International Journal of Innovative Computing, Information and Control
Volume 14, Number 4, August 2018
 ISSN 1349-4198
pp. 1327–1338

H_∞-CONTROL OF ROBUST LINEAR SYSTEMS WITH INTERVAL STATE AND INPUT DELAYS

Bayram Barış Kızılsaç* and Ulviye Başer

Department of Mathematics
Istanbul Technical University
Maslak, Istanbul 34469, Turkey
*Corresponding author: kizilsac@itu.edu.tr; baser@itu.edu.tr

Received August 2017; revised February 2018

Abstract. *In this paper, H_∞-control problem of uncertain linear systems with non-coincident time-varying state and input delays is considered. Both state and input delays are assumed to be in some given intervals. Contrary to the previous works, the lower bounds of these delays are not restricted to zero. By defining a suitable augmented Lyapunov-Krasovskii functional, a new delay-dependent sufficient condition is developed in terms of linear matrix inequalities to ensure H_∞-control of the system with minimum allowable disturbance attenuation level. The effectiveness and the advantages of the proposed method are illustrated on the various numerical examples.*
Keywords: H_∞-control, State delay, Input delay, Robust linear systems, Stability

1. **Introduction.** Stability analysis and the controller design for the system with time-delay have been widely studied in last decades. The earlier and recent references are [1, 2] and the references there in. Stability analysis of delay systems is achieved in delay independent or delay dependent cases. Results in delay independent case do not include any information on the size of delay, but delay dependent solutions include such information. Many works in delay dependent stability case try to enlarge the delay interval. In order to obtain less conservative results, a relaxed inequality to bound the cross terms is introduced in [3], zero equations with free variables to the solutions are added in [4, 5], the descriptor system approach is proposed in [6], and a new augmented Lyapunov-Krasovskii (L-K) functional is presented in [7, 8]. In [9] augmented L-K functional is applied for the stabilization of the uncertain systems with interval time-varying delays. In all these works, state delays are considered and the L-K functional approach and linear matrix inequality (LMI) based feedback design methods are used for the solutions of the problems such as stability, stabilization and H_∞ control.

As it can be seen from the literature, these problems are also studied on industrial systems having input delay, such as vehicle systems [10], seat suspension systems [11], networked systems [12], fuzzy systems [13], and systems with multiple delays [14]. In some dynamical systems, time-delays also effect the system state and control input simultaneously [15, 16, 17, 18, 19, 20]. In [15, 20] time-varying state and input delays are considered, but the derivative of state delay is restricted to less than 1. In [16, 18] constant and coincident state and input delays are considered. In [17] interval time-varying state and input delays are examined with respect to the midpoints of the intervals. In [19] augmented L-K functional is applied for the stabilization of linear time-varying non-coincident input and state delays with zero lower bounds. Because of some nonlinear terms

DOI: 10.24507/ijicic.14.04.1327

in the matrix inequalities given in both [8, 19], it is referred to the cone-complementary linearization algorithm suggested in [21].

As the knowledge of the authors, H_∞-control of systems with interval time-varying state and input delays has not been solved by augmented type L-K functional.

The main objectives of this paper are

• To obtain the less conservative results for the robust stability/H_∞-control of the systems with interval time-varying state and input delays;

• To improve an algorithm without using the cone-complementary method.

In this paper, a new augmented type L-K functional is first defined for the interval non-coincident time-varying state and input delays. Some zero terms are introduced to relax the solutions. A new delay-dependent sufficient condition is developed in terms of linear matrix inequalities to ensure robust stability and H_∞-control of the system with minimum allowable disturbance attenuation level without using the cone-complementary method. Besides that, the less conservative result in comparison to those of existing methods in the literature is presented for H_∞-control of uncertain linear systems with the interval time-varying state delay. Theorem 3.1 proposed here is extended for the neutral systems. The rest of this paper is organized as follows. The problem formulation is presented in Section 2. In Section 3, the sufficient conditions for the stability, stabilization and H_∞-control of the systems with interval time-varying state and also noncoincident state and input delays are established, and some concluding remarks are stated. Then, simulation studies illustrate the effectiveness of the proposed method in Section 4. Finally, the conclusions are given in Section 5.

In the sequel, the notations are fairly standard. $\mathbf{R^n}$ is the n-dimensional Euclidean space. $\mathbf{R^{m\times n}}$ denotes the the set of $m \times n$ real matrices. $X > 0$ denotes that X is real symmetric positive definite matrix.

2. Problem Statement and Preliminaries.

Consider the following system:

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-h_1(t)) + (B+\Delta B)u(t) \\ &\quad +(B_1+\Delta B_1)u(t-h_2(t)) + B_2 w(t), \qquad (1) \\ z(t) &= Cx(t), \qquad (2) \\ x(t) &= \phi(t), \quad \forall t \in \left[-\max\left\{\bar{h}_1, \bar{h}_2\right\}, 0\right], \qquad (3) \end{aligned}$$

where $x(t) \in \mathbf{R^n}$ and $u(t) \in \mathbf{R^m}$ are the state and control inputs, respectively, ϕ is a continuously differentiable initial function, w denotes the disturbance vector, A, A_1, B, B_1 and B_2 are known constant real matrices with appropriate dimensions and ΔA, ΔA_1, ΔB, ΔB_1 are the uncertainties of the system matrices of the form

$$\left[\begin{array}{cccc} \Delta A & \Delta A_1 & \Delta B & \Delta B_1 \end{array}\right] = DF(t)\left[\begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array}\right] \qquad (4)$$

in which the time-varying nonlinear function $F(t)$ satisfies

$$F^T(t)F(t) \le I, \ \forall t \ge 0. \qquad (5)$$

The delays $h_1(t)$ and $h_2(t)$ are time-varying continuous functions satisfying

$$0 \le \underline{h}_1 \le h_1(t) \le \bar{h}_1, \ 0 \le \underline{h}_2 \le h_2(t) \le \bar{h}_2, \qquad (6)$$

$$\dot{h}_1(t) \le \mu_1, \ \dot{h}_2(t) \le \mu_2. \qquad (7)$$

Now, define the following performance index

$$J(w(t)) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right] dt. \qquad (8)$$

Our goal is to find a memoryless state-feedback controller in the form of $u(t) = Kx(t)$, such that the closed-loop system

$$\dot{x}(t) = (A + BK)x(t) + A_1x(t - h_1(t)) + B_1Kx(t - h_2(t)) + B_2w(t) + Dp(t), \quad (9)$$
$$p(t) = F(t)q(t), \quad (10)$$
$$q(t) = (E_1 + E_3K)x(t) + E_2x(t - h_1(t)) + E_4Kx(t - h_2(t)) \quad (11)$$

is asymptotically stable and guarantees $J(w(t)) < 0$ under zero initial condition for all non-zero $w(t) \in L_2[0, \infty)$ and some prescribed $\gamma > 0$. In order to obtain the main results, we need the following lemmas.

Lemma 2.1. *For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, it follows that*

$$2a^Tb \leq a^TQa + b^TQ^{-1}b.$$

Lemma 2.2. [22] *For any constant-real matrix $P \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$, $P > 0$, scalar $\tau > 0$ and vector valued function $\chi : [0, \tau] \longrightarrow \mathbf{R}^{\mathbf{n}}$, the following inequality holds*

$$\tau \int_0^\tau \chi^T(s)P\chi(s)ds \geq \left(\int_0^\tau \chi(s)ds\right)^T P \left(\int_0^\tau \chi(s)ds\right). \quad (12)$$

3. **Main Results.** This section presents the delay-dependent stabilization conditions for system in (1)-(3) with interval time-varying delays.

Theorem 3.1. *Given positive scalars $\underline{h}_1$, $\bar{h}_1$, $\underline{h}_2$, $\bar{h}_2$, μ_1, μ_2 and ϵ. The closed-loop system in Equations (9)-(11) is asymptotically stable with disturbance attenuation γ, for any time-varying delays $h_1(t)$ and $h_2(t)$ satisfying (6) and (7) if there exist symmetric positive definite matrices P, M_i, N_i, Q_i and L_i, for $i = 1, 2$ and R_{ij}, for $i = 1, 2,\ j = 1, 2, 3$, and the matrices U, S_{ij}, for $i, j = 1, 2$, with appropriate dimensions satisfying the following LMI's*

$$\Sigma = \begin{bmatrix} \Theta & \sqrt{\mu}_1\Pi_1^T & \sqrt{\mu}_2\Pi_2^T & \Psi^TU \\ * & -L_1 & 0 & 0 \\ * & * & -L_2 & 0 \\ * & * & * & -\dfrac{1}{\epsilon}U \end{bmatrix} < 0, \quad (13)$$

where

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{33} \\ * & * & P_{33} \end{bmatrix} > 0,\ M_i = \begin{bmatrix} M_{i1} & M_{i2} \\ * & M_{i3} \end{bmatrix} > 0,\ N_i = \begin{bmatrix} N_{i1} & N_{i2} \\ * & N_{i3} \end{bmatrix} > 0,$$

for $i = 1, 2$,

$$\Pi_1 = \begin{bmatrix} P_{12}^T & P_{22}^T & P_{23} & 0 & \dots & 0 \end{bmatrix},\ \Pi_2 = \begin{bmatrix} P_{13}^T & P_{23}^T & P_{33} & 0 & \dots & 0 \end{bmatrix},$$

$\Theta_{1,1} = P_{12}+P_{12}^T+P_{13}+P_{13}^T+\bar{h}_1^2M_{11}+\bar{h}_2^2M_{21}-M_{13}-M_{23}+d_1^2N_{11}+d_2^2N_{21}+U^TA_K+A_K^TU^T+R_{11}+R_{21}+R_{12}+R_{22}+R_{13}+R_{23}+S_{11}+S_{11}^T+S_{21}+S_{21}^T+C^TC$, $\Theta_{1,2} = -P_{12}+M_{13}+UA_1-S_{11}+S_{12}^T$, $\Theta_{1,3} = -P_{13}+M_{23}+UB_1K-S_{21}+S_{22}^T$, $\Theta_{1,4} = P_{22}+P_{23}^T-M_{12}^T$, $\Theta_{1,5} = P_{23}+P_{33}-M_{22}^T$, $\Theta_{1,6} = P_{11} + \bar{h}_1^2M_{11} + \bar{h}_2^2M_{22} + d_1^2N_{12} + d_2^2N_{22} - U + A_k^TP^T$, $\Theta_{1,15} = U^TB_2$, $\Theta_{1,16} = U^TD$, $\Theta_{1,17} = -S_{11}$, $\Theta_{1,18} = -S_{21}$, $\Theta_{2,2} = \mu_1L_1 - 2M_{13} - 2N_{13} - (1 - \mu_1)R_{12} - S_{12} - S_{12}^T$, $\Theta_{2,4} = -P_{22} + M_{12}^T$, $\Theta_{2,5} = -P_{23}$, $\Theta_{2,6} = A_1^TU$, $\Theta_{2,7} = N_{13}^T$, $\Theta_{2,9} = M_{13} + N_{13}$, $\Theta_{2,11} = -M_{12}^T-N_{12}^T$, $\Theta_{2,13} = N_{12}^T$, $\Theta_{2,17} = -S_{12}$, $\Theta_{3,3} = \mu_2L_2-2M_{23}-2N_{23}-(1-\mu_2)R_{22}-S_{22}-S_{22}^T$, $\Theta_{3,4} = -P_{23}^T$, $\Theta_{3,5} = -P_{33} + M_{22}^T$, $\Theta_{3,6} = K^TB_1U^T$, $\Theta_{3,8} = N_{23}^T$, $\Theta_{3,10} = M_{23} + N_{23}$, $\Theta_{3,12} = -M_{22}^T - N_{22}^T$, $\Theta_{3,14} = N_{22}^T$, $\Theta_{3,18} = -S_{22}$, $\Theta_{4,4} = -M_{11}$, $\Theta_{4,6} = P_{12}^T$, $\Theta_{5,5} = -M_{21}$, $\Theta_{5,6} = P_{13}^T$, $\Theta_{6,6} = \bar{h}_1^2M_{13}+\bar{h}_2^2M_{23}+\bar{d}_1^2N_{13}+\bar{d}_2^2N_{23}-U-U^T+\bar{h}_1^2Q_1+\bar{h}_2^2Q_2$, $\Theta_{6,15} = U^TB_2$, $\Theta_{6,16} = U^TD$, $\Theta_{7,7} = -N_{13}-R_{11}$, $\Theta_{7,13} = -N_{12}^T$, $\Theta_{8,8} = -N_{23}-R_{21}$, $\Theta_{8,14} = -N_{22}^T$, $\Theta_{9,9} =$

$-M_{13}-N_{13}-R_{13}$, $\Theta_{9,11}=M_{12}^T+N_{12}^T$, $\Theta_{10,10}=-M_{23}-N_{23}-R_{23}$, $\Theta_{10,12}=M_{22}^T+N_{22}^T$, $\Theta_{11,11}=-M_{11}-N_{11}$, $\Theta_{12,12}=-M_{21}-N_{21}$, $\Theta_{13,13}=-N_{11}$, $\Theta_{14,14}=-N_{21}$, $\Theta_{15,15}=-\gamma^2 I$, $\Theta_{16,16}=-\epsilon U$, $\Theta_{17,17}=-Q_1$, $\Theta_{18,18}=-Q_2$.

Proof: Let us choose an L-K functional candidate as $V(x(t),t)=\sum_{i=1}^4 V_i(x(t),t)$, where

$$\begin{aligned}
V_1(x(t),t) &= \eta^T(t)P\eta(t),\\
V_2(x(t),t) &= \sum_{j=1}^2 \bar{h}_j \int_{-\bar{h}_j}^0 \int_{t+\theta}^t \xi^T(s)M_j\xi(s)dsd\theta\\
&\quad + \sum_{j=1}^2 \left(\bar{h}_j-\underline{h}_j\right) \int_{-\bar{h}_j}^{-\underline{h}_j} \int_{t+\theta}^t \xi^T(s)N_j\xi(s)dsd\theta,\\
V_3(x(t),t) &= \sum_{j=1}^2 \left(\int_{t-\underline{h}_j}^t x^T(s)R_{j1}x(s)ds + \int_{t-h_j(t)}^t x^T(s)R_{j2}x(s)ds \right.\\
&\quad \left. + \int_{t-\bar{h}_j}^t x^T(s)R_{j3}x(s)ds \right),\\
V_4(x(t),t) &= \sum_{j=1}^2 \bar{h}_j \int_{-\bar{h}_j}^0 \int_{t+\theta}^t \dot{x}^T(s)Q_j\dot{x}(s)dsd\theta, \qquad (14)\\
\eta(t) &= \left[\ x^T(t)\ \left(\int_{t-h_1(t)}^t x(s)ds\right)^T\ \left(\int_{t-h_2(t)}^t x(s)ds\right)^T \right]^T,\\
\xi(t) &= \left[\ x^T(t)\ \dot{x}^T(t)\right]^T.
\end{aligned}$$

Taking the time derivative of $V_i(x(t),t)$, $i=1,2,3,4$ along the trajectory of system (9) yields

$$\begin{aligned}
\dot{V}_1 &= 2\eta^T(t)P\dot{\eta}(t),\\
\dot{V}_2 &= \sum_{j=1}^2 \bar{h}_j^2\xi^T(t)M_j\xi(t) - \sum_{j=1}^2 \bar{h}_j \int_{t-\bar{h}_j}^t \xi^T(s)M_j\xi(s)ds\\
&\quad + \sum_{j=1}^2 \left(\bar{h}_j-\underline{h}_j\right)^2 \xi^T(t)N_j\xi(t) - \sum_{j=1}^2 \left(\bar{h}_j-\underline{h}_j\right) \int_{t-\bar{h}_j}^{t-\underline{h}_j} \xi^T(s)N_j\xi(s)ds,\\
\dot{V}_3 &= \sum_{j=1}^2 \left(x^T(t)R_{j1}x(t) - x^T\left(t-\underline{h}_j\right)R_{j1}x\left(t-\underline{h}_j\right) \right.\\
&\quad + x^T(t)R_{j2}x(t) - \left(1-\dot{h}_j(t)\right)x^T(t-h_j(t))R_{j2}x(t-h_j(t))\\
&\quad \left. + x^T(t)R_{j3}x(t) - x^T\left(t-\bar{h}_j\right)R_{j3}x\left(t-\bar{h}_j\right) \right),\\
\dot{V}_4 &= \sum_{j=1}^2 \left(\bar{h}_j^2\dot{x}^T(t)Q_j\dot{x}(t) - \bar{h}_j \int_{t-\bar{h}_j}^t \dot{x}^T(s)Q_j\dot{x}(s)ds \right).
\end{aligned}$$

For simplicity, denote $x(t)=:x$, $\dot{x}(t)=:\dot{x}$, $\eta(t)=:\eta$, $x(t-h_j(t))=:x_{h_j(t)}$ and $\int_{t-h_j(t)}^t x(s)ds=:i_{h_j(t)}$, for $j=1,2$. Then $\dot{\eta}(t)$ and $\dot{V}_1$ can be written as

$$\dot{\eta}=\eta_1+\dot{h}_1(t)\eta_2 x_{h_1(t)}+\dot{h}_2(t)\eta_3 x_{h_2(t)}, \qquad (15)$$

and

$$\dot{V}_1 = 2\eta^T P\eta_1 + 2\dot{h}_1(t)\eta^T P\eta_2 x_{h_1(t)} + 2\dot{h}_2(t)\eta^T P\eta_3 x_{h_2(t)}, \tag{16}$$

respectively, where

$$\eta_1 = \begin{bmatrix} \dot{x}^T & (x - x_{h_1(t)})^T & (x - x_{h_2(t)})^T \end{bmatrix}^T,\ \eta_2 = \begin{bmatrix} 0 & I & 0 \end{bmatrix}^T,\ \eta_3 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T.$$

By (7) and Lemma 2.1 there exist positive definite matrices L_1 and L_2 such that

$$\begin{aligned} 2\dot{h}_1(t)\eta^T P\eta_2 x_{h_1(t)} &\le \mu_1\Big(\eta^T P\eta_2 L_1^{-1}\eta_2^T P\eta + x_{h_1(t)}^T L_1 x_{h_1(t)}\Big), \\ 2\dot{h}_2(t)\eta^T P\eta_3 x_{h_2(t)} &\le \mu_2\Big(\eta^T P\eta_3 L_2^{-1}\eta_3^T P\eta + x_{h_2(t)}^T L_2 x_{h_2(t)}\Big), \end{aligned}$$

and so

$$\begin{aligned} \dot{V}_1 \le{}& 2\eta^T P\eta_1 + \mu_1\left(\eta^T P\eta_2 L_1^{-1}\eta_2^T P\eta + x_{h_1(t)}^T L_1 x_{h_1(t)}\right) \\ &+ \mu_2\left(\eta^T P\eta_3 L_2^{-1}\eta_3^T P\eta + x_{h_2(t)}^T L_2 x_{h_2(t)}\right). \end{aligned} \tag{17}$$

On the other hand, the integral parts of $\dot{V}_2$ can be written as the following way

$$\int_{t-\bar{h}_j}^{t} \xi^T(s) M_j \xi(s) ds = \int_{t-\bar{h}_j}^{t-h_j(t)} \xi^T(s) M_j \xi(s) ds + \int_{t-h_j(t)}^{t} \xi^T(s) M_j \xi(s) ds, \tag{18}$$

and

$$\int_{t-\bar{h}_j}^{t-\underline{h}_j} \xi^T(s) N_j \xi(s) ds = \int_{t-\bar{h}_j}^{t-h_j(t)} \xi^T(s) N_j \xi(s) ds + \int_{t-h_j(t)}^{t} \xi^T(s) N_j \xi(s) ds. \tag{19}$$

From (6) it is possible to write these inequalities as

$$\begin{aligned} -\sum_{j=1}^{2} \bar{h}_j \int_{t-\bar{h}_j}^{t} \xi^T(s) M_j \xi(s) ds \le{}& -\sum_{j=1}^{2} \left(\bar{h}_j - h_j(t)\right) \int_{t-\bar{h}_j}^{t-h_j(t)} \xi^T(s) M_j \xi(s) ds \\ &- h_j(t) \int_{t-h_j(t)}^{t} \xi^T(s) M_j \xi(s) ds, \end{aligned} \tag{20}$$

and

$$\begin{aligned} &-\sum_{j=1}^{2} \left(\bar{h}_j - \underline{h}_j\right) \int_{t-\bar{h}_j}^{t-\underline{h}_j} \xi^T(s) N_j \xi(s) ds \\ &\le -\sum_{j=1}^{2} \left(\bar{h}_j - h_j(t)\right) \int_{t-\bar{h}_j}^{t-h_j(t)} \xi^T(s) N_j \xi(s) ds \\ &\quad - \sum_{j=1}^{2} \left(h_j(t) - \underline{h}_j\right) \int_{t-h_j(t)}^{t-\underline{h}_j} \xi^T(s) N_j \xi(s) ds. \end{aligned} \tag{21}$$

Then, by Lemma 2.2 the right sides of the inequalities (20) and (21) can be written as

$$\begin{aligned} \le -\sum_{j=1}^{2} \Bigg[& \left(\int_{t-\bar{h}_j}^{t-h_j(t)} \xi^T(s) ds\right) M_j \left(\int_{t-\bar{h}_j}^{t-h_j(t)} \xi(s) ds\right) \\ &+ \left(\int_{t-h_j(t)}^{t} \xi^T(s) ds\right) M_j \left(\int_{t-\bar{h}_j}^{t-h_j(t)} \xi(s) ds\right)\Bigg], \end{aligned} \tag{22}$$

and

$$\leq -\sum_{j=1}^{2}\left[\left(\int_{t-\bar{h}_j}^{t-h_j(t)}\xi^T(s)ds\right)N_j\left(\int_{t-\bar{h}_j}^{t-h_j(t)}\xi(s)ds\right)\right.$$
$$\left.+\left(\int_{t-h_j(t)}^{t-\underline{h}_j}\xi^T(s)ds\right)N_j\left(\int_{t-h_j(t)}^{t-\underline{h}_j}\xi(s)ds\right)\right], \tag{23}$$

respectively. Now, let $\int_{t-\bar{h}_j}^{t-h_j(t)}x(s)ds =: a_j$, $x\left(t-\bar{h}_j\right) =: x_{\bar{h}_j}$, $\int_{t-h_j(t)}^{t-\underline{h}_j}x(s)ds =: b_j$, $x(t-\underline{h}_j) =: x_{\underline{h}_j}$. Then, by (22) and (23) $\dot{V}_2$ can be written as follows

$$\begin{aligned}\dot{V}_2 \leq \sum_{j=1}^{2}&\Big(\bar{h}_j^2\Big(x^TM_{j1}x+2x^TM_{j2}\dot{x}+\dot{x}^TM_{j3}\dot{x}\Big)-\Big(a_j^TM_{j1}a_j+2a_j^TM_{j2}x_{h_j(t)}\\
&-2a_j^TM_{j2}x_{\bar{h}_j}+x_{h_j(t)}^TM_{j3}x_{h_j(t)}-2x_{h_j(t)}M_{j3}x_{\bar{h}_j}+x_{\bar{h}_j}^TM_{j3}x_{\bar{h}_j}\Big)\\
&-\Big(i_{h_j(t)}^TM_{j1}i_{h_j(t)}+2x^TM_{j2}^Ti_{h_j(t)}+x^TM_{j3}x-2x^TM_{j3}x_{h_j(t)}\\
&-2x_{h_j(t)}^TM_{j2}^Ti_{h_j(t)}+x_{h_j(t)}^TM_{j3}^Tx_{h_j(t)}\Big)\\
&+\bar{d}_j^2\Big(x^TN_{j1}x+2x^TN_{j2}\dot{x}+\dot{x}^TN_{j3}\dot{x}\Big)-\Big(a_j^TN_{j1}a_j+2a_j^TN_{j2}x_{h_j(t)}\\
&-2a_j^TN_{j2}x_{\bar{h}_j}+x_{h_j(t)}^TN_{j3}x_{h_j(t)}-2x_{h_j(t)}N_{j3}x_{\bar{h}_j}+x_{\bar{h}_j}^TN_{j3}x_{\bar{h}_j}\Big)\\
&-\Big(b_j^TN_{j1}b_j+2b_j^TN_{j2}x_{\underline{h}_j}-2x_{h_j(t)}^TN_{j2}^Tb_j+x_{\underline{h}_j}^TN_{j3}x_{\underline{h}_j}\\
&-2x_{\underline{h}_j}^TN_{j3}x_{h_j(t)}+x_{h_j(t)}^TN_{j3}^Tx_{h_j(t)}\Big)\Big).\end{aligned} \tag{24}$$

Let $\int_{t-h_j(t)}^{t}\dot{x}(s)ds =: d_{h_j(t)}$, for $j=1,2$ and consider $\dot{V}_4$. Since $\bar{h}_j\int_{t-\bar{h}_j}^{t}\dot{x}^T(s)Q_j\dot{x}(s)ds$ can be written as Equation (20) and $Q_j>0$, then by Lemma 2.2 $\dot{V}_4$ can be written as

$$\dot{V}_4 \leq \sum_{j=1}^{2}\left(\bar{h}_j^2\dot{x}^TQ_j\dot{x}-d_{h_j(t)}^TQ_jd_{h_j(t)}\right). \tag{25}$$

Now, let U be an arbitrary matrix and consider the following zero terms

$$2\left(x^T+\dot{x}^T\right)U\left(-\dot{x}+A_Kx+A_1x_{h_1(t)}+B_1Kx_{h_2(t)}+B_2w+Dp(t)\right)=0, \tag{26}$$

$$\sum_{j=1}^{2}2\left[x^T(t)S_{j1}+x^T(t-h_j(t))S_{j2}\right]\left[x(t)-x(t-h_j(t))-\int_{t-h_j(t)}^{t}\dot{x}(s)ds\right]=0, \tag{27}$$

where $A+BK =: A_K$. As it is given in [9], since $p^T(t)p(t)\leq q^T(t)q(t)$ there exist a positive scalar ϵ and a positive definite matrix U such that

$$\chi(t)^T\Psi^T(\epsilon U)\Psi\chi(t)-p^T(t)(\epsilon U)p(t)\geq 0, \tag{28}$$

where

$$\chi(t)^T=\left[x^T\ x_{h_1(t)}^T\ x_{h_2(t)}^T\ i_{h_1(t)}^T\ i_{h_2(t)}^T\ \dot{x}^T\ x_{\underline{h}_1}^T\ x_{\underline{h}_2}^T\ x_{\bar{h}1}^T\ x_{\bar{h}2}^T\ a_1^T\ a_2^T\ b_1^T\ b_2^T\ w^T\ p^T\ d_{h_1(t)}^T\ d_{h_1(t)}^T\right],$$

$$\Psi=\left[\begin{array}{cccccccccccccccccc} E_1+E_3K & E_2 & E_4K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right].$$

Thus, by Equations in (7), (8), (17), (24), (25), (26), (27), (28) and $\dot{V}_3$ we have

$$\dot{V}+z^Tz-\gamma^2w^Tw\leq\chi(t)\left(\Theta+\Psi^T(\epsilon U)\Psi\right)\chi(t)+\mu_1P\eta_2L_1^{-1}\eta_2^TP+\mu_2P\eta_3L_1^{-1}\eta_3^TP<0, \tag{29}$$

and this inequality is equivalent to (13). □

Noting that the result in Theorem 3.1 is LMI for open-loop systems ($u(t) = 0$), but it is not an LMI for closed-loop systems ($u(t) \neq 0$). One cannot solve it directly using MATLAB LMI Toolbox for the closed-loop systems. So we present the following theorem to give the process of the controller design, which can be solved easily by LMI Toolbox.

Theorem 3.2. *Given positive scalars $\underline{h}_1$, $\bar{h}_1$, $\underline{h}_2$, $\bar{h}_2$, μ_1, μ_2 and ϵ. The system in (1)-(3) with state-feedback controller $u(t) = YX^{-1}x(t)$ is asymptotically stable with disturbance attenuation γ, for any time-varying delays $h_1(t)$ and $h_2(t)$ satisfying (6) and (7) if there exist symmetric positive definite matrices X, $\bar{P}_{ii}$, for $i = 1,2,3$, $\bar{M}_{ij}$ and $\bar{N}_{ij}$, for $i = 1,2$, $j = 1,3$, $\bar{R}_{ij}$, for $i = 1,2$, $j = 1,2,3$, $\bar{Q}_1$, $\bar{Q}_2$, $\bar{L}_1$, $\bar{L}_2$ and the matrices $\bar{P}_{12}$, $\bar{P}_{13}$, $\bar{P}_{23}$, $\bar{S}_{ij}$, for $i, j = 1,2$, $\bar{M}_{i2}$ and $\bar{N}_{i2}$, for $i = 1,2$ and Y with appropriate dimensions satisfying the following LMI's*

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Theta} & \sqrt{\mu_1}\bar{\Pi}_1^T & \sqrt{\mu_2}\bar{\Pi}_2^T & \bar{\Psi}^T & \bar{\Pi}_3^T \\ * & -\bar{L}_1 & 0 & 0 & 0 \\ * & * & -\bar{L}_2 & 0 & 0 \\ * & * & * & -\frac{1}{\epsilon}X & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{30}$$

where

$$\bar{\Pi}_1 = \begin{bmatrix} \bar{P}_{12}^T & \bar{P}_{22}^T & \bar{P}_{23} & 0 & \dots & 0 \end{bmatrix}, \tag{31}$$

$$\bar{\Pi}_2 = \begin{bmatrix} \bar{P}_{13}^T & \bar{P}_{23}^T & \bar{P}_{33} & 0 & \dots & 0 \end{bmatrix}, \tag{32}$$

$$\bar{\Pi}_3 = \begin{bmatrix} CX & 0 & \dots & 0 \end{bmatrix}, \tag{33}$$

$$\bar{\Psi} = \begin{bmatrix} E_1X + E_3Y & E_2X & E_4Y & \dots & 0 \end{bmatrix}, \tag{34}$$

$$\bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} & \bar{P}_{13} \\ * & \bar{P}_{22} & \bar{P}_{33} \\ * & * & \bar{P}_{33} \end{bmatrix} > 0, \ \bar{M}_j = \begin{bmatrix} \bar{M}_{j1} & \bar{M}_{j2} \\ * & \bar{M}_{j3} \end{bmatrix} > 0 \ , \bar{N}_j = \begin{bmatrix} \bar{N}_{j1} & \bar{N}_{j2} \\ * & \bar{N}_{j3} \end{bmatrix} > 0,$$

for $j = 1,2$ and
$\bar{\Theta}_{1,1} = \bar{P}_{12} + \bar{P}_{12}^T + \bar{P}_{13} + \bar{P}_{13}^T + \bar{h}_1^2\bar{M}_{11} + \bar{h}_2^2\bar{M}_{21} - \bar{M}_{13} - \bar{M}_{23} + d_1^2\bar{N}_{11} + d_2^2\bar{N}_{21} + AX + BY$ $X^TA^T + Y^TB^T + \bar{R}_{11} + \bar{R}_{21} + \bar{R}_{12} + \bar{R}_{22} + \bar{R}_{13} + \bar{R}_{23} + \bar{S}_{11} + \bar{S}_{11}^T + \bar{S}_{21} + \bar{S}_{21}^T$, $\bar{\Theta}_{1,2} = -\bar{P}_{12} + \bar{M}_{13} + A_1X - \bar{S}_{11} + \bar{S}_{12}^T$, $\bar{\Theta}_{1,3} = -\bar{P}_{13} + \bar{M}_{23} + B_1Y - \bar{S}_{21} + \bar{S}_{22}^T$, $\bar{\Theta}_{1,4} = \bar{P}_{22} + \bar{P}_{23}^T - \bar{M}_{12}^T$, $\bar{\Theta}_{1,5} = \bar{P}_{23} + \bar{P}_{33} - \bar{M}_{22}^T$, $\bar{\Theta}_{1,6} = \bar{P}_{11} + \bar{h}_1^2\bar{M}_{11} + \bar{h}_2^2\bar{M}_{22} + d_1^2\bar{N}_{12} + d_2^2\bar{N}_{22} - X + X^TA^T + Y^TB^T$, $\bar{\Theta}_{1,15} = B_2$, $\bar{\Theta}_{1,16} = DX$, $\bar{\Theta}_{1,17} = -\bar{S}_{11}$, $\bar{\Theta}_{1,18} = -\bar{S}_{21}$, $\bar{\Theta}_{2,2} = \mu_1\bar{L}_1 - 2\bar{M}_{13} - 2\bar{N}_{13} - (1 - \mu_1)\bar{R}_{12} - \bar{S}_{12} - \bar{S}_{12}^T$, $\bar{\Theta}_{2,4} = -\bar{P}_{22} + \bar{M}_{12}^T$, $\bar{\Theta}_{2,5} = -\bar{P}_{23}$, $\bar{\Theta}_{2,6} = X^TA_1^T$, $\bar{\Theta}_{2,7} = \bar{N}_{13}^T$, $\bar{\Theta}_{2,9} = \bar{M}_{13} + \bar{N}_{13}$, $\bar{\Theta}_{2,11} = -\bar{M}_{12}^T - \bar{N}_{12}^T$, $\bar{\Theta}_{2,13} = \bar{N}_{12}^T$, $\bar{\Theta}_{2,17} = -\bar{S}_{12}$, $\bar{\Theta}_{3,3} = \mu_2\bar{L}_2 - 2\bar{M}_{23} - 2\bar{N}_{23} - (1-\mu_2)\bar{R}_{22} - \bar{S}_{22} - \bar{S}_{22}^T$, $\bar{\Theta}_{3,4} = -\bar{P}_{23}^T$, $\bar{\Theta}_{3,5} = -\bar{P}_{33} + \bar{M}_{22}^T$, $\bar{\Theta}_{3,6} = Y^TB_1^T$, $\bar{\Theta}_{3,8} = \bar{N}_{23}^T$, $\bar{\Theta}_{3,10} = \bar{M}_{23} + \bar{N}_{23}$, $\bar{\Theta}_{3,12} = -\bar{M}_{22}^T - \bar{N}_{22}^T$, $\bar{\Theta}_{3,14} = \bar{N}_{22}^T$, $\bar{\Theta}_{3,18} = -\bar{S}_{22}$, $\bar{\Theta}_{4,4} = -\bar{M}_{11}$, $\bar{\Theta}_{4,6} = \bar{P}_{12}^T$, $\bar{\Theta}_{5,5} = -\bar{M}_{21}$, $\bar{\Theta}_{5,6} = \bar{P}_{13}^T$, $\bar{\Theta}_{6,6} = \bar{h}_1^2\bar{M}_{13} + \bar{h}_2^2\bar{M}_{23} + d_1^2\bar{N}_{13} + d_2^2\bar{N}_{23} - X - X^T + \bar{h}_1^2\bar{Q}_1 + \bar{h}_2^2\bar{Q}_2$, $\bar{\Theta}_{6,15} = B_2$, $\bar{\Theta}_{6,16} = DX$, $\bar{\Theta}_{7,7} = -\bar{N}_{13} - \bar{R}_{11}$, $\bar{\Theta}_{7,13} = -\bar{N}_{12}^T$, $\bar{\Theta}_{8,8} = -\bar{N}_{23} - \bar{R}_{21}$, $\bar{\Theta}_{8,14} = -\bar{N}_{22}^T$, $\bar{\Theta}_{9,9} = -\bar{M}_{13} - \bar{N}_{13} - \bar{R}_{13}$, $\bar{\Theta}_{9,11} = \bar{M}_{12}^T + \bar{N}_{12}^T$, $\bar{\Theta}_{10,10} = -\bar{M}_{23} - \bar{N}_{23} - \bar{R}_{23}$, $\bar{\Theta}_{10,12} = \bar{M}_{22}^T + \bar{N}_{22}^T$, $\bar{\Theta}_{11,11} = -\bar{M}_{11} - \bar{N}_{11}$, $\bar{\Theta}_{12,12} = -\bar{M}_{21} - \bar{N}_{21}$, $\bar{\Theta}_{13,13} = -\bar{N}_{11}$, $\bar{\Theta}_{14,14} = -\bar{N}_{21}$, $\bar{\Theta}_{15,15} = -\gamma^2I$, $\bar{\Theta}_{16,16} = -\epsilon X$, $\bar{\Theta}_{17,17} = -\bar{Q}_1$, $\bar{\Theta}_{18,18} = -\bar{Q}_2$, $\bar{h}_j - \underline{h}_j =: d_j$.

Proof: Consider the LMI Σ given in (13). In order to remove the nonlinearities in Σ, define $U^{-1} =: X$ and pre- and post-multiply Σ by the matrix $\Lambda = diag\{X, \dots, X, I, X, \dots, X\}$. The identity matrix I corresponds to the 15th row and column of Σ. Then, define also $XP_{ij}X =: \bar{P}_{ij}$, $XM_{ij}X =: \bar{M}_{ij}$, $XN_{ij}X =: \bar{N}_{ij}$, $XR_{ij}X =: \bar{R}_{ij}$, $XQ_iX =: \bar{Q}_i$, $XL_iX =: \bar{L}_i$, $XS_{ij}X =: \bar{S}_{ij}$, for suitable i, j. As a result from Schur complement, $\Lambda\Sigma\Lambda < 0$ is equivalent to the inequality in (30). $\square$

Remark 3.1. *The techniques used in the proof of Theorem 3.2 solve the problem in the case of non-coincident state and input delays. Besides that the results can be easily extended to the multi-delay, and also to the delay-partitioned cases. This technique has some similarities with the solution given in* [9], *but we observed that the results in* [9] *cannot be extended to non-coincident state and input delays case and also multi-delay case. The difficulties arise from some zero terms in the proof of Theorem 1 in* [9].

Remark 3.2. *The method in this article has no restrictions on the derivatives of the time-varying delays, while traditional design methods require the derivatives to be less than 1. So the proposed method can deal with fast time-varying delays. Noting that the sufficient condition in (30) of Theorem 3.2 does not contain any nonlinear terms and it can be solved easily by using Matlab's LMI Control Toolbox* [23].

Remark 3.3. *In order to solve the results in* [8, 19] *it is referred to the cone-complementarity linearization algorithm suggested in* [21]. *This algorithm performs the linearization process with some iterations and it runs until the acceptable error bounds, which are generally close to the dimensions of the inverse constraints, are obtained. In this work matrix inequalities do not involve any inverse constraints and there is no need to use this algorithm.*

In the following part, linear neutral systems [24], with interval state and input delays are considered and the solution of H_∞-control problem for such systems is examined. Now, consider the linear neutral systems

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-h_1(t)) + E\dot{x}(t-d) \\ &\quad + (B+\Delta B)u(t) + (B_1+\Delta B_1)u(t-h_2(t)) + B_2 w(t), && (35)\\ z(t) &= Cx(t), && (36)\\ x(t) &= \phi(t), \quad \forall t \in \left[-\max\left\{\bar{h}_1, \bar{h}_2, d\right\}, 0\right], && (37) \end{aligned}$$

where d is a constant delay and E is known real matrix with appropriate dimensions. Consider a difference operator $\mu(x_t) : C[-\tau, 0] \to \mathbf{R}^n$ given by $\mu(x_t) := x(t) - Ex(t-d)$ and also assume that $\| E \| < 1$, where $\| . \|$ denotes any matrix norm. This is sufficient condition for the asymptotic stability of $\mu(x_t) = 0$ independent of all delays, (see [25] for further details).

Theorem 3.3. *Given positive scalars $\underline{h}_1$, $\bar{h}_1$, $\underline{h}_2$, $\bar{h}_2$, μ_1, μ_2, d and ϵ. The system in (35)-(37) with state-feedback controller $u(t) = YX^{-1}x(t)$ is asymptotically stable with disturbance attenuation γ, for any time-varying delays $h_1(t)$ and $h_2(t)$ satisfying (6) and (7) if there exist symmetric positive definite matrices X, $\bar{P}_{ii}$, for $i = 1, 2, 3$, $\bar{M}_{ij}$ and $\bar{N}_{ij}$, for $i = 1, 2$, $j = 1, 3$, $\bar{R}_{ij}$, for $i = 1, 2$, $j = 1, 2, 3$, $\bar{Q}_1$, $\bar{Q}_2$, $\bar{L}_1$, $\bar{L}_2$, $\bar{T}$ and the matrices $\bar{P}_{12}$, $\bar{P}_{13}$, $\bar{P}_{23}$, $\bar{S}_{ij}$, for $i, j = 1, 2$, $\bar{M}_{i2}$ and $\bar{N}_{i2}$, for $i = 1, 2$ and Y with appropriate dimensions satisfying the following LMI's*

$$\bar{\Sigma}' = \begin{bmatrix} \bar{\Theta}' & \sqrt{\mu_1}\bar{\Pi}_1'^T & \sqrt{\mu_2}\bar{\Pi}_2'^T & \bar{\Psi}'^T & \bar{\Pi}_3'^T \\ * & -\bar{L}_1 & 0 & 0 & 0 \\ * & * & -\bar{L}_2 & 0 & 0 \\ * & * & * & -\frac{1}{\epsilon}X & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (38)$$

where $\bar{\Pi}_1' = \begin{bmatrix} \bar{\Pi}_1 & 0 \end{bmatrix}$, $\bar{\Pi}_2' = \begin{bmatrix} \bar{\Pi}_2 & 0 \end{bmatrix}$, $\bar{\Pi}_3' = \begin{bmatrix} \bar{\Pi}_3 & 0 \end{bmatrix}$, $\bar{\Psi}' = \begin{bmatrix} \bar{\Psi} & 0 \end{bmatrix}$, $\bar{\Theta}'_{ij} = \bar{\Theta}_{ij}$, $(i, j = 1, \ldots, 17)$ except $\bar{\Theta}'_{66} = \bar{\Theta}_{66} + \bar{T}$, $\bar{\Theta}'_{1,19} = EX$, $\bar{\Theta}'_{6,19} = EX$, $\bar{\Theta}'_{19,19} = -\bar{T}$ and $\bar{P}$, $\bar{M}_j$ and $\bar{N}_j$ $(j = 1, 2)$ are the matrices as in Theorem 3.2.

Proof: The proof of this theorem is achieved by the proof of Theorem 3.1. The L-K functional in (14) is used by adding the functional $V_5(x(t),t) = \int_{t-d}^{t} \dot{x}^T(s)T\dot{x}(s)ds$, so its derivative is $\dot{V}_5 = \dot{x}^T(t)T\dot{x}(t) - \dot{x}^T(t-d)T\dot{x}(t-d)$. In this case Equations (15)-(25) are the same but Equation (26) should be rearranged as

$$2\big(x^T + \dot{x}^T\big)U\big(-\dot{x} + A_K x + A_1 x_{h_1(t)} + B_1 K x_{h_2(t)} + E\dot{x}(t-d) + B_2 w + Dp(t)\big) = 0. \quad (39)$$

The difference between Equations (26) and (39) is the term

$$2\left(x^T + \dot{x}^T\right) UE\dot{x}(t-d) = 2x^T UE\dot{x}(t-d) + 2\dot{x}^T UE\dot{x}(t-d). \quad (40)$$

As a result of this, we consider the vector $\xi^T(t) = \begin{bmatrix} \chi^T(t) & \dot{x}^T(t-d) \end{bmatrix}$, and the matrices $\Theta'_{1,19} = UE$, $\Theta'_{6,19} = UE$, $\Theta'_{6,6} = \Theta_{6,6} + T$, $\Theta'_{19,19} = -T$, $\Theta'_{2,19} = \ldots = \Theta'_{18,19} = 0$, $\Theta'_{i,j} = \Theta_{i,j}$, for all $i,j = 1,\ldots,18$ except $\Theta'_{6,6} = \Theta_{6,6} + T$. In that case, the inequality in (29) can be written as

$$\dot{V} + z^T z - \gamma^2 w^T w \le \xi^T(t)\left(\Theta' + \Psi'^T(\epsilon U)\Psi'\right)\xi(t) + \mu_1 P\eta_2 L_1^{-1}\eta_2^T P + \mu_2 P\eta_3 L_1^{-1}\eta_3^T P < 0, \quad (41)$$

where $\Psi'^T = \begin{bmatrix} \Psi^T & 0 \end{bmatrix}$, and the following LMI is obtained

$$\Sigma' = \begin{bmatrix} \Theta' & \sqrt{\mu_1}{\Pi_1}'^T & \sqrt{\mu_2}{\Pi_2}'^T & \Psi'^T U \\ * & -L_1 & 0 & 0 \\ * & * & -L_2 & 0 \\ * & * & * & -\frac{1}{\epsilon}U \end{bmatrix} < 0, \quad (42)$$

where ${\Pi_1}' = \begin{bmatrix} \Pi_1 & 0 \end{bmatrix}$ and ${\Pi_2}' = \begin{bmatrix} \Pi_2 & 0 \end{bmatrix}$. Then by defining $U^{-1} = X$ and pre- and post-multiplying Σ' by $\Lambda' = diag\{\Lambda, X\}$ and by defining $\bar{T} =: XTX$, inequality (38) can be obtained.

Remark 3.4. *In this paper, for the first time, neutral systems with interval state and input delays are examined by the augmented matrix approach.*

4. Numerical Examples.

In this section, some numerical examples are prensented that demonstrate the validity of the method described above.

Example 4.1. *Consider the system* $\dot{x}(t) = Ax(t) + A_1 x(t-h_1(t)) + Bu(t) + B_1 u(t-h_2(t))$ *where*

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ -0.5 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} -2 & -0.5 & 0 & 0 \\ -0.2 & -1 & 0 & 0 \\ 0.5 & 0 & -2 & -0.5 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}^T,\ B_1 = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}^T,\ \mu = 0.$$

In this paper interval state and input delays are considered. Thus, the problem above is solved for nonzero lower bounds. Maximum value of the upper bound is calculated as $\bar{h}_1 = 0.715$, *for* $\bar{h}_2 = 0.3$ *and* $\underline{h}_1 = \underline{h}_2 = 0.1$. *In* [15, 19] *interval time delays are not considered. The same problem is solved by the method given in* [19] *for* $\underline{h}_1 = \underline{h}_2 = 0$ *and* $\bar{h}_2 = 0.1$ *and the upper bound for* $\bar{h}_1$ *is obtained as* 0.674. *As it is seen from Table 1 our result is less conservative than* $\bar{h}_1 = 0.674$ *and the results given in* [15]. *The state feedback matrix for* $\bar{h}_1 = 0.744$, $\bar{h}_2 = 0.1$ *is* $K = [-6.3\ \ -2.3\ 1.8\ \ -0.89]$.

In the following example, the robust stability for systems with input and state delay are considered.

Table 1. Example 4.1 for $\max \bar{h}_1$

	$\max \bar{h}_1$	$\bar{h}_2$	$\underline{h}_1$	$\underline{h}_2$
Zhang et al. [15]	0.56	0.1	0	0
Theorem 3.2	0.744	0.1	0	0
	0.682	0.6	0	0

Example 4.2. *Let A, A_1, B, B_1 as in Example 4.1 and the uncertainties are given by the matrices*

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ E_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = E_4, \ D = 0.1 * I, \ \mu = 0.$$

Robust stability problem is solved for this system by using Theorem 3.2 for various values of $\bar{h}_2$, $\underline{h}_1$ and $\underline{h}_2$. The results are shown in Table 2. The state feedback matrix is $K = [-2.72 \ -1.1 \ 0.5 \ -1.19]$, for $\bar{h}_1 = 0.655$, $\bar{h}_2 = 0.2$, $\underline{h}_1 = 0.1$ and $\underline{h}_2 = 0.1$.

Table 2. Example 4.2 for $\epsilon = 0.1$

	$\max \bar{h}_1$	$\bar{h}_2$	$\underline{h}_1$	$\underline{h}_2$
Theorem 3.2	0.671	0.1	0	0
	0.645	0.3	0	0
	0.655	0.2	0.1	0.1

Example 4.3. *Consider the following uncertain system with coincident state and input delay, where*

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \ A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ D = 0.2 * I, \ E_1 = E_2 = I, \ E_3 = E_4 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \ \mu = 0.$$

This problem is solved by cone-complementary algorithm in 83 steps in [19]*. However, in our method there is no need to use this algorithm. Applying Theorem 3.2 for $\gamma = 2.25$ and $\underline{h}_1 = \underline{h}_2$ we obtain the results in Table 3. As it is seen from this table, it is also possible to solve the same problem for nonzero lower bounds. Then, the feedback matrix, for $\bar{h}_1 = \bar{h}_2 = 0.9397$ and $\underline{h}_1 = \underline{h}_2 = 0.2$ is $K = [0 \ -7.2654]$. Also, in Table 4 we demonstrate the minimum values of γ, for $\underline{h}_1 = \underline{h}_2 = 0$ and $\bar{h}_1 = \bar{h}_2 = 1.036$. This value of γ is smaller than $\gamma = 2.25$. All these results illustrate the effectiveness of the proposed method.*

Table 3. Example 4.3

$\gamma = 2.25$	$\max\left(\bar{h}_1 = \bar{h}_2\right)$	$\underline{h}_1 = \underline{h}_2$
Parlakçı and Küçükdemiral [19]	0.776	0
Theorem 3.2 for $\epsilon = 0.1$	0.9397	0.2
Theorem 3.2 for $\epsilon = 0.2$	1.036	0

TABLE 4. Example 4.3

	$\bar{h}_1 = \bar{h}_2$	$\min\gamma$
Parlakçı and Küçükdemiral [19]	0.776	2.25
Theorem 3.2 for $\epsilon = 0.1$	0.776	0.057
Theorem 3.2 for $\epsilon = 0.2$	1.036	0.09

Example 4.4. *Consider the following system with input delay*

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 1 \\ 0.2 & 0.1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ D = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$E_1 = E_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},\ E_4 = [\ 1\ ],\ \mu = 0.$$

This example is borrowed from [17] *and in that work* $\bar{h}_1 = \bar{h}_2 = 4$*, and some feedback gains for different values of* ϵ *are given. By Theorem 3.2 we obtain* $\bar{h}_1 = \bar{h}_2 = 8$ *and the feedback gains* $K = [-0.012\ \ -0.0026]$*, for* $\epsilon = 0.1$ *and* $K = [-0.049\ \ -0.009]$*, for* $\epsilon = 0.2$*. So, the proposed method can be applied for a wider range of delays.*

Example 4.5. *Consider the following system without input delay*

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \end{bmatrix},\ \mu = 0.$$

This example is borrowed from [19] *and it reports that the minimum allowable* $\gamma = 0.0022$ *for* $\bar{h}_1 = 0.9710$*. As a result of the application of Theorem 3.2, the minimum allowable* γ *is found as* 0.000051 *for the same value of* $\bar{h}_1$*, and when* γ *is fixed at* 0.0022*, the maximum value of* $\bar{h}_1$ *is* 1.339*. From the above comparisons, it can be seen that the method in this article can lead to much less conservative results for this example.*

Example 4.6. *Consider Example 4.3 with the matrix* $E = \begin{bmatrix} -0.1 & 0.25 \\ 0.2 & 0.3 \end{bmatrix}$*. In this case,* $\bar{h}_1 = \bar{h}_2 = 0.9397$*,* $\underline{h}_1 = \underline{h}_2 = 0.2$ *and* $\gamma = 2.25$ *can be obtained for* $\epsilon = 0.1$*.*

5. Conclusions.

In this paper, new delay dependent sufficient conditions are proposed for the robust stability and H_∞ control of uncertain linear systems with non-coincident time-varying state and input delays. State and input delays are assumed to be in some intervals, in which the lower bounds of the delays are not restricted to zero. Proposed conditions do not include the cone-complementarity linearization algorithm. Less conservative sufficient conditions are obtained in terms of LMIs and some numerical examples are presented to illustrate the effectiveness of the proposed results. It is worth mentioning that the results of this paper may easily be extended to the multi-delay case. H_∞-control of norm-bounded and polytopic uncertain systems with interval state and input delays is another subject of the further research.

REFERENCES

[1] K. Gu, V. L. Kharitonov and J. Chen, *Stability of Time-Delay Systems*, Springer, Berlin, Germany, 2003.

[2] S. I. Niculescu, *Delay Effects on Stability: A Robust Approach*, Springer-Verlag, New York, 2002.

[3] P. Park, A delay-dependent stability criterion for systems with uncertain time-invariant delays, *IEEE Trans. Automatic Control*, vol.44, no.4, pp.876-877, 1999.

[4] S. Xu and J. Lam, Improved delay-dependent stability criteria for time-delay systems, *IEEE Trans. Automatic Control*, vol.50, pp.384-387, 2005.

[5] S. Y. Lee, W. I. Lee and P. Park, Improved stability criteria for linear systems with interval time-varying delays: Generalized zero equalities approach, *Applied Mathematics and Computation*, vol.292, pp.336-348, 2017.

[6] E. Fridman and U. Shaked, An improved stabilization method for linear time-delay systems, *IEEE Trans. Automatic Control*, vol.47, pp.1931-1937, 2002.

[7] J. Yoneyama, New delay-dependent approach to robust stability and stabilization for Takagi-Sugeno fuzzy time-delay systems, *Fuzzy Sets and Systems*, vol.158, pp.2225-2337, 2007.

[8] M. N. A. Parlakçı, Improved robust stability criteria and design of robust stabilizing controller for uncertain linear time-delay systems, *Int. J. Robust Nonlinear Control*, vol.16, pp.599-636, 2006.

[9] O. M. Kwon and J. H. Park, Delay-range-dependent stabilization of uncertain dynamic systems with interval time-varying delays, *Applied Mathematics and Computation*, vol.208, pp.58-68, 2009.

[10] N. Jalili, Optimum active vehicle suspensions with actuator time delay, *Journal of Dynamic Systems, Measurement and Control*, vol.123, pp.54-61, 2001.

[11] H. Gao, Y. Zhao and W. Sun, Input-delayed control of uncertain seat suspension systems with human-body model, *IEEE Trans. Control Systems Technology*, vol.18, pp.591-601, 2010.

[12] J. Xiong and J. Lam, Stabilization of linear systems over networks with bounded packet loss, *Automatica*, vol.43, pp.80-87, 2007.

[13] H.-L. Huang, Robust H_∞ control for fuzzy time-delay systems with parameter uncertainties – Delay dependent case, *International Journal of Innovative Computing, Information and Control*, vol.12, no.5, pp.1439-1451, 2016.

[14] S. K. Tadepalli and V. K. R. Kandanvli, Delay-dependent stability of discrete-time systems with multiple delay and nonlinearities, *International Journal of Innovative Computing, Information and Control*, vol.13, no.3, pp.891-904, 2017.

[15] X. M. Zhang, M. Wu, J. H. She and Y. He, Delay-dependent stabilization of linear systems with time-varying state and input delays, *Automatica*, vol.41, pp.1405-1412, 2005.

[16] X. Zhang, M. Li, M. Wu and J. She, Further results on stability and stabilization of linear systems with state and input delays, *International Journal of Systems Science*, vol.40, no.1, pp.1-10, 2009.

[17] T. Zhang, Y. Li, G. Liu, M. Li and J. She, Robust stabilization of uncertain systems with interval time-varying state and input delays, *International Journal of Systems Science*, vol.40, no.1, pp.11-20, 2009.

[18] B. Du, J. Lam and Z. Shu, Stabilization for state/input delay systems via static and integral output feedback, *Automatica*, vol.46, pp.2000-2007, 2010.

[19] M. N. A. Parlakçı and İ. Küçükdemiral, Robust delay-dependent H_∞-control of time-delay systems with state and input delays, *Int. J. Robust Nonlinear Control*, vol.21, pp.974-1007, 2011.

[20] L. V. Hien, T. D. Tran and H. M. Trinh, New H_∞ control design for polytopic systems with mixed time-varying delays in state and input, *International Journal of Innovative Computing, Information and Control*, vol.11, no.1, pp.105-121, 2015.

[21] L. E. Ghaoui, F. Oustry and M. A. Rami, A cone complementarity linearization algorithm for static output-feedback and related problems, *IEEE Trans. Automatic Control*, vol.42, pp.1171-1176, 1997.

[22] K. Gu, An integral inequality in the stability problem of time-delay systems, *Proc. of the 39th IEEE Conference on Decision Control*, Sydney, Australia, pp.2805-2810, 2000.

[23] P. Gahinet, A. Nemirovski, A. J. Laub and M. Chilali, *LMI Control Toolbox*, The Mathworks, Natik, MA, 1995.

[24] U. Baser and B. Kizilsac, Dynamic output feedback H_∞-control problem for linear neutral systems, *IEEE Trans. Automatic Control*, vol.52, pp.1113-1118, 2007.

[25] J. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, 1993.