## FAULT DETECTION FOR TIME-DELAYED NETWORKED CONTROL SYSTEMS WITH SENSOR SATURATION AND RANDOMLY OCCURRING FAULTS

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ABSTRACT. In this paper, the fault detection (FD) problem is investigated for networked control systems with random packet losses, stochastic time-delays, sensor saturation as well as randomly occurring faults. A series of random variables is utilized to describe the occurring probability of packet losses, time-delays and faults where all the variables are independent but satisfy the Bernoulli distribution. The measured output is subject to sensor saturation which is described by sector-nonlinearities. Then the mathematical model for networked control systems is established. The aim of this paper is to design an FD filter such that, for unknown input, the FD problem is converted into  $H_{\infty}$  filtering problem and, the error between the residual signal and the fault signal is made as small as possible. By Lyapunov stability theory and linear matrix inequalities (LMIs) method, sufficient conditions for the existence of the desired FD filter are established. Finally, a numerical simulation is presented to verify the effectiveness and usefulness of the designed method.

**Keywords:** Fault detection, Networked control systems, Time-delays, Sensor saturation, Random occurring faults, Linear matrix inequality (LMI)

1. Introduction. With the rapid developments of network technology in recent years, networked control systems (NCSs), which offer many advantages in terms of strong flexibility, easy installation and convenient sharing, have gained more popularity [1-3]. In NCSs, controllers, actuators, sensors and other system components are interconnected through the network. However, the insertion of communication networks will also lead to some problems such as packet losses, signal quantization and network-induced delays which will deteriorate the performance of systems and be a source of instability [4,5]. Therefore, many scholars have devoted themselves to investigating FD problems for NCSs.

Taking the increasing requirement for higher safety and reliability into consideration, fault detection (FD) problem for NCSs has been a hot topic [6-10]. In addition, due to the unpredictable network changes, the random occurring phenomena (ROP) in NCSs usually exist due primarily to the network size, limited battery storage, communication constraints and spatial deployment. ROP refer to those phenomena that appear intermittently in a random way based on a certain probability law [11]. Recently, ROP have been investigated in a wealth of literature. For example, by taking the random packet dropout into account, the FD filter design was investigated for discrete-time system [12]. Multiple randomly occurring nonlinearities were concerned in uncertain time-varying systems based on which the filter was designed [13]. To deal with the  $H_{\infty}$  fuzzy filtering problem

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for discrete-time Takagi-Sugeno (T-S) fuzzy systems with randomly occurring uncertainties and randomly occurring interval time-varying delays, a full-order fuzzy filter design method was discussed [14]. Unfortunately, an important class of network-induced randomly occurring faults has been largely overlooked in spite of its great importance in NCSs. To solve the FD problem, most literature assumes that the fault occurs definitely, which is different from the practical systems where fault occurs randomly and the occurrence probability can be estimated via statistical tests. Random faults may occur mainly because of the aging, disturbances, electromagnetic interference, and temporary failure of the sensors or actuators. If not properly coped with, the random faults would deteriorate the performance of systems or even cause the instability. In previous studies, only a limited number of results dealt with randomly occurring faults of networked control systems. For example, finite-horizon fault estimation problem for discrete time-varying systems with randomly occurring faults was investigated [15]. On the other hand, the finite-horizon fault estimation problem with randomly occurring fault was discussed for nonlinear time-varying systems [16]. To deal with the finite-time FD problem of nonlinear quantized large-scale networked systems with randomly occurring nonlinearities and faults, the mode-dependent observer-based finite-time fault detection filter was constructed [17]. In [18], fault was assumed to occur randomly and  $H_{\infty}$  fault estimator was designed for the time-varying systems with fading channels.

Moreover, saturation often exists in sensors due to the physical constraint such that sensors cannot provide unlimited signals. For some sensors in practical systems such as temperature sensor and image sensor, saturation is inevitable and could degrade the performance. A great deal of attention has been paid to various types of systems [19-23]. For example, the  $H_{\infty}$  filter has been designed for nonlinear networked systems with sensor saturations [21]. In [23], the FD problem has been investigated for Markovian jump systems with sensor saturation and randomly varying nonlinearities. Besides, the network-induced delays are inherently varying, random, and mutative [24]. However, most literature has been concerned with single time-varying delays which are assumed to occur and ignores the distributed, time-varying state delays which occur randomly in reality.

It is worth mentioning that FD problem with stochastic distributed time-varying delays, packet losses and randomly occurring faults for discrete networked systems subject to sensor saturation has not been researched to the best of our knowledge. For instance, in [25], FD problem for NCSs with randomly occurring faults was considered while stochastic time-delays and sensor saturation were not investigated. In [26], delays and faults were implicitly assumed to occur despite their randomness in practical systems. Summarizing the above discussion, in this article, we will deal with the FD filter design for NCSs involving packet losses, sensor saturation, stochastic distributed time-delays and randomly occurring faults. The purpose of this paper is to design the FD filter so that we can determine whether faults occur in NCSs. Sufficient conditions are established for the existence of the desired FD filter.

The remainder of this paper can be listed as follows. Section 2 formulates the problem under consideration. In Section 3, the  $H_{\infty}$  performance analysis and FD filter design are addressed. A numerical example is presented in Section 4. Finally, Section 5 concludes this article.

Notations: The notations used throughout the paper are as follows.  $\mathbb{R}^n$  denotes the *n*-dimensional Euclidean space; 0 and *I* represent the zero matrix and identity matrix with compatible dimensions. The notation P > 0 ( $\geq 0$ ) means that *P* is real symmetric and positive definite (semidefinite).  $L_2[0,\infty)$  is the space of square summable vectors. *E* is the mathematical expectation and  $prob\{\cdot\}$  means the occurrence probability of the event. In symmetric block matrices or complex matrix expressions, we use \* to represent a term

that is induced by symmetry and  $diag\{\cdot\}$  denotes the block diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

2. **Problem Statement and Preliminaries.** Consider the following discrete-time networked control system with randomly occurring faults and stochastic time-delays:

$$\begin{cases} x(k+1) = Ax(k) + B\sum_{i=1}^{q} b_i(k)x(k-\tau_i(k)) + D_1w(k) + \alpha(k)Ef(k) \\ y(k) = \sigma(Cx(k)) + D_2w(k) \end{cases}$$
(1)

where  $x(k) \in \mathbb{R}^n$  is the state vector;  $y(k) \in \mathbb{R}^m$  is the system output with saturation;  $w(k) \in \mathbb{R}^q$  is the disturbance input, which belongs to  $L_2[0,\infty)$ ;  $f(k) \in \mathbb{R}^l$  is the fault to be detected;  $\tau_i(k)$  (i = 1, 2, ..., q) are time-varying communication delays satisfying  $\tau_m \leq \tau_i(k) \leq \tau_M$ , where  $\tau_m$  and  $\tau_M$  are constant positive scalars representing the lower and upper bounds of the communication delays;  $A, B, C, D_1, D_2, E$  are known constant matrices with appropriate dimensions.

The stochastic variables  $b_i(k)$  (i = 1, 2, ..., q) and  $\alpha(k)$  are mutually uncorrelated Bernoulli distributed white-noise sequences and a natural assumption on the sequences  $b_i(k)$  (i = 1, 2, ..., q) and  $\alpha(k)$  is made as follows.

$$Prob\{b_{i}(k) = 1\} = E\{b_{i}(k)\} = \overline{b_{i}}$$

$$Prob\{b_{i}(k) = 0\} = E\{1 - b_{i}(k)\} = 1 - \overline{b_{i}}$$

$$Var\{b_{i}(k)\} = E\{(b_{i}(k) - \overline{b_{i}})^{2}\} = \overline{b_{i}}(1 - \overline{b_{i}})$$

$$Prob\{\alpha(k) = 1\} = E\{\alpha(k)\} = \overline{\alpha}$$

$$Prob\{\alpha(k) = 0\} = E\{1 - \alpha(k)\} = 1 - \overline{\alpha}$$
(3)

$$Var\{\alpha(k)\} = E\{(\alpha(k) - \overline{\alpha})^2\} = \overline{\alpha}(1 - \overline{\alpha})$$

where  $\overline{b_i} \in [0, 1]$  and  $\overline{\alpha} \in [0, 1]$  are known constants.

At the *k*th time point,  $\alpha(k) = 0$  indicates that the fault occurs in the system and  $\alpha(k) = 1$  shows that the system works normally. The greater the value of  $\overline{\alpha}$  is, the more probably the fault occurs.

Taking the phenomenon of sensor saturation into account, the saturation function  $\sigma(\cdot)$ :  $\mathbb{R}^m \to \mathbb{R}^m$  is defined as

$$\sigma(v) = \left[\sigma_1^T(v_1), \sigma_2^T(v_2), \dots, \sigma_m^T(v_m)\right]^T$$
(4)

where  $\sigma_i(v_i) = sign(v_i) \cdot \min \{v_{i,\max}, |v_i|\}, v_{i,\max}$  is the *i*-th element of the saturation level vector  $v_{\max}$ . It is worth noting that the notation of "sign" denotes the signum function.  $\sigma(\cdot)$  belongs to  $[L_1, L_2]$  with some given diagonal matrices  $L_1, L_2$ , where  $L_1 \ge 0, L_2 \ge 0$  and  $L_2 > L_1$ . And  $\sigma(\cdot)$  satisfies the following inequality:

$$\left[\sigma(Cx(k)) - L_1 Cx(k)\right]^T \left[\sigma(Cx(k)) - L_2 Cx(k)\right] \le 0$$
(5)

 $\sigma(Cx(k))$  can be divided into a linear and nonlinear part

$$\sigma(Cx(k)) = \phi(Cx(k)) + L_1 Cx(k)$$
(6)

The nonlinear vector-valued function  $\phi(Cx(k))$  satisfies  $\phi(Cx(k)) \in \Phi_S$  and  $\Phi_S$  is described as:

$$\Phi_s \stackrel{\Delta}{=} \left\{ \phi : \phi^T(Cx(k)) \left[ \phi(Cx(k)) - \overline{L}Cx(k) \right] \right\} \le 0$$
(7)

where  $\overline{L} = L_2 - L_1$ .

Then the system output with saturation can be described as:

$$y(k) = \phi(Cx(k)) + L_1 Cx(k) + D_2 w(k)$$
(8)

Assuming that there is a network channel between the sensor and the fault detection filter, packet losses are inevitably induced because of the limited bandwidth of the network. The measurement output is described as:

$$y_f(k) = \delta(k)(\phi(Cx(k)) + L_1Cx(k) + D_2w(k))$$
(9)

In (9), the stochastic variable  $\delta(k)$  is a Bernoulli distributed white-noise sequence with the probability distribution as follows:

$$Prob\{\delta(k) = 1\} = E\{\delta(k)\} = \overline{\delta}$$

$$Prob\{\delta(k) = 0\} = E\{1 - \delta(k)\} = 1 - \overline{\delta}$$

$$Var\{\delta(k)\} = E\{\left(\delta(k) - \overline{\delta}\right)^2\} = \overline{\delta}\left(1 - \overline{\delta}\right)$$
(10)

where  $\overline{\delta}$  is a known constant.

Select the following full-order FD filter:

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y_f(k) \\ r(k) = C_f \hat{x}(k) + D_f y_f(k) \end{cases}$$
(11)

where  $\hat{x}(k) \in \mathbb{R}^n$  denotes the state vector of the filter;  $r(k) \in \mathbb{R}^l$  is the residual that is compatible with the fault vector f(k);  $y_f(k)$  is the measurement output and the filter input;  $A_f$ ,  $B_f$ ,  $C_f$ ,  $D_f$  are appropriately dimensioned filter matrices to be determined.

Defining  $\xi(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T$ ,  $\theta(k) = \begin{bmatrix} w^T(k) & f^T(k) \end{bmatrix}^T$ , e(k) = r(k) - f(k), from (1), (9) and (11), the filtering error system can be obtained:

$$\begin{cases} \xi(k+1) = \left(\overline{A}_{1} + \tilde{\delta}_{k}\overline{A}_{2}\right)\xi(k) + \left(\overline{B}_{1} + \tilde{\delta}_{k}\overline{B}_{2}\right)\phi(Cx(k)) \\ + \left(\overline{C}_{1} + \tilde{\delta}_{k}\overline{C}_{2} + \tilde{\alpha}_{k}\overline{C}_{3}\right)\theta(k) + \sum_{i=1}^{q}\overline{A}_{di}\xi(k - \tau_{i}(k)) \\ + \sum_{i=1}^{q}\tilde{A}_{di}\xi(k - \tau_{i}(k)) \\ e(k) = \left(\overline{A}_{3} + \tilde{\delta}_{k}\overline{A}_{4}\right)\xi(k) + \left(\overline{B}_{3} + \tilde{\delta}_{k}D_{f}\right)\phi(Cx(k)) + \left(\overline{C}_{4} + \tilde{\delta}_{k}\overline{C}_{5}\right)\theta(k) \end{cases}$$
(12)

where

$$\begin{split} \tilde{\delta}_{k} &= \delta(k) - \bar{\delta}, \quad \tilde{\alpha}_{k} = \alpha(k) - \bar{\alpha}, \quad \overline{A}_{1} = \begin{bmatrix} A & 0 \\ \overline{\delta}B_{f}L_{1}C & A_{f} \end{bmatrix}, \quad \overline{A}_{2} = \begin{bmatrix} 0 & 0 \\ B_{f}L_{1}C & 0 \end{bmatrix}, \\ \overline{B}_{1} &= \begin{bmatrix} 0 \\ \overline{\delta}B_{f} \end{bmatrix}, \quad \overline{B}_{2} = \begin{bmatrix} 0 \\ B_{f} \end{bmatrix}, \quad \overline{C}_{1} = \begin{bmatrix} D_{1} & \overline{\alpha}E \\ \overline{\delta}B_{f}D_{2} & 0 \end{bmatrix}, \quad \overline{C}_{2} = \begin{bmatrix} 0 & 0 \\ \overline{\delta}B_{f}D_{2} & 0 \end{bmatrix}, \\ \overline{C}_{3} &= \begin{bmatrix} 0 & E \\ 0 & 0 \end{bmatrix}, \quad \overline{A}_{di} = \begin{bmatrix} \overline{b}_{i}B & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A}_{di} = \begin{bmatrix} (b_{i}(k) - \overline{b}_{i})B & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{A}_{3} &= \begin{bmatrix} \overline{\delta}D_{f}L_{1}C & C_{f} \end{bmatrix}, \quad \overline{A}_{4} = \begin{bmatrix} D_{f}L_{1}C & 0 \end{bmatrix}, \quad \overline{B}_{3} = \overline{\delta}D_{f}, \\ \overline{C}_{4} &= \begin{bmatrix} \overline{\delta}D_{f}D_{2} & -I \end{bmatrix}, \quad \overline{C}_{5} = \begin{bmatrix} D_{f}D_{2} & 0 \end{bmatrix} \end{split}$$

**Definition 2.1.** When  $\theta(k) = 0$ , system (12) is said to be exponentially mean-square stable if there exist constant  $\phi > 0$  and  $\tau \in (0, 1)$ , such that  $E\left\{\|\xi(k)\|^2\right\} \leq \phi \tau^k E\left\{\|\xi(0)\|^2\right\}$ , for all  $\xi(0) \in \mathbb{R}^n$ ,  $k \in I^+$ .

With this definition, the original FD filter design of the system (1) can be further converted into  $H_{\infty}$  filtering problem: design an FD filter (11) that makes the error between residual and fault signal as small as possible such that the following two requirements (Q1) and (Q2) are satisfied. By minimizing the  $H_{\infty}$  norm of the difference, the effect of the disturbance can be minimized and the sensitivity of the residual to fault can be maximized [27].

(Q1) The filtering error system (12) is exponentially mean-square stable;

(Q2) Under the zero-initial condition, the following  $H_{\infty}$  performance index is satisfied.

$$\sum_{k=0}^{\infty} E\left\{ \|e(k)\|^2 \right\} \le \gamma^2 E\left\{ \sum_{k=0}^{\infty} \|\theta(k)\|^2 \right\}$$
(13)

where  $\gamma$  is made as small as possible in the feasibility of (13).

The next step is to introduce a residual evaluation stage including an evaluation function J(k) and a threshold J(th) which are described as:

$$J(k) = E\left\{ \left[ \sum_{s=0}^{k} r^{T}(s)r(s) \right]^{1/2} \right\}, \quad J(th) = \sup_{\omega(k) \in l_{2}, f(k) = 0} J(L)$$
(14)

where L represents the maximum time step of J(k). We can detect the occurrence of fault by comparing J(k) with J(th) according to the following rule:

$$\begin{cases} J(k) > J(th) \Rightarrow \text{faults} \Rightarrow \text{alarm} \\ J(k) \le J(th) \Rightarrow \text{no faults} \end{cases}$$
(15)

After above treatments, the randomly occurring fault, stochastic time-delays, packet losses and sensor saturation are considered in the system (12). To deal with the filtering error system (12), we need to prove the stability and  $H_{\infty}$  performance.

3. Main Results. In this section, firstly, the conditions are investigated under which system (12) is exponentially mean-square stable and guarantees the performance defined in (13). Then the fault detection filter design problem will be discussed based on the results of the stability and  $H_{\infty}$  performance analysis. The following lemmas will be used in the derivation of our main results.

**Lemma 3.1.** [28]: let  $V(\xi(k))$  be a Lyapunov functional. If there exist real scalars  $\lambda \ge 0$ ,  $\mu > 0$ ,  $\nu > 0$ , and  $0 < \varphi < 1$ , such that

$$\mu \|\xi(k)\|^2 \le V(\xi(k)) \le \nu \|\xi(k)\|^2 \tag{16}$$

$$E\left\{V(\xi(k+1))|\xi(k)\right\} - V(\xi(k)) \le \lambda - \varphi V(\xi(k)) \tag{17}$$

Then the sequence  $\xi(k)$  satisfies

$$E\{\|\xi(k)\|^{2}\} \leq \frac{\nu}{\mu} \|\xi(0)\|^{2} (1-\varphi)^{k} + \frac{\lambda}{\mu\varphi}$$
(18)

**Lemma 3.2.** [29]: For matrices  $A, Q = Q^T$  and P > 0 such that  $A^T P A - Q < 0$  holds if and only if there exists a matrix G such that

$$\begin{bmatrix} -Q & A^T G \\ G^T A & P - G - G^T \end{bmatrix} < 0$$
(19)

3.1.  $H_{\infty}$  performance analysis. In the following theorem, we will present a sufficient condition such that the filtering error system (12) is exponentially stable in the mean square and has a guaranteed performance  $\gamma$ .

**Theorem 3.1.** For given positive scalars  $\overline{\alpha}$ ,  $\overline{b_i}$ ,  $\overline{\delta}$  and FD filter parameters  $A_f$ ,  $B_f$ ,  $C_f$ ,  $D_f$ , the filtering error system is exponentially stable in the mean square with a guaranteed performance  $\gamma > 0$  if there exist positive matrices P > 0 and  $Q_j > 0$  (j = 1, 2, ..., q), such that the following LMI (20) holds.

$$\Phi = \begin{bmatrix} \Phi_{11} + \overline{A}_3^T \overline{A}_3 + f_1^2 \overline{A}_4^T \overline{A}_4 & * & * & * \\ \Phi_{21} + \overline{B}_3^T \overline{A}_3 + f_1^2 D_f^T \overline{A}_4 & \Phi_{22} + \overline{B}_3^T \overline{B}_3 + f_1^2 D_f^T D_f & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} < 0$$
(20)

where

$$\begin{split} \Phi_{11} &= \overline{A}_{1}^{T} P \overline{A}_{1} + \sum_{j=1}^{q} (\tau_{M} - \tau_{m} + 1) Q_{j} - P + f_{1}^{2} \overline{A}_{2}^{T} P \overline{A}_{2} \\ \Phi_{21} &= \overline{B}_{1}^{T} P \overline{A}_{1} + f_{1}^{2} \overline{B}_{2}^{T} P \overline{A}_{2} + \overline{L} \hat{C}, \ \Phi_{22} &= \overline{B}_{1}^{T} P \overline{B}_{1} + f_{1}^{2} \overline{B}_{2}^{T} P \overline{B}_{2} - 2I, \ \Phi_{31} = \hat{Z}^{T} P \overline{A}_{1}, \\ \Phi_{32} &= \hat{Z}^{T} P \overline{B}_{1}, \ \Phi_{33} = \hat{Z}^{T} P \hat{Z} + diag \left\{ -Q_{1} + \tilde{A}_{1}, -Q_{2} + \tilde{A}_{2}, \dots, -Q_{q} + \tilde{A}_{q} \right\} \\ \tilde{A}_{i} &= \overline{b_{i}} \left( 1 - \overline{b_{i}} \right) \hat{A}_{d}^{T} P \hat{A}_{d}, \ \hat{A}_{d} &= \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}, \ \Phi_{41} = \overline{C}_{1}^{T} P \overline{A}_{1} + f_{1}^{2} \overline{C}_{2}^{T} P \overline{A}_{2} + \overline{C}_{4}^{T} \overline{A}_{3} + f_{1}^{2} \overline{C}_{5}^{T} \overline{A}_{4}, \\ \Phi_{42} &= \overline{C}_{1}^{T} P \overline{B}_{1} + f_{1}^{2} \overline{C}_{2}^{T} P \overline{B}_{2} + \overline{C}_{4}^{T} \overline{B}_{3} + f_{1}^{2} \overline{C}_{5}^{T} D_{f}, \ \Phi_{43} &= \overline{C}_{1}^{T} P \hat{Z}, \\ \Phi_{44} &= \overline{C}_{1}^{T} P \overline{C}_{1} + f_{1}^{2} \overline{C}_{2}^{T} P \overline{C}_{2} + f_{2}^{2} \overline{C}_{3}^{T} P \overline{C}_{3} + \overline{C}_{4}^{T} \overline{C}_{4} + f_{1}^{2} \overline{C}_{5}^{T} \overline{C}_{5} - \gamma^{2} I, \ f_{1} &= \sqrt{\overline{\delta} \left( 1 - \overline{\delta} \right)}, \\ f_{2} &= \sqrt{\overline{\alpha} \left( 1 - \overline{\alpha} \right)}, \ \hat{Z} &= \left[ \overline{A}_{d1}, \overline{A}_{d2}, \dots, \overline{A}_{dq} \right], \ \hat{C} &= \left[ \begin{array}{c} C & 0 \end{array} \right] \end{split}$$

**Proof:** Choose the Lyapunov functional as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k)$$
(21)

where

$$V_1(k) = \xi^T(k) P\xi(k) \qquad V_2(k) = \sum_{j=1}^q \sum_{i=k-\tau_j(k)}^{k-1} \xi^T(i) Q_j \xi(i)$$
$$V_3(k) = \sum_{j=1}^q \sum_{m=-\tau_M+1}^{-\tau_m} \sum_{i=k+m}^{k-1} \xi^T(i) Q_j \xi(i)$$

We will prove Theorem 3.1 from two aspects. On the one hand, we are ready to confirm the exponential stability of the system (12) with  $\theta(k) = 0$ ; on the other hand, we will move to the proof of the  $H_{\infty}$  performance for the system (12) with  $\theta(k) \neq 0$ .

When  $\theta(k) = 0$ , defining  $\Delta V(k) = V(k+1) - V(k)$  and taking mathematical expectation, we can get

$$E \{\Delta V_1(k)\} = E \{\xi^T(k+1)P\xi(k+1) - \xi^T(k)P\xi(k)\}$$
  
=  $E \{\left[\left(\overline{A}_1 + \tilde{\delta}_k \overline{A}_2\right)\xi(k) + \left(\overline{B}_1 + \tilde{\delta}_k \overline{B}_2\right)\phi(Cx(k)) + \sum_{i=1}^q \overline{A}_{di}\xi(k-\tau_i(k))\right]$ 

$$+\sum_{i=1}^{q} \tilde{A}_{di}\xi(k-\tau_{i}(k))\Big]^{T}P\Big[\Big(\overline{A}_{1}+\tilde{\delta}_{k}\overline{A}_{2}\Big)\xi(k)+\Big(\overline{B}_{1}+\tilde{\delta}_{k}\overline{B}_{2}\Big)\phi(Cx(k))$$

$$+\sum_{i=1}^{q} \overline{A}_{di}\xi(k-\tau_{i}(k))+\sum_{i=1}^{q} \tilde{A}_{di}\xi(k-\tau_{i}(k))\Big]\Big\}$$

$$=E\Big\{\xi^{T}(k)\left[\overline{A}_{1}^{T}P\overline{A}_{1}+f_{1}^{2}\overline{A}_{2}^{T}P\overline{A}_{2}-P\right]\xi(k)+2\phi^{T}(Cx(k))\left[\overline{B}_{1}^{T}P\overline{A}_{1}\right]$$

$$+f_{1}^{2}\overline{B}_{2}^{T}P\overline{A}_{2}\Big]\xi(k)+\phi^{T}(Cx(k))\left[\overline{B}_{1}^{T}P\overline{B}_{1}+f_{1}^{2}\overline{B}_{2}^{T}P\overline{B}_{2}\right]\phi(Cx(k))$$

$$+2\left(\sum_{i=1}^{q} \overline{A}_{di}\xi(k-\tau_{i}(k))\right)^{T}P\overline{A}_{1}\xi(k)+2\left(\sum_{i=1}^{q} \overline{A}_{di}\xi(k-\tau_{i}(k))\right)^{T}P\overline{B}_{1}\phi(Cx(k))$$

$$+\left(\sum_{i=1}^{q} \overline{A}_{di}\xi(k-\tau_{i}(k))\right)^{T}P\left(\sum_{i=1}^{q} \overline{A}_{di}\xi(k-\tau_{i}(k))\right)$$

$$+\sum_{i=1}^{q} \overline{b}_{i}\left(1-\overline{b}_{i}\right)\xi^{T}(k-\tau_{i}(k))\hat{A}_{d}^{T}P\hat{A}_{d}\xi(k-\tau_{i}(k))\Big\}$$

$$E\{\Delta V_{2}(k)\}$$

$$(22)$$

$$\leq E \left\{ \sum_{j=1}^{q} \left( \xi^{T}(k) Q_{j} \xi(k) - \xi^{T}(k - \tau_{j}(k)) Q_{j} \xi(k - \tau_{j}(k)) + \sum_{i=k-\tau_{M}+1}^{k-\tau_{m}} \xi^{T}(k) Q_{j} \xi(k) \right) \right\}$$
(23)

$$E\left\{\Delta V_3(k)\right\} \le E\left\{\sum_{j=1}^q \left((\tau_M - \tau_m)\xi^T(k)Q_j\xi(k) - \sum_{i=k-\tau_M+1}^{k-\tau_m}\xi^T(k)Q_j\xi(k)\right)\right\}$$
(24)

According to (7), we have

Denoting  $\varsigma(k) = \begin{bmatrix} \xi^T(k) & \phi^T(Cx(k)) & \xi^T(k - \tau_1(k)) & \dots & \xi^T(k - \tau_q(k)) \end{bmatrix}^T$  and combining (22)-(25), we have

$$E\left\{\Delta V(k)\right\} \le \varsigma^T(k)\Phi_1\varsigma(k) \tag{26}$$

where  $\Phi_1 = \begin{bmatrix} \Phi_{11} & * & * \\ \Phi_{21} & \Phi_{22} & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} \end{bmatrix}$ .

It is obvious that  $\Phi < 0$  represents  $\Phi_1 < 0$ . For nonzero  $\varsigma(k)$ ,  $\Phi_1 < 0$  indicates that  $E \{\Delta V(k)\} < 0$ , and then we can get

$$E \{\Delta V(k)\} = E \{V(k+1)\} - E \{V(k)\} \le -\lambda_{\min} (-\Phi_1) \varsigma^T(k)\varsigma(k)$$
  
$$< -\lambda_{\min} (-\Phi_1) \xi^T(k)\xi(k) < -\alpha\xi^T(k)\xi(k)$$
(27)

where  $0 < \alpha < \min\{\lambda_{\min}(-\Phi_1), \sigma\}, \sigma := \max\{\lambda_{\max}(P), \lambda_{\max}(Q_1), \dots, \lambda_{\max}(Q_q)\}$ 

$$\alpha \|\xi(k)\|^{2} < V(k) \le \sigma \|\xi(k)\|^{2}$$
(28)

$$\Delta V(k) < -\alpha \xi^T(k)\xi(k) < -\frac{\alpha}{\sigma}V(k) := -\psi V(k)$$
<sup>(29)</sup>

Therefore, according to Definition 2.1 and Lemma 3.1, we can conclude that system (12) is exponentially mean-square stable.

Next, we will prove that the performance defined in (13) is guaranteed.

When  $\theta(k) \neq 0$ , defining  $\eta(k) = [\varsigma^T(k) \quad \theta^T(k)]^T$ , the following inequality can be obtained according to (20):

$$E\{\Delta V(k+1)\} - E\{\Delta V(k)\} + E\{e^{T}(k)e(k)\} - \gamma^{2}\theta^{T}(k)\theta(k) = \eta^{T}(k)\Phi\eta(k) < 0 \quad (30)$$

Summing up (30) from 0 to  $\infty$  with respect to k yields:

$$\sum_{k=0}^{\infty} E\left\{ \|e(k)\|^2 \right\} \le \gamma^2 E\left\{ \sum_{k=0}^{\infty} \|\theta(k)\|^2 \right\} + E\left\{ \Delta V(0) \right\} - E\left\{ \Delta V(\infty) \right\}$$
(31)

Considering the zero initial condition  $\xi(0) = 0$ , we have

$$\sum_{k=0}^{\infty} E\left\{ \|e(k)\|^2 \right\} \le \gamma^2 E\left\{ \sum_{k=0}^{\infty} \|\theta(k)\|^2 \right\}$$
(32)

So we can know that the  $H_{\infty}$  performance constraint is achieved and the proof is complete.

3.2. **FD filter design.** Having proved that the system is exponentially mean-square stable and satisfies the  $H_{\infty}$  performance constraint based on Theorem 3.1, we are in a position to deal with the design of FD filter.

**Theorem 3.2.** For given positive scalars  $\overline{\alpha}$ ,  $\overline{b_i}$  and  $\overline{\delta}$ , the filtering error system is exponentially stable in the mean square with a guaranteed performance  $\gamma > 0$  if there exist positive matrices P > 0,  $Q_j > 0$  (j = 1, 2, ..., q), matrices G,  $\overline{A_f}$ ,  $\overline{B_f}$ ,  $\overline{C_f}$  and  $\overline{D_f}$  satisfying the following inequality:

$$\Xi = \begin{bmatrix} \Xi_1 & * & * \\ 0 & \Xi_2 & * \\ \Xi_3 & \Xi_4 & \Xi_5 \end{bmatrix} < 0$$
(33)

Moreover, if (33) is feasible, the parameters of the desired FD filter can be given by

$$\begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} = \begin{bmatrix} G_3^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{A}_f & \overline{B}_f \\ \overline{C}_f & \overline{D}_f \end{bmatrix}$$
(34)

where

$$\begin{split} P &= \left[ \begin{array}{c} P_1 & P_2 \\ P_2^T & P_3 \end{array} \right], \ G &= \left[ \begin{array}{c} G_1 & G_2 \\ G_3 & G_3 \end{array} \right], \ \Xi_1 = \left[ \begin{array}{c} -P + \sum_{j=1}^q (\tau_M - \tau_m + 1)Q_j & * \\ \overline{L}\hat{C} & -2I \end{array} \right] \\ \Xi_2 &= diag \left\{ -Q_1 + \tilde{A}_1, -Q_2 + \tilde{A}_2, \dots, -Q_q + \tilde{A}_q, -\gamma^2 I \right\} \\ \Xi_3 &= \left[ \begin{array}{c} \Xi_{311} & \Xi_{312} \\ \Xi_{321} & \Xi_{322} \\ \Xi_{331} & \Xi_{332} \\ \Xi_{341} & \Xi_{342} \\ 0 & 0 \end{array} \right], \ \Xi_4 &= \left[ \begin{array}{c} 0 & \Xi_{412} \\ 0 & \Xi_{422} \\ G^T \hat{Z} & \Xi_{432} \\ 0 & \Xi_{422} \\ \Xi_{422} & \Xi_{422} \\ 0 & \Xi_{422} \\ \Xi_{422} & \Xi_{422} \\ 0 & \Xi_{422} \\ \Xi_{422} & \Xi_{42} \\ \Xi_{42} &$$

$$\Xi_{342} = \begin{bmatrix} f_1 \overline{B}_f \\ f_1 \overline{B}_f \end{bmatrix}, \ \Xi_{412} = \begin{bmatrix} \overline{\delta D}_f D_2 & -I \end{bmatrix}, \ \Xi_{422} = \begin{bmatrix} f_1 \overline{D}_f D_2 & 0 \end{bmatrix}$$
$$\Xi_{432} = \begin{bmatrix} G_1^T D_1 + \overline{\delta B}_f D_2 & \overline{\alpha} G_1^T E \\ G_2^T D_1 + \overline{\delta B}_f D_2 & \overline{\alpha} G_2^T E \end{bmatrix}, \ \Xi_{442} = \begin{bmatrix} f_1 \overline{B}_f D_2 & 0 \\ f_1 \overline{B}_f D_2 & 0 \end{bmatrix}, \ \Xi_{452} = \begin{bmatrix} 0 & f_2 G_1^T E \\ 0 & f_2 G_2^T E \end{bmatrix}$$

**Proof:** Notice that (20) can be rewritten as follows

$$\begin{bmatrix} \Xi_{1} & * \\ 0 & \Xi_{2} \end{bmatrix} + \begin{bmatrix} \overline{A}_{3}^{T} \\ \overline{B}_{3}^{T} \\ 0 \\ \overline{C}_{4}^{T} \end{bmatrix} \begin{bmatrix} \overline{A}_{3}^{T} \\ \overline{B}_{3}^{T} \\ 0 \\ \overline{C}_{4}^{T} \end{bmatrix}^{T} + \begin{bmatrix} f_{1}\overline{A}_{4}^{T} \\ f_{1}D_{f}^{T} \\ 0 \\ f_{1}\overline{C}_{5}^{T} \end{bmatrix} \begin{bmatrix} f_{1}\overline{A}_{4}^{T} \\ f_{1}D_{f}^{T} \\ 0 \\ f_{1}\overline{C}_{5}^{T} \end{bmatrix}^{T} + \begin{bmatrix} \overline{A}_{1}^{T} \\ \overline{B}_{1}^{T} \\ \overline{C}_{1}^{T} \end{bmatrix} P \begin{bmatrix} \overline{A}_{1}^{T} \\ \overline{B}_{1}^{T} \\ \overline{C}_{1}^{T} \end{bmatrix}^{T} \\ \overline{C}_{1}^{T} \end{bmatrix} + \begin{bmatrix} f_{1}\overline{A}_{2}^{T} \\ \overline{C}_{1}^{T} \end{bmatrix}^{T} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_{1}\overline{C}_{5}^{T} \end{bmatrix} P \begin{bmatrix} f_{1}\overline{A}_{2}^{T} \\ \overline{C}_{1}^{T} \end{bmatrix}^{T} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_{2}\overline{C}_{3}^{T} \end{bmatrix} P \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_{2}\overline{C}_{3}^{T} \end{bmatrix}^{T} < 0$$

$$(35)$$

Based on Schur complement, we can transform (35) into the form of (36)

$$\Psi = \begin{bmatrix} \Xi_1 & * & * \\ 0 & \Xi_2 & * \\ \Psi_3 & \Psi_4 & \Psi_5 \end{bmatrix} < 0$$
(36)

where

$$\Psi_{3} = \begin{bmatrix} \overline{A}_{3}^{T} & f_{1}\overline{A}_{4}^{T} & \overline{A}_{1}^{T} & f_{1}\overline{A}_{2}^{T} & 0\\ \overline{B}_{3}^{T} & f_{1}D_{f}^{T} & \overline{B}_{1}^{T} & f_{1}\overline{B}_{2}^{T} & 0 \end{bmatrix}, \ \Psi_{4} = \begin{bmatrix} 0 & 0 & \hat{Z}^{T} & 0 & 0\\ \overline{C}_{4}^{T} & f_{1}\overline{C}_{5}^{T} & \overline{C}_{1}^{T} & f_{1}\overline{C}_{2}^{T} & f_{2}\overline{C}_{3}^{T} \end{bmatrix}$$
$$\Psi_{5} = \operatorname{diag}\left\{-I, -I, -P^{-1}, -P^{-1}, -P^{-1}\right\}$$

By the application of Lemma 3.2, we can notice that (36) holds if and only if there exists real matrix G, such that the following inequality (37) holds

$$\Gamma = \begin{bmatrix} \Xi_1 & * & * \\ 0 & \Xi_2 & * \\ \Gamma_3 & \Gamma_4 & \Xi_5 \end{bmatrix} < 0$$
(37)

where

$$\Gamma_{3} = \begin{bmatrix} \overline{A}_{3} & \overline{B}_{3} \\ f_{1}\overline{A}_{4} & f_{1}D_{f} \\ G^{T}\overline{A}_{1} & G^{T}\overline{B}_{1} \\ f_{1}G^{T}\overline{A}_{2} & f_{1}G^{T}\overline{B}_{2} \\ 0 & 0 \end{bmatrix}, \quad \Gamma_{4} = \begin{bmatrix} 0 & \overline{C}_{4} \\ 0 & f_{1}\overline{C}_{5} \\ G^{T}\hat{Z} & G^{T}\overline{C}_{1} \\ 0 & f_{1}G^{T}\overline{C}_{2} \\ 0 & f_{2}G^{T}\overline{C}_{3} \end{bmatrix}$$

Then let us partition P and G respectively as:

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_3 \end{bmatrix}$$
(38)

Denote

$$\overline{A}_f = G_3^T A_f, \quad \overline{B}_f = G_3^T B_f, \quad \overline{C}_f = C_f, \quad \overline{D}_f = D_f$$
(39)

After some conventional matrix operations, it can be clearly seen that (37) is equivalent to (33). Furthermore, from the condition (34), we know that  $G_3$  is invertible, and then the parameters of FD filter can be obtained from (39) immediately.

Therefore, if the conditions (33) and (34) hold, we can derive the conclusion that the obtained filter in the form of (11) makes the system (12) exponentially mean-square stable with an  $H_{\infty}$  performance  $\gamma$ . The proof is completed.

**Remark 3.1.** For the purpose of obtaining the parameters of FD filter, we use slack matrix variables  $G_1$ ,  $G_2$  and  $G_3$  so as to decouple Lyapunov matrix and the system matrices. The results obtained may be less conservative because the method here is better than special structure constraints on Lyapunov matrix adopted in the most existing literature.

**Remark 3.2.** (33) is an LMI over both the matrix variables and the scalar  $\gamma^2$ .  $\gamma^2$  can also be included as an optimization variable for LMI (33). Among these feasible solutions, the minimum attenuation level  $\gamma_{\min} = \sqrt{\gamma^2}$  for the FD dynamics (12) can be obtained and the sub-optimal FD filter can be readily found by solving the following convex optimization problem.

$$\begin{array}{l} \text{Minimize : } \gamma^2 \\ \text{subject to (33) over } P, \ Q_i, \ G, \ \overline{A}_f, \ \overline{B}_f, \ \overline{C}_f, \ \overline{D}_f \end{array} \tag{40}$$

4. Numerical Example. In this section, a numerical example is employed to verify the effectiveness and usefulness of the proposed method. Inspired by the model proposed in [20], we consider the discrete-time networked control system with the following parameters:

$$\begin{split} A &= \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & -0.3 & 0.1 \\ 0.1 & 0 & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & -0.3 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -0.2 & 0 & 0.1 \\ -0.1 & 0.1 & 0.1 \\ 0 & 0.2 & 0.1 \end{bmatrix}, \\ E &= \begin{bmatrix} 0.6 \\ 0.1 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 10 & 0.8 & 0.7 \\ -6 & 0.9 & 0.6 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.9 & -0.6 & 0.1 \\ 0.5 & 0.8 & 0.1 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \\ 2 &\leq \tau_i(k) \leq 3 \quad (i = 1, 2, \dots, q) \end{split}$$

Supposing that the stochastic parameters  $\overline{\alpha} = 0.4$ ,  $\overline{b}_1 = 0.4$ ,  $\overline{b}_2 = 0.6$ ,  $\overline{\delta} = 0.3$ , the sensor nonlinearity is given by

$$\phi(Cx(k)) = \frac{L_1 + L_2}{2}Cx(k) + \frac{L_2 - L_1}{2}\sin(x(k))$$

By using the MATLAB LMI toolbox, for system (12), from Theorem 3.2 we can obtain the minimal  $H_{\infty}$  performance index  $\gamma_{\min} = 1.603$  and the desired FD filter parameters as follows:

$$A_{f} = \begin{bmatrix} 0.4143 & 0.0423 & 0.1336\\ 0.1780 & -0.5254 & 0.0805\\ 0.2394 & -0.0476 & -0.1828 \end{bmatrix}, \quad B_{f} = \begin{bmatrix} -0.1433 & 0.2891 & 1.0350\\ 0.3507 & 0.0026 & -1.8620\\ 0.1199 & 0.2357 & -0.1808 \end{bmatrix}$$
$$C_{f} = \begin{bmatrix} -0.0937 & -0.0194 & -0.0749 \end{bmatrix}, \quad D_{f} = \begin{bmatrix} -0.0741 & -0.2072 & 0.3743 \end{bmatrix}$$

It should be pointed out that the obtained optimal performance index  $\gamma_{\min}$  will change as the values of  $\overline{\alpha}$ ,  $\overline{b}_1$ ,  $\overline{b}_2$  and  $\overline{\delta}$  change. Letting  $\overline{b}_1 = 0.4$  and  $\overline{\delta} = 0.3$ , it can be observed

$\gamma_{\min}$	$\overline{\alpha}=0.4$	$\overline{\alpha}=0.6$	$\overline{\alpha}=0.7$	$\overline{\alpha}=0.8$
$\overline{b}_2 = 0.5$	1.487	1.832	2.009	2.190
$\overline{b}_2 = 0.6$	1.603	1.991	2.191	2.395
$\overline{b}_2 = 0.8$	1.935	2.433	2.694	2.959
$\overline{b}_2 = 0.9$	2.174	2.747	3.048	3.355

TABLE 1.  $\gamma_{\min}$  for different  $\overline{\alpha}$  and  $\overline{b}_2$ 

TABLE 2.  $\gamma_{\min}$  for different  $\overline{\delta}$ 

	$\overline{\delta} = 0.1$	$\overline{\delta} = 0.3$	$\overline{\delta} = 0.7$	$\overline{\delta} = 0.8$
$\gamma_{\rm min}$	2.038	1.991	1.914	1.897

from Table 1 that the corresponding optimal performance, namely, the disturbance attenuation performance deteriorates with increased  $\overline{\alpha}$  and  $\overline{b}_2$  which is in consistent with the actual engineering application. In addition, the disturbance attenuation performance also degrades as the  $\overline{\delta}$  increases and can be observed from Table 2 by letting  $\overline{\alpha} = 0.6$ ,  $\overline{b}_1 = 0.4$  and  $\overline{b}_2 = 0.6$ .

From Tables 1 and 2, we can see that it makes sense to study the randomness of packet losses, time-delays and faults which have an important impact on NCSs.

The initial states are selected as  $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ ,  $\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ . Without loss of generality, the disturbance input and the fault signal are supposed to be

$$w(k) = \begin{bmatrix} e^{-0.02k} \sin(0.2k) \\ e^{-0.03k} \sin(0.3k) \\ e^{-0.04k} \sin(0.4k) \end{bmatrix}, \quad f(k) = \begin{cases} 0.6 + 0.2 \sin(k), & 70 \le k \le 150 \\ 0, & \text{else} \end{cases}$$

Under the conditions of  $\overline{\alpha} = 0.4$ ,  $\overline{b}_1 = 0.4$ ,  $\overline{b}_2 = 0.6$  and  $\overline{\delta} = 0.3$ , the residual response r(k) and evolution of residual evaluation function J(k) are shown in Figures 1 and 2. According to (14), we select the threshold  $J(th) = \sup_{f=0} E\left\{\left[\sum_{k=0}^{400} r^T(k)r(k)\right]^{1/2}\right\}$ . An average value J(th) = 1.2184 is obtained by using 400 Monte Carlo simulations and the



FIGURE 1. Residual signal with  $\overline{\alpha} = 0.4$ 



FIGURE 2. Residual evaluation function with  $\overline{\alpha} = 0.4$ TABLE 3. Thresholds and time steps for different  $\overline{\alpha}$ 

	$\overline{\alpha} = 0.4$	$\overline{\alpha} = 0.6$	$\overline{\alpha} = 0.7$	$\overline{\alpha} = 0.9$	$\overline{\alpha} = 1$
Thresholds	1.2184	1.0236	0.7779	0.5472	0.5114
Time steps	15	10	7	4	2
3	1 1		I	1 1	
2.5 -		ſ	Г		ise
				<b>– – –</b> J(th)	
		1			



FIGURE 3. Residual evaluation function with  $\overline{\alpha} = 0.6$ 

fault can be detected in 15 time steps after its occurrence. By letting  $\bar{b}_1 = 0.4$ ,  $\bar{b}_2 = 0.6$  and  $\bar{\delta} = 0.3$ , for different values of  $\bar{\alpha}$ , the values of J(th) and time steps are listed in Table 3 and the evolutions of residual evaluation function are shown in Figures 3-6.

It is worth mentioning that the fault occurs definitely when  $\overline{\alpha} = 1$  and can be quickly detected in 2 time steps. Obviously, the more probably the fault occurs, the shorter the detecting time of a fault would be. The designed FD filter is sensitive to the occurrence of fault and the simulation results demonstrate the usefulness of the method presented in this paper.



FIGURE 4. Residual evaluation function with  $\overline{\alpha} = 0.7$ 



FIGURE 5. Residual evaluation function with  $\overline{\alpha}=0.9$ 



FIGURE 6. Residual evaluation function with  $\overline{\alpha} = 1$ 

5. Conclusions. In this paper, we have investigated the FD problem for NCSs with random packet losses, stochastic distributed time-varying delays, sensor saturation and randomly occurring faults. Different from the existing literature, the time-delays are assumed to be varying and random, and fault is supposed to occur randomly as well. Sufficient conditions have been derived such that the filtering error system is exponentially mean-square stable and satisfies the  $H_{\infty}$  performance constraint. FD filter has been designed to be the residual generator and we determine whether fault occurs on the basis of residual evaluation stage. A numerical example has been given to demonstrate the effectiveness and usefulness of the addressed method. Further, the closed-loop networked control systems with random sensor saturation and nonlinear perturbations will be our future topic of research.

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