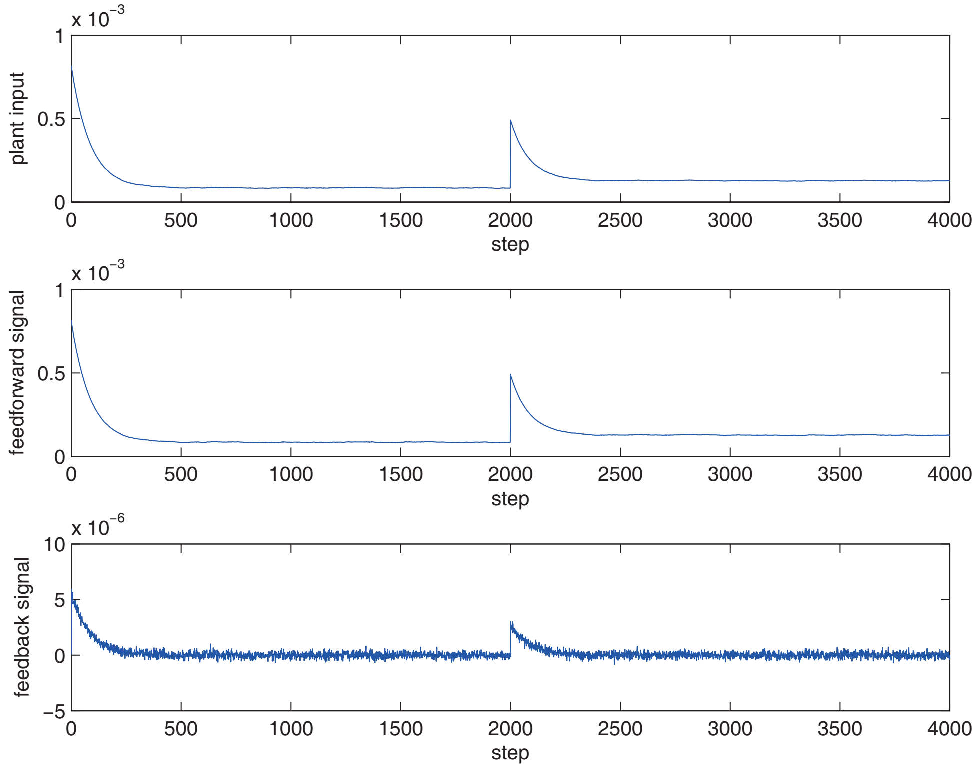

FIGURE 4. Proposed method (input)

Moreover, Figure 3 and Figure 4 show the control inputs $u(t)$ (upper figure of each figure), the feedforward signals (middle ones), which are expressed as $C_1\left(z^{-1}\right)w(t)$ described in (19), and the feedback signals $C_2\left(z^{-1}\right)y(t)$ (lower ones). In Figure 4, the proposed controller shows that the feedback signal appears in order to follow the reference signal, and disappears (becomes almost zero) when the control object was achieved. These figures show that the on-demand type feedback controller is effective in noisy environment.

5. **Conclusion.** This paper studied on-demand type feedback control system under noisy environment. A new design parameter to reduce the noise influence was considered. The numerical example was given to verify the effectiveness of the proposed method, whose feedback signal appears in order to follow the reference signal, and disappears when the control object was achieved. As future works, there is an extension to multi-input multi-output systems using the proposed method. Moreover, a selection method of design polynomial $H\left[z^{-1}\right]$ will be considered.

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