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WEIGHTS AGGREGATED MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZER FOR OPTIMAL POWER FLOW CONSIDERING THE GENERATION COST, EMISSION, TRANSMISSION LOSS AND BUS-VOLTAGE PROFILE

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Abstract. *This study proposes weights aggregated multi-objective particle swarm optimizer with static penalty constraint handling technique, linearly altered upper and lower velocity bounds and objectives handled via weights aggregation approach, to solve for the multi-objective optimal power flow problem in power systems for simultaneous minimization of active transmission loss, load buses volt deviation, cost and thermal emissions of active power generation units while acknowledging different operational and security constraints forced by the system and due to the network limited abilities. These objective functions conflict with one another, so a fuzzy based mechanism is represented to extract the best trade-off point among non-dominated solutions, obtained via multiple runs of the proposed algorithm with different weight settings. The multi-objective problem is converted to single objective optimization problem and standard particle swarm optimization with linearly modified upper and lower velocity limits is applied to come up with a final narrower range during iterative procedure for better local exploration of search space. The proposed method of solution is implemented to IEEE 30-bus benchmark system using MATLAB and compared with other nature inspired methods. The obtained results show better performance of the proposed algorithm over the other presented methods while strictly following all the system model constraints.*
Keywords: Particle swarm optimization, Multi-objective optimal power flow, Weights aggregation method, Static penalty constraint method, Linearly time varied velocity limits

1. **Introduction.** Optimal power flow (OPF) is a standard mathematical problem in the power system engineering and optimization in which the optimal settings of the power system operation point are determined. It was first introduced by Carpentier in 1962 [1] that in general aims to optimize certain objectives subject to the network power flow equations and system operation and security limits. The main objective of the OPF problem is to find the steady state optimized operating point in a multidimensional problem for mainly an economic and secure operation, where the multidimensional nature comes from the number of available control variables available in the system. It has gained importance over the years in the power system operation and control because of the increased drifting of electricity market towards the deregulation state, incorporation of the FACTS (Flexible AC Transmission) devices and an increasing trend towards the renewable energy resources.

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There are many traditional and non-traditional (modern heuristic algorithms) that have been developed and applied successfully for the purpose of solving the OPF problem for a quality solution. The traditional classical methods include, e.g., interior point methods [2,3], linear programming [4], Newton method [5], and quadratic programming [6] methods. As far as the modern artificial methods are concerned, they offer many advantages over traditional methods and they exist in numerous number developed over past decade. Sepulveda and Lazo applied the simulated annealing (SA) for the solution of OPF [7]. The simulations are done on IEEE-6 and IEEE-30 bus test systems. The application of ant colony optimization for minimization of total fuel cost of thermal generation units has been studied in [8]. The modified version of artificial bee colony is found in [9] for multi-objective minimization that is based on mutation and crossover operation of differential evolution. The particle swarm optimization that is developed in 1995, later on is also applied for the OPF solution as studied in [10,11]. Niknam et al. [12,13] presented the modified shuffle frog leaping algorithm in order to minimize the generation fuel cost and thermal generation emissions simultaneously and modified honey bee mating optimization to solve the dynamic OPF while taking account of the valve-point effects. The evolutionary algorithms' implementation has been studied in, e.g., [14,15] to obtain the optimal values of control variables. Similarly, the application of differential evolution method is studied in, e.g., [16,17] for cost minimization, maximization of voltage stability index and voltage profile improvement. Besides this the gravitational search algorithm, e.g., in [18], biogeography based optimization, e.g., [19], tabu search algorithm [20], flower pollination algorithm [21] to minimize the generation costs, transmission power loss and emissions from generation units while maximizing the voltage stability index, ant lion optimizer [22] for reactive dispatch problem in order to improve the voltage stability index and minimize the power losses, vortex search algorithm [23], cuckoo search algorithm [24], dance bee colony [25], moth flame optimization [26], dynamic population artificial bee colony [27] in which the population size varies over time, modified imperialist competitive algorithm [28], and genetic algorithm [29], are also applied successfully. The main goal of applying these modern intelligent methods is to obtain the high quality multidimensional solution point with non-continuous and non-differential objective functions and constraints that is generally not achievable through the use of classical methods as they offer various drawbacks for solving highly complex non-linear functions with non-linear constraints.

Particle swarm optimization (PSO) was first developed by Eberhart and Kennedy in 1995 [30]. It is one of the best known artificial based methods which has many local variants and its improved versions have been successfully applied to various non-linear problems with better results, e.g., as studied in [31]. Initially it was developed for the single function optimization but later on multi-objective (MO) problems are also solved with certain modifications in the basic model. As far as the multi-objective particle swarm optimization (MOPSO) approaches are concerned, these can be broadly classified into two categories. The first one is related to the designing of algorithms in which each particle is evaluated only for one objective function and selection of best positions is similar to the single function minimization. All objective functions are converted into a single function via linear summation of properly selected weight factors [32] based on the importance level of each objective function also sometimes known as the weights aggregation method (WAM). The application of WAM in MOPSO for the solution of OPF is studied in, e.g., [33-37]. Hadji et al. [33] solved the OPF problem for simultaneous minimization of generation fuel cost and emissions by converting the MO problem into single objective problem through the use of weight factors. Similarly, in a most recent research [36], Teeparthi and Kumar employed this method to model the total production cost of thermal and wind generators. The second approach is based on evaluation of all functions against each particle and finding the non-dominated Pareto optimal front [38]. [10,11,39-41] solve the OPF by applying the Pareto dominance concept in MOPSO.

The extraction of non-dominated points (also called as the leaders), however, is not straightforward as it demands the proper manipulation of information for choosing the best positions and guiding the swarm to the global optimal solution unlike in the weights aggregation approach where the determination of best positions is easy and straightforward. There is no absolute global best of the swarm and personal bests of particles but there exist a set of non-dominated points equally good with respect to all objectives, so selection of global best in a swarm and personal best for each particle, in this approach becomes a non-trivial task as stated in [41]. Poor selection of global and personal best guides the swarm to a sub-optimal solution. On the other hand, WAM with proper settings of weight factors provides a simple, robust and reliable solution technique as it provides straightforward update of the swarm and best positions. It also has been investigated in the literature review that the OPF problem has not been widely addressed using WAM for more than two objectives so this research addresses WAM with MOPSO for four objectives. To further enhance the local investigation capability of the PSO algorithm in the specified search space, linearly time varied upper and lower velocity limits have been incorporated during the iterative procedure of optimization.

To certify the proposed approach's feasibility, the weights aggregated multi-objective particle swarm optimization (WA-MOPSO) with linearly time varied upper and lower velocity vectors is applied on IEEE-30 bus test system. The obtained simulation results verify that the proposed method is feasible for the solution of the standard static optimal power flow problem. The rest of the research article is presented as follows. Section 2 presents the problem's mathematical formulation that consists of objective functions and the relevant system constraints. Section 3 details the necessary supportive mathematics that is applied towards the problem's solution. In Section 4, the complete algorithm is presented. Section 5 presents the detailed simulations on IEEE-30 bus system and finally the conclusions are carried out in Section 6.

2. **Problem Formulation and Constraints.** The general mathematical formulation of the MO OPF can be expressed as follows.

$$\text{Minimize} \quad (F_1(x,u), F_2(x,u), \ldots, F_q(x,u)) \tag{1}$$

Subject to:

$$g_1(x,u), g_2(x,u), \ldots, g_m(x,u) = 0; \quad h_1(x,u), h_2(x,u), \ldots, h_n(x,u) \leq 0 \tag{2}$$

where x represents the vector of state or dependent variables consisting of all load bus voltages V_L, real power generation at slack bus P_{gslack}, reactive power generations at all generator buses Q_{gk}, and line flows in transmission lines S_{Lp}, and u is the vector of control or independent variables consisting of real power generations at generator buses P_{gk} except at slack bus P_{gslack}, voltages at generator buses V_k, shunt reactive power injections Q_{ci}, and transformer tap settings T_i. So mathematically x and u can be formulated as shown below.

$$\begin{aligned} x^T &= \left[V_{L1} \ldots V_{Lj}, P_{gslack}, Q_{g1} \ldots Q_{gk}, S_{L1} \ldots S_{Lp}\right], \\ u^T &= \left[P_{g2} \ldots P_{gk}, V_1 \ldots V_k, T_1 \ldots T_i, Q_{c1} \ldots Q_{ci}\right] \end{aligned} \tag{3}$$

The proposed mathematical model of the system is presented below.

A. Generation fuel cost minimization. The fuel cost of the real power generations can be mathematically modeled by Equation (4) as found in [35].

$$\text{Minimize } C = \sum_{k=1}^{N_g} \left(a_k + b_k P_{gk} + c_k P_{gk}^2 \right) \ \$/\text{hr} \tag{4}$$

where C is the total fuel cost of all generation units, N_g is the number of generation units, and P_{gk} and a_k, b_k, c_k are the real power generations and fuel cost coefficients respectively, at generator bus index k.

B. Thermal generation emission minimization. The conventional thermal power plants produce toxic gases which include carbon oxides, sulphur oxides and nitrogen oxides that are extremely harmful. Power producing utilities are bound not only to produce energy at minimum cost but also with lowest possible emission level, according to the US Clean Air Act Amendments of 1990 [42]. Thus, in this research the emission function is mathematically modeled by summing up the quadratic and the exponential terms by taking account of the two types of emissions, i.e., NO_x and SO_x as studied in [35].

$$\text{Minimize } E = \sum_{k=1}^{N_g} \left(\alpha_k + \beta_k P_{gk} + \gamma_k P_{gk}^2 + \eta_k \exp(\lambda_k P_{gk}) \right) \ \text{ton/hr} \tag{5}$$

where E is the total emission in ton/hr from the thermal generation units and α_k, β_k, γ_k, η_k, λ_k are the emission coefficients of the k_{th} generator.

C. Power loss minimization in transmission lines. The real power loss reduction in transmission network is an effective method to decrease the generation cost. The researchers try to minimize extra power that is wasted as loss in order to lower the total power generation, thus reducing the total fuel cost and increasing the profit margins. The power loss is modeled as given in Equation (6).

$$\text{Minimize } P_L = \sum_{p=1}^{L} P_{Lossp} \tag{6}$$

where P_L represents the total power loss in L number of transmission lines and P_{Lossp} is the power loss in the P_{th} line.

D. Voltage deviation minimization at load buses. The power flow solution provides all the magnitudes of load bus voltages. Thus, the voltage deviation at the load buses in the power system distribution network can be mathematically written as given below in Equation (7) [43].

$$\text{Minimize } V_d = \sum_{j=1}^{N_L} |V_j - V_{ref}| \tag{7}$$

where V_d represents the total voltage deviation of N_L number of load buses in the power system, V_j is the voltage magnitude at each load bus j and V_{ref} is the pre-specified reference voltage magnitude. The V_{ref} is usually set as 1.0 per unit [37,43].

E. Problem constraints. The equality and inequality constraints represent the power balance conditions, power system security and network limitations for smooth and balance operation.

Real and reactive power balance at each and every bus in the system.

$$P_i = P_{gi} - P_{di} = 0; \quad Q_i = Q_{gi} - Q_{di} = 0 \tag{8}$$

where P_{gi} and P_{di} are the net real power generation and net real power demand at the ith bus respectively, and Q_{gi} and Q_{di} are the net reactive power injection and net reactive power demand at the bus i respectively.

The inequality constraints are modeled by the respective equations shown below.

1) Real power generations P_{gi}, reactive power generations Q_{gi}, and voltage magnitudes V_{gi} limits.

$$P_{gi\min} \leq P_{gi} \leq P_{gi\max}; \quad Q_{gi\min} \leq Q_{gi} \leq Q_{gi\max}; \quad V_{gi\min} \leq V_{gi} \leq V_{gi\max} \tag{9}$$

where $P_{gi\min}$, $Q_{gi\min}$ and $V_{gi\min}$ are lower limit values and $P_{gi\max}$, $Q_{gi\max}$ and $V_{gi\max}$ are the upper limit values of the real power generations, reactive power generations and generator bus volt magnitudes respectively.

2) Load bus voltages V_{Li} limits.

$$V_{Li\min} \leq V_{Li} \leq V_{Li\max} \tag{10}$$

where $V_{Li\min}$ and $V_{Li\max}$ are the minimum and maximum limit values of load bus voltages respectively.

3) Transformers' tap settings T_i limits.

$$T_{i\min} \leq T_i \leq T_{i\max} \tag{11}$$

where $T_{i\min}$ and $T_{i\max}$ are the minimum and maximum limit values of transformers' tap settings respectively.

4) Switchable VAR compensators Q_{ci} limits.

$$Q_{ci\min} \leq Q_{ci} \leq Q_{ci\max} \tag{12}$$

where $Q_{ci\min}$ and $Q_{ci\max}$ are the minimum and maximum limit values of reactive powers injections respectively.

5) Transmission lines loading S_{Lp} limits.

$$S_{Lp} \leq S_{Lp\max} \tag{13}$$

where $S_{Lp\max}$ is the maximum limit values of the powers transmitted in transmission lines.

3. **Method of Solution.** For implementation of the proposed method, each position of the particle represents a vector of control variables.

$$u_d = \left[V_1 \ldots V_k, T_1 \ldots T_i, Q_{c1} \ldots Q_{ci}, P_{g2} \ldots P_{gk}\right]^T \tag{14}$$

where $d = 1, \ldots, N_p$, and N_p represents the population size of the swarm.

The lower and upper limits of the particle's position are lower and upper limits of the respective control variables whereas the lower and upper limits of the velocities are defined as follows.

$$v_{d\max} = \frac{(x_{d\max} - x_{d\min})}{t}; \quad v_{d\min} = -v_{d\max} \tag{15}$$

where t is the current iteration number.

The positions of the particles and their respective velocities are randomly initialized within their limits as follows.

$$x_d^{(0)} = x_{d\min} + r_1 \times (x_{d\max} - x_{d\min}); \quad v_d^{(0)} = v_{d\min} + r_2 \times (v_{d\max} - v_{d\min}) \tag{16}$$

where r_1 and r_2 are the random numbers in range $[0 \ \ 1]$.

In the iterative procedure, the positions and their respective velocities' limits are enforced according to the following equation.

$$x_d^{new} = \begin{pmatrix} x_{d\max} \text{ for } x_d > x_{d\max} \\ x_{d\min} \text{ for } x_d < x_{d\min} \end{pmatrix}; \quad v_d^{new} = \begin{pmatrix} v_{d\max} \text{ for } v_d > v_{d\max} \\ v_{d\min} \text{ for } v_d < v_{d\min} \end{pmatrix} \tag{17}$$

The inertia weight in the iterative process is updated according to the following equation as found in [33].

$$w_t = \frac{(w_{\max} - w_{\min}) \times (\text{iter}_{\max} - t)}{\text{iter}_{\max}} + w_{\min} \tag{18}$$

The particle's velocity and position are updated using Equation (19).

$$v_{i,d}^{t+1} = w_t v_{i,d}^t + c_1 r_3 \Big(pbest_{i,d}^t - x_{i,d}^t \Big) + c_2 r_4 \Big(gbest_{i,d}^t - x_{i,d}^t \Big); \quad x_{i,d}^{t+1} = v_{i,d}^{t+1} + x_{i,d}^t \tag{19}$$

where $pbest_{i,d}^t$ is the current personal best positions set of the particles, $gbest_{i,d}^t$ is the current global best positions set of the swarm, t is the iteration number, $x_{i,d}^{t+1}$ and $v_{i,d}^{t+1}$ are the new positions and their respective velocities respectively, c_1 and c_2 are the learning factors, r_3 and r_4 are the random numbers in range $[0 \;\; 1]$ and $x_{i,d}^t$ is the set of current iteration's positions of the N_p particles.

The weights aggregation method for multi-objective problem as studied in, e.g., [32,43] is modeled as shown below in Equation (20).

$$F(x) = \sum_{i=1}^{N_{obj}} w_i \times f_i(x) \tag{20}$$

where w_i are the non-negative weights such that,

$$\sum_{i=1}^{N_{obj}} w_i = 1 \tag{21}$$

The optimization is performed on the newly transformed $F(x)$. The penalized objective function is found using the exterior penalty function method as given in Equation (22) and found in [44].

$$f_p(x) = f(x) \pm \left(\sum_{n=1}^{N_{ueq}} r_n \times G_n + \sum_{m=1}^{N_{eq}} c_m \times L_m \right) \tag{22}$$

where

$$\begin{cases} G_n = \max[0, h_n(x, u)]^k \\ L_m = |g_m(x, u)|^r \end{cases} \tag{23}$$

where k and r are normally 1 or 2. N_{ueq} and N_{eq} are the number of inequality and equality constraints respectively. r_n and c_m are the positive constants and are normally known as the penalty factors. In this paper, the equality constraints of the proposed multi-objective problem are handled during power flow analysis employing Newton Raphson technique of solving power flow networks, so there is no need to incorporate penalty functions for equality constraints. The inequality constraints of the control variables are made satisfied by defining the boundary of the search space at the start of the optimization process whereas the inequality constraints on the state variables are handled through penalty function technique as defined by Equation (22). The value of k is chosen as 2 resulting in a quadratic nature. The penalty factors are chosen static; thus they do not depend on current generation number.

In multi-objective optimization problems, there is more than one objective to be optimized and usually these objectives conflict each other (i.e., optimization of one objective cannot be achieved without degradation of another objective). So therefore, there exists no longer a single solution (as in single objective optimization) but exist a group of trade-off solutions called the Pareto optimal solutions and thus the decision making for the best compromise multidimensional point is not straightforward. So in this research, the fuzzy mechanism is adopted as it can handle the multi-objective problems with conflicting objectives. It assigns to each objective function, a membership function $\mu_i(X)$ as represented in Equation (24) and found in [35].

$$\mu_i(X) = \begin{cases} 1 & \text{if } F_i(X) \le F_{i\,\min} \\ \dfrac{F_{i\,\max} - F_i(X)}{F_{i\,\max} - F_{i\,\min}} & \text{if } F_{i\,\min} < F_i(X) < F_{i\,\max} \\ 0 & \text{if } F_i(X) \ge F_{i\,\max} \end{cases} \tag{24}$$

where $\mu_i(X)$ is the membership value of the ith objective function, $F_{i\,\min}$ and $F_{i\,\max}$ are their lower and upper limits respectively and are found from the optimization of each objective individually. As the values of $\mu_i(X)$ get higher and higher, the greater solution satisfaction is achieved. With $\mu_i(X) = 1$, the solution is of complete satisfaction whereas $\mu_i(X) = 0$ represents completely unsatisfied solution. For a total number of M Pareto points, the membership function $\mu_i(X)$ can be normalized as shown below.

$$\mu^k = \frac{\sum_{j=1}^{N_{obj}} \mu_j}{\sum_{k=1}^{M} \mu_j \sum_{j=1}^{N_{obj}} \mu_j} \tag{25}$$

The Pareto optimal point having the maximum μ^k value can be selected as the best trade-off point.

4. **Proposed Algorithm in Steps.** This section demonstrates the application of the proposed WA-MOPSO with iteratively altering the velocity bounds to a final narrower range for better local investigation of the search zone for multiple objectives and constraints handled via static penalty constraint method. Following steps should be done in order to apply the proposed method to the OPF problem.

Step 1: As a first step, according to Equations (9)-(13), initialize all input data which consist of upper and lower limits on control variables (including all generator bus voltages, all generator real powers except real power at slack bus, all transformer tap settings, all reactive powers compensation values), upper and lower limits on state variables (including real power flows in transmission lines, all generator reactive powers, all load bus voltages, and slack bus real power), fuel cost coefficients and emission coefficients of all generation units.

Step 2: Define all PSO parameters (including inertia weight maximum $w_{\max}$ and minimum $w_{\min}$, learning factors values c_1 and c_2, swarm size N_p and maximum generations size $\text{iter}_{\max}$, penalty values for inequality boundaries violation of all dependent variables (including penalty value for load buses volt deviation violations P_v, slack bus real power generation violation P_s, transmission lines real power flow violations P_l, and generators reactive power generations violations P_g), and corresponding weight factors for all the objective functions w_1, w_2, w_3 and w_4 based on the prominent level of the respective objective.

Step 3: Generate the random initial population $x_{i,d}^{(0)}$ with desired number of N_p particles and their respective velocities $v_{i,d}^{(0)}$ as shown below in Equation (26). Define upper and lower limits on velocity vector using Equation (15). The format of the particles shown is chosen according to the format as shown below. All the values in the position vector satisfy their upper and lower bounds as chosen during the algorithm.

$$\left[V_{g1}\ V_{g2}\ V_{g5}\ V_{g8}\ V_{g11}\ V_{g13}\ T_{6\text{-}9}\ T_{6\text{-}10}\ T_{4\text{-}12}\ T_{27\text{-}28}\ Q_{c10}\ Q_{c24}\ P_{g2}\ P_{g5}\ P_{g8}\ P_{g11}\ P_{g13}\right]$$

$[V_{g1}\ V_{g2}\ V_{g5}\ V_{g8}\ V_{g11}\ V_{g13}\ T_{6\text{-}9}\ T_{6\text{-}10}\ T_{4\text{-}12}\ T_{27\text{-}28}\ Q_{c10}\ Q_{c12}\ Q_{c15}\ Q_{c17}$
$Q_{c20}\ Q_{c21}\ Q_{c23}\ Q_{c24}\ Q_{c29}\ P_{g2}\ P_{g5}\ P_{g8}\ P_{g11}\ P_{g13}]$

Step 4: Using the static penalty constraint handling technique, alter the constrained problem to an unconstrained one using Equation (27). The $F_1(X)$, $F_2(X)$, $F_3(X)$ and $F_4(X)$ are the penalized objective functions and $C(u,x)$, $E(u,x)$, $P_L(u,x)$, and $V_d(u,x)$ represent the proposed objectives as defined by Equations (4)-(7) respectively. Since the constraints have to be met, the values of the penalty factors are chosen high to be the 10,000.

$$x_{i,d}^{(0)} = \begin{pmatrix} x_{1,d}^{(0)} \\ x_{2,d}^{(0)} \\ \cdots \\ x_{N_p,d}^{(0)} \end{pmatrix}; \qquad v_{i,d}^{(0)} = \begin{pmatrix} v_{1,d}^{(0)} \\ v_{2,d}^{(0)} \\ \cdots \\ v_{N_p,d}^{(0)} \end{pmatrix} \tag{26}$$

$$\begin{aligned} F_i(X) &= \begin{pmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \\ F_4(X) \end{pmatrix} \\ &= \begin{pmatrix} C(u,x) + P_s \times (P_{g1} - P_{g1\lim})^2 + P_v \times \sum_{j=1}^{N_L} (V_j - V_{j\lim})^2 \\ \quad + P_q \times \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi\lim})^2 + P_l \times \sum_{j=1}^{L} (P_{Linej} - P_{Linej\max})^2 \\ E(u,x) + P_s \times (P_{g1} - P_{g1\lim})^2 + P_v \times \sum_{j=1}^{N_L} (V_j - V_{j\lim})^2 \\ \quad + P_q \times \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi\lim})^2 + P_l \times \sum_{j=1}^{L} (P_{Linej} - P_{Linej\max})^2 \\ P_L(u,x) + P_s \times (P_{g1} - P_{g1\lim})^2 + P_v \times \sum_{j=1}^{N_L} (V_j - V_{j\lim})^2 \\ \quad + P_q \times \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi\lim})^2 + P_l \times \sum_{j=1}^{L} (P_{Linej} - P_{Linej\max})^2 \\ V_d(u,x) + P_s \times (P_{g1} - P_{g1\lim})^2 + P_v \times \sum_{j=1}^{N_L} (V_j - V_{j\lim})^2 \\ \quad + P_q \times \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi\lim})^2 + P_l \times \sum_{j=1}^{L} (P_{Linej} - P_{Linej\max})^2 \end{pmatrix} \end{aligned} \tag{27}$$

The limiting values of the state variables in Equation (27) are found by the following equation.

$$x_{\lim} = \begin{cases} x_{\max} & \text{for } x > x_{\max} \\ x_{\min} & \text{for } x < x_{\min} \\ x & \text{otherwise} \end{cases} \tag{28}$$

Step 5: Convert the multi-objective problem to single objective problem using the linear summation of weight factors according to Equation (29).

$$F'(X) = \sum_{i=1}^{N_{obj}} w_i \times F_i(X) = w_1 \times F_1(X) + w_2 \times F_2(X) + w_3 \times F_3(X) + w_4 \times F_4(X) \tag{29}$$

where N_{obj} is the number of objective functions to be minimized. The values of weight factors are chosen based on the importance level of each function and hence prior knowledge to the problem is required. In order to get the desirable set of non-dominated solution points, the algorithm is run multiple times with different weight setting values for each simulation run. For a specific set of weight factors, only single global optimum solution is obtained.

Step 6: Against each particle in the swarm, perform power flow analysis using the Newton Raphson method and solve the power flow network for all state variables. Calculate $F'(X)$ using Equation (29) and find the minimum value of function $F'_{\min}(X)$ for each particle.

Step 7: Assign $x_{i,d}^{(0)}$ to the best positions of particles $pbest_{i,d}^{(0)}$, and position of the particle corresponding to $F'_{\min}(X)$ to $gbest_d^{(0)}$, initially. Set iteration counter $t = t + 1$.

Step 8: Update inertia weight w_t and $v_{d\max}$, $v_{d\min}$ according to Equations (18) and (15) respectively.

Step 9: Calculate new velocities $v_{i,d}^{t+1}$, and update the positions $x_{i,d}^{t+1}$ of all particles in the swarm using Equation (19). Enforce the limits of velocities and positions of particles according to Equation (17).

Step 10: Perform load flow analysis using Newton Raphson method against newly obtained positions and calculate $F'(X)$ using Equation (29).

Step 11: To find the best fitness function value of each particle up to the current iteration, update the $pbest_{i,d}^{t+1}$ by comparing the $pbest_{i,d}^{t}$ to the positions corresponding to $F'(X)^{(t+1)}$. Update the global best value by finding the minimum of $pbest_{i,d}^{t+1}$ and thus set it to the $gbest_d^t$.

Step 12: Update counter $t = t + 1$, and return to Step 8. Repeat until $t \leq \text{iter}_{\max}$.

Step 13: Extract best trade-off solution point among non-dominated solution points that are obtained during multiple runs of the proposed algorithm, using Equations (24) and (25). Print the results.

5. **Simulations and Results.**

To illustrate the validation and performance of the proposed weights aggregated multi-objective particle swarm optimization (WA-MOPSO) algorithm with linearly decreasing the upper velocity bound and increasing the lower velocity bound at the same time to a final narrower range of velocity limit vector for making the particles to make smaller and smaller jumps from points to points towards the end of the algorithm, during the iterative process of optimization, for better local investigation of the search zone, it is coded at MATLAB programming platform and is applied on standard IEEE 30-bus system which has non-linear characteristics. It has 6 generators at buses 1, 2, 5, 8, 11 and 13, 4 tap changing transformers at lines 6-9, 6-10, 4-12 and 27-28, 41 transmission lines and 24 load buses. The system's total real and reactive demand is 2.834 (p.u.) and 1.262 (p.u.) respectively on the basis of 100MVA in all simulations. The fuel cost and emission coefficients of the generation units that are used in this research can be found in [35]. The detailed system bus and transmission line data can be seen in, e.g., [45]. For solving the power flow network for all unknown system variables, Newton Raphson iterative method is used.

The proposed algorithm is tested while employing two different sets of shunt compensators. In single function minimization and multi-objective optimization of two objectives, 2 shunt compensators at buses 10 and 24 with total of 17 control variables are used whereas in the multi-objective optimization considering all objectives simultaneously, 9 shunt compensators are installed at buses 10, 12, 15, 17, 20, 21, 23, 24 and 29 with total of 24 control variables. These vectors are also shown in Section 4 above, Step 3. The

TABLE 1. Algorithm's adjusted parameters for all cases

Parameters	Cost	Emission	Volt Deviation	Power Loss	Power Loss & Cost	All Objectives
N_p	20	20	20	50	20	20
c_1	2.0	2.0	2.0	2.0	2.0	2.0
c_2	2.0	2.0	2.0	2.0	2.0	1.6
$w_{\max}$	0.90	0.90	0.90	0.90	0.90	0.90
$w_{\min}$	0.40	0.40	0.40	0.20	0.40	0.40
k_p	10000	10000	10000	10000	10000	10000
k_q	10000	10000	10000	10000	10000	10000
k_s	10000	10000	10000	10000	10000	10000
k_v	10000	10000	10000	10000	10000	10000
$\mathbf{iter}_{\max}$	200	200	200	200	200	200
w_c	–	–	–	–	0.5000	0.0101
w_E	–	–	–	–	–	0.3300
w_{V_d}	–	–	–	–	–	0.3299
w_{P_L}	–	–	–	–	0.5000	0.3300

default boundary limit values that are chosen for generator bus voltages and load bus voltages are kept in range [0.95 1.10] p.u. and [0.95 1.05] p.u. respectively whereas for the reactive shunt compensators and transformers taps, they are chosen to be [0 0.05] p.u. and [0.90 1.10] respectively. The reader is supposed to assume these default input ranges unless any other limit range is specified in particular simulation case. The summary of algorithm's parameters that are adjusted for different case runs is given in Table 1. The maximum iteration size and the population size of the swarm are chosen to be 200 and 20 respectively in all simulations except in the power loss minimization case where the population size is 50. The other parameters of the algorithm in each case are adjusted accordingly while having in mind the prime objective of obtaining a better optimized value. In order to obtain a constraints satisfied and guaranteed global solution the penalty factor values are chosen high, i.e., 10,000 and kept constant throughout the algorithm run in all simulation cases. The penalty factors are not dependent on the iteration number since they are constants. The corresponding weight factors in multi-objective optimization are selected based on the functions prominent levels. A small weight factor value for the cost function is because of the larger values of the cost in dollars per hour over the other objective function values. If the weight factor for cost is increased, its value go dramatically down in the MO optimization but at the cost of another objective function (trade-off) so proper and balance control settings are very compulsory for balance results.

As a first step, the algorithm is applied to the single function minimization in order to minimize each objective individually that is summarized through Case 1a $\sim$ Case 1d. During individual optimization of each objective, $F_{i\max}$ and $F_{i\min}$ are obtained over 20 independent runs that are needed for extraction of best compromise point using fuzzy logic technique. In order to investigate about the algorithm's convergence, few other tabular results have been compiled and are shown and described below. The results obtained in single objective minimization as well as in multi-objective optimization are compared with other existing methods in literature.

Case 1: Minimizing each objective individually

Case 2: Minimizing cost and power loss simultaneously for economic dispatch

Case 3: Minimizing all objectives simultaneously

Case 1a: Minimizing fuel cost of the generation units. In this section, the particle swarm optimization algorithm with linearly decreasing $V_{d\max}$ and increasing $V_{d\min}$ at the same time is simulated for individual minimization of the total fuel cost of the generators as defined by Equation (4) while ignoring the other objective functions of the presented model. The upper and lower velocity bounds are iteratively altered to a narrower final range for exploring the search zone more efficiently. According to this approach, as the algorithm converges to the optimal solution iteration after iteration, the particles of the swarm make smaller and smaller jumps from location to location towards the end of the algorithm. The final results obtained in this case are compared to tabu search algorithm (TSA) [20], modified differential evolution (MDE) [17], enhanced genetic algorithm (EGA) [29], improved particle swarm optimization (IPSO) [10], and ant colony optimization (ACO) [8] as shown in Table 2. The minimum fuel cost obtained is 800.975 \$/hr. The reactive shunt compensator values are set between [0.00 0.50] p.u. for this case only, at buses 10 and 24 of the IEEE power network. Table 2 clearly validates the effectiveness of the proposed algorithm to obtain the better optimized cost as compared to other heuristic methods. It is necessary to note that all the values of the control variables of global best particle fall in their allowed limits.

Table 2. Cost function (\$/hr) comparison results with other methods

	TSA	**EGA**	**MDE**	**IPSO**	**ACO**	**Proposed PSO**
$\boldsymbol{P_{g1}}$, **MW**	176.04	176.20	176.009	177.0431	177.8635	177.1862
$\boldsymbol{P_{g2}}$, **MW**	48.76	48.75	48.801	49.2090	43.8366	48.9508
$\boldsymbol{P_{g5}}$, **MW**	21.56	12.44	21.334	21.5135	20.8930	21.3774
$\boldsymbol{P_{g8}}$, **MW**	22.05	21.95	22.262	22.6480	23.1231	22.5380
$\boldsymbol{P_{g11}}$, **MW**	12.44	12.42	12.460	10.4146	14.0255	10.6210
$\boldsymbol{P_{g13}}$, **MW**	12.00	12.02	12.000	12.0000	13.1199	12.0000
cost, \$/hr	802.29	802.06	802.376	801.9780	803.1230	**800.9750**

Case 1b: Minimizing active power loss in the transmission lines. The Case 1b presents the active power loss minimization as defined by Equation (6). The results obtained are compared with genetic algorithm (GA), strength Pareto evolutionary algorithm (SPEA) [46], improved particle swarm optimization (IPSO) [10] and Pareto optimal particle swarm optimization (POPSO) [40]. The best power transmission loss obtained through the application of the proposed algorithm is 3.1727 MW, which is better than the other mentioned methods as shown in Table 3. The convergence curves of algorithm for an instance are also shown for Case 1a and Case 1b in Figure 1 over 200 iterations.

Case 1c: Minimizing emission of the generation units. In this case, minimization of thermal plants' emissions is considered as defined by Equation (5) while ignoring the rest of the objectives of the model problem. The results obtained via the application of the proposed PSO with lowering $V_{d\max}$ and increasing $V_{d\min}$ in this section are compared with particle swarm optimization (PSO), genetic algorithm (GA), shuffle frog leaping algorithm (SFLA) and modified shuffle frog leaping algorithm (MSFLA) [12], constriction learning particle swarm optimization (CLPSO) [11] and improved particle swarm optimization (IPSO) [10] as shown in Table 4. It is evident that the proposed algorithm is able to achieve better power generation setting values in order to lower the emissions of the thermal power generation plants including NO_x and SO_x and thus resulting in reduction in atmospheric pollution which enhances human and social welfare.

TABLE 3. Comparison of power loss function (MW)

	GA	SPEA	IPSO	POPSO	Proposed PSO
V_{g1}, p.u.	–	–	1.0470	1.1000	1.0536
V_{g2}, p.u.	1.0300	1.0440	1.0440	1.0772	1.0508
V_{g5}, p.u.	1.0000	1.0230	0.9760	1.0712	1.0331
V_{g8}, p.u.	1.0000	1.0220	1.0350	1.0761	1.0390
V_{g11}, p.u.	1.0200	1.0420	0.9840	1.0499	1.0468
V_{g13}, p.u.	1.0400	1.0430	1.0420	1.0322	1.0692
$T_{6\text{-}9}$	1.0000	1.0900	1.0290	1.0100	0.9848
$T_{6\text{-}10}$	1.0100	0.9000	0.9800	1.0500	0.9452
$T_{4\text{-}12}$	1.0000	1.0200	1.0100	1.0100	1.0004
$T_{27\text{-}28}$	1.0400	0.9600	0.9700	0.9800	0.9644
Loss, MW	5.3513	5.1995	5.0732	4.5190	**3.1727**

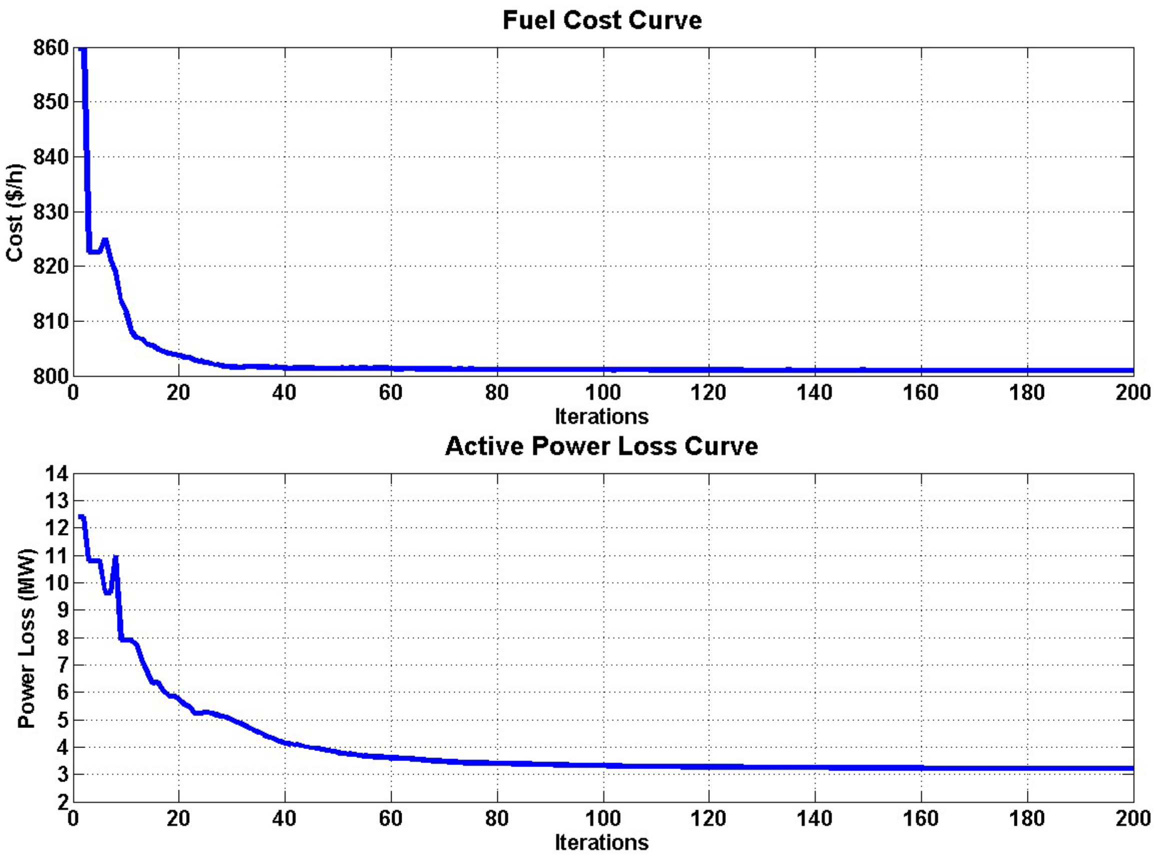

FIGURE 1. Algorithm's convergence curves for Cases 1a and 1b

Case 1d: Minimizing voltage deviation at the load buses. This section shows the minimization of voltage deviation at load buses as a primary objective function as described by Equation (7) while setting the corresponding weight values of the other objective functions to zero. The minimum voltage deviation value extracted is 0.0602 p.u., which is then compared with biogeography based optimization (BBO) [19], differential evolution (DE) [16], artificial bee colony (ABC), modified artificial bee colony (MABC) [9], non-dominated sorting genetic algorithm (NSGA-II) and hybrid multi-objective particle swarm optimization (HMOPSO) [47]. Table 5 clearly validates that the proposed PSO

TABLE 4. Comparison of emission (ton/hr) with other algorithms

	PSO	**GA**	**SFLA**	**MSFLA**	**CLPSO**	**IPSO**	**Proposed PSO**
P_{g1}, **MW**	59.8079	78.2885	64.4840	65.7798	65.3083	67.0400	59.2400
P_{g2}, **MW**	80.0000	68.1602	71.3807	68.2688	72.9975	68.1400	72.8205
P_{g5}, **MW**	50.0000	46.7848	49.8573	50.0000	50.0000	50.0000	50.0000
P_{g8}, **MW**	35.0000	33.4909	35.0000	34.9999	34.0000	35.0000	35.0000
P_{g11}, **MW**	27.1398	30.0000	30.0000	29.9982	28.0000	30.0000	30.0000
P_{g13}, **MW**	40.0000	36.3713	39.9729	39.9970	40.0000	40.0000	40.0000
Emission	0.2096	0.21170	0.2063	0.2056	0.2059	0.2058	**0.1944**

TABLE 5. Comparison of voltage deviation (p.u.)

	BBO	**DE**	**ABC**	**MABC**	**NSGA-II**	**HMOPSO**	**Proposed PSO**
V_d	0.1020	0.1357	0.1040	0.0841	0.1274	0.0913	**0.0602**

TABLE 6. Proposed algorithm's characteristics over 20 independent runs (Case 1)

	Cost, C	**Emission, E**	**Volt Deviation, V_d**	**Power Loss, P_L**
Maximum	802.3912	0.2189	0.1545	3.9014
Minimum	800.9750	0.1944	0.0602	3.1727
Mean	801.3399	0.1972	0.0853	3.2869
Variance	0.26275	0.000058	0.00135	0.0480
Standard Deviation	0.5126	0.0076	0.0367	0.2191

algorithm is capable of improving the bus voltage profile by obtaining the minimum voltage deviation at the load buses.

During individual minimization of each objective, the maximum and minimum values are extracted over 20 independent runs in order to apply the fuzzy logic method for best trade-off point for multi-objective optimization. The minimum and maximum values of the objective functions are presented in Table 6 along with the mean values and the standard deviation for each case of individual optimization. The algorithm is applied with the same parameter adjustments as shown in Table 1.

Table 7 shows the convergence of the algorithm for single objective minimization. The table depicts approximately the linear relationships between the population size and the number of iterations for all objectives as far as the convergence to the optimal point is concerned. This fact is also illustrated in dual-axis Figure 2. As the population size is increased, the algorithm converges in less number of iterations; thus the curve for each function is approximate linear with a little deviation about the mean.

Case 2: Minimizing cost and power loss simultaneously for economic dispatch. In this section of the simulations, the proposed WA-MOPSO method is tested for simultaneous minimization of generation's fuel cost and active power loss for economic dispatch, in order to minimize overall cost of the system operation. The reduction of the active power loss is another way of decreasing system active power generation or ideally producing the only amount of power that could serve the load demand and thus resulting in less fuel consumption and increasing profit margin. The results obtained in this case

TABLE 7. Convergence of the algorithm (population size vs. iterations): Case 1

Simulations	**Parameters**		**Single Objective Optimization**			
Serial no.	Chosen Population Size	No. of Iterations, Algorithm converged	\$/hr	Ton/hr	MW	p.u.
1	10	200	801.0474	0.2059	3.6356	0.0970
2	20	100	801.1721	0.2049	3.2852	0.0870
3	30	67	801.5430	0.2051	3.2815	0.0916
4	40	50	801.3413	0.2097	3.3662	0.1119
5	50	40	801.7939	0.2074	3.5056	0.0945
6	60	34	801.1394	0.2073	3.6720	0.0825
7	70	29	801.3248	0.2057	3.4873	0.0764
8	80	25	802.2514	0.2074	3.6051	0.1051
9	90	23	801.5683	0.2074	3.4870	0.1105
10	100	20	802.2247	0.2091	3.5493	0.0826

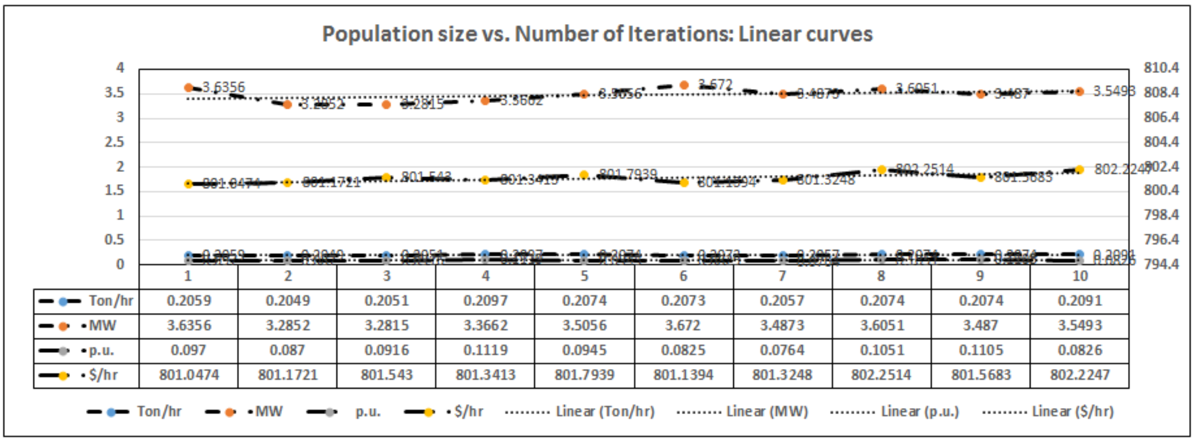

FIGURE 2. Objective functions' curves obtained over 10 simulations

TABLE 8. Comparison of cost and power loss: Case 2

Output Powers	**SA**	**EP**	**WA-MOPSO**
P_1, **MW**	188.02	173.848	171.74
P_2, **MW**	47.45	49.998	48.67
P_5, **MW**	19.77	21.386	21.57
P_8, **MW**	13.40	22.630	25.38
P_{11}, **MW**	11.25	12.928	12.83
P_{13}, **MW**	14.09	12.000	12.00
P_L, **MW**	10.58	9.390	**8.7969**
Cost, \$/hr	804.4300	802.6200	**801.4478**

are summarized in Table 8 and compared with simulated annealing (SA) [7] and evolutionary programming (EP) [48]. It can be seen that the proposed algorithm is capable of obtaining better cost value and a decrease in power loss in transmission lines as compared to other mentioned algorithms. The minimum cost achieved is 801.4478 \$/hr and the

TABLE 9. Function evaluations against different weight values: Case 2

Serial No.	w_{PL}	w_c	Power Loss, MW	Cost, $/h
1	0.999	0.001	3.2575	968.0383
2	0.990	0.010	3.5916	922.2120
3	0.900	0.100	6.9511	867.6569
4	0.800	0.200	7.9867	803.7814
5	0.600	0.400	8.7288	801.6631
6	0.500	0.500	8.7969	801.4478
7	0.400	0.600	9.1427	801.3705
8	0.200	0.800	9.1917	801.3410
9	0.100	0.900	9.6414	801.3345
10	0.010	0.990	9.7210	801.2661

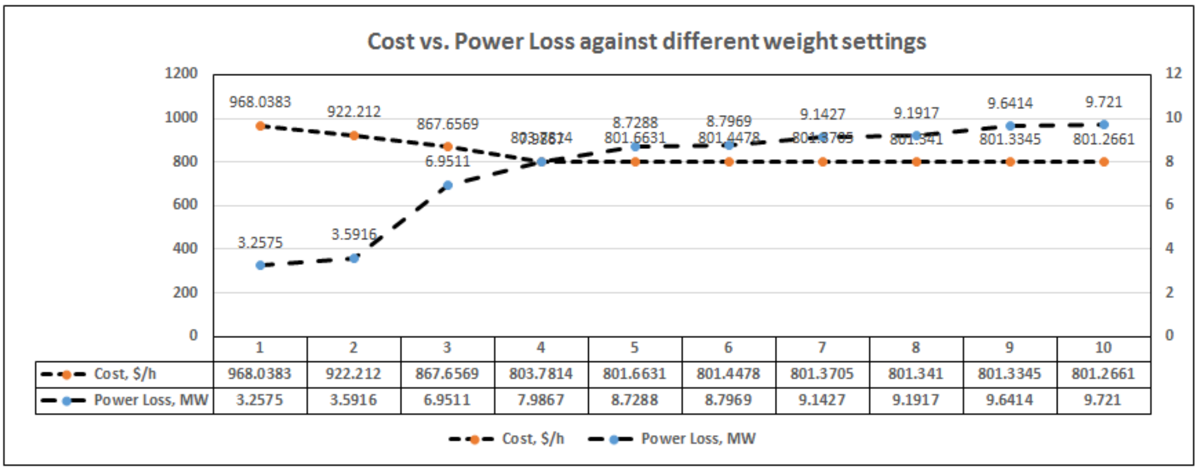

FIGURE 3. Cost and power loss curves against different weight settings

minimum power loss is 8.7969 MW. The dimension of the optimization problem in Case 1 and Case 2 is 17 as there are only two shunt capacitors that are installed at the buses 10 and 24. The obtained values of shunt capacitors for Case 2 are 1.26 MVAR and 5.00 MVAR.

The proposed algorithm for Case 2 is also simulated against different values of the objectives weights. The cost and power loss functions evaluations are shown in Table 9 for different weight settings representing non-dominated solutions that are equally good with respect to both objectives. The global best solution with $w_c = 0.5$ and $w_{PL} = 0.5$ can be selected as the best compromise solution. It is obvious from Table 9 that as the weight of the respective function is getting higher, its value is decreasing and vice versa which means that increasing the weight value for the particular objective increases its importance level during the optimization procedure. The optimal point for Case 2 according to the format in Section 4 is: [1.0754 1.0522 1.0282 1.0387 1.0671 1.0720 1.0026 0.9430 1.0181 0.9885 0.0126 0.0500 0.4867 0.2157 0.2538 0.1283 0.1200].

Figure 3 also illustrates that as the one objective function's value decreases the other function value increases as there is trade-off in MO optimization. The two curves cross each other at the simulation no. 4 which can also be chosen as the best compromise solution manually.

Case 3: Optimizing all objectives simultaneously. In this case, 9 shunt compensators are used at buses 10, 12, 15, 17, 20, 21, 23, 24 and 29 with total of 24 control variables and emission function without exponential term is employed. The minimum and maximum values of the objective functions obtained through Case 1a ∼ Case 1d are used to calculate the membership values $\mu_i(X)$ of the objective functions. The best values of generation fuel cost, emission, transmission loss and voltage deviation are compared with artificial bee colony (ABC) [9] as given in Table 10. It is obvious that the demonstrated algorithm is capable of finding the better values of objective functions. The minimum values of fuel cost, emission, power loss and volt deviation obtained are 851.8040 $/hr, 0.2221 ton/hr, 4.9120 MW and 0.3157 p.u., respectively. The optimized values of control variables (multidimensional optimal point) according to the format chosen are: [1.0620 1.0497 1.0250 1.0328 1.0220 1.0342 0.0011 0.0348 0.0189 0.0342 0.0232 0.0410 0.0166 0.0393 0.0304 0.9693 1.0207 1.0341 0.9847 0.5374 0.3594 0.3500 0.300 0.2631]. It is important to note that all the values of the control variables representing a position vector lie in their allowed upper and lower limits and are represented in per unit system on the basis of 100 MVA except transformer tap setting values which are basically the off-nominal tap ratios in range [0.90 1.10].

Table 10. Optimized functions' values (Case 3)

Method	$/hr	Ton/hr	MW	p.u.
ABC	852.5226	0.2224	4.9129	0.3474
WA-MOPSO	**851.8040**	**0.2221**	**4.9120**	**0.3157**

In order to examine the convergence of the algorithm in multi-objective optimization, it is simulated in two different ways on the IEEE 30-bus test system. Table 11 shows the multi-objective results for varied population size vs. fixed number of iterations in the first few rows and fixed population size vs. varied number of maximum iterations in the last few rows. It can be seen that as the number of particles in a swarm is increased for fixed iterations, the quality of the solutions gets better; however, there is another factor involved in MO, i.e., as there is trade-off between the solutions in multi-objective optimization a better value of one objective is also at the cost of the another objective. The last few rows of Table 11 are to investigate the effect of constant swarm size vs. varied maximum number of iterations, on the solutions in Case 2 and Case 3. It can be observed that here also the solutions get better while increasing the number of iterations for fixed population size which again verifies the approximate linear relationship between

Table 11. Convergence of the algorithm (multi-objective optimization)

Parameters		**Case 2**		**Case 3**			
N_p	$\text{iter}_{\max}$	**$/hr**	**MW**	**MW**	**Ton/hr**	**$/hr**	**p.u.**
10	50	818.0762	8.4635	5.4815	0.2262	855.9636	0.2675
20	50	803.8942	8.9928	6.0441	0.2365	836.7591	0.1853
30	50	802.6221	8.7440	5.0298	0.2233	852.6422	0.2907
40	50	802.0788	8.9661	5.0355	0.2238	849.4924	0.2977
50	50	801.8248	9.0863	4.9657	0.2233	851.2627	0.3258
20	30	813.3160	8.4784	5.4035	0.2332	842.4860	0.3210
20	40	804.4891	9.5069	5.0605	0.2178	855.6477	0.1739
20	60	801.7543	9.0682	5.4298	0.2281	848.8490	0.3118
20	70	803.4524	8.2316	4.9298	0.2210	852.7502	0.3181
20	80	801.3172	8.8707	5.0192	0.2196	853.8948	0.2148

the population size and the number of iterations. It should be kept in mind that the quality of a particular solution is also dependent on the algorithm's control adjustments.

The improvement in one objective function in MO is the result of degradation of another objective function. Based on the analysis and results obtained via application of the proposed algorithm it is evident that the weights aggregation method for multi-objective particle swarm optimization is well capable of solving optimization problems with more than one objective to be optimized. It is an alternate method than Pareto dominance based methods, that is robust and its implementation is also straightforward. The global and personal best particles can be easily updated by comparing the current function evaluations to the previous ones. The external archive is also not needed and fuzzy MO mechanism at the end is an efficient method for obtaining the best trade-off particle. Furthermore, the proposed linearly altering the lower and upper bounds on velocity vector is a technique to enhance the algorithm's search ability. It helps the algorithm to search the feasible zone for better results. The smaller and smaller jumps from points to points in the multidimensional search area towards the end of the algorithm, is an efficient technique to search the global best position with improved function values.

6. **Conclusions.** The main goal achieved by the proposed weights aggregated multi-objective particle swarm optimization (WA-MOPSO) is to get the advised set of points in the power system's operation and control that satisfies the power system security and operational constraints, in addition satisfying the environmental and economic conditions simultaneously. It is observed in the literature review of the OPF problem that the weights aggregation method has not been applied extensively on the multi-objective problems and its efficiency has not been widely investigated for more than two objectives, so in this research work it is applied to minimization of four objective functions simultaneously, i.e., power generation cost, thermal power plants' emissions, load buses volt deviation from pre-specified value and transmission lines' active power loss. The major strength of this method is its robustness with proper control adjustments and straightforward implementation like the single objective optimization with little modifications; however, the disadvantage of WAM is that the algorithm needs to be run multiple times for desired number of non-dominated solutions. The proposed technique of time varied upper and lower bounds on velocity vector is an efficient method for improving the PSO search ability in multidimensional search zone. It helps the algorithm to search the global solution with more refined results. Furthermore, the fuzzy logic method provides an alternative way to choose the best trade-off solution instead of choosing the best particle manually among non-dominated solutions set that is obtained via applying the proposed algorithm repeatedly with combination of different weight settings. For both single objective and multi-objective optimization cases, the proposed method is shown as feasible to get the better function values while comparing to other algorithms in literature. Due to its simplicity, robustness with proper adjustments and straightforward implementation, this method is suggested for solving the single as well as multi-objective optimization problems even for more than four objectives regarding the solution of standard static optimal power flow problem.

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