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CHARACTERISTIC SIMILARITY OF PRODUCTION KEY ELEMENTS GREATLY AFFECTING PROFIT OF A PRODUCTIVE BUSINESS

Kenji Shirai[1] and Yoshinori Amano[2]

[1]Faculty of Information Culture
Niigata University of International and Information Studies
3-1-1, Mizukino, Nishi-ku, Niigata 950-2292, Japan
shirai@nuis.ac.jp

[2]Kyohnan Elecs Co., LTD.
8-48-2, Fukakusanishiura-cho, Fushimi-ku, Kyoto 612-0029, Japan
y_amano@kyohnan-elecs.co.jp

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Abstract. *In this study, we illustrate that the lead time, inventory, and rate of return deviation time series for production processes exhibit similar power-law distribution characteristics by analyzing the actual data. Lead time and inventory are considered to be autonomous distributed systems involving multiple oscillators, and the Kuramoto model is used to represent the synchronization phenomena. As an evidence for the existence of power-law characteristics, we depict that their behaviors demonstrate fluctuations and on-off intermittency that can be represented using Langevin dynamics. Finally, we verify that all the three parameters depict power-law distributions using actual data.*
Keywords: Lead time, Inventory, Rate of return deviation, Power-law distribution, Langevin type equation

1. **Introduction.** For an improvement of the production processes, the operation research approach (OR approach) traditionally dominated the research topics. To improve production lead time, shortening production throughput is a major challenge [1, 2]. Moreover, various theories have been applied to improving production processes and increasing productivity. An analysis that uses the queuing model and applies a log-normal distribution to modeling a system in the steel industry is described [3].

On the other hand, fluctuations in the supply chain and market demand and the changes in the production volume of suppliers are propagated to other suppliers, and their effects are amplified. Therefore, because the amounts of stock are large, an increase or decrease of the suppliers' stock is modeled using differential equation. This differential equation is said as Billwhip model, representing a stock congestion [4, 5]. These studies are very interesting contents.

The theory of constraints ("TOC") describes the importance of avoiding bottlenecks in production processes [6]. When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment may lead to delays. "TOC" gives important suggestions for increasing efficiency of production projects. There is no research that mathematically models propagation of production density.

(1) Reducing the lead time, improving the throughput, and synchronizing the production process by the TOC.

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(2) Sharing the demand information and performing mathematical evaluations.
(3) Analyzing the reduction and fluctuating demands of the subsystem (using nonlinear vibration theory).
(4) Basing the inventory management approach on stochastic demand.

When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment itself may lead to delays. The improvement of production processes was presented that the "Synchronization with pre-process" method was the most desirable in practice using the actual data in production flow process based on the cash flow model by using the stochastic differential equation (SDE) of log-normal type [8]. In essence, we have proposed the best way, which is a synchronous method using the Vasicek model for mathematical finance [9].

With respect to a mathematical modeling, we reported that a production process is a mathematical approach, which corresponds to physical diffusion. Using this approach, the diffusion equation, which dominates the production process, was shown. Moreover, we clarified that the production process was dominated a diffusion equation [10]. The production density of each process corresponding to the physical propagation of heat was introduced in our previous study [10].

To evaluate a production process, the lead time of production system in the production stage by using a stochastic differential equation of the log-normal type, which is derived from its dynamic behavior, is modeled [11]. The use of a mathematical model that focuses on the selection process and adaptation mechanism of the production lead time is used [11]. Using this model and risk neutral integral, the evaluation equation for the compatibility condition of the production lead time is defined and then calculated. Furthermore, it is clarified that the throughput of the production process was reduced [11, 12]. To determine a throughput rate, an expected value and volatility of throughput of the whole process period are estimated by utilizing Kalman filter theory having been used for a state estimation problem in the control theory [12].

With respect to our previous research for improvement of lead time, we modeled mathematically and evaluated the method of the production processes. We reported that the synchronization method is superior for improving throughput in production processes, which is used by a production flow process [8]. The production flow process is utilized for production of high-mix low-volume equipments, which are produced through several stages in the production process. This method is good for producing specific control equipment such as semiconductor manufacturing equipment in our experience. Then, we have reported that the production flow process has nonlinear characteristics in our previous study [13]. Moreover, a working-time delay is propagated through the stages in the production process. Its delays are due to volatility in the model. The actual data indicated that in the production flow process, the delays were propagated to the successive stages [10, 13].

With respect to a rate of return deviation (RoRD), we reported that RoRD has power-law characteristics [14]. Generally, disnormality of RoRD in business is well known about a stock price fluctuation model, although with conditions. For example, there exists widely-known Levy process [16]. However, almost all of the reported actual data was entirely limited to stock price data. As another example, also in applying real option, many of the return fluctuation models are of a log-normal stochastic differential equation, and there is also one that handles a jump process [17].

However, we think that, as far as the present writers and the like know, there has been no report that handles power-law distribution focusing attention on RoRD of a privately-owned company of equipment manufacturing business. Further, regarding a make-to-order production department (production-number based manufacturing system), in relation between RoRD and a sales amount in the case of recent production departments, a model of RoRD becomes Langevin type.

Inventory control method is an important research topic in the operations research (OR) field, and most of the methods are stochastic and statistical in nature. We have previously presented an approach based on stochastic differential equations; however, no dynamic method has been developed yet that utilizes methods from mathematical finance [18]. For example, the stochastic relations between the levels of different inventory items are assumed to be derived from the fluctuations in demand.

We model inventory management as a stochastic diffusion process, which describes stock management using a partial differential equation of log-normal type [18]. Effectively managing inventory at the end of the fiscal year is crucial for small- and medium-sized manufacturers, and this could be achieved by utilizing the path-dependent option evaluation theory from mathematical finance [18].

We analyze the lead time by representing its behavior using an autonomous distributed system that comprises multiple oscillators using the Kuramoto model to represent the synchronization phenomena.

In this study, all of the lead time, inventory control and RoRD all represent stochastic fluctuations and have a great influence on production business revenues. We report that these three key elements in the production processes exhibit power distribution characteristics, fluctuations and on-off intermittency [14, 22, 26]. Finally, we verify that the lead time, inventory management and RoRD show power-law distribution based on real data. Of course, we also report together that these key elements have fluctuation and on-off intermittency characteristics. To the best of our knowledge, this is the first study to investigate the widths of phase transitions in the manufacturing industry.

2. Production Systems in the Manufacturing Equipment Industry.

The production methods used in manufacturing equipment are briefly covered in this paper. More information is provided in our report [14]. This system is considered to be a "Make-to-order system with version control", which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Figure 1(A), the "Customer side" refers to an ordering company and "Supplier (D)", which is ordered from the manufacturing sector (C), means the target company in this paper. The product manufacturer, which is the source of the ordered production equipment, presents an order that takes account of the market price. In Figure 1(B), the market development department at the customer's factory receives the order through the sale contract based on the predetermined strategy.

A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow process, which manufactures low volumes of a wide variety of products, is produced through several stages in the production process. In Figure 2, the process consists of six stages. In each step S1-S6 of the manufacturing process, materials are being produced.

The direction of the arrows represents the direction of the production flow. Production materials are supplied through the inlet and the end-product is shipped from the outlet [8].

For this flow production system, we make the following two assumptions.

Assumption 2.1. *The production structure is nonlinear.*

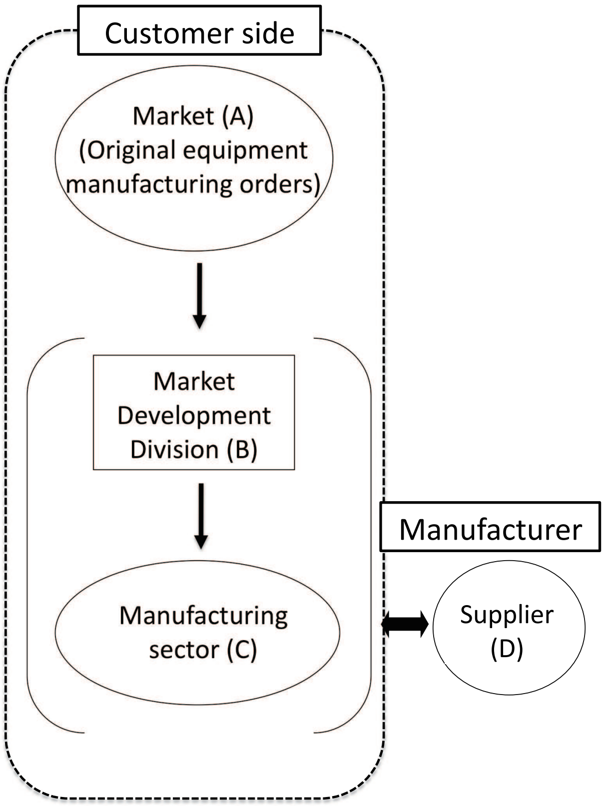

FIGURE 1. Business structure of company of research target

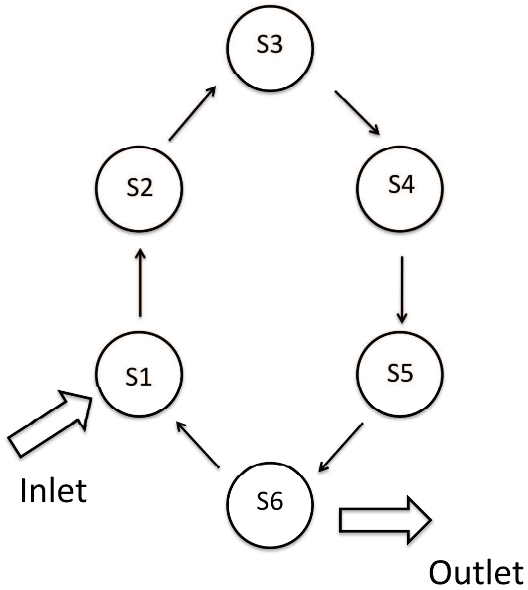

FIGURE 2. Production flow process

Assumption 2.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system).*

Assumption 2.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption 2.1 reflects the realistic production structure and is somewhat valid. Assumption 2.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 2.2 is reasonable because new production starts from S1. Please refer to Appendix A.

3. **Kuramoto Model.** Consider a single oscillator, which is represented as a point $e^{i\theta}$ on the unit circle in the complex plane, with a frequency of ω. The behavior of such an oscillator can be represented as follows [19].

$$\frac{d\theta}{dt} = \omega \tag{1}$$

Here, we are focusing only on the phase θ and are neglecting factors such as the oscillator's amplitude or shape. In terms of a production process, the oscillator corresponds to the lead time at each stage of the process. A system in which several such oscillators are coupled nonlinearly is called a coupled oscillator system. This is the basis of the Kuramoto model, which can be presented as follows.

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_i) \tag{2}$$

where $i = 1, 2, \ldots, N$. ω_i and K (> 0) denote a natural frequency at vibrator of the i-th order and a coupling strength between vibrators respectively.

Definition 3.1. *An order parameter* z

$$z = re^{i\phi} = \frac{1}{N}\sum_{i=1}^{K} e^{i\theta_k} \tag{3}$$

where r represents the degree of phase synchronization and the larger r indicates that the degree of synchronization is higher. Therefore, $0 \le r \le 1$, $r = 0$ and $r = 1$ denote asynchronous and synchronous respectively.

To perform accurate analysis, it is convenient to consider a continuous system. Therefore, let the fraction of oscillators with frequency ω be $g(\omega)$, and let the probability density of the phase θ at time t be $h(\theta, \omega, t)$. An equation for the density h can further be obtained using Equation (2) as follows.

$$\frac{\partial h}{\partial t} + \frac{\partial(\rho h)}{\partial \theta} = \xi(\theta, t) \tag{4}$$

where $\xi(\theta, t)$ denotes a white noise and is derived as follows:

$$\langle \xi(\theta, t) \cdot \xi(\theta', t') \rangle = \xi\delta(\theta - \theta') \times \delta(t - t') \tag{5}$$

If the production process is synchronized, the lead time is minimized. Here, the density of synchronization is $g(\omega)$. We obtain the equation in case of $N \to \infty$ in Equation (3) as follows:

$$z = re^{i\phi} = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} e^{i\theta} h(\theta, \omega, t) g(\omega) d\omega d\theta \tag{6}$$

The solution for an incoherent state where all the oscillators are moving randomly depicts absolutely no correlations among them. However, if the phase distributions of the oscillator population are uniform, all the oscillators are in statically stable states although the phases of the individual oscillators continue to change according to their own frequencies ω. Sufficiently large values of K create a completely synchronized solution where all of the oscillators depict a common frequency and different phases. In partially synchronized states, only some of the oscillators with similar natural frequency are observed to be synchronized, while the others have moved apart. We call the rate at which the oscillators are synchronized as the "flux", which is depicted by ρ and can be defined as follows.

$$\rho(\theta, \omega, t) \simeq \omega + Kr\sin(\theta - \theta') \approx \omega + Kr\Delta\theta \equiv \beta \tag{7}$$

where $\Delta\theta = \theta - \theta'$. β is a constant.

$$\frac{\partial h}{\partial t} + \beta\frac{\partial h}{\partial \theta} = \xi(\theta, t) \tag{8}$$

Equation (9) is known to follow the Fokker Planck equation as follows [18, 20]:

$$\frac{\partial h}{\partial t} + \beta\frac{\partial h}{\partial \theta} = \xi\frac{\partial^2 h}{\partial h^2} \tag{9}$$

3.1. **Mathematical modeling of a lead time and an inventory by using Kuramoto's model.** The vibrator in the Kuramoto's model corresponds to each process in processes. h_i corresponds to a lead time or an inventory. An inventory is indispensable for reducing lead time. It is important to consider logistics. Here, the logistics are considered as part of securing inventory parts.

In Figure 3, process $(i-1)$ and process $(i+1)$ are uncorrelated and θ_i denotes the phase at process i. Let the deviation of phase between processes $h_{i-1} = \theta_{i-1} - \theta_i$ and $h_i = \theta_i - \theta_{i+1}$. In Figure 4, there exists correlation between the processes proximate to one another in the production. In other words, the autocorrelation of $h_i(t)$ only is enabled. As mentioned in our previous study, the RoRD in the production business can be described as a Langevin type equation [22]. Figure 5 shows an equivalent model of flow-shop type production processes. Let $h_i \equiv d_i - d_{i-1}$ and $d\theta_i/dt = d_i$.

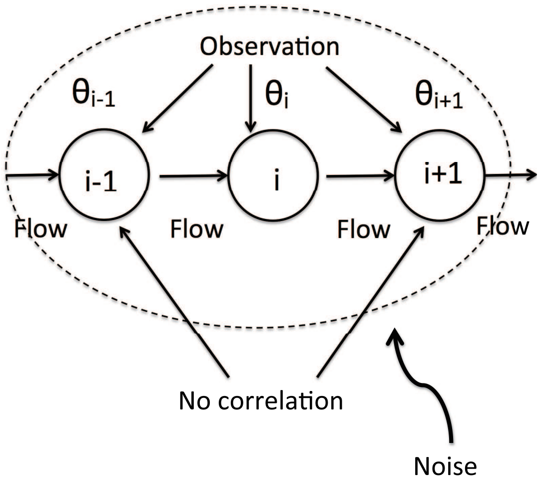

FIGURE 3. Throughput propagation under noise

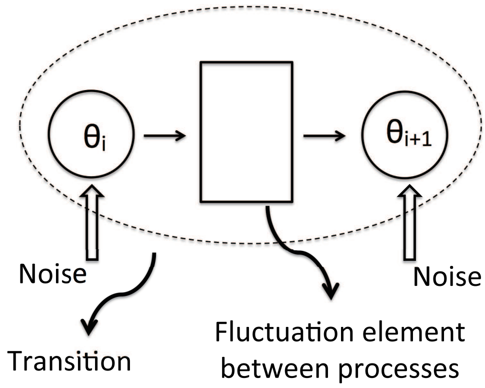

FIGURE 4. Fluctuation between processes

We call θ_i the phase parameter of the processes. A, B, C are coupling coefficients. d_i, d_{i-1} denote equivalents to the potential energies of processes. N denotes a node in the circuit. $E(t) = E_0 + e_m \sin(wt - \varphi)$ has an alternating current.

With respect to $d_i - d_{i-1} = \Delta d_i$, $\Delta d_i = 0$ basically impossible by means of the coupling coefficients A, B, C with no harmony. Therefore, the deviation signal, h_i, undergoes fluctuations. In Figure 5, asynchronous phenomena are realistically evoked in the processes due to fluctuations affected by the variable parameter C. A detailed analysis is omitted here.

h_i is represented by Langevin type equation as follows:

$$\frac{dh_i}{dt} = f_i(h_i, t) + \sqrt{H} r_i(t) \tag{10}$$

where $f_i(h_i; t)$ denotes a probability throughput. $h \in [h_1, h_2, \ldots, h_{N'}]$, and $\sqrt{H} r_i(t)$ denotes the noise term.

Figure 7 shows a lead time at work start time. $h_i(t)$, t_i and $1, 2, \ldots, n$ denote a normalized lead time, work start time and work start number respectively. Figures 8-11 are annual transition data of lead time and inventory at work start time. The month in which almost appropriate inventory is secured has a short lead time. This indicates that there is a close relationship between lead time and inventory. We will describe that the normalized lead time data and the inventory data have a power-law distribution characteristics.

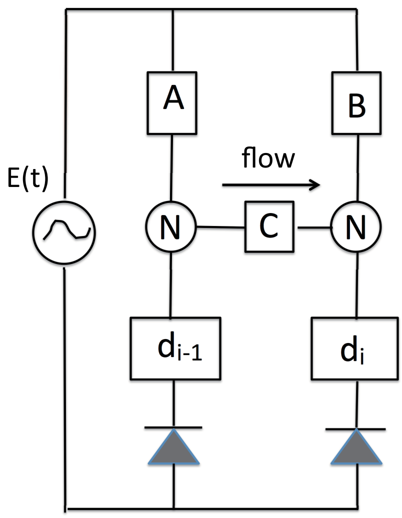

FIGURE 5. Equivalent model of throughput propagation

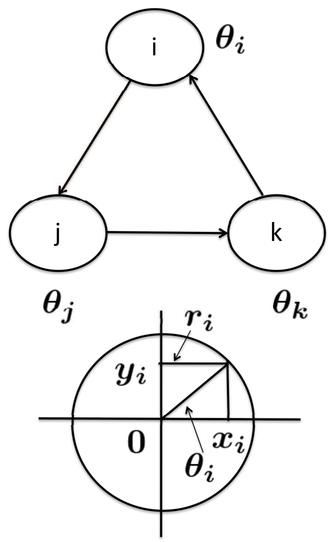

FIGURE 6. Production flow process by polar coordinate

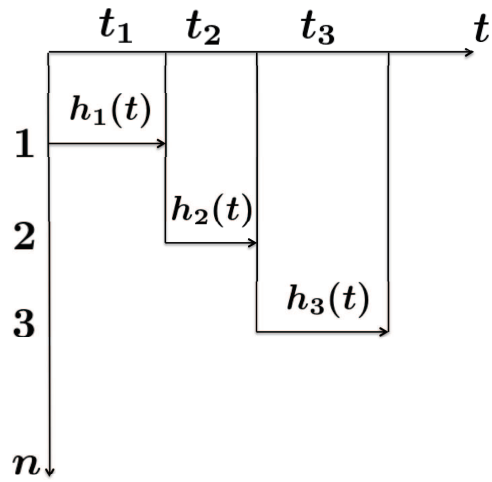

FIGURE 7. Lead time at work start time

3.2. **Stochastic model with lower lead time and inventory limits.** In this section, we analyze the lead time and inventory based on actual normalized data. A lead time, an inventory, a rate of return deviation are affected stochastically by the impact of logistics, supplier delays and so on. The reason for assuming the Wiener process is because the probability distribution is based on the normal type or the lognormal type.

Assumption 3.1. *Two elements' model of the i-th process*

$$dh_i(t) = \mu_i h_i(t)dt + \sigma_i dW_i(t) \tag{11}$$

where $h_i(t)$, μ_i, σ_i and $W_i(t)$ denote two elements, average, volatility and Wiener process respectively.

In Equation (12) that is provided below, changes in the order size do not change the manufacturing lead time, i.e., the power factor B does not change. Here, the lead times are normalized by dividing them using the maximum lead time in the series. If we omit the subscripts i in Equation (12), we obtain the following.

$$dh(t) = \mu h(t)dt + \sigma dW(t) \tag{12}$$

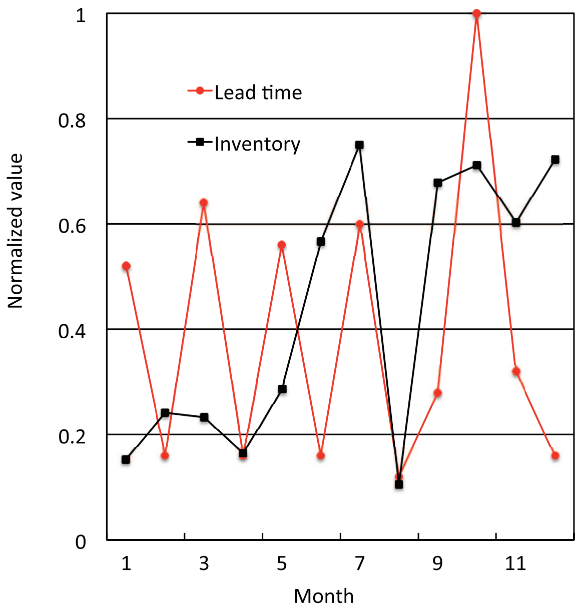

FIGURE 8. Annual normalized data transition of lead time and inventory (2007)

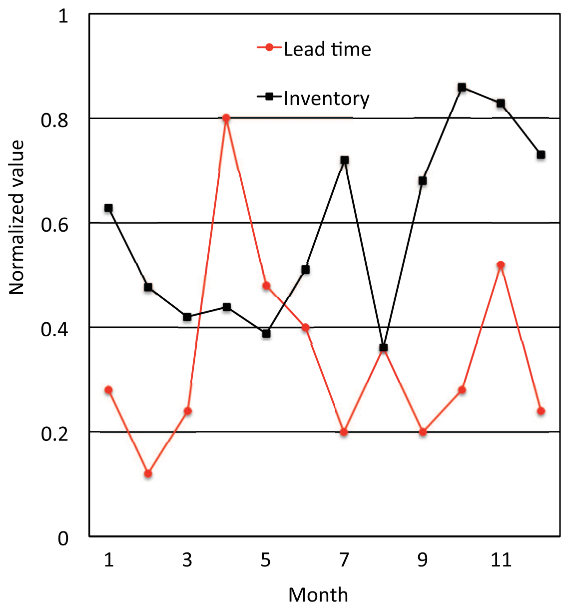

FIGURE 9. Annual normalized data transition of lead time and inventory (2008)

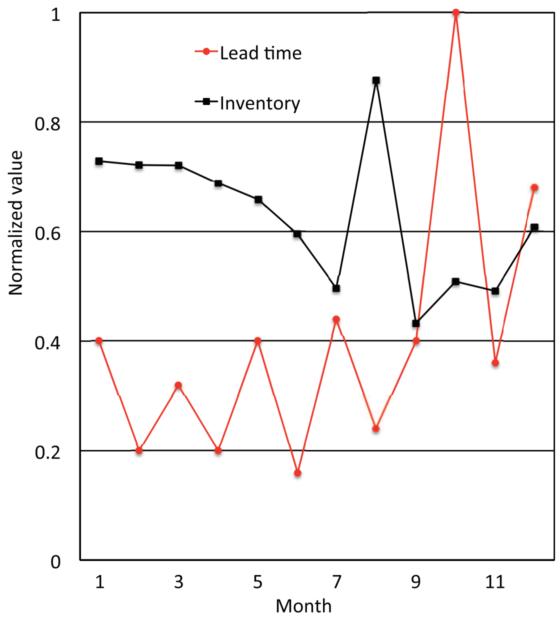

FIGURE 10. Annual normalized data transition of lead time and inventory (2009)

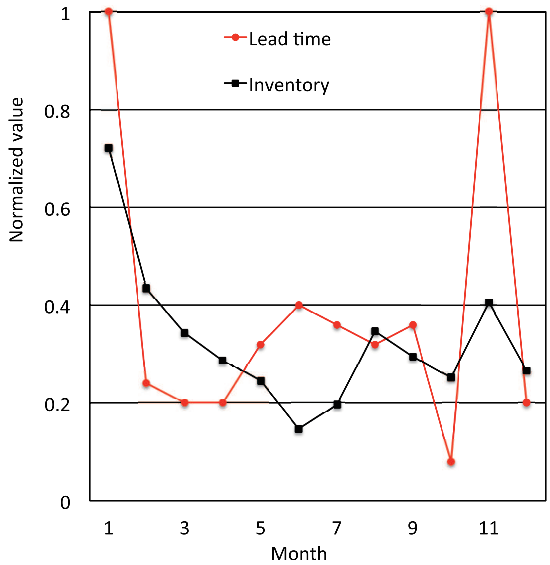

FIGURE 11. Annual normalized data transition of lead time and inventory (2010)

The solution of $h(t)$ is derived as follows:

$$h(t) = h_0 + \mu \int_0^t h(s)ds + \sigma \int_0^t h(s)dW(s) = h_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W(t)\right\} \tag{13}$$

At this time, let $h(t)$ be the minimum value.

Assumption 3.2.

$$h(t) \leq h_{\min} \tag{14}$$

where $h_{\min}$ is a minimum value.

The probability $P(h_{\min} \leq h(t))$ as $t \to \infty$ is as follows:

$$P(h_{\min} \leq h(t)) = 1 - \exp\left[-\left(1 - \frac{2\mu}{\sigma^2}\right)(h(t) - h_{\min})\right]$$
$$= 1 - \exp\left[-\zeta(h(t) - h_{\min})\right], \quad \zeta = 1 - \frac{2\mu}{\sigma^2} \tag{15}$$

From Equation (15), h converges to exponential distribution. As a result, we obtain as follows:

$$P(h_{\min} \leq h(t)) = \left(\frac{h(t)}{h_{\min}}\right)^{-\zeta} \tag{16}$$

The cumulative probability distribution $P(> |\Delta h|)$ for the absolute value of the deviation $|\Delta h|$ is as follows:

$$P(> |\Delta h|) = |\Delta h|^{-\zeta} \tag{17}$$

From actual data, ζ can be calculated as follows:

$$\zeta = 1 - \frac{2\mu}{\sigma^2} = 1 - \frac{0.732}{0.055} \approx -12.30 \tag{18}$$

4. **Power-Law Distributions for the Lead Time, Inventory, and RoRD.** We can use actual data to obtain the power-law distributions for the lead time, inventory, and RoRD. The time series where the items start together (in a batch) clearly depicts a similar type of log-normal distribution as sets of normalized lead time data.

4.1. **Power-law distributions for the lead time and inventory.** Figure 12 shows that a throughput is proportional to a rate of return in production processes. Then, we introduce the lead-time function so that we can analyze a production process [21]. The lead time of production equipment is proportional to the RoR. Therefore, we determined that the probability density function (PDF) of lead time was also the same PDF of RoR. The lead time function $f(y)$ is assumed to be a log-normal probability density function. Therefore, we can calculate the lead time by computing the continuous expected value, as depicted in Figure 13.

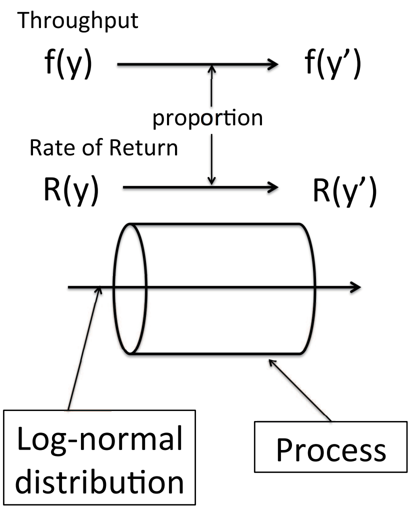

FIGURE 12. Throughput fluctuation in a process distribution amount

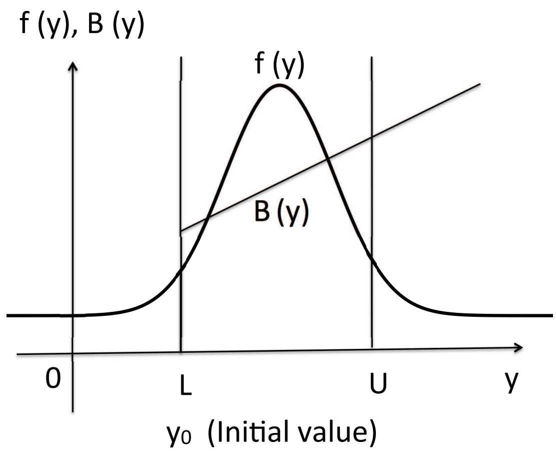

FIGURE 13. Lead time function $f(y)$ and loss function $B(y)$

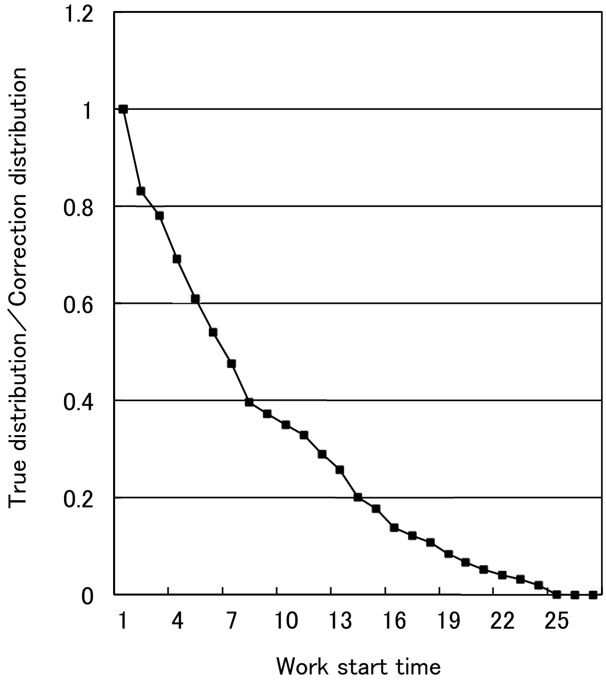

FIGURE 14. Power-law distribution of lead time and inventory from 2007 to 2008

Assumption 4.1. *Lead time function for a log-normal probability density function.*

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{-\frac{(\ln(y/y_0) - \mu)^2}{2\sigma^2}\right\} \tag{19}$$

where μ is an average value, σ is a volatility and y_0 is an initial lead time.

4.2. **Power-law distribution of RoRD.** Here, it is shown that data of RoRD of a certain control equipment manufacturing company collected by us conforms to power-law distribution. About that company, we calculated the return of 10 years from April, 1999 to March, 2008 on a month-by-month basis to calculate RoRD. The result is shown in Figure 15. Here, given that the return of the n-th month is S_n, a rate of return was defined by the following formula. From data for observed monthly RoRD, we calculate a probability density function (Figure 15) [14]. Results indicate that it conforms to a log-normal distribution (Figure 15, Theoretical). Our previous study provides further information.

$$f(x) = \frac{1}{\sqrt{2\pi}(x - k_p)\sigma_p} \exp\left\{-\frac{1}{2}\left(\frac{(\ln x - k_p) - m}{\sigma_p}\right)^2\right\} \tag{20}$$

where k_p is a displacement of x, σ_p is a volatility and m is an average.

$$D_{n+1} = \frac{S_{n+1} - S_n}{S_n} \tag{21}$$

where S_n, $n = 1, 2, \ldots$ indicates monthly return, and D_n indicates a monthly rate of return.

Using this rate of return, the following RoRD is defined.

Definition 4.1. *Definition of RoRD in Equation (22)*

$$\Delta D(n) \equiv D_{n+1} - D_n \tag{22}$$

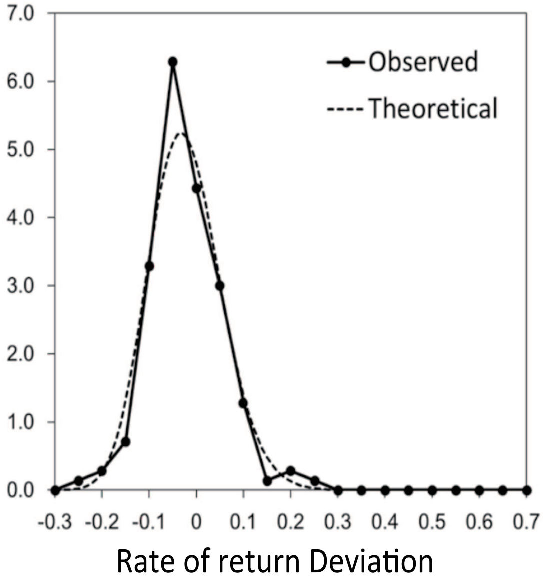

Figure 15. Probability density function of RoRD: actual data (solid line) and data based on theoretical formula (dotted line)

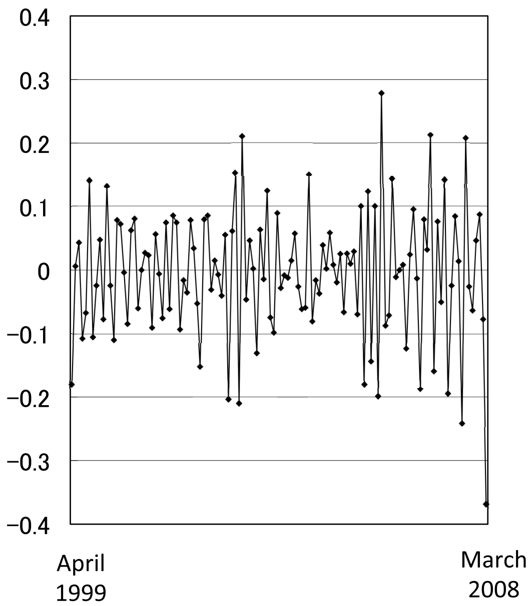

Figure 16. RoRD data

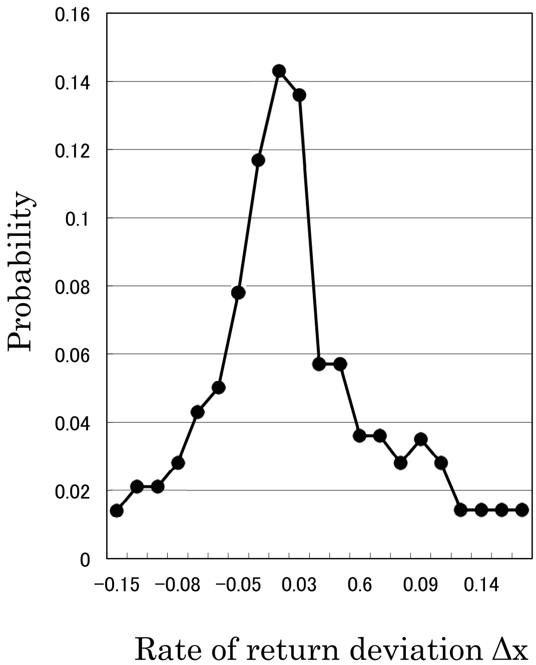

Figure 17. Probability density function of rate of return ΔD

RoRD ΔD can be considered as a random variable fluctuating on a monthly basis. As a result of examining distribution of an appearance frequency of this RoRD ΔD, the probability density function was as shown in Figure 17 and the distribution was as shown in Figure 18.

Here, the distribution was obtained as follows. A range within which ΔD varies is divided into a plurality of zones, and let $n(\Delta D, \Delta D + \delta D)$ be the number of pieces of data included in a zone of width $\delta D[\Delta D, \Delta D + \delta D]$. Assuming that the number of pieces of data is M, the probability density function $P(\Delta D)$ can be defined by the following

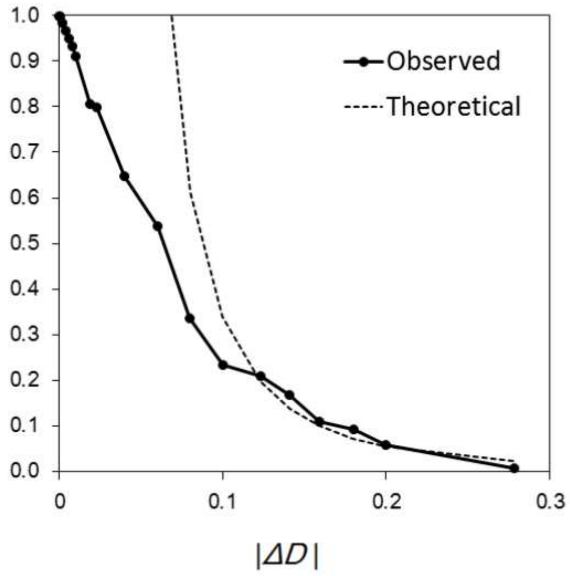

FIGURE 18. Cumulative distribution about ΔD

formula.

$$P(\Delta D) = \frac{n(\Delta D, \Delta D + \delta D)}{M \cdot \delta D} \tag{23}$$

In this paper, distribution $P(> \Delta D)$ is defined by the following formula.

$$P(> \Delta D) = \frac{N(\Delta D)}{M} = \int_{\Delta D}^{\infty} d\Delta D P(\Delta D) \tag{24}$$

where $N(\Delta D)$ is the rank order of ΔD. In stock price fluctuation models and the like, it is known that a skirt of distribution of this cumulative distribution $P(> |\Delta D)$ follows the following power-law distribution [23, 24].

$$P(> |\Delta D|) \propto |\Delta D|^{-\beta} \tag{25}$$

Therefore, as a result of performing fitting to the formula of power-law distribution using the observed data of $|\Delta D| > 0.1$ corresponding to a skirt of distribution by MS-Excel,

$$P(> |\Delta D|) = 0.0008|\Delta D|^{-2.63} \tag{26}$$

was obtained (Figure 18, Theoretical, $R^2 = 0.926$). As above, a fluctuation model of RoRD is of self-similarity, and it will show fractal nature [24, 25]. Also, this power-law distribution characteristic has fluctuation nature in phase transition [24, 25].

5. **On-Off Intermittency Verification by Actual Data.** The current business style is a complete make-to-order production system and the production process is a batch process. Figures 21 and 22 show the deviation of the lead time of a batch production standardization. C1, C2, D1, D2 in Figure 19 become C1', C2', D1', D2' by moving the work start time respectively. P1 in Figure 20 represents the movement of the working power to W3. P2 in Figure 20 represents the movement of the working power to W5.

Therefore, Figure 19 and Figure 20 show the production processes before and after on-off intermittency, respectively. Our strategy was to change the start time of the production and reduce the worker's variance, as shown in Figure 20. After reconstructing the process shown in Figure 20, we could handle sudden orders by appropriately managing the available manpower and prevented opportunity loss. As a result, we increased monthly shipments.

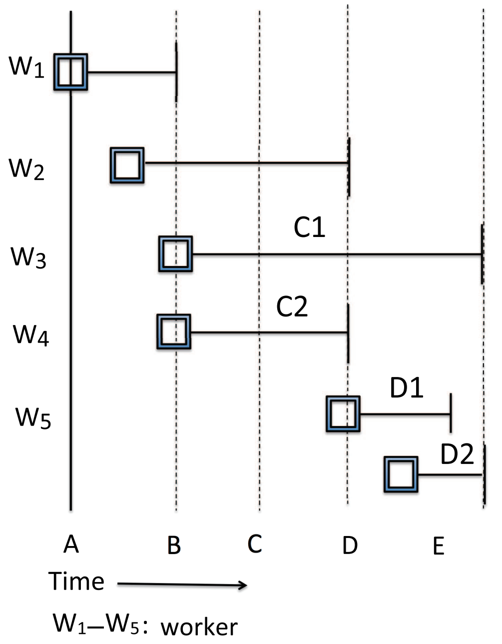

FIGURE 19. Process before managing of on-off intermittency

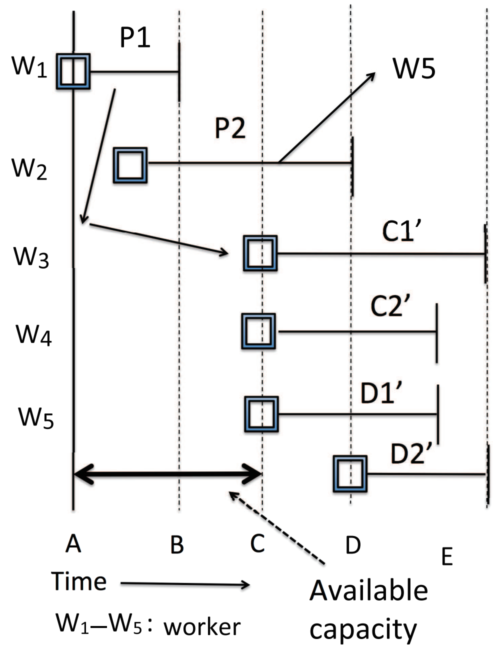

FIGURE 20. Process after managing of on-off intermittency

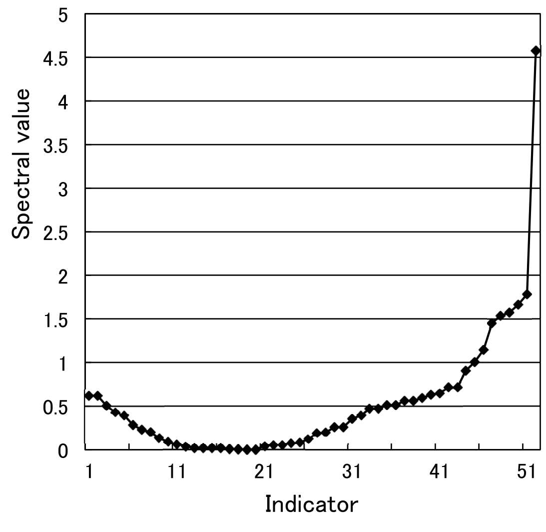

FIGURE 21. Fluctuation spectrum of the batch production normalized lead time and the inventory

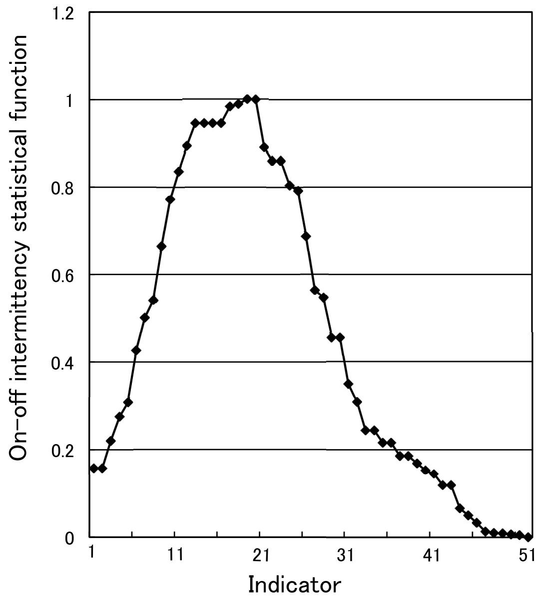

FIGURE 22. On-off intermittency statistical function $\exp(-t \cdot S(z))$ of the batch production normalized lead time and the inventory

From Table 1 and Table 2, no change was observed in the actual lead times. However, the variance is reduced. In the process diagram, the time allocated for the operator is the first half of the entire process. We could offer a sudden customer support for the production. As a result, the production throughput also was improved [26]. For further details please refer to our previous study [26]. We describe the summary as follows.

TABLE 1. Normalized lead time/variance before reclassification of production process

Lead time	Variance
9.67	0.236

TABLE 2. Normalized lead time/variance after reclassification of production process

Lead time	Variance
9.67	0.165

Definition 5.1. *On-off intermittency indicator*

$$h_T^n = \frac{\tilde{h}_T^n}{h_{LT}} \tag{27}$$

where h_{LT} is a longest lead time, and $\tilde{h}_T^n$ is the lead time of the n-th project.

The lead time spectrum is represented by

$$S\left(\tilde{z}_n\right) = \frac{1}{2}\left(\tilde{z}_n + \frac{1}{2}\right) \tag{28}$$

where $\mu = 1$, $\tilde{z}_n$ is the ordinal data of z_n, and represents a relative indicator. As a result, we obtain as follows [26]:

$$P\left(\tilde{z}_n, t\right) \approx \exp\left[-t \cdot S\left(\tilde{z}_n\right)\right] \tag{29}$$

From Figure 21 and Figure 22, we can see that the spectral fluctuation value at around time indicator = 20 is converging to the true average value. In other word, the lead time fluctuates around the average value. The fluctuation average data exists at around 20

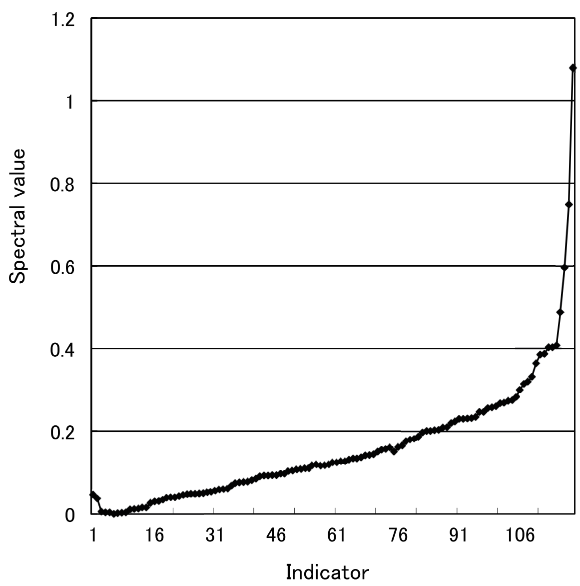

FIGURE 23. Fluctuation spectrum of RoRD

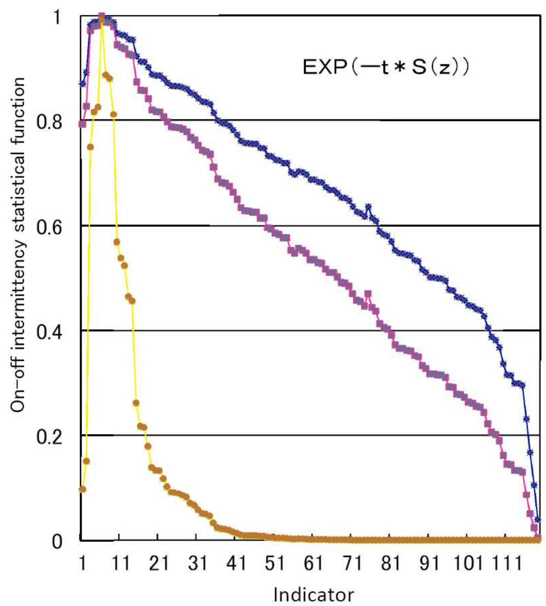

FIGURE 24. On-off intermittency statistical function $\exp(-t \cdot S(z))$ of RoRD

of time series outset in Figure 21 and Figure 22. Figure 23 and Figure 24 are similar indicators for on-off intermittency. We can see that the spectral fluctuation value at around time indicator = 7 is converging to the true average value.

6. **Conclusion.** This study has depicted that the lead time, inventory, and RoRD can be handled uniformly using the following definitions. Another significant contribution is that we have investigated the qualitative understanding that has been obtained using real data. Finally, we have depicted that these three elements exhibit on-off intermittency and fluctuations. With respect to the propagation between stages, we would like to propose a mathematical model that considers constraints on propagation on the upstream and downstream sides as a delay.

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Appendix A. Analysis of the Testrun Results.

- (Testrun1): Because the throughput of each process (S1-S6) is asynchronous, the overall process throughput is asynchronous. In Table 4, we list the manufacturing time (min) of each process. In Table 5, we list the volatility in each process performed by the workers. Finally, Table 4 lists the target times. The theoretical throughput is obtained as $3 \times 199 + 2 \times 15 = 627$ (min). In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. In Figure 25, we plot the measurement data listed in Table 4, which represents the total working time of each worker (K1-K9). In Figure 26, we plot the data contained in Table 4, which represents the volatility of the working times.
- (Testrun2): Set to synchronously process the throughput. The target time listed in Table 6 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 7 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Testrun3): Introduce a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 8, the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 9 presents the volatility of each working process (S1-S6) for each worker (K1-K9). On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput (T'_s) can be obtained using the "Synchronization with preprocess" method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20 \times 6 \text{ (First cycle)} + 17 \times 6 \text{ (Second cycle)} \\ &\quad + 20 \times 6 \text{ (Third cycle)} + 20 \text{ (Previous process)} + 8 \text{ (Idol-time)} \\ &\sim 370 \text{ (min)} \end{aligned} \tag{30}$$

  The full synchronous throughput in one stage (20 min) is

$$T'_s = 3 \times 120 + 40 = 400 \text{ (min)} \tag{31}$$

  Using the "Synchronization with preprocess" method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed "Synchronization with preprocess" method is realistic and can be applied in flow production systems. Below, we represent for a description of the "Synchronization with preprocess".

In Table 8, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time. Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{32}$$

where the throughput of the preprocess is set to 20 (min). In Equation (32), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) × 3 (cycles).

In Table 3, Testrun3 indicates a best value for the throughput in the three types of theoretical working time. Testrun2 is ideal production method. However, because it is difficult for talented worker, Testrun3 is a realistic method.

The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.
Testrun1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73,
Testrun2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92,
Testrun3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95.
Volatility data represent the average value of each Testrun.

Table 3. Correspondence between the table labels and the Testrun number

	Table Number	Production process	Working time	Volatility
Testrun1	Table 4	Asynchronous process	627 (min)	0.29
Testrun2	Table 6	Synchronous process	500 (min)	0.06
Testrun3	Table 8	"Synchronization with preprocess" method	470 (min)	0.03

Table 4. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 5. Volatility of Table 4

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

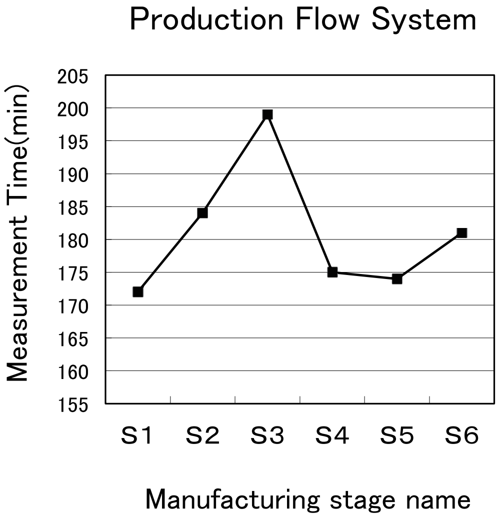

Figure 25. Total work time for each stage (S1-S6) in Table 4

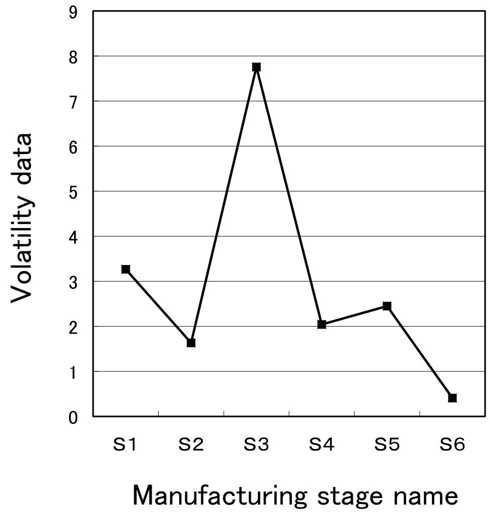

Figure 26. Volatility data for each stage (S1-S6) in Table 4

Table 6. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

Table 7. Volatility of Table 6

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

Table 8. Total manufacturing time at each stage for each worker, K5(*): Preprocess

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

Table 9. Volatility of Table 8, K5(*): Preprocess

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0