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ADAPTIVE NEURAL CONTROL FOR MIMO NONLINEAR SYSTEMS WITH UNKNOWN DEAD ZONE BASED ON OBSERVERS

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Abstract. *This paper discusses the problem of adaptive neural control for a class of multiple-input multiple-output (MIMO) nonlinear systems with unknown dead-zone inputs. By applying radial basis function (RBF) neural networks (NNs) to identify the unknown functions and constructing observers, a backstepping control scheme is developed. The proposed control method requires only one adaptive law for every subsystem. And the designed controllers can ensure that all the signals in the closed-loop systems are bounded, and the target signals can be tracked within a small error as well. At last, the simulation example is provided to show the effectiveness of the proposed scheme.*
Keywords: Adaptive control, Backstepping, Neural networks, Output feedback, Unknown dead zone

1. **Introduction.** In modern industrial field, many practice systems usually require multiple control signals, which are called multiple-input multiple-output (MIMO) systems. Because of the increase of the control loops and the existence of the coupling phenomenon between various inputs, the structure of MIMO systems is more complex than that of single input and single output systems. On the other hand, as unknown nonlinearities are inherent in practical systems, the problem of control and stabilization analysis for MIMO systems with nonlinearities and uncertainties becomes more academically challenging. In order to deal with the problem, approximation-based adaptive control methods are widely used, such as [1, 2, 3, 4, 5] for state feedback and [6, 7, 8] for output feedback. In [9], by inducing high dimensional integral Lyapunov functions, adaptive state feedback control and adaptive output feedback control are both derived. In these works, neural networks or fuzzy logical systems (FLSs) are applied as approximators to identifying unknown nonlinearities by making use of their self-studying ability. Because NNs and FLSs have the capacity of nonlinear function approximating, the approximation-based adaptive control strategy is not only used to realize the high-precision control for complex systems, but also suitable for the systems which are difficult to describe by mathematical models. Especially in recent years, this method has been successfully applied to various engineering fields. For example, in [10], fuzzy controllers ensure the stability of MIMO nonlinear industrial processes with no reliable models. In [11], the proposed adaptive neural control scheme is evaluated on a two-link robot manipulator.

Additionally, notice dead zone as one of non-smooth nonlinear characteristics usually occurs in many practical systems and may influence the performance of control systems. Many scholars devote themselves to the study of nonlinear systems with dead-zone nonlinearities, for example, see [12, 13, 14, 15, 16, 17, 18] for state feedback and [19, 20, 21, 22]

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for output feedback in the present of dead zones. Thereinto, given the effects of the dead-zone and multiple control signals, the authors of [15] present a decentralized variable structure control for a class of uncertain large-scale systems with known dead-zones. In [16], the need for known parameter bounds of dead zones is removed. In [17], the tracking problem for a class of nonlinear systems with nonsymmetric dead-zone input is discussed, but the considered dead-zone is linear. In [18], the authors investigate the unknown nonlinear dead-zone, and the mean-value theorem is first applied to deriving a formulation of the perturbed nonlinear dead-zone. So the dead-zone can be treated as the system nonlinearities. The above mentioned works are based on an assumption that state variables are available. In [19, 20, 21, 22] the assumption is removed and state observers are designed to estimate the unknown states. However, most existing observer-based adaptive output feedback control approaches depend on some nonlinear matrix inequalities to complete the analysis of observation error dynamic systems. For example, in [23], inequality $\lambda_{\min}(Q) - \frac{1}{\gamma}\|P\|^2 - n > 0$ and $PA + A^TP + Q < 0$ must be satisfied where P and Q are positive definite matrices and A is defined as in (9). Generally, it is hard to solve these nonlinear matrix inequalities. However, this difficulty can be overcome by line matrix inequalities just as done in [24].

With these observations, we further consider the control problem for a class of MIMO nonlinear systems with unknown dead-zone inputs. By applying neural networks as approximators and constructing state observers, a Lyapunov-based control strategy is investigated, which can ensure the overall signals are bounded and the tracking errors converge to a small neighborhood of the origin. The main contributions of this paper can be summarized as follows. (1) The problems of stability analysis and control design are carried out with linear matrix inequalities and convex combination method. This method can reduce the difficulty of simulation and simplify calculations. (2) The nonlinearly parameterized adaptation is applied for function approximation, and only one adaptive law is established for every subsystem. So the designed controllers are more convenient to implement in the real industrial process, and the control method proposed in this note has certain practical significance.

The rest contents are as follows. Section 2 introduces the preliminaries. An adaptive output-feedback neural control scheme is presented in Section 3. Section 4 presents a simulation example, followed by Section 5 which concludes the work.

2. Problem Statement and Preliminaries.

For $i = 1, 2, \ldots, N$, consider the following uncertain MIMO nonlinear systems:

$$\begin{cases} \dot{x}_{i,j} = f_{i,j}(\bar{x}_{i,j}) + x_{i,j+1} + d_{i,j}(x), \quad 1 \le j \le n_i - 1, \\ \dot{x}_{i,n_i} = f_{i,n_i}(x) + u_i + d_{i,n_i}(x), \\ y_i = x_{i,1}, \end{cases} \tag{1}$$

where $x = \left[x_1^T, \ldots, x_N^T\right]^T$, $x_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in R^{n_i}$, $\bar{x}_{i,j} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j}]^T \in R^j$, $(i = 1, \ldots, N;\ j = 1, \ldots, n_i)$ are the state variables. $d_{i,j}(\cdot)$ is the external disturbance satisfying $|d_{i,j}| \le \bar{d}_{i,j}$, and $\bar{d}_{i,j}$ is a positive constant. $y_i \in R$ denotes the control output variable of the ith nonlinear subsystem which can be measured directly only. $f_{i,j}(\bar{x}_{i,j})$ is an unknown smooth nonlinear function with $f_{i,j}(0) = 0$. $u_i \in R$ is the output of the dead-zone nonlinearity and also the actual control input to the ith subsystem. The dead-zone characteristic can be modeled as follows [19].

$$u_i = D(v_i) = \begin{cases} g_{ri}(v_i - b_{ri}), & v_i \ge b_{ri} \\ 0, & -b_{li} < v_i < b_{ri}, \\ g_{li}(v_i + b_{li}), & v_i \le -b_{li}, \end{cases} \tag{2}$$

where $D(\cdot)$ denotes the considered dead zone. $v_i(t) \in R$ is the input of the dead-zone, and also the control signal to be designed. $g_{li} > 0$, $g_{ri} > 0$, $b_{li} > 0$ and $b_{ri} > 0$ are unknown constants, which represent the right slope, left slope, right breakpoint and left breakpoint of dead-zone, respectively. To facilitate the control design, the following assumptions will be used in the subsequent developments.

Assumption 2.1. *There exist positive constants* W_i *such that* $|v_i| \leq W_i$.

Assumption 2.2. *For the system function* $f_{i,j}(\cdot)$, *there exist known constants* $\underline{a}_{pq}$, $\overline{a}_{pq}$ *such that* $\underline{a}_{pq} \leq \frac{\partial f_{i,j}}{\partial x_{m,n}} \leq \overline{a}_{pq}$, $1 \leq i$, $m \leq N$, $1 \leq j \leq n_i$, $1 \leq n \leq n_m$, *where* n_i *and* n_m *stand for the number of state variables in the* i*th and* j*th subsystems, respectively.* $p = \sum_{k=0}^{i-1} n_k + j$ *and* $q = \sum_{k=0}^{m-1} n_k + n$ *with* $n_0 = 0$.

Remark 2.1. *Since* $f_{i,j}(x) = \left[\frac{\partial f_{i,j}}{\partial x_{1,1}}, \ldots, \frac{\partial f_{i,j}}{\partial x_{N,n_N}}\right] x$. *By Assumption 2.2, there exist constants* $h_{i,j} > 0$ *such that* $|f_{i,j}(x)| \leq h_{i,j}||x||$. *It means that the monotonically increasing function* $\rho_{i,j}(w) = h_{i,j}w$, *with* $w \in R$ *is the bounding function of* $f_{i,j}(\cdot)$.

Assumption 2.3. *For* $i = 1, \ldots, N$, *there exists a positive constant* $\bar{y}_{di}$ *such that* $|y_{di}| < \bar{y}_{di}$ *and* $\left|y_{di}^{(k)}\right| < \bar{y}_{di}$, *with* $y_{di}^{(k)}$ *being the k-order derivative of* y_{di}.

Thus, the dead zone (2) can be expressed as

$$u_i = g_i(t)v_i(t) + m_i(t), \tag{3}$$

where

$$g_i(t) = \begin{cases} g_{ri}, & v_i > 0, \\ g_{li}, & v_i \leq 0, \end{cases}, \quad m_i(t) = \begin{cases} -g_{ri}b_{ri}, & v_i \geq b_{ri}, \\ -g_i(t)v_i(t), & -b_{li} < v_i(t) < b_{ri}, \\ g_{li}b_{li}, & v_i \leq -b_{li}. \end{cases}$$

Let $m_i(t) \leq \bar{m}_i$ with $\bar{m}_i = \max\{g_{li}b_{li}, g_{ri}b_{ri}\}$. Define $\bar{\beta}_i = \max\{g_{li}, g_{ri}\}$ and $\underline{\beta}_i = \min\{g_{li}, g_{ri}\}$, and one has $\frac{g_i(t)}{\underline{\beta}_i} = 1 + \rho_i(t)$ with $\rho_i(t)$ being a piecewise positive bounded function. Using the above equations, one has $\rho_i(t) \leq \frac{\bar{\beta}_i}{\underline{\beta}_i} - 1$. So, dead zone can be further expressed as

$$u_i = \underline{\beta}_i(1 + \rho_i(t))v_i(t) + m_i(t). \tag{4}$$

Combining with (4), the systems (1) can be rewritten as the state-space form (for $1 \leq i \leq N$)

$$\begin{cases} \dot{x}_i = A_i x_i + L_i y_i + F_i(x) + d_i + B_i\underline{\beta}_i v_i(t) + B_i\left[\underline{\beta}_i \rho_i(t)v_i(t) + m_i(t)\right], \\ y_i = C_i^T x_i, \end{cases} \tag{5}$$

where $F_i(x) = [f_{i,1}(x_{i,1}), \ldots, f_{i,n_i}(x)]^T$, $L_i = [l_{i,1}, \ldots l_{i,n_i}]^T$, $B_i = [0, \ldots, 0, 1]^T_{n_i\times 1}$, $C_i^T = [1, 0, \ldots, 0]_{1\times n_i}$, $d_i = [d_{i,1}, \ldots, d_{i,n_i}]^T$, and $A_i = \begin{bmatrix} L_{n_i-1} & I_{n_i-1} \\ -l_{i,n_i} & 0 \end{bmatrix}$ with $L_{n_i-1} = [-l_{i,1}, \ldots, -l_{i,n_i-1}]^T$. The vector L_i is chosen suitably such that A_i is a strict Hurwitz matrix.

3. **Control Design and Stability Analysis.** First, we should design the state observer, which can be expressed as follows:

$$\begin{cases} \dot{\hat{x}}_{i,j} = \hat{x}_{i,j+1} + l_{i,j}(y_i - \hat{x}_{i,1}), & 1 \leq i \leq N,\ 1 \leq j \leq n_i - 1, \\ \dot{\hat{x}}_{i,n_i} = v_i + l_{i,n_i}(y_i - \hat{x}_{i,1}). \end{cases} \tag{6}$$

Similarly, the state observer (6) can be converted into the state-space form:

$$\dot{\hat{x}}_i = A_i\hat{x}_i + L_i y_i + B_i v_i, \tag{7}$$

where $\hat{x}_{i,j}$ denotes the estimation of $x_{i,j}$. Define the estimation error as $e_{i,j} = x_{i,j} - \hat{x}_{i,j}$.

From (5) and (7), we get the observer error equation for the ith subsystem:

$$\dot{e}_i = A_i e_i + F_i(x) + d_i + B_i\left(\underline{\beta}_i - 1\right)v_i(t) + B_i\left[\underline{\beta}_i \rho_i(t)v_i(t) + m_i(t)\right], \tag{8}$$

where $e_i = [e_{i,1},\ldots,e_{i,n_i}]^T$. And let $e = \left[e_1^T,\ldots e_N^T\right]^T$, $A = diag[A_1,\ldots,A_N]$, $F(x) = \left[F_1^T(x),\ldots,F_N^T(x)\right]^T$, $B = diag[B_1,\ldots,B_N]$, $D = \left[d_1^T,\ldots,d_N^T\right]^T$ and $\bar{v} = [\bar{v}_1,\ldots,\bar{v}_2]^T$ with $\bar{v}_i = \left(\underline{\beta}_i - 1\right)v_i(t)$, $\underline{v} = [\underline{v}_1,\ldots,\underline{v}_2]^T$ with $\underline{v}_i = \underline{\beta}_i\rho_i(t)v_i(t) + m_i(t)$, for the whole MIMO systems, the whole observer-error equation can be expressed by

$$\dot{e} = Ae + F(x) + D + B\left(\bar{v} + \underline{v}\right). \tag{9}$$

Next up, an adaptive backstepping design method for the systems (1) can be proposed. The neural control signals are required as follows (for $i = 1,\ldots,N$; $j = 1,\ldots,n_i$)

$$\alpha_{i,j} = -\frac{1}{2a_{i,j}^2}z_{i,j}\hat{\theta}_i - \frac{1}{2}z_{i,j} - k_{i,j}z_{i,j}, \tag{10}$$

where $z_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}$ with $\alpha_{i,0} = y_{di}$. $k_{i,j}$ and $a_{i,j}$ are positive design parameters. $\hat{\theta}_i$ is the estimation of θ_i that will be specified later and the evaluated error is $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. Notice that $\alpha_{i,n_i} = v_i$ is the real control signal for the ith subsystem. And for $2 \le j \le n_i$, one has

$$-\dot{\alpha}_{i,j-1} = -\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}\left(\hat{x}_{i,k+1} + l_{i,k}e_{i,1}\right) - \frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i - \sum_{k=0}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial y_{di}{}^{(k)}}y_{di}^{(k+1)}. \tag{11}$$

The adaptive laws are defined as the solution of the following differential equations:

$$\dot{\hat{\theta}}_i = \sum_{j=1}^{n_i}\frac{r_i}{2a_{i,j}^2}z_{i,j}^2 - \sigma_i\hat{\theta}_i, \tag{12}$$

where r_i and σ_i are positive design parameters.

Remark 3.1. *For any initial condition $\hat{\theta}_i(t_0) \ge 0$, the solution $\hat{\theta}_i(t) \ge 0$ holds for $t \ge t_0$. In the following text, it is assumed that $\hat{\theta}_i(t) \ge 0$.*

At this stage, the main results of this note are summarized in the following theorem.

Theorem 3.1. *For the resulting nonlinear MIMO system (1) with unknown dead-zones, under Assumptions 2.1-2.3 and based on the following inequality (13), the proposed control scheme with the control signals $\alpha_{i,j}$ (10), state observer systems (6) and adaptive laws θ_i (12), ensure that all the signals in the close-loop nonlinear MIMO systems remain bounded and the outputs converge to a small neighborhood of the reference signals. The above conclusions are valid, then the following inequality should be satisfied.*

$$PA + A^TP + PJ_{pq} + J_{pq}^TP + (\varepsilon_0 + 3\tau)I + \gamma < 0, \quad 1 \le p,q \le g, \tag{13}$$

where $g = \sum_{i=1}^N n_i$ and P is a definitive positive matrix. J_{pq} is a constant matrix which element at the pth row and the qth column is $\bar{a}_{pq}$ or $\underline{a}_{pq}$ and others are zero.

Proof: For the whole nonlinear systems, consider the Lyapunov function candidate as $V = V_0 + \sum_{i=1}^{N} V_1 + \sum_{i=1}^{N} V_2$ with $V_0 = e^T P e$, $V_1 = \frac{1}{2}\sum_{j=1}^{n_i} z_{i,j}^2$, $V_2 = \frac{1}{2r_i}\tilde{\theta}_i^2$.

The derivative of V_0, which is relative to the observer error systems, can be calculated as

$$\begin{aligned} \dot{V}_0 = e^T \left(PA + A^T P\right) e + 2e^T P \left(F(x) - F(\hat{x})\right) \\ + 2e^T PF\left(\hat{x}\right) + 2e^T PD + 2e^T PB\left(\bar{v} + \underline{v}\right). \end{aligned} \tag{14}$$

Additionally, with the fact $P > 0$, the following inequality holds

$$2e^T P \left(F(x) - F(\hat{x})\right) = 2e^T PJe \leq e^T \left[PJ + J^T P\right] e, \tag{15}$$

where $J = \left[\frac{\partial f_{i,j}}{\partial x_{m,n}}\right]$ is a Jacobian matrix, which has g rows and g columns. According to Assumption 2.2, every nonzero element in the Jacobian matrix has its own upper and lower bounds. Namely, there exists a function $0 \leq \mu_{pq}(t) \leq 1$ such that $\frac{\partial f_{i,j}}{\partial x_{m,n}} = \mu_{pq}\underline{a}_{pq} + (1-\mu_{pq})\bar{a}_{pq}$. Thus, J can be reformulated as the following form:

$$J = \sum_{p=1}^{g}\sum_{q=1}^{g}\left[\mu_{pq}\underline{F}_{pq} + (1-\mu_{pq})\overline{F}_{pq}\right], \quad 0 < \alpha_{pq} < 1, \tag{16}$$

where $\underline{F}_{pq}$ and $\overline{F}_{pq}$ are constant matrixes and they have only one nonzero element $\underline{a}_{pq}$ and $\bar{a}_{pq}$ at their pth row and qth column, respectively. In order to overcome this difficulty coming from time-varying Jacobian matrix J, a group of linear matrix inequalities (LMIs) is applied to subsequent procedures. Furthermore, from Remark 1, Assumption 2.3 and Lemma 3 in [25], one has:

$$2e^T PF(\hat{x}) \leq \varepsilon_0 e^T e + c\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i}|z_{i,j}|^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right)\right) + \sum_{i=1}^{N} c_0\bar{y}_{di} \tag{17}$$

with $c_0 = \varepsilon_0^{-1}\|P^2\|\sum_{j=1}^{n_i-1} h_{i,j}^2$ and $c = gc_0$.

Next up, for any positive constant τ, based on Assumption 2.1, one has

$$2e^T PD \leq \tau e^T e + \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\sum_{j=1}^{n_i}\bar{d}_{ij}^2, \tag{18}$$

$$2e^T PB\bar{v} \leq \tau e^T e + \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\left(\underline{\beta}_i - 1\right)^2 W_i^2, \tag{19}$$

$$2e^T PB\underline{v} \leq \tau e^T e + \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\left[\left(\bar{\beta}_i - \underline{\beta}_i\right)W_i + m_i\right]^2. \tag{20}$$

Consequently, substituting (17), (18), (19) and (20) into (14) we have

$$\dot{V}_0 \leq e^T\left(PA + A^T P + PJ + J^T P + (\varepsilon_0 + 3\tau)I\right)e + \delta_0 + c\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i} z_{i,j}^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right)\right), \tag{21}$$

where $\delta_0 = \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\sum_{j=1}^{n_i}\bar{d}_{ij}^2 + \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\left(\underline{\beta}_i - 1\right)^2 W_i^2 + \frac{1}{\tau}\|P\|^2\sum_{i=1}^{N}\left[\left(\bar{\beta}_i - \underline{\beta}_i\right)W_i + m_i\right]^2 + \sum_{i=1}^{N} c_0\bar{y}_{di}$. Next, we calculate the derivative of V_1.

$$\begin{aligned}\dot{V}_1 &= z_{i,1}\left(z_{i,2}+\alpha_{i,1}+l_{i,1}e_{i,1}-\dot{y}_{di}\right)+\sum_{j=2}^{n_i-1}z_{i,j}\left(\alpha_{i,j}-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}\hat{x}_{i,k+1}-\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right)\\ &+\sum_{j=2}^{n_i-1}z_{i,j}e_{i,1}\left(l_{i,j}-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}l_{i,k}\right)+\sum_{j=2}^{n_i-1}z_{i,j}z_{i,j+1}+z_{i,n_i}\left(v_i-\dot{\alpha}_{n_i-1}+l_{i,n_i}e_{i,1}\right).\end{aligned} \tag{22}$$

By using the completion of squares, one has

$$z_{i,1}l_{i,1}e_{i,1}\leq\frac{1}{2\gamma_{i,1}}l_{i,1}^2z_{i,1}^2+\frac{\gamma_{i,1}}{2}e_{i,1}^2, \tag{23}$$

$$z_{i,j}e_{i,1}\left(l_{i,j}-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}l_{i,k}\right)\leq\frac{1}{2\gamma_{i,j}}z_{i,j}^2\left(l_{i,j}-\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}l_{i,k}\right)^2+\frac{1}{2}\gamma_{i,j}e_{i,1}^2. \tag{24}$$

Define functions $\bar{f}_{i,j}(z_{i,j})$ as

$$\begin{aligned}\bar{f}_{i,1}(Z_{i,1}) &= \frac{1}{2\gamma_{i,1}}z_{i,1}l_{i,1}^2+cz_{i,1}\phi_{i,1}^2\left(\hat{\theta}_{i,1}\right)-\dot{y}_{di},\\ \bar{f}_{i,j}(Z_{i,j}) &= -\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}\hat{x}_{i,k+1}-\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i+\frac{1}{2\gamma_{i,j}}z_{i,j}\left(\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{i,k}}l_{i,k}-l_{i,j}\right)^2\\ &\quad+z_{i,j-1}+cz_{i,j}\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right),\quad 1\leq j\leq n_i-1,\\ \bar{f}_{i,n_i}(Z_{i,n_i}) &= -\dot{\alpha}_{i,n_i-1}+l_{i,n_i}e_{i,1}+z_{i,n_i-1}+cz_{i,n_i}\phi_{i,n_i}^2\left(\hat{\theta}_{i,n_i}\right),\end{aligned}$$

where $Z_{i,j}=\left[\hat{x}_{i,1},\ldots,\hat{x}_{i,j},\hat{\theta}_i,y_{di},\dot{y}_{di},\ldots,y_{di}^{(j)}\right]^T$, and $y_{di}^{(j)}$ is the jth derivative of y_{di}. Now, substituting the inequalities (23), (24) into (22) and considering these defined functions $f_{i,j}(\cdot)$, one has:

$$\dot{V}_1\leq\sum_{j=1}^{n_i}z_{i,j}\left(\alpha_{i,j}+\bar{f}_{i,j}(z_{i,j})\right)+\sum_{j=1}^{n_i-1}\frac{1}{2\gamma_{i,j}}e_{i,1}^2-c\sum_{j=1}^{n_i}z_{i,j}^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right). \tag{25}$$

RBF NNs approximators are employed to identify these lumped unknown dynamics as done in [3, 26]. For any given $\varepsilon_{i,j}>0$, there exists a neural network $W_{i,j}^TS_{i,j}(Z_{i,j})$ such that

$$\bar{f}_{i,j}(Z_{i,j})=W_{i,j}^TS_{i,j}(Z_{i,j})+\delta_{i,j}(Z_{i,j}),$$

where $\delta_{i,j}\leq\varepsilon_{i,j}$ is the approximation error. $S_{i,j}(Z_{i,j})$ is the basis function vector of the neural networks. Notice the fact $S_{i,j}^T(Z_{i,j})S_{i,j}(Z_{i,j})\leq N_{i,j}$ with $N_{i,j}$ being the dimension of $S_{i,j}$ and using completion of squares again, we have

$$z_{i,j}\bar{f}_{i,j}\leq\frac{1}{2a_{i,j}^2}z_{i,j}^2\theta_i+\frac{1}{2}a_{i,j}^2+\frac{1}{2}z_{i,j}^2+\frac{1}{2}\varepsilon_{i,j}^2,\ 1\leq i\leq N,\ 1\leq j\leq n_i, \tag{26}$$

where the unknown constant $\theta_i=\max_{1\leq j\leq n_i}\{\theta_{i,j}\}$ with $\theta_{i,j}=N_{i,j}\|W_{i,j}\|^2$. Then substituting (10) and (26) into (25) obtains:

$$\begin{aligned}\dot{V}_1\leq &-\sum_{j=1}^{n_i}k_{i,j}z_{i,j}^2+\sum_{j=1}^{n_i}\frac{1}{2a_{i,j}^2}z_{i,j}^2\tilde{\theta}_i+\sum_{j=1}^{n_i}\frac{1}{2}\left(a_{i,j}^2+\varepsilon_{i,j}^2\right)\\ &+\sum_{j=1}^{n_i-1}\frac{1}{2}\gamma_{i,j}e_{i,1}^2-c\sum_{j=1}^{n_i}z_{i,j}^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right).\end{aligned} \tag{27}$$

It is easy to get $\dot{V}_2 = -\frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i$ and taking (21) and (27) into account, we can get

$$\begin{aligned}\dot{V} \leq\ & e^T\left[PA + A^TP + PJ + J^TP + (\varepsilon_0 + 3\tau)I + \gamma\right]e - \sum_{i=1}^{N}\sum_{j=1}^{n_i} k_{i,j}z_{i,j}^2 + \delta_0 \\ & + \sum_{i=1}^{N}\frac{\tilde{\theta}_i}{r_i}\left(\sum_{j=1}^{n_i}\frac{r_i}{2a_{i,j}^2}z_{i,j}^2 - \dot{\hat{\theta}}_i\right) + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right),\end{aligned} \tag{28}$$

where $\gamma = diag\left[\sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\frac{1}{2}\gamma_{i,j}, 0, \ldots, 0\right]$. According to the inequality (16) and Lemma 3 in [27], (13) is equivalent to the following inequity

$$PA + A^TP + PJ + J^TP + (\varepsilon_0 + 3\tau)I + \gamma < 0. \tag{29}$$

The above inequality (29) means that there exists a constant $\mu > 0$, such that

$$e^T\left[PA + A^TP + PJ + J^TP + (\varepsilon_0 + 3\tau)I + \gamma\right]e < -\frac{\mu}{\lambda_M(P)}e^TPe, \tag{30}$$

where $\lambda_M(P)$ is the maximal eigenvalue of matrix P. Next, using $\tilde{\theta}\hat{\theta} \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2$ and submitting (12), (30) into (28), we can obtain:

$$\begin{aligned}\dot{V} \leq\ & -\frac{\mu}{\lambda_M(P)}e^TPe - \sum_{i=1}^{N}\sum_{j=1}^{n_i}k_{i,j}z_{i,j}^2 - \sum_{i=1}^{N}\frac{\sigma_i}{2r_i}\tilde{\theta}_i^2 \\ & + \sum_{i=1}^{N}\frac{\sigma_i}{2r_i}\theta_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right) + \delta_0.\end{aligned} \tag{31}$$

Now, defining $a_0 = \min\left\{\frac{\mu}{\lambda_M(P)}, 2k_{i,j}, \sigma_i, 1 \leq i \leq N, 1 \leq j \leq n_i\right\}$ and $b_0 = \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left[\frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right)\right] + \sum_{i=1}^{N}\frac{\sigma_i}{2r_i}\theta_i^2 + \delta_0$, then (31) can be rewritten as

$$\dot{V(t)} \leq -a_0V(t) + b_0. \tag{32}$$

Solving the above inequality, one has

$$V(t) \leq \left(V(0) - \frac{b_0}{a_0}\right)e^{-a_0t} + \frac{b_0}{a_0}, \tag{33}$$

which means that for $t \geq 0$, $z = \left[z_{1,1}, \ldots, z_{N,n_N}, \tilde{\theta}_1, \ldots, \tilde{\theta}_N\right]^T$ belongs to the compact set $\Omega = \left\{z|V(z(t)) \leq V(0) + \frac{b_0}{a_0}\right\}$. Therefore, all the signals in the closed-loop system are bounded. In addition, from (33), we can also obtain $\lim_{t\to\infty} z_{i,1}^2 \leq \frac{2b_0}{a_0}$, for $i = 1, \ldots, N$. It implies that the tracking errors $z_{i,1}$ will converge to the circle domain with $\sqrt[2]{\frac{2b_0}{a_0}}$ being its radius. Because a_0 and b_0 are unknown, an explicit estimation of the tracking errors is impossible. However, based on the definitions of a_0 and b_0, by reducing $a_{i,j}$, $\varepsilon_{i,j}$ and σ_i meanwhile increasing r_i, one can get smaller tracking errors. At the present stage, the proof is complete.

4. **Simulation Example.**

Example 4.1. *Consider the following MIMO systems with unknown dead-zone inputs to verify the effectiveness of the proposed method.*

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(x_{i,1}) + d_{i,1}(x), \\ \dot{x}_{i,2} = u_i + f_{i,2}(x) + d_{i,2}(x), \\ y_i = x_{i,1}, \end{cases}$$

where $i = 1, 2$, $f_{1,1} = 0.25\cos(x_{11})\sin(x_{11})$, $f_{1,2} = 0.7\sin(x_{1,2})$, $f_{2,1} = 0.25\sin^2(x_{21})$, $f_{2,2} = \sin(x_{2,2})$, *and* $d_{1,1} = \sin(x_{21})\sin(x_{11})$, $d_{1,2} = \sin(x_{11}x_{12}x_{21}x_{22})\cos(x_{11})$, $d_{2,1} = \sin(x_{22})\cos(x_{11})$, $d_{2,2} = \sin(x_{1,2})$. *Choosing* $\varepsilon_0 + 3\tau = 0.01$, $\gamma = 0.01I$, *and for given* J_{pq}, *solving LMIs (29) one can get* $l_{1,1} = 15$, $l_{1,2} = 31.7$, $l_{2,1} = 16.5$, $l_{2,2} = 62$ *and obtain* P *as*

$$P = \begin{bmatrix} 215.6 & -95.6 & -1.9 & 0.7 \\ -95.6 & 51.6 & 0.005 & -0.02 \\ -1.9 & 0.005 & 224.3 & -68.3 \\ 0.7 & -0.02 & -68.3 & 25.1 \end{bmatrix}.$$

For $i = 1, 2$; $j = 1, 2$, *the design parameters are set as* $k_{i,j} = 10$, $a_{i,j} = 0.5$, $\sigma_i = 1$, $r_i = 7.5$, *and the initial conditions are chosen as* $x_{1,1}(0) = 0.2$, $x_{2,1}(0) = 0.5$, $x_{i,2}(0) = 0.1$. *The other initial conditions are chosen as zeros. In the simulation, the width of the Gaussian function is chosen as two. The parameters in the dead-zone model are selected as* $b_{r1} = 1.5$, $g_{r1} = b_{l2} = 2$, $g_{li} = g_{r2} = b_{l1} = b_{r2} = 1$. *Then the simulation results are shown in Figures 1-6. Figure 1 and Figure 2 are the trajectories of* y_1, y_{d1}, y_2 *and* y_{d2}, *which show that the output of considered nonlinear systems can trace the referenced signals perfectly. Figure 3 shows the responses of state variable* $x_{i,2}$ *and* $\hat{x}_{i,2}$ *(for* $i = 1, 2$*). Figure 4 shows the boundedness of adaptive parameters* $\hat{\theta}_1$ *and* $\hat{\theta}_2$. *Figure 5 and Figure 6 display the control input signals* v_i *and the output signals of dead-zones* u_i. *From these simulation results, it is observed that the good tracking performance is achieved. Meanwhile, all signals involved in the closed-loop system are bounded. So the proposed controllers in this paper are effectiveness.*

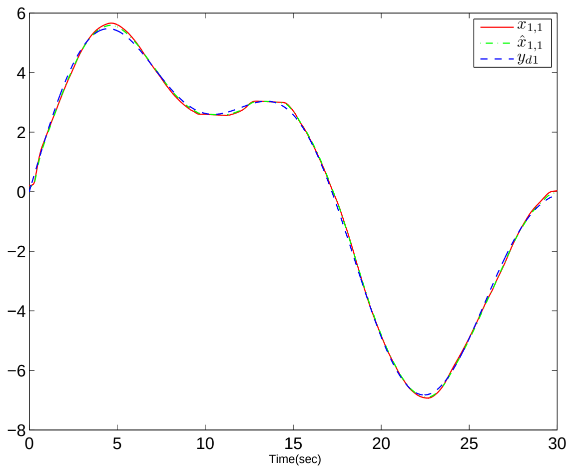

FIGURE 1. $x_{1,1}$, y_{d1} and $\hat{x}_{1,1}$

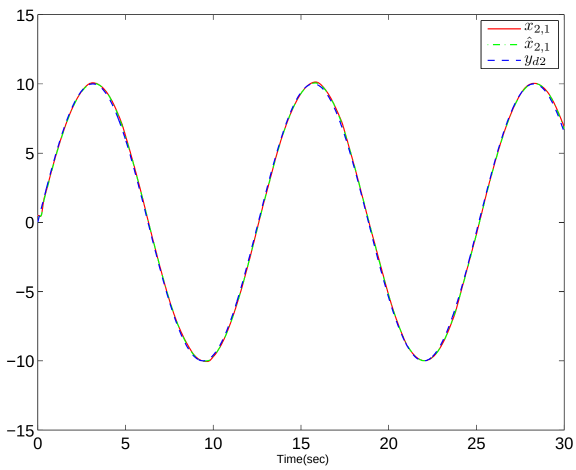

FIGURE 2. $x_{2,1}$, y_{d2} and $\hat{x}_{2,1}$

5. **Conclusions.** In this paper, a neural adaptive scheme is investigated for a class of uncertain MIMO nonlinear systems with unknown dead zone inputs. The state observers are designed to overcome the problem of unobtainable states. Based on the adaptive backstepping method and using the approximation performance of neural networks, the adaptive controllers with parameter adaptive laws are developed, and the stability of the resulting systems is proved. At the same time, the outputs of closed-loop systems can be

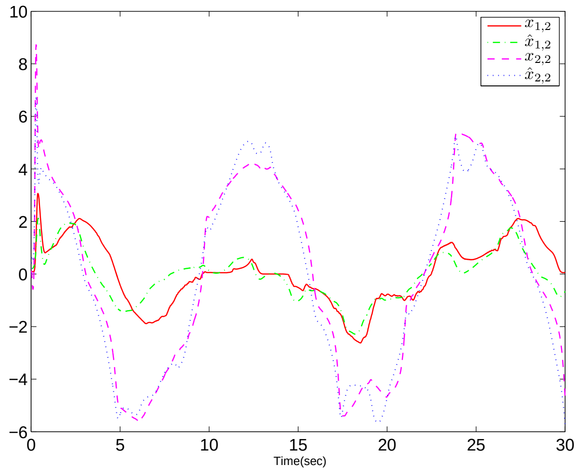

FIGURE 3. $x_{i,2}$ and $\hat{x}_{i,2}$ $(i = 1, 2)$

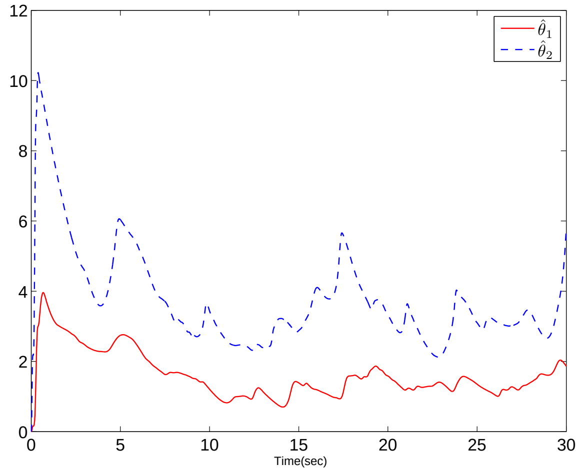

FIGURE 4. $\hat{\theta}_i$ $(i = 1, 2)$

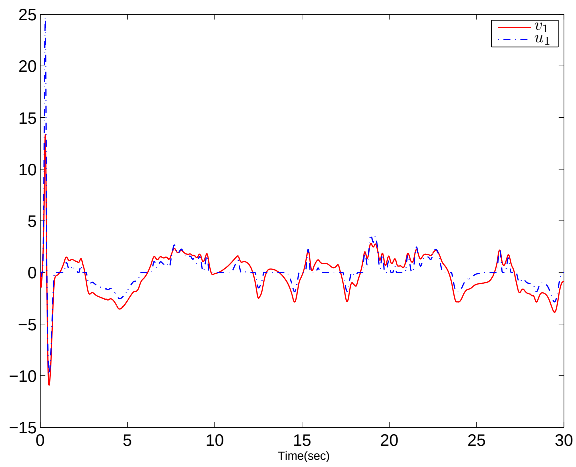

FIGURE 5. v_1 and u_1

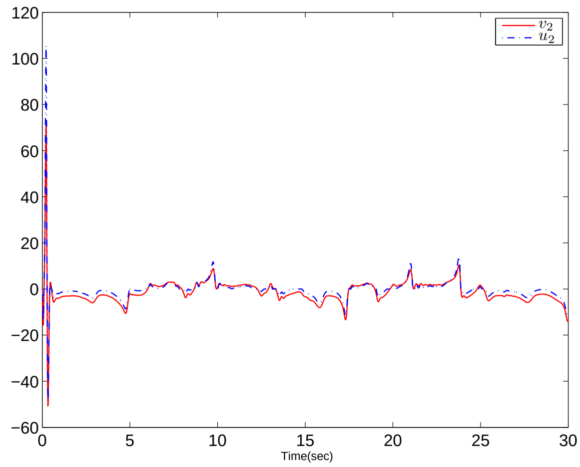

FIGURE 6. v_2 and u_2

guaranteed to track the reference signals very well by the proposed control scheme. In addition, the combination of convex combination method and linear matrix inequalities helps us overcome the obstacle coming from nonlinear matrix inequalities. As a result, the difficulty of simulation is reduced and the computational burden is significantly alleviated. Therefore, the developed control algorithm is more suitable for practical systems. In future work, time-varying delays, which often occur in various kinds of practical applications, will be considered in the design process with a view to further improve the control scheme proposed in this paper.

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