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FAULT DETECTION FILTER DESIGN FOR DISCRETE-TIME SWITCHED TIME-DELAY SYSTEMS WITH STATIC QUANTIZER AND QUANTIZED FEEDBACK

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Abstract. *This study solves the problem of the fault detection filter design for switched time-delay systems with quantization effects. We introduce the static quantizer, and the quantizer error is concerned about in this study. Subsequently, we design fault detection filters as residual generators to make exponential stability of augmented switched systems. Then, for the disturbances and the faults input, the estimation error between the faults and residuals is minimized. The fault detection (FD) problem is translated into the problem of H_∞ filtering. Lyapunov-Krasovskii function is constructed with average dwell-time switching rules, sufficient conditions are established for FD filter based on linear matrix inequality (LMI), and the filter gains are obtained by solving the problem of convex optimization. Finally, we give an example to demonstrate the theoretical developments by numerical simulations.*

Keywords: Fault detection, Time-delay, Switched systems, Static quantizer

1. **Introduction.** With the development of technology, control problems encountered in modern industrial process are becoming more and more complex [1]. As a special type of hybrid systems, switched systems have a lot of engineering applications, such as aircraft control system [2], robot control system [3] and electric power system [4]. A typical switched system is a control system consisting of a set of continuous (or discrete) time subsystems and a switching rule that decides how to switch between subsystems [5-7].

The system with time-delay refers to the current trend of development obviously depends on the state of past history [8]. At present, time domain method is mainly used for delay dependent problems in the world. The time domain method of time-delay systems stability analysis is based on Lyapunov-Krasovskii function and LMIs [9].

There is a time delay which is in a subsystem model or switched signal called a switched time-delay system. It has a strong practical background, such as power systems and network control systems [10]. At present, the main research methods of stability are single Lyapunov function method, multiple Lyapunov functions and ADT approach [11].

In the network control system, the signal is transmitted through the network. Therefore, there must be quantization processors to convert analog signals and digital signals into one another. At present, the research on quantitative control theory mainly focuses on two aspects: static quantizer and dynamic quantizer. Static quantizer mainly includes two forms: uniform quantizer and logarithmic quantizer [12]. The advantage of static quantizer is that it is simple in structure and easy to be maintained in practice. Because the logarithm quantizer saves the bandwidth compared with the uniform quantizer, most

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scholars use a logarithmic quantizer to quantify the signal [13]. In view of the above advantages of the logarithm quantizer, the logarithm quantizer is used in this paper.

On the other hand, FD technique has been the international attention of the automatic control community. There are three main types of FD approaches: the model-based method, the signal-based method and the knowledge-based method [14]. The model-based fault detection method has become more important in the last decades [15]. Such methods of FD are generally in two steps: in order to generate residual signals, an observer or a filter is constructed; tracking the residual signal for realizing the FD. Along with the development of robust control technology, the approach of FD using H_∞ optimization method is attracting more and more attention.

However, it should be pointed out that the FD of the current switched time-delay systems with quantizer is still in its initial stage [16, 17].

It is noted that the addition of the quantizer will introduce the quantization error. Hence, new methods are required to deal with the problem of quantizer error. The traditional modeling method, which does not consider the time-delay and quantization condition, is no longer suitable for network transmission requirements. Therefore, the research of FD problem for switched time-delay systems with quantizer is of great theoretical and practical significance.

In view of the above problems, this paper studies the fault detection filter design problem for switched time-delay systems with quantization effects [18]. First, a static logarithmic quantizer is designed. Subsequently, a robust FD filter is designed which takes static quantizer error into account. Meanwhile, LMIs are established to satisfy the sufficient conditions of a certain performance, and characterize the filter gains by solving a convex linear matrix. The main contributions of this paper are as follows. The proposed FD filter design considers the requirements of network transmission, the systems output is quantized, the switched time-delay is considered, and a method to overcome the quantization error is proposed.

This paper is structured in the following order. In Section 2, the problem of the FD switched time-delay systems with quantization effects and the design objectives is introduced. Sufficient conditions are illustrated for tracking system performances and details of the FD filter design method are presented in Section 3. Section 4 presents an example to illustrate the given approach. Section 5 provides the conclusions of this study.

2. Problem Formulation.

2.1. Problem statement. Consider discrete-time switched time-delay systems as follows

$$\begin{aligned} x(k+1) &= \sum_{j=1}^{N} \zeta_j(k)(A_j x(k) + A_{dj} x(k-d) + B_j f(k) + E_j w(k)) \\ y(k) &= \sum_{j=1}^{N} \zeta_j(k)(C_j x(k) + C_{dj} x(k-d) + D_j f(k) + F_j w(k)) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $y(k) \in R^m$ is the measured output, and $f(k) \in R^{n_d}$ and $w(k) \in R^{n_d}$ are the fault vector and disturbance input respectively, which belongs to $l_2[0\ \infty)$. The positive integer d and N stand for the state delay and the number of subsystems respectively. $\zeta_j(k)$ is switching signal which represents the j-th subsystem when $\zeta_j(k) = 1$. $\zeta_j(k)$: $k \in Z^+$, $j \in \mathcal{L} = \{1, \ldots, N\}$, $Z^+ \to \{0, 1\}$ and $\sum_{j=1}^{N} \zeta_j(k) = 1$. The matrices A_j, A_{dj}, B_j, E_j, C_j, C_{dj}, D_j and F_j are of appropriate dimensions for each subsystem, and switch with μ at the switching instant.

The following logarithmic quantizer is used for quantization [18]: when the quantizer input is a scalar, the set of the logarithmic quantizer $q(\cdot)$ quantization series has the following description: $U = \{\pm u(i) : u(i) = \rho^i u(0), i = 0, \pm 1, \pm 2, \dots\}$, $u(0) > 0$, $0 < \rho < 1$ and mapping relationship $q(\cdot)$ satisfies

$$q(y) = \begin{cases} u(i) & \dfrac{1}{1+\delta}u(i) < y \leq \dfrac{1}{1-\delta}u(i) \\ 0 & y = 0 \\ -q(-y) & y < 0 \end{cases}$$

where y is the input of quantizer, u is the output of quantizer, δ is the parameter, and ρ is the quantization density, and satisfies $\delta = \frac{1-\rho}{1+\rho}$.

Then, the output $y(k)$ with quantizer can be written as $y_q(k) = q(y(k))$, where the quantizer error is $e(k) = y_q(k) - y(k)$.

Remark 2.1. *In the network control system, we need to pay attention to data transmission in the limited bandwidth of communication channel intervention. Therefore, the system needs to design the quantizer for signal transmission. There are two types: static quantizer and dynamic quantizer* [19, 20]. *In this paper, the authors use the logarithmic quantizer of static quantizer.*

The design of FD filters is as follows

$$\begin{aligned} x_f(k+1) &= \sum_{j=1}^{N} \zeta_j(k)(A_{fj}x_f(k) + B_{fj}y_q(k)) \\ r(k) &= \sum_{j=1}^{N} \zeta_j(k)(C_{fj}x_f(k) + D_{fj}y_q(k)) \end{aligned} \tag{2}$$

where $x_f(k)$ is the filters' state, $r(k)$ is residual signal for switched system (1) and the matrices A_{fj}, B_{fj}, C_{fj}, and D_{fj} are appropriate dimensions' filter parameters which are to be determined.

To detect, there is no need to estimation fault $f(k)$ directly. The fault signals of a certainly frequency band are more useful for the study; therefore, the fault signals can be weighed. The weighted fault denotes $\hat{f}(k) = W_f(z)f(k)$ with a presented weighted matrix $W_f(z)$. The minimal realization of $W_f(z)$ is supposed to be

$$\begin{aligned} x_w(k+1) &= \sum_{j=1}^{N} \zeta_j(k)(A_{wj}x_w(k) + B_{wj}f(k)) \\ \hat{f}(k) &= \sum_{j=1}^{N} \zeta_j(k)(C_{wj}x_w(k) + D_{wj}f(k)) \end{aligned} \tag{3}$$

where $x_w(k)$ is the weighted fault's state, $f(k)$ and $\hat{f}(k)$ are the original fault and the weighted fault, respectively. Matrices A_{wj}, B_{wj}, C_{wj} and D_{wj} ($j \in \mathcal{L}$) are known.

Remark 2.2. *By introducing the $W_f(z)$, it limits the frequency range of the fault signal, but it can improve the performance of the system and capture the frequency domain characteristics that reflect different frequency characteristics. Moreover, $\hat{f}(k)$ can get the appropriate dimension by choosing $W_f(z)$.*

Denoting the augmented state vector $\tilde{x}(k) = \left[x^T(k),\ x_{fj}^T(k),\ x_w^T(k)\right]^T$, $\tilde{x}(k-d) = \left[x^T(k-d),\ 0,\ 0\right]^T$, $\tilde{w}(k) = \left[w^T(k),\ e^T(k),\ f^T(k)\right]^T$ and $r_e(k) = r(k) - \hat{f}(k)$, where

$e(k) = y_q(k) - y(k)$, the augmented switched systems are obtained as follows

$$\begin{aligned} \tilde{x}(k+1) &= \sum_{j=1}^{N} \zeta_j(k) \left(\tilde{A}_j \tilde{x}(k) + \tilde{A}_{dj} \tilde{x}(k-d) + \tilde{B}_j \tilde{w}(k) \right) \\ r_e(k) &= \sum_{i=1}^{N} \zeta_j(k) \left(\tilde{C}_j \tilde{x}(k) + \tilde{C}_{dj} \tilde{x}(k-d) + \tilde{D}_j \tilde{w}(k) \right), \ j \in \mathcal{L} \end{aligned} \tag{4}$$

where

$$\left[\begin{array}{c|c|c} \tilde{A}_j & \tilde{A}_{dj} & \tilde{B}_j \\ \hline \tilde{C}_j & \tilde{C}_{dj} & \tilde{D}_j \end{array}\right] = \left[\begin{array}{ccc|ccc|ccc} A_j & 0 & 0 & A_{dj} & 0 & 0 & E_j & 0 & B_j \\ B_{fj}C_j & A_{fj} & 0 & B_{fj}C_{dj} & 0 & 0 & B_{fj}F_j & B_{fj} & B_{fj}D_j \\ 0 & 0 & A_{wj} & 0 & 0 & 0 & 0 & 0 & B_{wj} \\ \hline D_{fj}C_j & C_{fj} & -C_{wj} & D_{fj}C_{dj} & 0 & 0 & D_{fj}F_j & D_{fj} & D_{fj}D_j - D_{wj} \end{array}\right]$$

Remark 2.3. *It should be pointed out that the estimation of weighted fault $\hat{f}(k)$ is provided by the residual signal $r(k)$. Since the weighted matrix $W_f(z)$ is given, the weighted fault $\hat{f}(k)$ changes when system fault $f(k)$ happens. According to $r_e(k) = r(k) - \hat{f}(k)$, the residual signal $r(k)$ tracks the weighted fault signal $\hat{f}(k)$. Therefore, measuring $r(k)$ can achieve the purpose of detecting the fault $f(k)$. That is, we only need to focus on the relationship between $r_e(k)$ and $\hat{f}(k)$.*

The following task is the design of the FD filter: for a given system (1), a switching signal satisfying the average dwell time is designed; meanwhile, the FD filters are designed to make augmented system (4) satisfy exponential stability and minimize the influence of interference $\tilde{w}(k)$ on the error $r_e(k)$. The interference signal includes input disturbance $w(k)$ and quantizer error $e(k)$. Under zero-initial condition, let the scales $\gamma > 0$ and $0 < \alpha < 1$, the performance weighted gain γ's infimum is made small in the feasibility of

$$\sum_{k=0}^{\infty} e^{-\alpha k} r_e^T(k) r_e(k) \le \gamma^2 \sum_{k=0}^{\infty} \tilde{w}^T(k) \tilde{w}(k) \tag{5}$$

Remark 2.4. *In a network control system, the signal needs to be transmitted over the network. Therefore, the signal needs to be quantized before transmission. According to augmented system (4), the FD filter design becomes an H_∞ filter problem satisfying the performance index (5). Afterward, it minimizes the influence of interference $\tilde{w}(k)$ and makes the residual signal $r(k)$ sensitive to the fault.*

Since designing A_{fj}, B_{fj}, C_{fj}, D_{fj} of the FD filters, evaluating the generated residual is the next objective. In this study, we choose the residual evaluation function $J_r(k)$ as follows

$$J_r(k) = \sqrt{\frac{1}{k} \sum_{s=1}^{k} r^T(s) r(s)},$$

where k represents the evaluation time step. Then, we choose the threshold as follows

$$J_{th}(k) = \sup_{\substack{w(k) \in l_2[0,\infty) \\ f(k)=0, j \in \mathcal{L}}} J_r(k)$$

Remark 2.5. *The threshold J_{th} is the residual evaluation function $J_r(k)$'s supremum with the disturbance input belonging to $l_2[0,\infty)$ and the fault free. In practice, the J_{th} is the estimation of the $J_r(k)$ without the fault.*

As a result, we can detect faults on the basis of the logical rules as follows

$$\begin{cases} \|J_r(k)\| \leq J_{th}, & \text{no fault} \\ \|J_r(k)\| > J_{th}, & \text{fault detected with alarm} \end{cases}$$

2.2. **Preliminaries.** At the end of this section, some known lemmas and definitions are introduced.

Definition 2.1. *For any $t_0 < t_s < t_v$, denote $N_{\sigma(t)}(t_s, t_v)$ as the switching number during the time period (t_s, t_v). If $N_{\sigma(t)}(t_s, t_v) \leq N_0 + (t_v - t_s)/\tau_a$, for $\tau_a > 0$, $N_0 \geq 0$, τ_a goes by the name of average dwell time (ADT).*

Definition 2.2. *For there exist constants $a > 0$ and $b > 0$, $\sigma(t)$ is the switching signal. If the solution of the system satisfies $\|x(k)\| \leq a\|x(0)\|_c e^{-bk}$, $\forall k \in \mathcal{L}$, where $\|x(0)\|_c = sup_{-h\leq\theta\leq 0}\|x(\theta)\|$, system (4) satisfies globally uniformly exponentially stable (GUES).*

Lemma 2.1. [21]. *Assuming that a symmetric matrix $\Theta \in R^{n\times n}$, $\mathcal{M}$ and $\mathcal{H}$ of column dimension n are given matrices, there are necessary and sufficient conditions for matrix $\mathcal{F}$ to establish inequality $\Theta + \mathcal{M}^T\mathcal{F}\mathcal{H} + \mathcal{H}^T\mathcal{F}^T\mathcal{M} < 0$ which holds: $\mathcal{N}_{\mathcal{M}}^T\Theta\mathcal{N}_{\mathcal{M}} < 0$, $\mathcal{N}_{\mathcal{H}}\Theta\mathcal{N}_{\mathcal{H}}^T < 0$, where $\mathcal{N}_{\mathcal{M}}$ and $\mathcal{N}_{\mathcal{H}}$ represent arbitrarily bases of null space of $\mathcal{M}$ and $\mathcal{H}$, respectively.*

3. **The Fault Detection Filter Design.** The authors will investigate the aforementioned FD problems in this section. Firstly, following lemma is given, and the expected inequality conditions are constructed.

3.1. **Conditions for FD filter with quantization effects.**

Lemma 3.1. *Considering augmented system (4), let $\gamma > 0$, $0 < \alpha < 1$ and $\mu \geq 1$. If the Lyapunov-Krasovskii function $V_{\sigma(k)}\big(\tilde{x}(k)\big)$ exists, the following inequalities are satisfied:*

$$V_{\sigma(k)}\big(\tilde{x}(k)\big) \leq e^{-\alpha}V_{\sigma(k)}\big(\tilde{x}(k-1)\big) - \Gamma(k-1) \tag{6}$$

$$V_{\sigma(k)}\big(\tilde{x}(k)\big) \leq \mu V_{\sigma(k-1)}\big(\tilde{x}(k)\big) \tag{7}$$

where $\Gamma(k) \triangleq r_e^T(k)r_e(k) - \gamma^2\tilde{w}^T(k)\tilde{w}(k)$.

For any switching signal with the ADT

$$\tau_a \geq \tau_a^* = ceil\left[\frac{\ln\mu}{\alpha}\right] \tag{8}$$

where the function ceil(ν) represents rounding real number ν to the nearest integer greater than or equal to ν.

Then augmented system (4) is GUES satisfying weighted l_2 performance with gain γ.

Moreover, the state decay can be given by

$$\|x(k)\| \leq \sqrt{\frac{b}{a}}e^{-0.5\left(\alpha-\frac{\ln\mu}{\tau_a}\right)k}\|x(0)\| \tag{9}$$

where $a = \lambda_{\min}\big(P_{\sigma(k_q)}\big)$, $b = \lambda_{\max}\left(P_{\sigma(k_q)}\right) + \lambda_{\max}\big(Q_{\sigma(k_q)}\big)$, $\forall\sigma(k_q) \in \mathcal{L}$.

Proof: Firstly, we consider the stability of system (4) when $\tilde{w}(k) = 0$. The following Lyapunov-Krasovskii function for the jth ($\sigma(k_q) = j$) subsystem of system (4) is chosen:

$$V_j(\tilde{x}(k)) = \tilde{x}^T(k)P_j\tilde{x}(k) + \sum_{s=k-d}^{k-1} e^{\alpha(s-k+1)}\tilde{x}^T(s)Q_j\tilde{x}(s) \tag{10}$$

Along trajectory of the augmented switched system (4), we have

$$\begin{aligned}
&V_j(k+1) - e^{-\alpha}V_j(k) \\
=\ & \tilde{x}^T(k+1)P_j\tilde{x}(k+1) + \sum_{s=k-d+1}^{k} e^{\alpha(s-k)}\tilde{x}^T(s)Q_j\tilde{x}(s) \\
& - \left(e^{-\alpha}\tilde{x}^T(k)P_j\tilde{x}(k) + e^{-\alpha}\sum_{s=k-d}^{k-1} e^{\alpha(s-k+1)}\tilde{x}^T(s)Q_j\tilde{x}(s) \right) \\
=\ & \tilde{x}^T(k+1)P_j\tilde{x}(k+1) - e^{-\alpha}\tilde{x}^T(k)P_j\tilde{x}(k) + \tilde{x}^T(k)Q_j\tilde{x}(k) \\
& - e^{-\alpha d}\tilde{x}^T(k-d)Q_j\tilde{x}(k-d) \\
=\ & \begin{bmatrix} \tilde{x}(k) \\ \tilde{x}(k-d) \end{bmatrix}^T \Pi_j \begin{bmatrix} \tilde{x}(k) \\ \tilde{x}(k-d) \end{bmatrix}
\end{aligned} \tag{11}$$

where

$$\Pi_j = \begin{bmatrix} \tilde{A}_j^T P_j \tilde{A}_j - e^{-\alpha}P_j + Q_j & \tilde{A}_j^T P_j \tilde{A}_{dj} \\ * & \tilde{A}_{dj}^T P_j \tilde{A}_{dj} - e^{-\alpha d}Q_j \end{bmatrix}.$$

Hence, if $\Pi_j \leq 0$ is established, it follows from (11) that

$$V_j(\tilde{x}(k+1)) \leq e^{-\alpha}V_j(\tilde{x}(k)) \tag{12}$$

By iterating over (12), we get, for $k \in [k_q, k_{q+1})$

$$V_j\big(\tilde{x}(k)\big) \leq e^{-\alpha\left(k-k_q\right)}V_j\big(\tilde{x}(k_q)\big) \tag{13}$$

For any given integer $k > 0$, we let $0 = k_0 < k_1 < \cdots < k_q = k_{N_{\sigma(0,k)}}$ denote the switching instants of $\sigma(k_q)$ over the interval $[0, k)$. According to (7) and (10), we can easily obtain

$$V_j\left(\tilde{x}(k_q)\right) \leq \mu V_j\left(\tilde{x}\big(k_q^-\big)\right) \tag{14}$$

Combining (13) and (14), then one can derive that

$$\begin{aligned}
V_{\sigma(k_q)}\left(\tilde{x}(k)\right) &\leq e^{-\alpha(k-k_q)}V_{\sigma(k_q)}\big(\tilde{x}(k_q)\big) \\
&\leq \mu e^{-\alpha(k-k_q)}V_{\sigma(k_{q-1})}\big(\tilde{x}(k_q^-)\big) \\
&\leq \mu e^{-\alpha(k-k_q)}e^{-\alpha(k_q-k_{q-1})}V_{\sigma(k_{q-1})}\big(\tilde{x}(k_{q-1})\big) \\
&\leq \mu^2 e^{-\alpha(k-k_{q-2})}V_{\sigma(k_{q-2})}\big(\tilde{x}(k_{q-2})\big) \\
&\leq \cdots \\
&\leq \mu^{N_\sigma(k_0,k)}e^{-\alpha k}V_0\big(\tilde{x}(k_0)\big)
\end{aligned} \tag{15}$$

As Definition 2.1 of the ADT $N_\sigma(k_0, k) \leq N_0 + (k - k_0)/\tau_a$ and (8), (15) can be written as $V_{\sigma(k_q)}\left(\tilde{x}(k)\right) \leq e^{-\left(\alpha - \frac{\ln\mu}{\tau_a}\right)k}V_0\big(\tilde{x}(k_0)\big)$.

Denote $a = \lambda_{\min}\big(P_{\sigma(k_q)}\big)$, $b = \lambda_{\max}\big(P_{\sigma(k_q)}\big) + \lambda_{\max}\big(Q_{\sigma(k_q)}\big)$, $\forall\sigma(k_q) \in \mathcal{L}$, it yields that

$$V_{\sigma(k_q)}\big(\tilde{x}(k)\big) \geq a\|\tilde{x}(k)\|^2 \tag{16}$$

$$V_{\sigma(k_q)}\big(\tilde{x}(0)\big) \le b\|\tilde{x}(0)\|^2 \tag{17}$$

Combining (16) and (17), we obtain $\|\tilde{x}(k)\| \le \sqrt{\frac{b}{a}}e^{-0.5\left(\alpha-\frac{\ln\mu}{\tau_a}\right)k}\|\tilde{x}(0)\|$.

Thus, system (4) is GUES satisfying the ADT for any switching signal.

Secondly, we verified that the system satisfies the l_2 performance. The weighted l_2 performance is established for augmented system (4) for zero initial condition $\tilde{x}(k_0)$ and any nonzero $\tilde{w}(k) \in l_2\,[0,\ \infty)$. Let $\Gamma(k) \triangleq r_e^T(k)r_e(k) - \gamma^2\tilde{w}^T(k)\tilde{w}(k)$, and we have

$$V_{\sigma(k_q)}(\tilde{x}(k)) \le e^{-\alpha}V_{\sigma(k_q)}(\tilde{x}(k_q-1)) - \Gamma(k-1) \tag{18}$$

By iterating over (18), we obtain

$$\begin{aligned}
V_{\sigma(k_q)}\big(\tilde{x}(k)\big) &\le e^{-\alpha(k-k_q)}V_{\sigma(k_q)}\big(\tilde{x}(k_q)\big) - \sum_{s=k_q}^{k-1} e^{-\alpha(k-s-1)}\Gamma(s) \\
&\le \mu e^{-\alpha(k-k_q)}V_{\sigma(k_{q-1})}\big(\tilde{x}(k_q^-)\big) - \sum_{s=k_q}^{k-1} e^{-\alpha(k-s-1)}\Gamma(s) \\
&\le \cdots \\
&\le \mu^{N_\sigma(k_0,k)}e^{-\alpha k}V_0\big(\tilde{x}(k_0)\big) - \sum_{s=0}^{k-1} \mu^{N_\sigma(s,k)}e^{-\alpha(k-s-1)}\Gamma(s).
\end{aligned}$$

Then, under the zero initial condition and due to the fact that $V(\tilde{x}(k)) \ge 0$, we obtain

$$\sum_{s=0}^{k-1} \mu^{N_\sigma(s,k)}e^{-\alpha(k-s-1)}\Gamma(s) \le 0 \tag{19}$$

Since $\Gamma(k) \triangleq r_e^T(k)r_e(k) - \gamma^2\tilde{w}^T(k)\tilde{w}(k)$, (19) can be written as

$$\sum_{s=k_0}^{k-1}\left(\mu^{N_\sigma(s,k)}e^{-\alpha(k-s-1)}\right)r_e^T(s)r_e(s) \le \sum_{s=k_0}^{k-1}\left(\mu^{N_\sigma(s,k)}e^{-\alpha(k-s-1)}\right)\gamma^2\tilde{w}^T(s)\tilde{w}(s) \tag{20}$$

Multiplying both sides of (20) by $e^{-N_\sigma(k_0,k)\ln\mu}$, one can obtain

$$\sum_{s=k_0}^{k-1}\left(\mu^{-N_\sigma(k_0,s)}e^{-\alpha(k-s-1)}\right)r_e^T(s)r_e(s) \le \sum_{s=k_0}^{k-1}\left(\mu^{-N_\sigma(k_0,s)}e^{-\alpha(k-s-1)}\right)\gamma^2\tilde{w}^T(s)\tilde{w}(s).$$

Moreover, it follows from (8) that $0 \le N_\sigma(k_0,s) \le \frac{s-k_0}{\tau_a} \le \frac{(s-k_0)\alpha}{\ln\mu}$.

From the above and $\mu \ge 1$, then we have

$$\begin{aligned}
\sum_{s=k_0}^{k-1}\left(\mu^{-\frac{(s-k_0)\alpha}{\ln\mu}}e^{-\alpha(k-s-1)}\right)r_e^T(s)r_e(s) &\le \sum_{s=k_0}^{k-1}\left(\mu^{-N_\sigma(k_0,s)}e^{-\alpha(k-s-1)}\right)r_e^T(s)r_e(s) \\
&\le \sum_{s=k_0}^{k-1}\left(e^{-\alpha(k-s-1)}\right)\gamma^2\tilde{w}^T(s)\tilde{w}(s).
\end{aligned}$$

Since $\mu^{-\frac{(s-k_0)\alpha}{\ln\mu}} = e^{-\alpha(s-k_0)}$, then we obtain

$$\sum_{s=k_0}^{k-1} e^{-\alpha(s-k_0)}e^{-\alpha(k-s-1)}r_e^T(s)r_e(s) \le \sum_{s=k_0}^{k-1} e^{-\alpha(k-s-1)}\gamma^2\tilde{w}^T(s)\tilde{w}(s) \tag{21}$$

By integrating both sides of (21) from $k = 1$ to ∞, it arrives at

$$\sum_{k=1}^{\infty}\sum_{s=k_0}^{k-1} e^{-\alpha(s-k_0)}e^{-\alpha(k-s-1)}r_e^T(s)r_e(s) \le \sum_{k=1}^{\infty}\sum_{s=k_0}^{k-1} e^{-\alpha(k-s-1)}\gamma^2\tilde{w}^T(s)\tilde{w}(s).$$

Redistrict double integral interval, and we have

$$\sum_{k=s+1}^{\infty}\sum_{s=k_0}^{\infty} e^{-\alpha(s-k_0)}e^{-\alpha(k-s-1)}r_e^T(s)r_e(s) \le \sum_{k=s+1}^{\infty}\sum_{s=k_0}^{\infty} e^{-\alpha(k-s-1)}\gamma^2\tilde{w}^T(s)\tilde{w}(s) \tag{22}$$

Translate double integral into repeated integral in (22), and then we obtain

$$\sum_{k=s+1}^{\infty} e^{-\alpha(k-s-1)}\sum_{s=k_0}^{\infty} e^{-\alpha(s-k_0)}r_e^T(s)r_e(s) \le \sum_{k=s+1}^{\infty} e^{-\alpha(k-s-1)}\sum_{s=k_0}^{\infty}\gamma^2\tilde{w}^T(s)\tilde{w}(s) \tag{23}$$

Since $\sum_{k=s+1}^{\infty} e^{-\alpha(k-s-1)} = \frac{1}{1-e^{-\alpha}}$, (23) can become

$$\frac{1}{1-e^{-\alpha}}\sum_{s=k_0}^{\infty} e^{-\alpha(s-k_0)}r_e^T(s)r_e(s) \le \frac{1}{1-e^{-\alpha}}\sum_{s=k_0}^{\infty}\gamma^2\tilde{w}^T(s)\tilde{w}(s).$$

Since $0 < \alpha < 1$, we have $\sum_{s=k_0}^{\infty} e^{-\alpha(s-k_0)}r_e^T(s)r_e(s) \le \sum_{s=k_0}^{\infty}\gamma^2\tilde{w}^T(s)\tilde{w}(s)$.

It illustrates further that $\sum_{k=0}^{\infty} e^{-\alpha k}r_e^T(k)r_e(k) \le \sum_{k=0}^{\infty}\gamma^2\tilde{w}^T(k)\tilde{w}(k)$. Therefore, we have the conclusion that augmented switched system (4) is GUES and satisfies the l_2 gain γ with ADT satisfying (8) for any switching signal. □

Remark 3.1. *In Lemma 3.1, we are not concerned about the condition when the delay time includes switching points, that is $k - k_q < d$, where the state delay d is a positive integer. For the jth ($\sigma(k_q) = j$) subsystem of the system, we have Lyapunov-Krasovskii function: $V_j(\tilde{x}(k)) = \tilde{x}^T(k)P_j\tilde{x}(k) + \sum_{s=k-d}^{k-1} e^{\alpha(s-k+1)}\tilde{x}^T(s)Q_j\tilde{x}(s)$. When $k - k_q < d$, we obtain*

$$\begin{aligned}&V_j(k+1) - e^{-\alpha}V_j(k)\\ &= \begin{bmatrix}\tilde{x}(k)\\ \tilde{x}(k-d)\end{bmatrix}^T \begin{bmatrix}\tilde{A}_j^TP_j\tilde{A}_j - e^{-\alpha}P_j + Q_j & \tilde{A}_j^TP_j\tilde{A}_{dj}\\ * & \tilde{A}_{dj}^TP_j\tilde{A}_{dj} - e^{-\alpha d}Q_j\end{bmatrix}\begin{bmatrix}\tilde{x}(k)\\ \tilde{x}(k-d)\end{bmatrix}\end{aligned}$$

That is, Equation (11) remains unchanged. Therefore, the condition we are considering is contained in Lemma 3.1.

Theorem 3.1. *Let constants $\gamma > 0$, $0 < \alpha < 1$, $\mu > 1$. If there exist matrix variables $\hat{A}_{fj}$, $\hat{B}_{fj}$, $\hat{C}_{fj}$, $\hat{D}_{fj}$, Y_{aj}, M_{aj}, N_j and symmetric positive-definite matrices*

$$P_j = \begin{bmatrix}P_{j1} & P_{j2} & P_{j3}\\ * & P_{j5} & P_{j6}\\ * & * & P_{j9}\end{bmatrix} > 0,\ Q_j = \begin{bmatrix}Q_{j1} & Q_{j2} & Q_{j3}\\ * & Q_{j5} & Q_{j6}\\ * & * & Q_{j9}\end{bmatrix} > 0,\ j \in \mathcal{L},$$

satisfying the following inequalities

$$\begin{bmatrix}\Xi_{j1} & \Xi_{j2} & \Xi_{j3} & \Xi_{j4} & \Xi_{j5} & 0\\ * & P_{j9} - He(Z) & \Xi_{j6} & 0 & \Xi_{j7} & 0\\ * & * & -e^{-\alpha}P_j + Q_j & 0 & 0 & \Xi_{j8}\\ * & * & * & -e^{-\alpha d}Q_j & 0 & \Xi_{j9}\\ * & * & * & * & -\gamma^2 I & \Xi_{j10}\\ * & * & * & * & * & -I\end{bmatrix} < 0 \tag{24}$$

$$P_j - \mu P_i \leq 0,\ Q_j - \mu Q_i \leq 0 \qquad j,\ i \in N,\ i \neq j \tag{25}$$

where

$$\Xi_{j1} = \begin{bmatrix} P_{j1} - He(Y_j) & P_{j2} - M_j - N_j^T \\ * & P_{j5} - aHe(N_j) \end{bmatrix},\ \Xi_{j2} = \begin{bmatrix} P_{j3} \\ P_{j6} \end{bmatrix},$$

$$\Xi_{j3} = \begin{bmatrix} Y_j^T A_j + \hat{B}_{fj} C_j & \hat{A}_{fj} & 0 \\ M_j^T A_j + a\hat{B}_{fj} C_j & a\hat{A}_{fj} & 0 \end{bmatrix},\ \Xi_{j4} = \begin{bmatrix} Y_j^T A_{dj} + \hat{B}_{fj} C_{dj} & 0 & 0 \\ M_j^T A_{dj} + a\hat{B}_{fj} C_{dj} & 0 & 0 \end{bmatrix},$$

$$\Xi_{j5} = \begin{bmatrix} Y_j^T E_j + \hat{B}_{fj} F_j & \hat{B}_{fj} & Y_j^T B_j + \hat{B}_{fj} D_j \\ M_j^T E_j + \hat{B}_{fj} F_j & a\hat{B}_{fj} & M_j^T B_j + \hat{B}_{fj} D_j \end{bmatrix},\ \Xi_{j6} = \begin{bmatrix} 0 & 0 & Z_j^T A_{wj} \end{bmatrix},$$

$$\Xi_{j7} = \begin{bmatrix} 0 & 0 & Z_j^T B_{wj} \end{bmatrix},\ \Xi_{j8} = \begin{bmatrix} \hat{D}_{fj} C_j & \hat{C}_{fj} & -C_{wj} \end{bmatrix}^T,$$

$$\Xi_{j9} = \begin{bmatrix} \hat{D}_{fj} C_{dj} & 0 & 0 \end{bmatrix}^T,\ \Xi_{j10} = \begin{bmatrix} \hat{D}_{fj} F_j & \hat{D}_{fj} & \hat{D}_{fj} D_j - D_{wj} \end{bmatrix}^T,$$

then augmented switched system (4) is GUES and satisfies the weighted l_2 performance with ADT satisfying (8) for any switching signal. In addition, if (24) and (25) are workable, then we can give the following FD filter gains in form of (2)

$$\begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} = \begin{bmatrix} N_j^T & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}_{fj} & \hat{B}_{fj} \\ \hat{C}_{fj} & \hat{D}_{fj} \end{bmatrix}.$$

Proof: Along the trajectory of switched system (4) and according to Lemma 3.1, we obtain

$$\begin{aligned} & V_j(k+1) - e^{-\alpha} V_j(k) - \gamma^2 \tilde{w}(k)^T \tilde{w}(k) + r_e(k)^T r_e(k) \\ &= \begin{bmatrix} \tilde{x}(k) \\ \tilde{x}(k-d) \\ \tilde{w}(k) \end{bmatrix}^T \Theta_j \begin{bmatrix} \tilde{x}(k) \\ \tilde{x}(k-d) \\ \tilde{w}(k) \end{bmatrix}, \end{aligned}$$

where

$$\Theta_j = \begin{bmatrix} \begin{matrix} \tilde{A}_j^T P_j \tilde{A}_j - e^{-\alpha} P_j \\ + Q_j + \tilde{C}_j^T \tilde{C}_j \end{matrix} & \tilde{A}_j^T P_j \tilde{A}_{dj} + \tilde{C}_j^T \tilde{C}_{dj} & \tilde{A}_j^T P_j \tilde{B}_j + \tilde{C}_j^T \tilde{D}_j \\ * & \tilde{A}_{dj}^T P_j \tilde{A}_{dj} - e^{-\alpha d} Q_j + \tilde{C}_{dj}^T \tilde{C}_{dj} & \tilde{A}_{dj}^T P_j \tilde{B}_j + \tilde{C}_{dj}^T \tilde{D}_j \\ * & * & \tilde{B}_j^T P_j \tilde{B}_j - \gamma^2 I + \tilde{D}_j^T \tilde{D}_j \end{bmatrix}$$

and

$$V_j(\tilde{x}(k)) - \mu V_i(\tilde{x}(k)) = \tilde{x}(k)^T (P_j - \mu P_i) \tilde{x}(k) + \sum_{s=k-d}^{k-1} e^{\alpha(s-k+1)} \tilde{x}^T(s) (Q_j - \mu Q_i) \tilde{x}(s),$$

$$j,\ i \in \mathcal{L},\ i \neq j.$$

Hence, if the following inequalities are established

$$\Theta_j < 0 \tag{26}$$

$$P_j \leq \mu P_i,\ Q_j \leq \mu Q_i,\ j,\ i \in \mathcal{L},\ j \neq i,$$

then switched system (4) is GUES with ADT satisfying (8) for any switching signal and satisfies a weighted l_2 performance according to Lemma 3.1.

The convex conditions will be established, and we write (26) as follows

$$\begin{bmatrix} \tilde{A}_j & \tilde{A}_{dj} & \tilde{B}_j \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} P_j & 0 & 0 \\ * & -e^{-\alpha}P_j + Q_j & 0 \\ * & * & -e^{-\alpha d}Q_j \end{bmatrix} \begin{bmatrix} \tilde{A}_j & \tilde{A}_{dj} & \tilde{B}_j \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} \tilde{C}_j^T\tilde{C}_j & \tilde{C}_j^T\tilde{C}_{dj} & \tilde{C}_j^T\tilde{D}_j \\ * & \tilde{C}_{dj}^T\tilde{C}_{dj} & \tilde{C}_{dj}^T\tilde{D}_j \\ * & * & -\gamma^2 I + \tilde{D}_j^T\tilde{D}_j \end{bmatrix} < 0 \tag{27}$$

Denote

$$\Upsilon_j = \begin{bmatrix} P_j & 0 & 0 & 0 \\ * & -e^{-\alpha}P_j + Q_j + \tilde{C}_j^T\tilde{C}_j & \tilde{C}_j^T\tilde{C}_{dj} & \tilde{C}_j^T\tilde{D}_j \\ * & * & -e^{-\alpha d}Q_j + \tilde{C}_{dj}^T\tilde{C}_{dj} & \tilde{C}_{dj}^T\tilde{D}_j \\ * & * & * & -\gamma^2 I + \tilde{D}_j^T\tilde{D}_j \end{bmatrix},$$

and then we can get that (27) can be obtained

$$\begin{bmatrix} \tilde{A}_j & \tilde{A}_{dj} & \tilde{B}_j \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T \Upsilon_j \begin{bmatrix} \tilde{A}_j & \tilde{A}_{dj} & \tilde{B}_j \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \tag{28}$$

On the other hand, it is equivalent to

$$\begin{bmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T \Upsilon_j \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \tag{29}$$

According to Lemma 2.1, (28) can be written as

$$\Upsilon_j + He\left(\begin{bmatrix} -I \\ \tilde{A}_j^T \\ \tilde{A}_{dj}^T \\ \tilde{B}_j^T \end{bmatrix} W_j \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} \right) < 0 \tag{30}$$

where W_j introduced by Lemma 2.1 is an additional appropriate dimensions' matrix variable. The partitioning of W_j in form is given by

$$W_j = \begin{bmatrix} Y_j & M_j & 0 \\ N_j & aN_j & 0 \\ 0 & 0 & Z_j \end{bmatrix}.$$

According to Schur complement, (30) is equivalent to

$$\begin{bmatrix} P_j - He(W_j) & W_j^T\tilde{A}_j & W_j^T\tilde{A}_{dj} & W_j^T\tilde{B}_j & 0 \\ * & -e^{-\alpha}P_j + Q_j & 0 & 0 & \tilde{C}_j^T \\ * & * & -e^{-\alpha d}Q_j & 0 & \tilde{C}_j^T \\ * & * & * & -\gamma^2 I & \tilde{D}_j^T \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{31}$$

Define $\hat{A}_{fj} = N_j A_{fj}$, $\hat{B}_{fj} = N_j B_{fj}$, $\hat{C}_{fj} = C_{fj}$, $\hat{D}_{fj} = D_{fj}$, and then (31) becomes (24). Thus, if inequalities (24) and (25) are established, augmented switched system (4) for any nonzero $w(k) \in l_2[0, \infty)$ is GUES and satisfies a weighted l_2 performance gain γ. Therefore, the proof is completed. □

3.2. **Algorithm.** It should be noted that the conditions (24) and (25) are both convex. Thus, the problem of FD filters design can be converted into the problem of optimization directly as follows:

$$\begin{aligned} &\min \quad \gamma \\ &\text{s.t.} \quad (24),\ (25), \quad j,\ i \in \mathcal{L} \end{aligned} \tag{32}$$

Remark 3.2. *The l_2 gain γ is related to μ and α, and we can obtain $\hat{A}_{fj}$, $\hat{B}_{fj}$, $\hat{C}_{fj}$, $\hat{D}_{fj}$, N_j and calculate the admissible ADT for the system by getting the solution of the optimization problem.*

The filters' gain matrices A_{fj}, B_{fj}, C_{fj}, D_{fj} can be derived from a standard procedure. As follows, the $\hat{A}_{fj}$, $\hat{B}_{fj}$, $\hat{C}_{fj}$, $\hat{D}_{fj}$, N_j are expressed as the optimal solution of (32):

$$A_{fj} = N_j^{-1}\hat{A}_{fj},\ B_{fj} = N_j^{-1}\hat{B}_{fj},\ C_{fj} = \hat{C}_{fj},\ D_{fj} = \hat{D}_{fj} \tag{33}$$

4. **Example.** Two examples are given in this section. Example 4.1 is given to illustrate the feasibility of the design approach, and Example 4.2 is given for comparison to show the advantage of this work.

Example 4.1. *We give an example to illustrate the feasibility of the design approach in Example 4.1. Switched system (1) with two subsystems is considered, and subsystem parameters are given separately. The specific example is referenced from* [22], *which studied the controller design for a switched time-delay systems with quantized feedback. The discrete-time state feedback controller is of the form $u(k) = K_j x(k)$, where K_j is the state feedback gain, $u(k)$ is the control input, and $x(k)$ is the state in* [22].

In order to get stable systems, the parameters are set as $A_j = A_j' + B_j' K_j$, $C_j = C_j' + D_j' K_j$, where A_j' and B_j' are the matrices of state, and C_j' and D_j' are the matrices of control input in [22]. *B_j and D_j are given by authors, which are matrices with appropriate dimensions of the fault vector, and other parameters are the same as* [22].

$$\left[\begin{array}{c|c|c|c} A_1 & A_{d1} & B_1 & E_1 \\ \hline C_1 & C_{d1} & D_1 & F_1 \end{array}\right] = \left[\begin{array}{cc|cc|c|c} 0.36346 & -0.1129 & -0.2 & 0.1 & 1.3 & 0.4 \\ -0.15481 & 0.18065 & 0.2 & 0.15 & 1.6 & 0.5 \\ \hline -0.15481 & -0.11935 & 0.02 & 0 & 1.4 & 0.1 \end{array}\right],$$

$$\left[\begin{array}{c|c|c|c} A_2 & A_{d2} & B_2 & E_2 \\ \hline C_2 & C_{d2} & D_2 & F_2 \end{array}\right] = \left[\begin{array}{cc|cc|c|c} -0.37225 & 0.64933 & -0.06 & 0.04 & 1.5 & 0.2 \\ 0.1185 & 0.06622 & 0.02 & 0.06 & 1.2 & 0.6 \\ \hline 0.0185 & 0.06622 & 0 & 0.06 & 1.5 & 0.2 \end{array}\right].$$

Choose the matrices of fault weighting systems in the form of (2) as follows

$$A_{w1} = A_{w2} = -0.6,\ B_{w1} = B_{w2} = 0.5,\ C_{w1} = C_{w2} = 0.9,\ D_{w1} = D_{w2} = -0.4.$$

Let $\alpha = 0.05$, $\mu = 1.1$, $d = 4$, $a_1 = a_2 = 2$. The optimal performance gain is $\gamma = 0.9754$ by solving the problem of convex optimization. And FD filters' gain matrices are obtained

$$\left[\begin{array}{c|c} A_{f1} & B_{f1} \\ \hline C_{f1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c} 0.0025 & -0.0008 & -0.0002 \\ -0.0008 & 0.0033 & -0.0045 \\ \hline 0.1537 & 0.0902 & -0.5791 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{f2} & B_{f2} \\ \hline C_{f2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c} -0.0019 & 0.0028 & -0.0008 \\ 0.0023 & -0.0014 & -0.0021 \\ \hline -0.0103 & -0.1310 & -0.5712 \end{array}\right].$$

We choose $w_1(k) = w_2(k) = 0.8\sin(0.2\pi k)e^{-0.02k}$ *as the disturbance of each subsystem to demonstrate simulation results of FD purposes. The minimal ADT* τ_a^* *must be satisfying* $\tau_a^* = ceil\left[\frac{\ln\mu}{\alpha}\right] = 2$. *In this paper, we choose the ADT of switching signal* $\tau_a > 2$, *which is shown in Figure 1.*

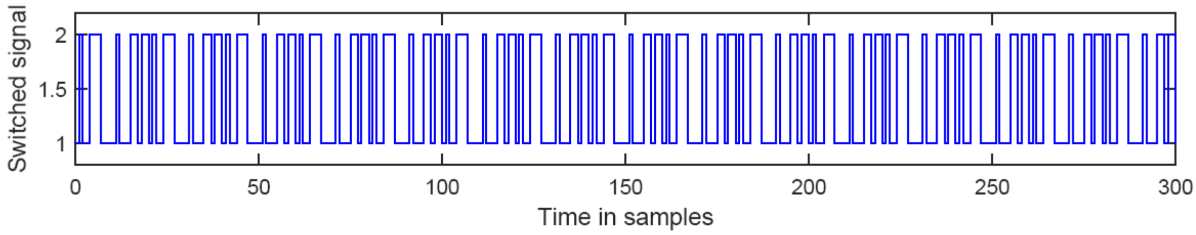

FIGURE 1. Switching signal with ADT $\tau_a > 2$

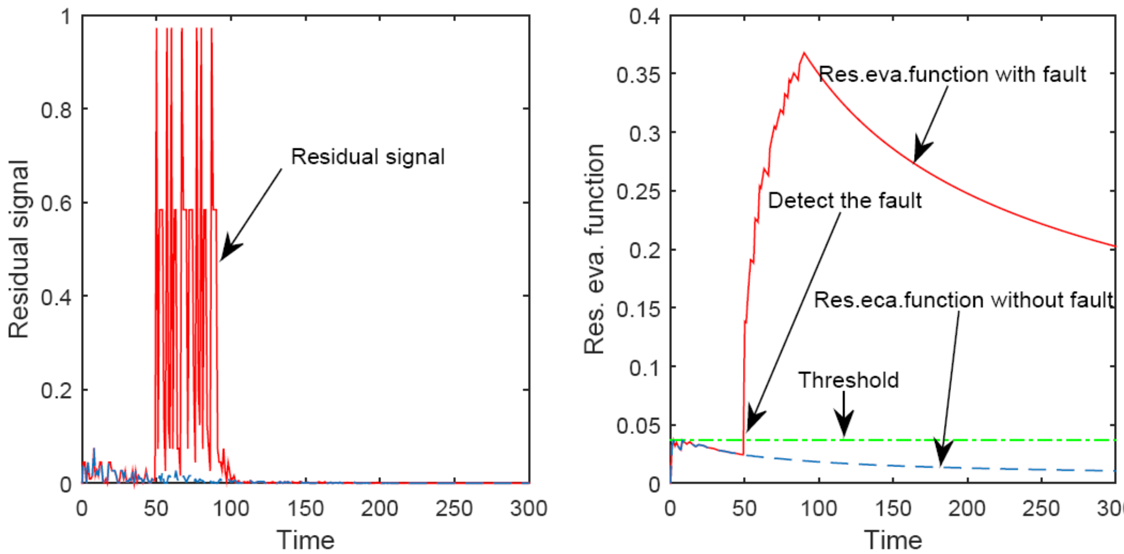

FIGURE 2. Residual signal of the filers $r(k)$ (left) and the residual evaluation function $J_{r(k)}$ (right) for case (1)

Case (1): The fault for the 1st subsystem occurs from 50 steps to 90 steps, and the fault here is a unit amplitude. In the left chart of Figure 2, the dotted line is the residual signal $r(k)$ *with no fault and the solid line is the residual signal* $r(k)$ *with fault. When the fault occurs, the residual signal* $r(k)$ *changes sharply for the residual signal tracks the fault signal. In the right chart of Figure 2, the straight dotted line is the threshold* $J_{th_{r(k)}}$, *the dotted line is the residual evaluation function* $J_r(k)$ *with no fault and the solid line is the residual evaluation function* $J_r(k)$ *with fault. When the residual evaluation function* $J_r(k)$ *is greater than the threshold* $J_{th_{r(k)}}$, *the fault is detected. Through the simulation results, we can see that when the fault of the 1st subsystem occurs at* $k = 50$, $J_r(k) > J_{th_{r(k)}}$ *at* $k = 50$ *steps. Thus, we can detect the fault of the 1st subsystem.*

Case (2): The fault for the 2nd subsystem occurs from 100 steps to 140 steps, and the fault here is a unit amplitude. In the chart of Figure 3, the meaning of curve' form is the

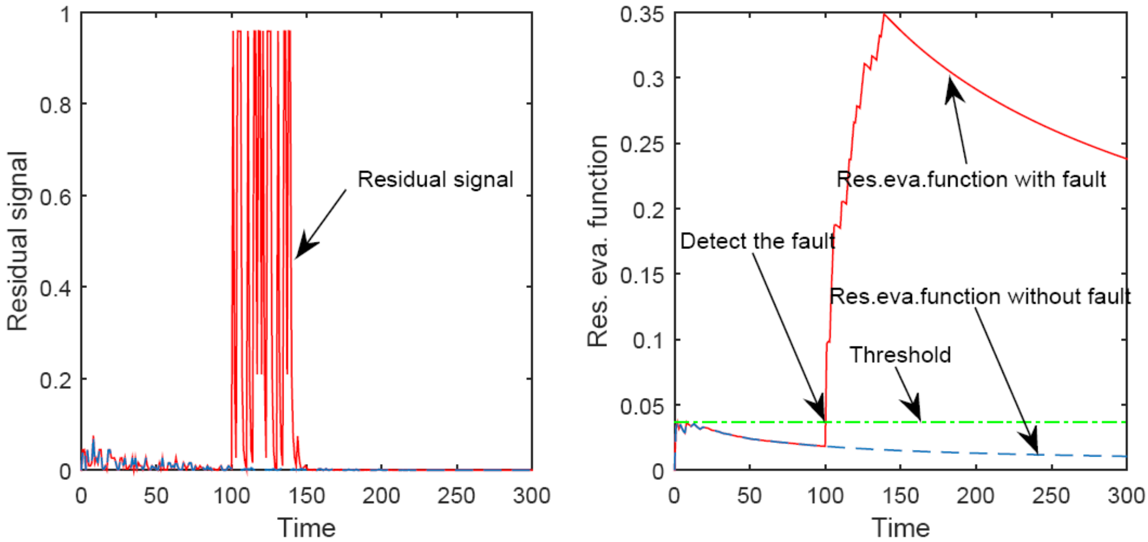

FIGURE 3. Residual signal of the filers $r(k)$ (left) and the residual evaluation function $J_{r(k)}$ (right) for case (2)

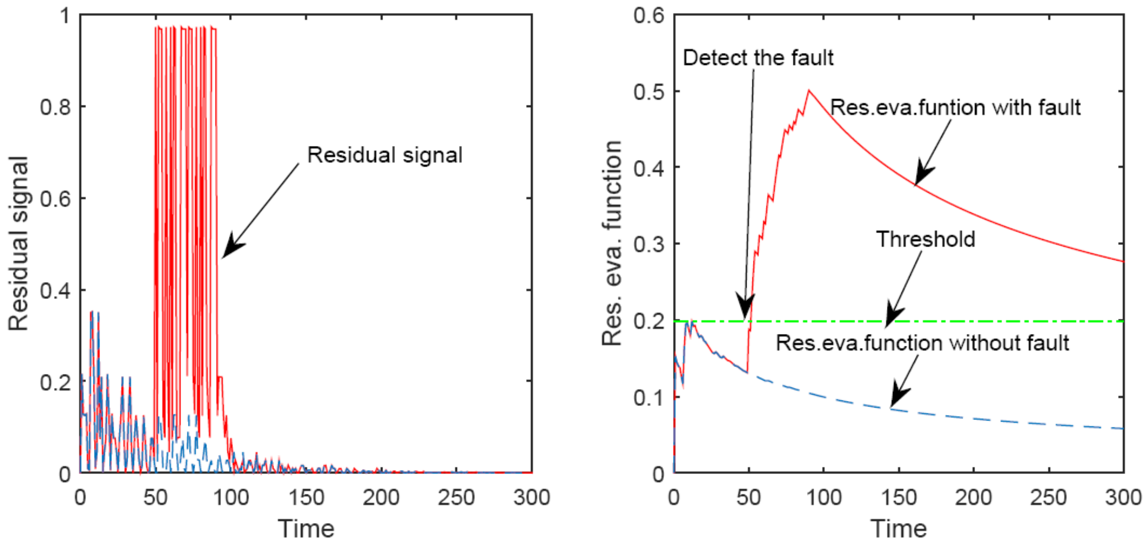

FIGURE 4. Residual signal of the filers $r(k)$ (left) and the residual evaluation function $J_{r(k)}$ (right) in Example 4.2 for case (1)

same as Figure 2. Through the simulation results, when the fault of the 2nd subsystem occurs at $k = 100$*,* $J_r(k) > J_{th_{r(k)}}$ *at* $k = 100$ *steps. Thus, the fault of 2nd subsystem is detected.*

Example 4.2. *Example 4.2 is referenced from* [23]*, which studied robust FD for switched systems with time-delay, but the requirements of network transmission are not considered. Switched system (1) with two subsystems is considered, and subsystem parameters are given in* [23]*.*

In Example 4.2, the disturbance of each subsystem $w_1(k)$*,* $w_2(k)$ *and the ADT parameter setting are the same as Example 4.1. The FD filters are designed.*

The fault for the 1st subsystem occurs from 50 steps to 90 steps, the simulation results are shown in Figure 4. For purposes of comparison, the simulation results which use the matrices of FD filters in [23] *are shown in Figure 5. The fault here is a unit amplitude.*

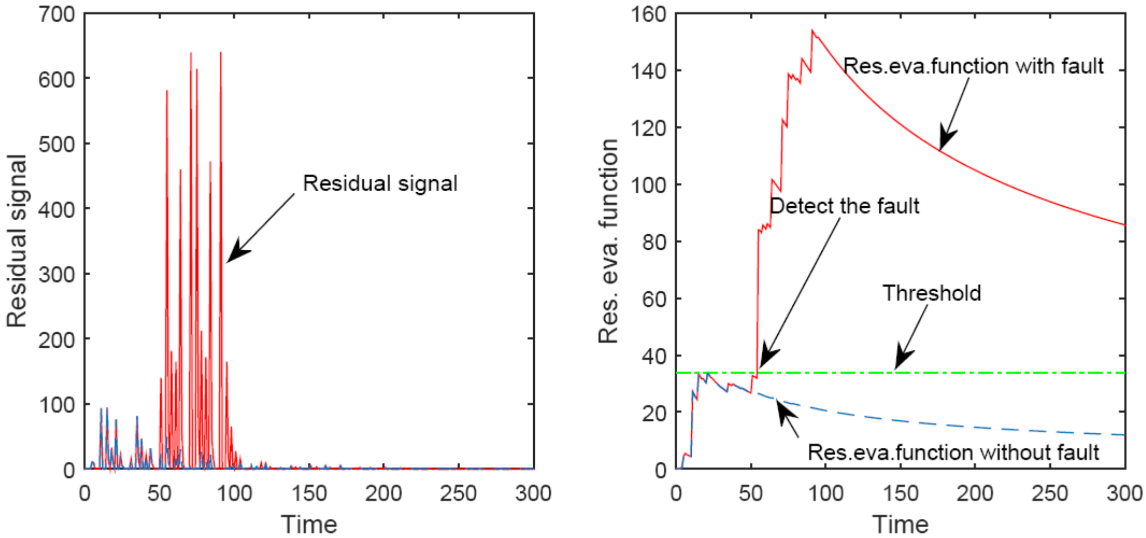

FIGURE 5. Residual signal of the filers $r(k)$ (left) and the residual evaluation function $J_{r(k)}$ (right) in [23] for case (1)

The curves of different forms represent the same meaning as Example 4.1. The simulation results of the 1st are shown in Figure 4, when the fault occurs at $k = 50$, $J_r(k) > J_{th_{r(k)}}$ *at* $k = 51$ *steps. Then, it is easy to draw from Figure 5 that the fault of the 1st subsystem can be detected at* $k = 53$. *By comparing the simulation results of Figure 4 and Figure 5, although* [23] *can also achieve the purpose of FD, it is clear that the speed of FD is slow in Figure 5.*

5. **Conclusions.** The problem of the FD filter design for discrete-time switched time-delay systems with quantization effects has been studied in this paper. First of all, an l_2 performance for the system has been proposed. Subsequently, sufficient conditions have been obtained to character given performance. The FD filters are represented by a formula as LMI conditions, and by searching for solutions of convex optimization problems, we can get the gains of the filters. Finally, an example is given to demonstrate the feasibility of the proposed approach. The future work is to consider the effect of packet dropout on the FD design for switched systems with quantizer.

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