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ITERATIVE METHOD FOR CONTROLLING THE SWAY OF A PAYLOAD ON TOWER (SLEWING) CRANES USING A COMMAND PROFILE APPROACH

Roberto Paolo Luigi Caporali

Mathematics for Applied Physics di Roberto Caporali
P.O. Box 1255176, Roma Prima Porta, Rome 00188, Italy
robertopaolo.caporali@gmail.com

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Abstract. *This work describes a novel method able to reduce the sway of a suspended load during the slewing motion of a tower (or jib) crane. The proposed method is based on an iterative calculation of the sway angle and of the corresponding applied velocity profiles as input to the crane motors. The "command smoothing" method is used for reducing the sway, that is an additional damping is introduced into the system in order to control the sway, obtaining the values of the coefficients k with the observer method. A detailed mathematical model is developed, taking account of also the non-linear components of the forces, as centrifugal and Coriolis forces, on the system. Simulation results show that the proposed anti-sway method is able to damp the oscillations of the suspended load during the contemporary movement along the three perpendicular Cartesian axes, that is the two horizontal axes (said trolley and slewing axes) and the third vertical axis (said also hoisting axis). Furthermore, as a consequence of being an iterative method, it is possible to obtain a complete damping of the sway at the end of the movement, also if the velocity set is changed many times by the human operator, during the control of the crane movement: that is possible even if the previous damping of the oscillations was again in progress and not completely finished.*
Keywords: Anti-sway, Open-loop control, Iterative method, Tower cranes

1. **Introduction.** In a general way, it is possible to divide cranes into two categories: industrial cranes and slewing cranes.

For the first category of cranes, an anti-sway method is easier to develop as consequence of the decoupling among the equations of the movement. In fact, industrial cranes (gantry or overhead cranes) exhibit a one-degree-of-freedom sway in the solution equations.

On the contrary, slewing cranes, such as tower cranes or self-erecting cranes or harbor cranes, are subjected to the condition that the load-line attachment undergoes to a rotation. In this case, the equations of the movement are necessarily coupled. Conventionally, point-to-point operations using slewing cranes are performed by operators with the goal to not excite the oscillation modes of their payload and crane structure. Typically, the pendulum modes exhibit low frequencies.

The two fundamental motions of the payload in the horizontal plane, resulting from the operator commands produce a sway tangential to the arc of a circle defined by the tip of the crane jib and a sway radial to the same rotation due to the centrifugal acceleration of the payload.

Currently, speaking about slewing cranes, the swing of the payload is not compensated in any automatic way. Only for pre-planned movements where the desired start and stop positions of the payload are well specified it is possible to define an automatic anti-sway

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method. However, that is not the typical situation of slewing crane for the building sector. In this case, today, only the high experience of the crane operator allows bringing the payload to a stopping point without a swing.

The movements of the crane cause sway of the load. There are extensive researches on anti-sway for industrial (overhead traveling) cranes. That is due, in this case, to the possibility to decouple the movements and, as a consequence of this, to use linear models to solve the motion equations.

Instead of that, slewing crane movements are characterized by the appearance of Coriolis and Centrifugal forces. So the equations become non-linear and the application of linear control theory leads to several problems.

In this work, in order to solve this non-linear problem, we make considerations concerning non-linear force interactions, using the Lagrange equations governing the physical system given by a tower (or jib) crane.

The fundamental movements of a tower crane typically are:

1) A rotation (or slewing) of the jib. The jib is connected to the top of a vertical shaft and has a suspension point for the payload that is suspended through cables. Jib performs a rotation or slewing movement about a vertical axis z (slewing movement).

2) A linear movement of the suspension point of the load along the jib. This second movement can be referred to a translation movement. The translation movement is performed along the horizontal axis x of the jib (trolley movement).

3) A device able to hoist the load. This device is defined typically using a cable which passes through the crane. The length of the cable is variable in order to move the load on a vertical axis z. This third possible movement along the axis z is usually named as the vertical movement (or hoisting movement).

It is obviously desirable to reduce the sway of the payload in order to obtain two positive effects: increase the safety of the load transfer and reduce the operational time to the shorter possible time.

To difference of the sway generated from linear movements (trolley movements only), the sway due to a rotation, as a consequence of the centrifugal force, remains present even if the tangential component of the acceleration during the rotation movement is zero.

Several solutions to the problem to reduce the swing of the suspended load were proposed in the past. However, as before said, almost all the solutions are related to sway generated by industrial cranes (that is gantry and overhead cranes), where there is not the complication due to the slewing movement. Instead, with reference to works relative to tower cranes, as consequence of the complication due to the coupling of movements, not many papers were published in the past.

Some papers and patents were published, in the recent years, relatively to the solution of the payload sway in a tower crane. However, the proposed solutions were related to some different ways to obtaining a reduction of the swing.

The work of Abdel-Rahman et al. [1] and the recent work of Ramli et al. [2] allow having an exhaustive literature review of the control strategies relative to the cranes.

Historically, solutions with “input shaping”, with “adaptive control” and with “model predictive control” were investigated at first.

Particularly important for the control of slewing cranes, in the paper of Palis and Palis [3] the method of adaptive control was developed for a tower crane. Due to the difference in dynamics, the general motion of the crane was divided into slow and fast subsystems, which can be optimized independently. That leads to have an approximate solution of the anti-sway problem.

Relevant recent studies relative to the “adaptive control” for tower cranes, recently come from Sun et al. [4] and Fujioka et al. [5].

Instead, the recent papers of Barisa et al. [6] and Böck and Kugi [7] developed some "model predictive controls" for slewing cranes. Nevertheless, this last kind of method is optimal in automatic control, but not with a manual operator, when a target position is established before starting the movement.

Recently, it was noted for the possible excellent performances of the filtering method for the control of slewing cranes.

Really, the first work was obtained with the U.S. Patent 5,908,122 to Robinett et al. [8], where the sway correction was given filtering crane input signals. The filtering method uses a model representing the dynamics of the rotary crane with a matrix of nonlinear equations of motion, and after linearizing the matrix with respect to the radial sway angle and to the tangential sway angle. However, in that work, the equations of the movement were linearized. In this way, all the effects relative to the centrifugal force and to the Coriolis force are not completely considered.

An evolution in the use of the "filtering" method was obtained in the recent European Patent EP2896590 to Caporali [9]. In that invention the calculation of the system gain is obtained introducing filtered inputs, being these filtered inputs used in order to produce the damping of the profiles.

Nevertheless, the most important limitation of the filter technique is due to the fact that it is not able to consider possible sequential commands without that the previous damping of the sway was totally obtained.

More recently, a kind of solution was introduced in order to control the sway, providing an additional damping into the system: this method is defined also "command smoothing".

In this approach, the crane acceleration is typically given in the general form: $\ddot{x} = r + k_1\vartheta + k_2\dot{\vartheta}$ or similar expression. Here above, r is a function of the time that depends on the movement profile of the crane. The obtained system will have the wanted damping ratio and the natural frequency using the convenient values of the coefficients k_1 and k_2.

For example, the "command smoothing" method was used in the U.S. Patent Application 2011/0218714 A1 of Stanchev and Velachev [10] for a tower crane.

In this invention, the calculation step uses a mathematical model of the pendulum with damping. However, the limit of this invention is in the fact that the correction coefficients (necessary to define the correct damping of the sway) are defined experimentally according to the different possible lengths of the suspension cables of the load. So, these coefficients have to be obtained only after experimentation, without a way to have a complete theory of their computation.

Moreover, in the same invention, the effects relative to the Coriolis force are not considered. That leads to a loose of the precision of the sway correction.

Singhose et al. [11] developed a global comparison between the input-shaped step function and a command smoothing by using an S-curve profile for a portable tower crane.

Also the present work is part of this set of solutions, defined as "command smoothing".

On the one side, this actual work represents a strong evolution of the previous patent application [9]. While in the patent the solution was obtained with the "filtering" method, and in the present work the equation solutions are obtained using an iterative method of calculation.

The method presented in this paper uses a kind of "command smoothing" for reducing the sway, introducing an additional damping into the system in order to control the sway. As a consequence of the fact that this method is iterative, it is possible to obtain the total damping of the sway changing many times the velocity set, also if the previous damping control of the oscillations is again in progress. In other words, it is not necessary that the target velocity has been reached in order to carry out a subsequent command towards a subsequent target velocity. That is fundamental, because the usual control of the tower crane is manual, by a human operator: he is used for changing very often the target velocity using the remote controller.

Furthermore, in this way, the profiles of the speed can be modulated, with either smooth or strong gradient.

The presented method can be considered innovative compared to previous works above described in some fundamental aspects.

In this work, the cable flexibility and the relative damping are introduced: in this way, this method allows to obtain more performances as regards to the previous methods where these data are not taken into account. That was not considered in the previous patent [9].

This novel work is based on the calculation of the complete dynamical equations for the crane system. In this way, also the continuous variation of the hoisting height for the suspension point of the payload and the variation of the corresponding velocity and acceleration are considered.

Besides, in the present work, two fundamental effects are considered: both the centrifugal and Coriolis forces that produce the non-linear terms in the movement equations. So the present method is really able to consider all the non-linear effects of the crane movements.

Furthermore, the mathematical model is developed without the need to obtain in an experimental way the correction coefficients in the movement equations. So this work is directly applicable on any length of the cable relative to the payload, considering the continuous variation of the hoisting height for the suspension point of the payload and the variation of the corresponding velocity and acceleration. Therefore, the application of the method is able to eliminate sway also during a continuous variation of the length of the pendulum.

The calculation uses the fundamental equations obtained by the Lagrange equations in order to define the successive steps. The control process is realized by a control device and includes a calculation step for determining the angles of oscillation of the load and the time derivative of the sway angles. This method does not require means to measuring the sway angle, but the control device includes means to obtaining the "effective length" l_{eff} of the suspension cable. The values of the coefficients k, which need to introduce an additional damping, are obtained with the observer method.

The velocities of the trolley movement and of the slewing movements are obtained directly from the reference velocities supplied by the control device. It is assumed that the drive controls the speed references with high rapidity.

This paper is organized as follows.

The dynamic model of the tower crane is presented in Section 2, describing in detail the geometric representation of the tower crane and the cable flexibility. From Lagrange equations, the dynamical equation relative to the sway angle is obtained, where the functional dependence by the acceleration of both the slewing and trolley movements and the coupling of the two equations relative to the sway angles are highlighted. The equation solutions are obtained with an iterative method, where the estimated values of the sway angles and the slewing and trolley acceleration are obtained step by step. The need to have a trolley movement, also in absence to a corresponding trolley command, in order to compensate the centrifugal effect of the slewing movement, is highlighted. After that, designing an observer method, it is possible to synchronize this estimated model with the real crane, verifying that the system is observable.

In Section 3 an implementation of the method and the most relevant results of the model simulation are presented, with particular emphasis to the fact that total damping is obtained changing many times the velocity set, also during the previous ramp.

At the end, in Section 4 concluding remarks are given.

2. **Dynamic Model of the Tower Crane System and Solution Description.** The goal of this method is to assist the control of the hoisting machine able to perform a translation movement along an axis x and a rotation movement of the suspension point together with the hoisting movement of the load along the axis z.

Referring to Figure 1, a slewing crane includes a multi-body system with three independent degrees-of-freedom for positioning a spherical-pendulum as the payload of mass m. Specifically, the slewing crane includes a translating load-line, having length L and a payload of mass m attached to a translating trolley with mass M. A crane operator or a computer positions the payload using some available commands and changing the load length L.

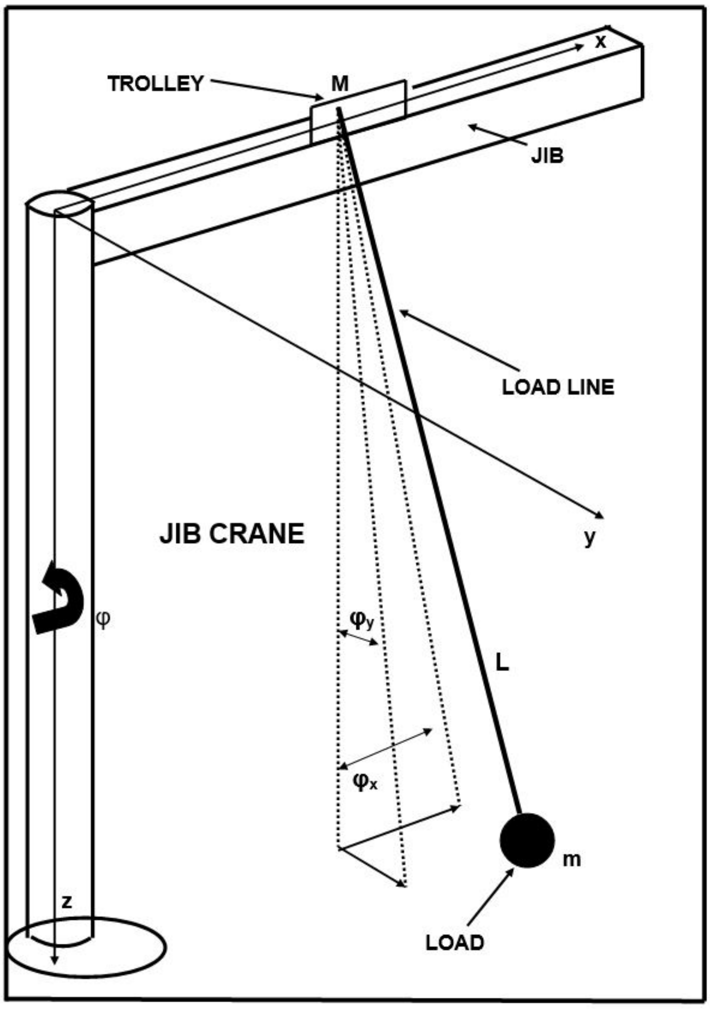

FIGURE 1. Geometric description of the tower crane system. The jib is connected to the top of a vertical shaft that represents the vertical structure of the crane. The trolley (with mass M) moves on the direction x and has a suspension point for the payload with mass m. It is suspended through a cable of length L. Jib performs a rotation about a vertical axis z that corresponds to the vertical shaft of the crane.

The crane configuration is characterized by one vertical axis (in Figure 1, axis z), a first horizontal axis, the radial axis (in Figure 1, axis x) in the direction of the trolley displacement, and a second horizontal axis, perpendicular to the radial axis x (in Figure 1, axis y). There are three controlled motions for slewing crane corresponding to these three axes: a trolley translation along the axis x, a rotation of the crane about his vertical axis z (slewing motion) and a vertical motion along the axis z with a variation of the length L.

In order to describe the motion of the slewing crane, an equivalent kinematics scheme with concentrated masses is represented in Figure 1.

Dynamics of the generalized slewing crane is represented as a multi-body system with 5 independent degrees of freedom, described by 5 Lagrangian coordinates q_i:

$q_1 = r$: radial position of the mass center of the trolley on the axis x

$q_2 = \varphi$: slewing angle (rotation around the z axis)

$q_3 = l$: hoisting position of the crane (along the z axis)

$q_4 = \varphi_x$: sway angle tangential to the radial direction

$q_5 = \varphi_y$: sway angle normal to the radial direction

If we refer to the energy, the following balance can be defined.

2.1. **Potential energy.** The potential energy of the load with mass m is given:

$$V_m = -mg \cdot z + C \tag{1}$$

where z is the position along the vertical, g is the gravity acceleration and C is an arbitrary constant. Therefore, we have (with reference to Figure 1):

$$z = l \cos\varphi_x \cos\varphi_y \tag{2}$$

And, therefore, it is obtained:

$$V_m \equiv \delta L = -mgl \cos\varphi_x \cos\varphi_y \tag{3}$$

$$U_m = -V_m = mgl \cos\varphi_x \cos\varphi_y \tag{4}$$

2.2. **Kinetic energy.** The kinetic energy of the slewing crane is the sum of the corresponding terms for the tower T_T (with momentum of inertia J_T), for the boom (said also "jib") T_B (with mass m_B and position of the mass center r_B), for the trolley T_R (with mass m_R and position of the mass center r) and for the payload T_L (with mass m_L and velocity of the mass center v_L)

$$T = T_T + T_B + T_R + T_L \tag{5}$$

$$T_T = \frac{1}{2} J_T \dot{\varphi}^2 \tag{6}$$

$$T_B = \frac{1}{2} m_B r_B^2 \dot{\varphi}^2 \tag{7}$$

$$T_R = \frac{1}{2} m_R \left(r^2 \varphi^2 + \dot{r}^2 \right) \tag{8}$$

$$T_L = \frac{1}{2} m_L v_L^2 \tag{9}$$

In order to simplify the final system of Lagrange equations, we take account of only small sway angles. That means the following assumptions:

$$\sin\varphi_x \approx \varphi_x, \quad \sin\varphi_y \approx \varphi_y, \quad \cos\varphi_x \approx 1, \quad \cos\varphi_y \approx 1 \tag{10}$$

With this assumption, the velocity v_L of the payload can be obtained defining his expression in terms of the Lagrangian coordinates. In this way, it is possible to obtain, for the term T_L, the following expression:

$$T_L = \frac{1}{2} m_L \left\{ \begin{array}{l} \dot{\varphi}^2 (l\varphi_x + r)^2 + \left(\varphi_x \dot{l} + \dot{\varphi}_x l + \dot{r}\right)^2 + (l\dot{\varphi}\varphi_y)^2 + \left(\dot{l}\varphi_y + l\dot{\varphi}_y\right)^2 + \dot{l}^2 \\ + (l\dot{\varphi}_x \varphi_x)^2 + (l\dot{\varphi}_y \varphi_y)^2 + 2\dot{\varphi}(l\varphi_x + r)\left(\dot{l}\varphi_y + l\dot{\varphi}_y\right) \\ -2\dot{\varphi}\left(\varphi_x \dot{l} + \dot{\varphi}_x l + \dot{r}\right)(l\varphi_y) - 2\dot{\varphi}_x \varphi_x l\dot{l} - 2\dot{\varphi}_y \varphi_y l\dot{l} + 2\dot{\varphi}_x \dot{\varphi}_y \varphi_x \varphi_y l^2 \end{array} \right\} \tag{11}$$

2.3. **Lagrange equations.** Establishing the Lagrange function as:

$$L = T + U \tag{12}$$

it is possible to apply the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{13}$$

Defining the five components for the generalized forces Q_i in the following way:

$$Q_1 = F_r, \quad Q_2 = M, \quad Q_3 = F_z, \quad Q_4 = F_x, \quad Q_5 = F_y \tag{14}$$

where F_r, M, F_z, F_x, F_y are the generalized, non-conservative, dissipative and active forces working on the system. Particularly F_x and F_y represent the components, on the axes x and y, of air resistance forces acting on the payload and on the hoisting system, the components of the wind force, the components of the forces due to the damping of the rotation movement and due to the structure's elasticity of the hoisting system.

At the end of our calculations, we are able to obtain the five Lagrange equations in the corresponding five Lagrangian coordinates and their derivative with respect to time:

$$\begin{array}{l} (m_R + m_L)\ddot{r} - m_L l\varphi_y \ddot{\varphi} + m_L \varphi_x \ddot{l} + m_L l\ddot{\varphi}_x \\ -\dot{\varphi}^2 \left[m_R r + m_L (r + l\varphi_x)^2\right] + 2m_L\left(\dot{\varphi}_x \dot{l} - \varphi_y \dot{l}\dot{\varphi} - \varphi_y l\dot{\varphi}\right) = F_r \end{array} \tag{15}$$

$$\begin{array}{l} J_T \ddot{\varphi} - m_L l\varphi_y \ddot{r} + m_L r\varphi_y \ddot{l} + m_L r l\ddot{\varphi}_y \\ +2\left\{m_L\left(l\varphi_x \dot{r}\dot{\varphi} + l\dot{\varphi}_x r\dot{\varphi} + \dot{l}\varphi_x r\dot{\varphi} + r\dot{r}\dot{\varphi} + \dot{l}\dot{\varphi}_y r\right) + m_R r\dot{r}\dot{\varphi}\right\} = M \end{array} \tag{16}$$

$$\varphi_x \ddot{r} + \ddot{l} + r\varphi_y \ddot{\varphi} - r\varphi_x \dot{\varphi}^2 + 2\dot{r}\dot{\varphi}\varphi_y - g = F_z/m_L \tag{17}$$

$$\begin{array}{l} \left\{\ddot{r} - l\varphi_y \ddot{\varphi} + l\ddot{\varphi}_x - (r + l\varphi_x)\dot{\varphi}^2 + 2\left(\dot{l}\dot{\varphi}_x - l\dot{\varphi}_y \dot{\varphi} - \dot{l}\varphi_y \dot{\varphi}\right) + g\varphi_x\right\} \\ -\left(\dot{l}^2 \varphi_x\right)/l = F_x/m_L \end{array} \tag{18}$$

$$\begin{array}{l} \left\{(r + l\varphi_x)\ddot{\varphi} + l\ddot{\varphi}_y - l\varphi_y \dot{\varphi}^2 + 2\left(\dot{l}\dot{\varphi}_y + \dot{r}\dot{\varphi} + l\dot{\varphi}_x \dot{\varphi} + \dot{l}\varphi_x \dot{\varphi}\right) + g\varphi_y\right\} \\ -\left(\dot{l}^2 \varphi_y\right)/l = F_y/m_L \end{array} \tag{19}$$

So, the multiple degrees of freedom rotary crane system results defined by the five differential equations where the terms relative to the centrifugal and Coriolis forces logically appear.

2.4. **Pendulum with cable flexibility.** Although the model of the pendulum is able to describe the real physical situation of the payload attached to the slewing crane, this model is not able to perform really well if compared with the realistic situation. Typically, this model estimates again the period time too low for all cable lengths. A too small period time can be the cause of too small cable lengths and very responsive cable forces.

It is therefore likely that the assumption of an infinite stiffness of the cable is the major factor to consider for this difference.

In general, considering the equations of the motion modeling the crane dynamics, there exist some damping in the response of the model. When we take a look at the non-linear equations of motion, velocity dependent terms can be seen in the second order time-derivatives of the constraint equations, which are part of the system. These terms are the cause of the damping. However, this damping is relatively small and not a good approximation of the damping in a real crane.

We must switch to another model, that is exactly the same as the one presented in the previous section but now extended with a certain stiffness k and damping b of the cable material, as represented in Figure 2 where the cable has a parallel spring and damper. In this case, the period time T of the pendulum is increased compared to the model without cable flexibility. Part of this increase is due to the elongation of the cable introduced by the finite cable stiffness. The cable stiffness and damping are given by:

$$k_{cable} = \frac{k}{L} \tag{20}$$

$$b_{cable} = \frac{b}{L} \tag{21}$$

Being the cable stiffness and damping dependent from the length, the correction on the period time T is not the same for every length. When the cable length is short, the stiffness will be larger and the model will approximate the model without cable flexibility.

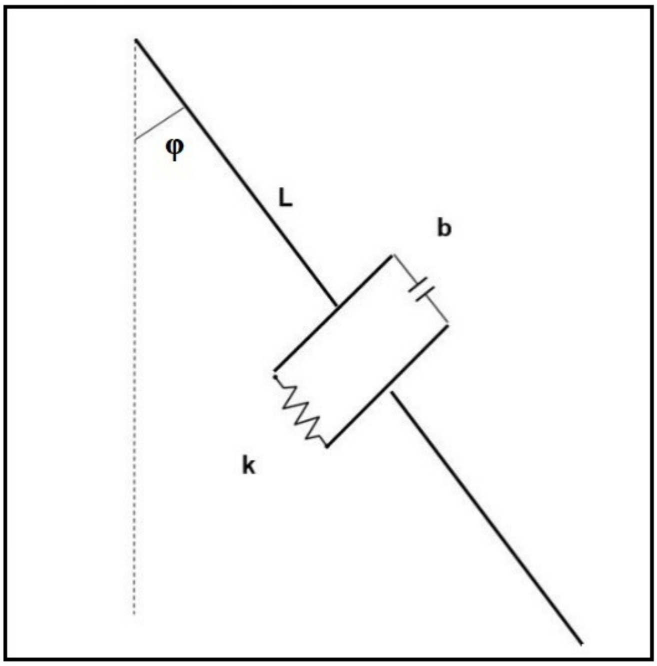

FIGURE 2. Representation of the cable flexibility. The cable has cable stiffness k, damping b, and length L. φ is the sway angle (tangential or radial) with respect to the vertical.

However, the elongation of the cable is not the only cause of the increase in period time. Typically, the elongation of the cable with classic load parameters at a length of 20 m is only 0.15 m.

Therefore, the biggest part of the increase in period time comes from the phase delay introduced by the cable stiffness. That is due to the fact that, as a consequence of the inclination of the cable, it also exerts a horizontal force on the load. When the cable has a finite stiffness, this force will be applied with a phase delay that will cause an increase of the period time with respect to an infinite cable stiffness.

Also the damping parameter of the cable is an important factor in the total damping of the movement and has to be absolutely considered.

Relative to the term of the potential energy corresponding to the contribution of the elongation, we can obtain, in the first approximation of a linear pendulum (being k the cable stiffness coefficient and u the cable elongation due to the finite stiffness):

$$\frac{\partial V_u}{\partial \varphi} \equiv \frac{1}{2} k u^2 = k l^2 \varphi \tag{22}$$

2.5. **Equations solution.** Because the goal is to obtain a strategy in order to reduce the oscillatory motion of slewing crane payload, we will concentrate on the last Equations (18) and (19) involving the two variables φ_x and φ_y corresponding to the two oscillatory degrees of freedom.

In fact, in contrast to the linear movements, a rotation movement generates two components of sway. This sway angle exhibits non-zero components φ_x and φ_y corresponding to the two oscillatory degrees of freedom in the two perpendicular directions x and y. The second component φ_y along the axis y is generated by the variation of the velocity of the suspension point and can be eliminated by acting only on the control of the rotation movement. Instead of this, the other component φ_x along the axis x is generated by the centrifugal force. This force causes a movement of the load that is directed along a perpendicular plane xz. Therefore, the component φ_x cannot be eliminated by acting on the control of the rotation movement, but it involves acting also on the control the translation movement along the axis x, that is acting on the trolley movement.

As consequence of that, the centrifugal force produces the movement of the load along the axis x, even when the rotation movement takes place at a constant velocity.

Disregarding the terms of superior order in (18) and (19) and considering all the trigonometric terms, we obtain:

$$\ddot{\varphi}_x = {}^1\!/\!_l \left\{ \begin{array}{l} -g \sin \varphi_x - \ddot{r} \cos \varphi_x + \varphi^2 \cdot (r + l \sin \varphi_x) \cos \varphi_x \\ + \left(\dot{l} - k_f \right) \cdot \dot{\varphi}_x + kl \sin \varphi_x + [(k_{w,x} \cdot \dot{r})/m_l] \end{array} \right\} \tag{23}$$

$$\ddot{\varphi}_y = {}^1\!/\!_l \left\{ \begin{array}{l} -g \sin \varphi_y - r\ddot{\varphi} \cos \varphi_y + \varphi^2 \cdot (l \sin \varphi_y) \cos \varphi_y \\ + \left(\dot{l} - k_f \right) \cdot \dot{\varphi}_y + kl \sin \varphi_y + [(k_{w,y} \cdot r\dot{\varphi})/m_l] \end{array} \right\} \tag{24}$$

where k_f is the friction coefficient with a fixed value, at prior fixed depending on the considered axis. k is the cable stiffness coefficient. $k_{w,x}$ is the wind coefficient (positive or negative) due to the force of the wind in the x direction. $k_{w,y}$ is the wind coefficient (positive or negative) due to the force of the wind in the y direction. $\dot{l}$ is the derivative of the length l (that is the velocity of the vertical movement along the axis Z). $\ddot{r}$ is the acceleration of the trolley movement along the axis x, $\ddot{\varphi}$ is the angular acceleration of the slewing movement along the axis y. $\dot{r}$ is the acceleration of the trolley movement along the axis x, and $\dot{\varphi}$ is the angular velocity of the slewing movement along the axis y. g obviously is the gravity acceleration and m_l is the mass of the payload.

In order to obtain the control of the load sway, this method requires a control device. This control device calculates the angles φ_x and φ_y of the oscillation of the load in the directions x and y, and the angular velocities $\dot{\varphi}_x$ and $\dot{\varphi}_y$ of the same angles of sway. In order to obtain them, the control device uses only the information of the length of the pendulum, the information of the speed and acceleration of the trolley movement along the axis x, the information of the speed and acceleration of the slewing movement along the axis y and the information relative to the wind speed.

The control process is realized by the control device and includes a calculation step for determining the angles of oscillation of the load and the velocities of the sway angles, starting from the knowledge only of the information above described.

Practically, the control method calculates the described data through an iterative process using the velocity and acceleration of the sway angles.

The calculation uses (23) and (24) in order to define the successive step. This method does not require means to measuring the sway angle.

The control device includes means to obtaining the length l of the suspension cable and means to obtaining information relative to the distance r between the suspension point of the load and the rotation axis z. As a first option, the distance r can be obtained by integration starting from the reference velocity $V_{Ref(x)}$ that is the input to the drive able to control the motor causing the movement of the trolley in the direction x. Obviously, it is assumed that the drive is able to control the speed reference with high rapidity. As a second option, the distance r can be obtained by means of a sensor as a rotating encoder on the shaft of the translation motor, or as an absolute encoder, or as a potentiometer along the jib.

The control device also includes means to obtaining information relative to the velocity $\dot{\varphi}$ of the movement of the slewing along the direction y. As a first option, the velocity $\dot{\varphi}$ of the movement of the slewing can be obtained directly from the reference velocity $V_{Ref(y)}$ that is the input to the drive able to control the motor causing the movement of the slewing in the direction y. Obviously, it is assumed that the drive controls the speed reference with high rapidity. As a second option, the velocity $\dot{\varphi}$ of the movement of the trolley can be obtained by a velocity estimation generated in the variable speed drive responsible to control the motor causing the movement of the slewing in the direction y. As a third option, the velocity of the movement of the trolley $\dot{\varphi}$ can be obtained by a measurement of the real translational velocity of the slewing. However, this option needs the use of a motion sensor.

The device is able to regulate the system and must include an estimator module and a correction module.

The estimator module receives as input the length l of the cable, the velocity $\dot{\varphi}$ of the movement of the slewing relative to the y direction (velocity of the angular rotation) and the distance r between the suspension point of the load and the rotation axis z.

In order to compute the sway angle and the velocity of the sway angle, this method uses (23) and (24) representing a mathematical model of a rotational pendulum with damping.

Starting from (23), φ_x, $\dot{\varphi}_x$ and $\ddot{\varphi}_x$ are internal variables which are obtained with an iterative method that can be represented, at any time t, in the following way:

$$\dot{r}_t = (r_t - r_{t-1}) \cdot \Delta t \tag{25}$$

$$\ddot{r}_t = (\dot{r}_t - \dot{r}_{t-1}) \cdot \Delta t \tag{26}$$

$$\dot{l}_t = (l_t - l_{t-1}) \cdot \Delta t \tag{27}$$

$$\ddot{\varphi}_{x,t} = {}^{1}\!/_{l_t} \left\{ \begin{array}{l} -g \sin\varphi_{x,t} - \ddot{r}_t \cos\varphi_{x,t} + \varphi_t^2 \cdot (r_t + l_t \sin\varphi_{x,t}) \cos\varphi_{x,t} \\ + \left(\dot{l}_t - k_f\right) \cdot \dot{\varphi}_{x,t} + k l_t \sin\varphi_{x,t} + [(k_{w,x} \cdot \dot{r}_t)/m_l] \end{array} \right\} \tag{28}$$

$$\dot{\varphi}_{x,t} = \dot{\varphi}_{x,t-1} + \ddot{\varphi}_{x,t-1} \cdot \Delta t \tag{29}$$

$$\varphi_{x,t} = \varphi_{x,t-1} + \dot{\varphi}_{x,t-1} \cdot \Delta t \tag{30}$$

In these equations, $\varphi_{x,t}$ and $\varphi_{x,t-1}$ represent the sway angles along the x direction respectively at the time t and at the previous time $t-1$. $\dot{\varphi}_{x,t}$ and $\dot{\varphi}_{x,t-1}$ represent the velocities of the sway angle along the x direction respectively at the time t and at the previous time $t-1$. $\ddot{\varphi}_{x,t}$ and $\ddot{\varphi}_{x,t-1}$ represent the accelerations of the sway angle along the x direction respectively at the time t and at the previous time $t-1$. $\ddot{r}_t$ represents the acceleration of the movement of the trolley relative to the x direction at the time t. $\dot{r}_t$ and $\dot{r}_{t-1}$ represent the velocities of the movement of the trolley relative to the x direction respectively at the time t and at the previous time $t-1$. $\dot{l}_t$ represents the velocity of the movement of the hoisting relative to the z vertical direction at the time t. l_t and l_{t-1} represent the length of the cable respectively at the time t and at the previous time $t-1$. Δt represents the time difference between the instant t and the instant $t-1$.

The iterative process starts with the hypothesis that at the initial instant the values of the sway angle φ_x, the velocity of the sway angle $\dot{\varphi}_x$ and the acceleration of the sway angle $\ddot{\varphi}_x$ are equal to zero, that is the pendulum is initially in quiet conditions.

Therefore, for $t = 0$, we have:

$$\varphi_{x,0} = \dot{\varphi}_{x,0} = \ddot{\varphi}_{x,0} = 0 \tag{31}$$

The considerations defined for (23) can be repeated for (24) relative to the tangential sway angle φ_y.

In fact, starting from (24), φ_y, $\dot{\varphi}_y$ and $\ddot{\varphi}_y$ are internal variables which are obtained with an iterative method that can be represented, at any time t, in the following way:

$$\ddot{\varphi}_t = (\dot{\varphi}_t - \dot{\varphi}_{t-1}) \cdot \Delta t \tag{32}$$

$$\dot{l}_t = (l_t - l_{t-1}) \cdot \Delta t \tag{33}$$

$$\ddot{\varphi}_{y,t} = {}^1\!/\!_{l_t} \left\{ \begin{array}{l} -g \sin\varphi_{y,t} - r_t \ddot{\varphi}_t \cos\varphi_{y,t} + \varphi_t^2 \cdot (l_t \sin\varphi_{y,t}) \cos\varphi_{y,t} \\ + \left(\dot{l}_t - k_f\right) \cdot \dot{\varphi}_{y,t} + k l_t \sin\varphi_{y,t} + [(k_{w,y} \cdot r_t \dot{\varphi}_t)/m_l] \end{array} \right\} \tag{34}$$

$$\dot{\varphi}_{y,t} = \dot{\varphi}_{y,t-1} + \ddot{\varphi}_{y,t-1} \cdot \Delta t \tag{35}$$

$$\varphi_{y,t} = \varphi_{y,t-1} + \dot{\varphi}_{y,t-1} \cdot \Delta t \tag{36}$$

In these equations, $\dot{\varphi}_{y,t}$ and $\dot{\varphi}_{y,t-1}$ represent the velocities of the sway angle along the y direction respectively at the time t and at the previous time $t-1$. $\ddot{\varphi}_{y,t}$ and $\ddot{\varphi}_{y,t-1}$ represent the accelerations of the sway angle along the y direction respectively at the time t and at the previous time $t-1$. $\ddot{\varphi}_t$ represents the angular acceleration of the movement of the slewing axis relative to the y direction at the time t. $\dot{\varphi}_t$ and $\dot{\varphi}_{t-1}$ represent the angular velocities of the movement of the slewing axis relative to the y direction respectively at the time t and at the previous time $t-1$. $\dot{l}_t$ represents the velocity of the movement of the hoisting relative to the z vertical direction at the time t. l_t and l_{t-1} represent the length of the cable respectively at the time t and at the previous time $t-1$. Δt represents the time difference between the instant t and the instant $t-1$.

The iterative process starts with the hypothesis that at the initial instant the values of the sway angle φ_y, the velocity of the sway angle $\dot{\varphi}_y$ and the acceleration of the sway angle $\ddot{\varphi}_y$ are equal to zero, that is the pendulum is initially in quiet conditions.

Therefore, for $t = 0$, we have:

$$\varphi_{y,0} = \dot{\varphi}_{y,0} = \ddot{\varphi}_{y,0} = 0 \tag{37}$$

The control device is able to control, at the same time, the movements along the axes x, y and z, being the axis z the vertical axis relative to the hoisting movement.

It must be underlined that (23) contains a specific important term:

$$\varphi^2 \cdot (r + l \sin \varphi_x) \cos \varphi_x.$$

As it can be seen, this term is always positive when the angular velocity φ is different from zero. This term corresponds to the physical fact of the existence of the centrifugal force. Therefore, as soon as a rotation is affected (also when the angular acceleration $\ddot{\varphi}$ is equal to zero) a sway angle φ_x is obtained in direction x perpendicular to the direction of the instantaneous angular rotation. It is important to establish that the goal of the control is not to eliminate this sway during the rotation movement because this perpendicular sway is a natural part of the rotational movement, but to arrive at an equilibrium position with a non-zero sway (in x direction) of the load corresponding to a non-zero angle during the rotation. Obviously, at the end of the rotational movement, the sway angle φ_x must be equal to zero. Therefore, during the rotational movement, we will have a $\varphi_{x,eq}$ angle with a non-zero value. The goal will be to not cancel this angle but to stabilize the load without oscillations with a fixed inclination corresponding to the $\varphi_{x,eq}$ angle.

In a first approximation, this equilibrium angle is obtained as:

$$\varphi_{x,eq} = r\varphi^2 \big/ \left(g - l\varphi^2\right) \tag{38}$$

As described above, the iterative method allows to the estimator module to calculate in real time the estimated values of the sway angle and of the velocity of the sway angle.

To be able to synchronize this model with the real crane an observer has to be designed, in order to verify that the system is observable.

Using the observer method (see, i.e., Srivastava et al. [12]) applied to (23) and (24), we can determine the eigenvectors and corresponding eigenvalues. As consequence, the observer gain matrix can be obtained.

Specifically, using an explicit form of control, we obtain the correction $\Delta\dot{r}$ of the velocity of the movement along the axis X according to the following equation:

$$\Delta\dot{r} = K_{0,r} \cdot (\varphi_x - \varphi_{x,eq}) + K_{1,r} \cdot \dot{\varphi}_x \tag{39}$$

wherein $K_{0,r}$ and $K_{1,r}$ are the observer gains applied respectively to the effective sway angle $(\varphi_x - \varphi_{x,eq})$ and to the velocity $\dot{\varphi}_x$ of the sway angle along the axis x, and $\Delta\dot{r}$ is the correction signal that is added to the velocity set-point $\dot{r}_{set}$ along the axis x.

In other words, $\dot{x}_{set}$ is the set-point velocity defined either by the crane's operator during a manual controlled movement or by the automatic definition of the velocity during an automatic controlled movement.

Therefore, the reference of velocity applied as an input to the inverter driving the motor relative to the axis x is the velocity set $\dot{r}_{ref}$, that is obtained in the following way:

$$\dot{r}_{ref} = \dot{r}_{set} + \Delta\dot{r} \tag{40}$$

The $K_{0,r}$ and $K_{1,r}$ observer gains are functions of the cable length, in order to optimize the velocity corrections according to the length of the pendulum.

Exactly the same considerations can be applied to the correction $\Delta\dot{\varphi}$ of the angular velocity of the movement along the axis y according to the following equation:

$$\Delta\dot{\varphi} = K_{0,\varphi} \cdot \varphi_y + K_{1,\varphi} \cdot \dot{\varphi}_y \tag{41}$$

wherein $K_{0,\varphi}$ and $K_{1,\varphi}$ are the observer gains applied respectively to the sway angle φ_y and to the velocity $\dot{\varphi}_y$ of the sway angle along the axis y, and $\Delta\dot{\varphi}$ is the correction signal that is added to the velocity set-point $\dot{\varphi}_{set}$ along the axis y.

Therefore, the reference of velocity applied as an input to the inverter driving the motor relative to the axis y is the angular velocity set $\dot{\varphi}_{ref}$, that is obtained in the following way:

$$\dot{\varphi}_{ref} = \dot{\varphi}_{set} + \Delta\dot{\varphi} \tag{42}$$

The $K_{0,\varphi}$ and $K_{1,\varphi}$ observer gains are functions of the cable length, in order to optimize the velocity corrections according to the length of the pendulum.

In a general way, as previously described, during the rotational movement more than the tangential sway angle φ_y, also a sway angle φ_x in a direction perpendicular to the slewing movement is obtained; this angle in direction x can be deleted only by acting on the translation movement of the trolley. However, if no translation movement is requested, the final position of the suspension point in direction x should be equal to the initial position. In order to obtain that, it can be possible to memorize the initial position r of the trolley. After that, we could define a correction velocity to the trolley in order to come back the suspension point of the payload on the trolley to the initial position. That is important, above all, in the automatically controlled movements.

3. **Implementations and Results.** As consequence of his simplicity, the control device can be integrated in a Plc.

This iterative method does not require any preliminary stage of modeling in order to know parameters as a measure of the sway angle or a measure of the current flux of the motor, with the aim to define a transfer function between the velocity of the crane and the sway angle.

Only, the correction method uses the calculated values of the sway angles and of the sway angles velocities in order to compute the correction at the trolley and slewing velocity to add to the set-point velocities (arriving from the operator or defined in an automatic way), obtaining the speed profiles.

A drive controller generates the speed profiles of the movements relative to the trolley (trolley movement) and relative to the slewing (slewing movement) and it supplies the information of the speed profile to the drive able to control the corresponding motor.

Figure 3 shows a simplified diagram of the control device according to this method relative to the movement of a load along the horizontal axis (trolley or/and slewing axes). In general the control device 10 is made with an estimator module 11 connected with an input-output module 12. Practically, the estimator module 11 receives the inputs from the input-output module 12. These inputs are the length l of the cable, the vertical velocity of the cable, the information relative to the non-conservative component of the generalized force Q_φ (for example, the k_f friction coefficient), the speed reference of the trolley $V_{\text{Set Trolley}}$ and of the crane slewing $V_{\text{Set Slew}}$ from the operator or from the controller. After that, the estimator module 11 computes the speed profile to be supplied to the motors in order to obtain the anti-sway functionality.

The estimator module was practically realized with a Plc. In fact, in order to verify the theoretical results of the investigation, the whole system of governing equation was simulated in Codesys V3.5 SP7. That is because Codesys, in structured language, is actually the most common way to realizing function blocks in industrial motion control environment.

The cyclic task, where the function block of the used Plc was realized, had a time of updating equal to 30 ms.

The Function Blocks (FB's) used were two: the first FB computes the speed profiles necessary to obtain the anti-sway functionality and the corresponding actual sway angles, and the second FB is used to compute the actual length l of the cable and the corresponding vertical speed using the data coming from an external unit (i.e., encoders) connected to the motor of the corresponding movement.

The speed reference is the target value to which the speed must reach, defined from the crane operator or from the automatic control. Usually, it is defined in Hz, as a consequence of the way in which the electric motors are defined. At the speed in Hz on the fast shaft

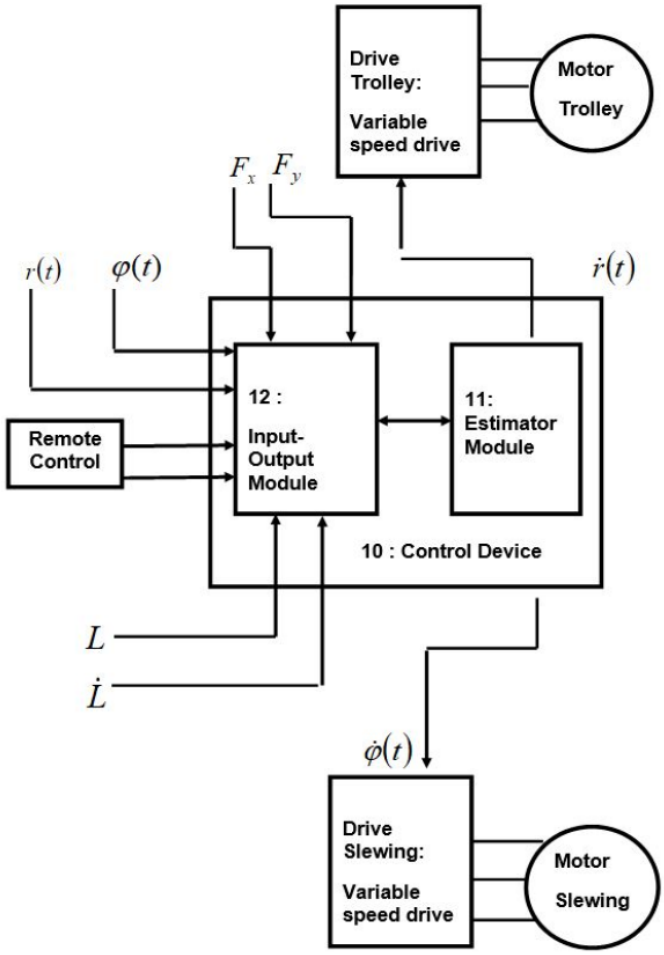

FIGURE 3. Diagram describing the control device, the input-output, the drives and corresponding motors controlled by the control device

(that is on the motor) a velocity corresponds on the slow shaft (that is on the wheels moving on the rails of the trolley or on the shaft controlling the slewing motion of the jib), depending on the reduction gearing from the motor to the wheel. Typically, the max speed of a trolley can be from 0.2 m/s to also more 2 m/s. The max speed of a slewing motion (measured in rad/s) can arrive to 0.5 rad/s.

The ramp set is the value of the time that would be necessary for the linear motor ramp to reach the speed reference. Based on the speed reference, the estimator module computes the real speed profile in order to have the anti-sway effect. Obviously, the speed profile is longer of the linear ramp set.

The cable length has a very important influence on the speed profile, because the greater the height is (and so the cable length), the greater the time of the speed profile is. In a general way, it is very important to reduce the time of the speed profile in order to obtain a fast answer to the commands of the tower crane movement.

In Figure 4, we can see the profiles of the angular velocity profile for the slewing movement and the corresponding profile of the tangential sway angle.

Instead, in Figure 5, the profiles of the velocity profile of the radial movement and the corresponding profile of the perpendicular sway angle are shown. That in correspondence of the same basic data describes in the figures. In this way, a fundamental difference stands out: we can see in Figure 5 that the perpendicular sway angle φ_x does not go to zero, but tends to a constant value, the $\varphi_{x,eq}$ angle, as described previously. In fact, the

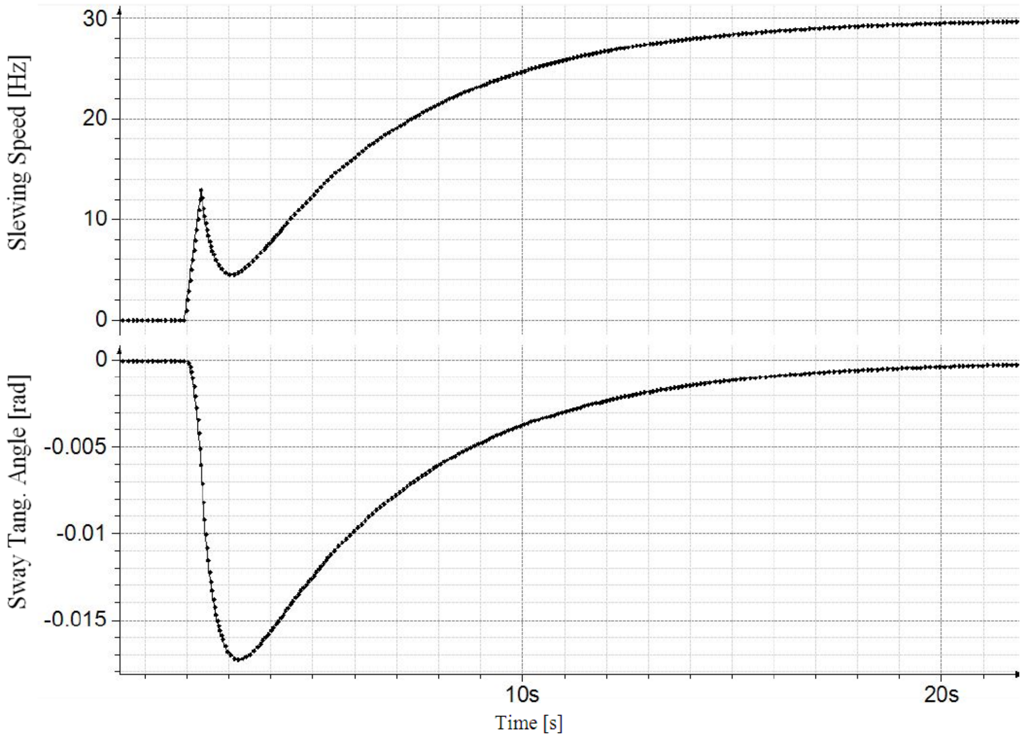

FIGURE 4. A graph relative to the angular velocity profile $\dot{\varphi}_{ref}$ of the slewing movement and the corresponding profile of the tangential sway angle φ_y. That is in correspondence to a command relative to the only slewing movement. The specific values are: speed reference = 30 Hz, ramp set = 1.5 s, cable length = 10.5 m, cycle time of the Plc = 30 ms.

goal of the control is not to eliminate this sway during the rotation movement, because this perpendicular sway is a natural part of the rotational movement, but to arrive at an equilibrium position with a non-zero sway (in x direction) of the load, corresponding to a non-zero angle.

Some subsequent commands (4 commands) are shown to go to different velocities (but not to zero velocity) for the slewing movement in Figure 6. We can see that the corresponding sway angles, at the end of the movement, go to zero.

Therefore, we see that, with this method, it is possible to obtain the total damping of the sway changing many times the velocity set, also if the previous damping control of the oscillations is again in progress (see Figure 6). That is fundamental because the usual control of the tower crane is typically manual, by a human operator: typically, he will change often the target velocity with his command.

This characteristic is typical of the "command profile" method, as the method developed in this work. For example, the works of Barisa et al. [6] or of Böck and Kugi [7], using the "model predictive control", are optimized for the automatic control, where there is a pre-defined position target. However, when the control is manual, the method defined in this work is preferable.

An important characteristic is to be highlighted in Figure 7. Varying the basic parameters for the profiles, the same profiles for the velocity can be very fast, particularly corresponding to a very fast stop with anti-sway. This is a relevant result in order to control the crane with high performance for the answer to the command. It is possible to note that, in order to obtain the wanted fast stop profile, the trolley speed has to take negative values. That corresponds to little slow movements in opposite direction to

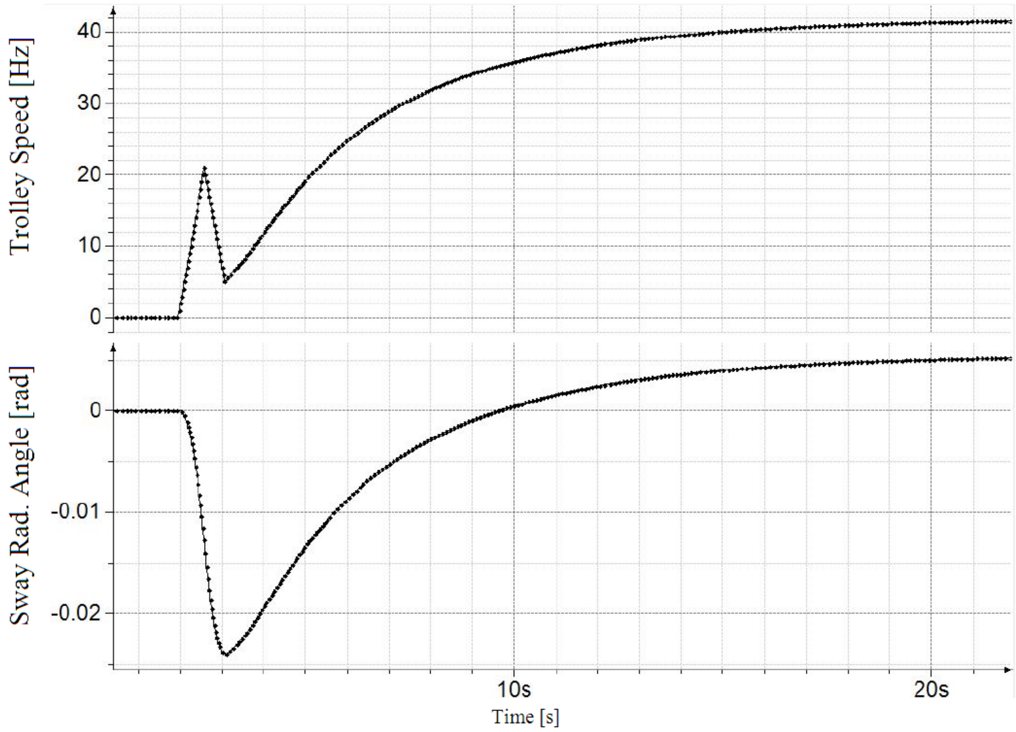

FIGURE 5. A graph relative to the velocity profile $\dot{r}_{ref}$ of the radial movement and the corresponding profile of the perpendicular sway angle φ_x. That is in correspondence to a command relative to the only radial movement. In other words, only a trolley command is given. The specific values are: speed reference = 35 Hz, ramp set = 1.5 s, cable length = 10.5 m, cycle time of the Plc = 30 ms.

that of the commanded movement, necessary so that the corresponding sway angle profile reduces to zero in a fast way (as it is possible to see in Figure 7).

Also in the work of of Stanchev and Velachev [10], the "command smoothing" method was used. Nevertheless, in that work, profiles for the velocity were not defined as very fast. In fact, in that work the possibility to have a negative velocity for the trolley or for the slewing (with the goal to have a very fast stop) was not hypothesized. Nevertheless, this characteristic is highly important for tower cranes.

A further relevant aspect can be considered examining Figure 8. In that figure, not only the profile for the slewing velocity is very short (that is with a few times, as in the previous Figure 7), but, above all, the trolley velocity (despite being little, only some Hz) is not zero, although the trolley speed reference is zero: that is, also without a trolley command by the operator. That corresponds to the fact described previously: during the rotational movement, beyond that the tangential sway angle φ_y, also a sway angle φ_x in a direction perpendicular to the slewing movement is formed. This angle in direction x, due to the tangential acceleration caused by the variation of the slewing speed, can be deleted only by acting on the translation movement of the trolley.

4. **Conclusions.** In this paper, the sway control for a tower (jib) crane was investigated. A solution for the effective non-linear equations of motion is obtained defining an iterative method for the solution of the movement equation. It was considered also the cable flexibility and stiffness that cause a sensible variation of the period time for the pendulum relative to the oscillations of the payload.

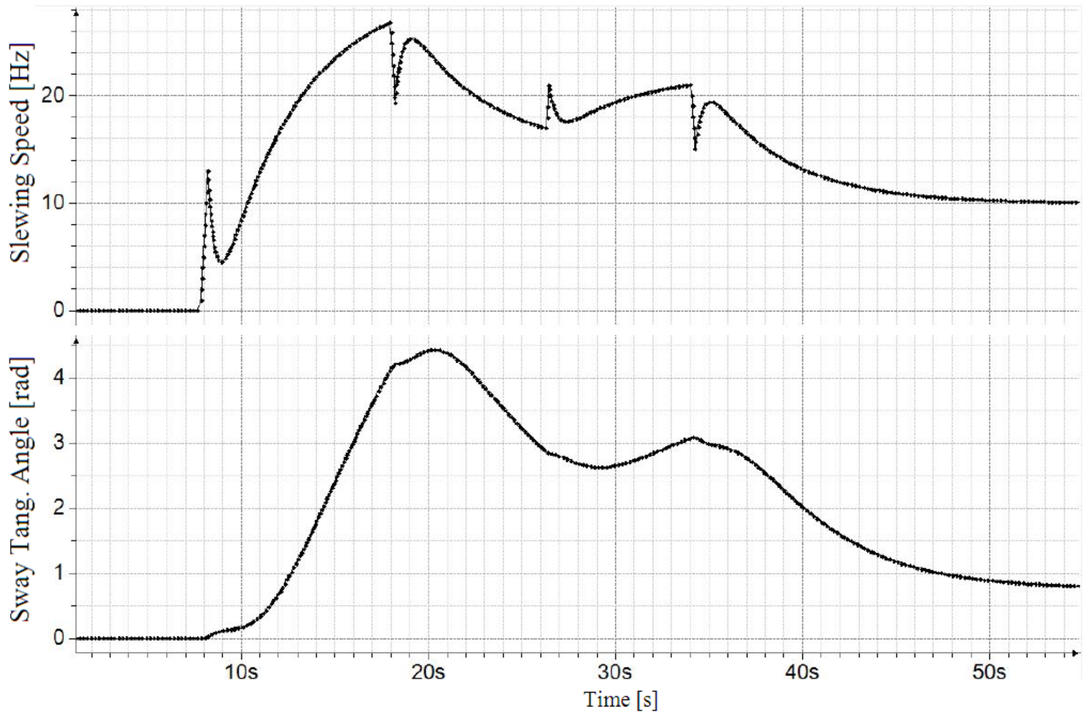

FIGURE 6. A graph relative to the angular velocity profile $\dot{\varphi}_{ref}$ of the slewing movement and the corresponding profile of the tangential sway angle φ_y. That is in correspondence to some subsequent commands of the slewing movement by the operator. The specific values are: initial slewing speed reference = 30 Hz, final slewing speed reference = 10 Hz, ramp set = 1.5 s, cable length = 10.5 m, cycle time of the Plc = 30 ms.

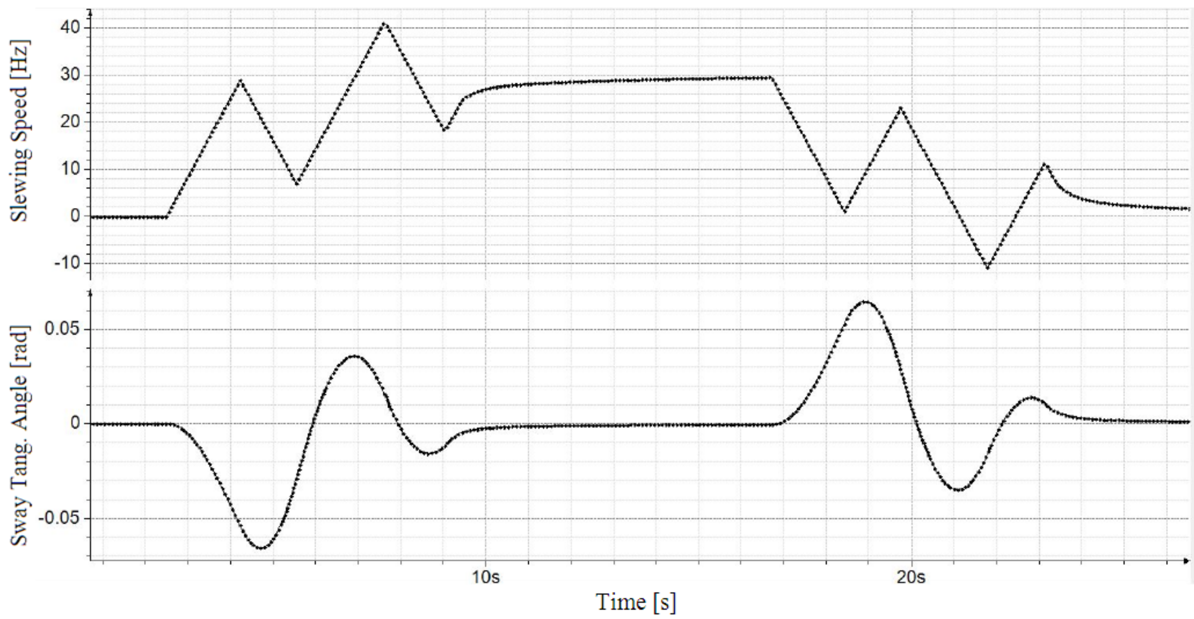

FIGURE 7. A graph relative to the angular velocity profile $\dot{\varphi}_{ref}$ of the slewing movement and the corresponding profile of the tangential sway angle φ_y. That is in correspondence to a command relative to the only slewing movement. The specific values are: speed reference = 30 Hz, ramp set = 3.0 s, cable length = 10.5 m, cycle time of the Plc = 30 ms.

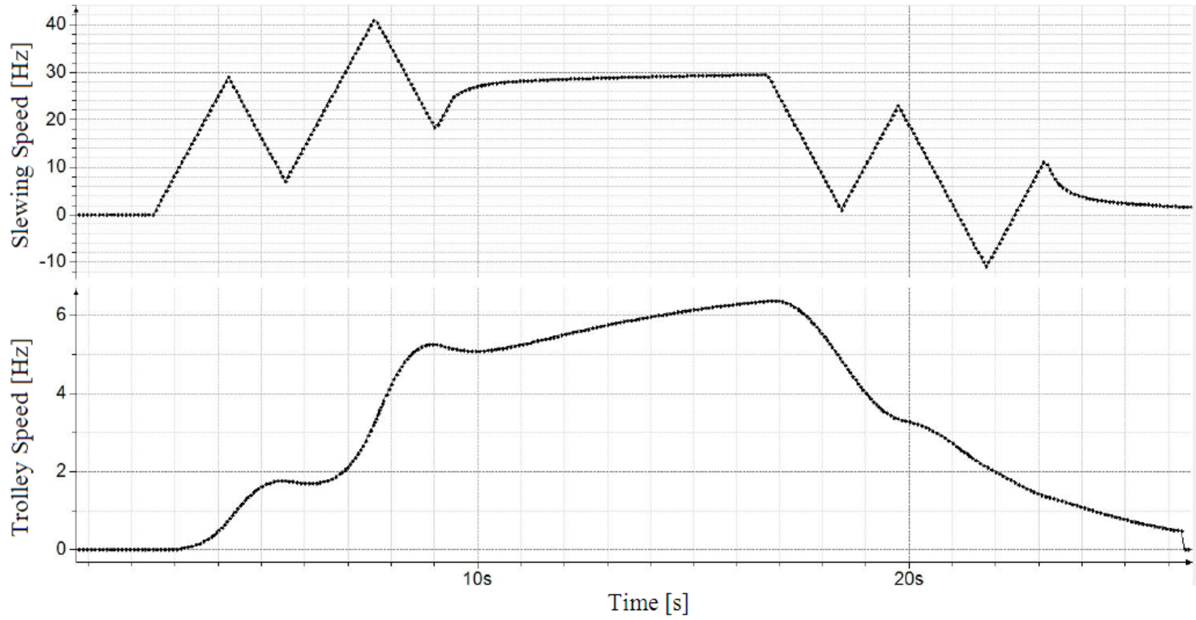

FIGURE 8. A graph relative to the angular velocity profile $\dot{\varphi}_{ref}$ of the slewing movement and the velocity profile $\dot{r}_{ref}$ of the radial movement. That is in correspondence to only a command of the slewing movement. The specific values are: slewing speed reference = 30 Hz, trolley speed reference = 0 Hz, ramp set = 3.0 s, cable length = 10.5 m, cycle time of the Plc = 30 ms. There is also a command to stop the movement (Final slewing speed reference = 0 Hz).

Besides, the present work considered also the continuous variation of the hoisting height for the suspension point of the payload and the variation of the corresponding velocity and acceleration.

A detailed mathematical model is developed which considered also the non-linear components of the forces, as centrifugal and Coriolis forces on the system.

The obtained velocity profile can generate a very fast stop while the anti-sway functionality is activated: that allows fast control from the operator in case of manual control.

Possible future developments of this work may concern two fundamental aspects. On the one side, it should be important to obtain a solution that takes account of, in some way, the most important vibrational modes of the tower structure: they can influence the sway of the payload, at least in certain situations. On the other side, the effect of external disturbances such as wind is to be studied in a deep way, because they have very significant effects on the tower crane's performances.

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