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DYNAMIC PROGRAMMING TECHNIQUE IN MULTI UAVS FORMATION ANOMALY DETECTION

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Abstract. *To deal with the problem of anomaly detection in multi UAVs formation, and simplify the complexity of hypothesis testing or probability inequalities, the anomaly detection problem can be transformed to identify some unknown parameters process. To avoid a statistical description on measurement noise, a worthwhile alternative is the bounded noise characterization. In the presence of bounded noise, the projection algorithm with dead zone and its modified form are proposed to identify the unknown parameters, such that the robustness of projection algorithm can be enhanced by increasing a dead zone. Furthermore, dynamic programming technique is introduced to balance the desire for lower present cost with the undesirability of high future cost in determining the anomaly detector, and then the cost of collecting new observations and the higher probability of accepting the wrong hypothesis can be compensated. A numerical example illustrates the characteristic of the anomaly detection problem.*
Keywords: Multi UAVs formation, Anomaly detection, Projection algorithm, Dynamic programming, Dead zone

1. **Introduction.** Unmanned aerial vehicles (UAVs) are used as efficient detection, attack and defense devices, and have demonstrated great superiority in our military field in recent years. To improve the detection and attack efficiency and extend the applied range of UAVs, one novel concept of multi UAVs formation is proposed in America. Multi UAVs formation is the use of many UAVs to fly in a special flight mode, not throughout the entire task completion. The aim of this formation flight is to maintain the relation position and attitude of each UAV, thus expanding the vision field, achieving the full ground or controlling a target, and improving the integration efficiency. Multi UAVs formation is the basis and prerequisite for the UAV mission. When formation task or battlefield environment changes with time, the entire multi UAVs formation needs to be adjusted. This adjustment is called formation autonomous reconfiguration. An overview about the auxiliary role of the multi UAVs formation trajectory design in a cooperative investigative process is given in [1].

Research regarding multi UAVs formation has primarily concentrated on formation control, formation aerodynamic coupling and formation communication positioning systems. These three categories can be further divided into trajectory planning, trajectory maintenance, trajectory control and autonomous reconfiguration. However, little research has been reported on the detection of multi UAVs formation anomalies. To the best of our knowledge, the problem of anomaly detection corresponding to multi UAVs formation

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flight is firstly proposed by the author. This problem lies in the judgement of the behavior and characteristics of a certain number of subsystems which differ from most other subsystems in large scale systems. The idea of anomaly detection is derived from statistical signal processing which solves the decision or hypothesis testing problem. Commonly after the application of some probability inequalities to compute probability levels for each hypothesis hold, then the hypothesis can be determined by comparing the respective probability level. M. Gevers and L. Ljung [2] introduce some statistical hypothesis testing methods including the cluster method, classification method, nearest neighbor method, information entropy method and the spectral analysis method. The cluster method is used for image compression sensing [3]. J. H. Braslavsky [4] considers the credibility of applying minimum variance control in Gaussian communication channels and the information entropy method is used to determine the probability of failure with the application of that minimum variance control strategy. In O. C. Imer and S. Yuksel [5], the spectral analysis method is proposed to judge the hypothesis testing probability regarding system stability when data drop information is added to the communication channel. In V. Gupta and D. Spanos [6], the optimal control of dynamical systems over unreliable communication links is considered, it is pointed out that when the number of hypotheses is greater than two, it becomes difficult to solve multi hypothesis testing problem, and the number of hypotheses will increase with the number of systems increasing [7]. Therefore, the process of anomaly detection is proposed here to use multi hypothesis testing to determine the statistical probability, which is employed to judge whether one certain system is abnormal. The anomaly detection more widely exists in the fault identification and credibility corresponding to some hypotheses.

Here in this paper we study the problem of multi UAVs formation anomaly detection. Firstly let us give an explicit description of this problem. This is a very relevant problem, and of highest interest for safety in aeronautics. Assume the formation flight consists of N UAVs, and these N UAVs show similar dynamics. In fact, airplanes are constantly gathering data and being monitored for this exact reason. Many data are collected through ground control station to supervise these N UAVs and enhance the flight safety of formation. Classical methods used to detect anomaly need the knowledge about the number of the abnormal UAVs k, and then from statistical signal processing theory [8], anomaly detection is achieved by one multi hypothesis testing in which k UAVs are chosen from a set of N UAVs, leading to a total of C_N^k possible hypotheses. Through comparing observed value and model predictive value, statistical testing operation is used to judge the hypothesis. However, here in order to avoid classical statistical knowledge, we propose to apply the system identification method to achieving the goal with no any knowledge of those N UAVs. This system identification method uses only the input and output data to achieve the anomaly detection of multi UAVs formation.

From flight dynamic analysis and the idea that each nonlinear function can be approximated by one appropriate linear function, one unknown linear relation is determined to describe the measurable quantities of the N UAVs. After choosing a certain period of time as the total number of observations, the linear input-output relations corresponding to N UAVs are obtained. These unknown linear relations show the relationship among the angle of attack, velocity, dynamic pressure, elevator deflection and the stabilizer deflection angle [9]. The concept behind this anomaly detection problem means that the difference of each UAV corresponds to the difference of one unknown parameter value in the unknown linear relation. By the use of this idea, the former anomaly detection problem is changed to identify these N unknown parameters using only input-output data, as each unknown parameter corresponds to each UAV. To solve this identification problem, the common maximum likelihood estimation is used to identify these N unknown parameters. Generally when the number of abnormal UAVs is known as k in prior, then those k parameters with large errors among N parameters are regarded as abnormal states, and other $N-k$ states are normal states. The above anomaly detection process is similar to data clustering. Due to that N UAVs have similar dynamics, then all N unknown parameters must be equal or nearly equal. After conducting cluster analysis with respect to these identified N parameters estimations the parameter estimations with errors in the admissible range are clustered as first class, while another second class includes the parameter estimations with large errors. This second class can be regarded as the abnormal states of UAVs. During the process of identifying the unknown parameters, the centralized or distributed strategy can be employed. As the concept of anomaly detection in multi UAVs formation is proposed originally by the author, the interested reader can refer to two main references [10,11]. [12] constructs one non convex sparse optimization problem to identify a linear unknown parameter vector, and further the optimum necessary condition and the fast gradient algorithm are respectively proposed to estimate the optimum values. Furthermore in [13], one unknown nonlinear relation is established to describe the measurable quantities, and then after expanding it by the basis functions, one bias compensated least squares method is applied to solving the unknown parameter vector. As the problem of anomaly detection for multi UAVs is not considered yet in research field, it is very necessary to continue this topic deeply based on our above papers.

Here in this paper, let us continue to study the anomaly detection in multi UAVs formation deeply on the basis of our two papers [10,11], where experimental data are usually obtained by means of measurement procedures that are known to be affected by noise. Most of the results available in [10,11] are based on a statistical description of the uncertainty or noise affecting the measurement data. As we all know the least squares method and maximum likelihood method are suited for identification accuracy under one condition that the considered noise may be a zero mean random signal that is statistically independent on the input. However, zero mean random white noise is an ideal case, and it will not exist in engineering. In addition, deriving statistical properties of noise is often very difficult in practice as it is usually not possible to measure noise directly. A worthwhile alternative to the statistical description is the so-called bounded error characterization where measurement noises are assumed to be unknown but bounded. Such a description may be more appropriate in many cases where a priori statistical information is not available. Based on the unknown but bounded description of the measurement noise, we investigate one projection algorithm with dead zone in the presence of bounded noise. This unknown but bounded noise is considered in set membership identification widely. The robustness of projection algorithm with dead zone can be enhanced by increasing a dead zone in the parameter update equation. Moreover, we turn to the main idea of dynamic programming that is proposed to consider a hypothesis testing problem in anomaly detection. In this dynamic programming technique, a decision maker can make observation, at a risk of cost each, relating to two hypotheses. Given a new measurement data, one can either accept one of hypotheses or delay the decision, pay the risk of cost, and obtain a measurement data. The reason of introducing dynamic programming into anomaly detection is that the risk of cost is more actual than theory, and the optimality of the decision policy can be guaranteed.

This paper is organized as follows. In Section 2, the problem of anomaly detection for multi UAVs formulation is formulated. In Section 3, based on the unknown but bounded description of the measurement noise, one projection algorithm with dead zone is investigated, and the robustness of projection algorithm with dead zone can be enhanced by increasing a dead zone in the parameter update equation. In Section 4, dynamic programming strategy is proposed to consider a hypothesis testing problem in anomaly detection, and further the main idea of using dynamic programming strategy is to compute the risk of decision corresponding to each choice policy by using probability inequalities, and determine those abnormal UAVs with lower risk. In Section 5, one simulation example illustrates the effectiveness of our proposed theories. Section 6 ends the paper with final conclusion.

2. **Problem Description.** Assume that formation flight system in Figure 1 includes N UAVs, and all UAVs exhibit similar dynamics.

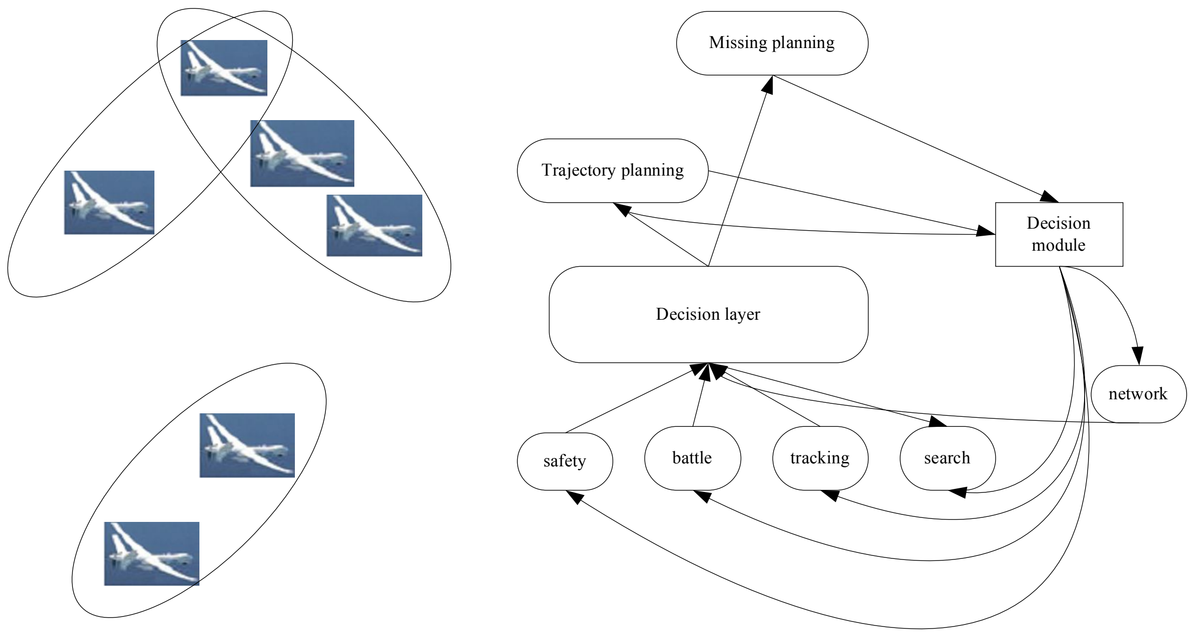

FIGURE 1. Structure of multi UAVs formation flight system

Using the stress analysis process of flight control systems [12], one nonlinear dynamical model of N UAVs is established. This nonlinear relation indicates that when force is applied to one UAV, the acceleration of each UAV is a function with respect to certain variables such as aircraft control surface position, velocity, dynamic pressure, thrust acting on UAV and other flight operation qualities. The nonlinear relation is expanded around one time instant by using a Taylor series expansion, and then a linear relation is obtained to approximate the former nonlinear relation by neglecting some high order terms. It means the following linear unknown relation holds.

$$y_i(t) = \varphi_i(t)^T\theta_i + e_i(t) \qquad i = 1, 2, \ldots, N \tag{1}$$

where t is time instant, i denotes one UAV, $y_i(t) \in R$ is the observed output at time instant t corresponding to UAV i, and $\varphi_i(t)$ is one regression vector at time instant t. θ_i is an unknown parameter with respect to UAV i, and $e_i(t)$ is the external noise. Consider each UAV i, $\{y_i(t), \varphi_i(t)\}_{t=1}^{M}$ denotes one data set collected by ground control station at time instant t, and M is the total number of time instant t. For one single flight, a number of sensors record values during flight, for example, an airspeed sensor for a single flight contains the speed of the aircraft over the entire flight. These recorded values are all included in data set $\{y_i(t), \varphi_i(t)\}_{t=1}^{M}$. Among these N UAVs, for $i = 1, 2, \ldots, N$, as all UAVs have similar dynamics and each UAV i corresponds to each parameter θ_i, then all of these N parameters must be equal to each other, i.e., it holds that

$$\theta_1 = \theta_2 = \cdots = \theta_N = \theta_0 \tag{2}$$

where θ_0 is a true or nominated parameter value, if Equation (2) holds, then all N UAVs are in normal states. On the contrary, if one parameter θ_j deviates from the nominated parameter θ_0, then UAV j is deemed as abnormal state. Due to the number of abnormal UAVs k is not known in prior, and the total number of possible choice of choosing k UAVs from a set of N UAVs leads to C_N^k.

$$C_N^k = \frac{N!}{(N-k)!k!}$$

From above description about anomaly detection problem, the main contribution of applying dynamic programming technique is to compute the risk of decision corresponding to each choice policy by using probability inequalities, and determine which k UAVs are in abnormal states with lower risk.

From Equations (1) and (2) given above, we see that after all unknown parameters θ_i, $i = 1, 2, \ldots, N$ are identified, that special nominated parameter value θ_0 can be given as the average value of these N parameters.

$$\theta_0 = \frac{\theta_1 + \theta_2 + \cdots + \theta_N}{N}$$

Also those UAVs with parameters deviated from the nominated parameter θ_0 are abnormal. The above anomaly detection problem description in multi UAVs formation can be solved by the maximum likelihood method [9], where noise $e_i(t)$ in Equation (1) is assumed to be one Gaussian white noise with zero mean and unknown variance.

3. **Projection Algorithm with Dead Zone.** As white noise is an ideal case, and it does not exist in engineering, the above assumption on noise $e_i(t)$ is very strict. To relax this strict assumption on noise, a worthwhile alternative is the bounded noise characterization. By using this bounded description of measurement noise, we propose one projection algorithm with dead zone and its modified form as the least squares algorithm with dead zone in the presence of bounded noise.

Consider again Equation (1), now noise $e_i(t)$ denotes a bounded noise term such that $sup|e_i(t)| \leq \Delta$, where Δ is a bound. Our goal is to identify each unknown parameter θ_i by using data set $\{y_i(t), \varphi_i(t)\}_{t=1}^{M}$ in the presence of bounded noise $e_i(t)$.

Introduce the following projection algorithm with dead zone to identify unknown parameter θ_i.

$$\hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \frac{a_i(t)\varphi_i(t)}{c_i + \varphi_i(t)^T\varphi_i(t)}\left[y_i(t) - \varphi_i(t)^T\hat{\theta}_i(t-1)\right] \tag{3}$$

where $\hat{\theta}_i(0)$ is given, $c_i > 0$ and

$$a_i(t) = \begin{cases} 1 & \left|y_i(t) - \varphi_i(t)^T\hat{\theta}_i(t-1)\right| > 2\Delta \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

When the prediction error is smaller than the size of the noise, the choice of $\{a_i(t)\}$ is to terminate the iterative algorithm. To give an explicit analysis on this projection algorithm with dead zone, we subtract the nominated parameter θ_o from both sides of (3) and give

$$\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_o$$

$$\tilde{\theta}_i(t) = \tilde{\theta}_i(t-1) - \frac{a_i(t)\varphi_i(t)}{c_i + \varphi_i(t)^T\varphi_i(t)}\left[\varphi_i(t)^T\tilde{\theta}_i(t-1) + e_i(t)\right] \tag{5}$$

Take square operation on both sides of (5) and note that $a_i(t) = 0$ or 1.

$$\begin{aligned}\left\|\tilde{\theta}_i(t)\right\|^2 &= \left\|\tilde{\theta}_i(t-1)\right\|^2 - \frac{2a_i(t)\left[w_i(t) - e_i(t)\right]w_i(t)}{c_i + \varphi_i(t)^T\varphi_i(t)} + \frac{a_i(t)^2\varphi_i(t)^T\varphi_i(t)w_i(t)^2}{\left[c_i + \varphi_i(t)^T\varphi_i(t)\right]^2} \\ &\le \left\|\tilde{\theta}_i(t-1)\right\|^2 + \frac{a_i(t)}{c_i + \varphi_i(t)^T\varphi_i(t)}\left[2w_i(t)e_i(t)\right] - \frac{a_i(t)w_i(t)^2}{c_i + \varphi_i(t)^T\varphi_i(t)}\end{aligned} \tag{6}$$

where $w_i(t)$ is model error, i.e.,

$$w_i(t) = y_i(t) - \varphi_i(t)^T\hat{\theta}_i(t-1) = -\varphi_i(t)^T\tilde{\theta}_i(t-1) + e_i(t) \tag{7}$$

Due to inequality $2ab \le ka^2 + \frac{b^2}{k}$, for any k, then

$$\begin{aligned}\left\|\tilde{\theta}_i(t)\right\|^2 &= \left\|\tilde{\theta}_i(t-1)\right\|^2 - \frac{a_i(t)w_i(t)^2}{c_i + \varphi_i(t)^T\varphi_i(t)} + \frac{a_i(t) + \varphi_i(t)^T\varphi_i(t)w_i(t)^2}{c_i + \varphi_i(t)^T\varphi_i(t)}\left[\frac{w_i(t)^2}{2} + 2w_i(t)^2\right] \\ &\le \left\|\tilde{\theta}_i(t-1)\right\|^2 - \frac{1}{2}\frac{a_i(t)w_i(t)^2}{c_i + \varphi_i(t)^T\varphi_i(t)} + \frac{2a_i(t) + \Delta^2}{c_i + \varphi_i(t)^T\varphi_i(t)}\end{aligned} \tag{8}$$

From (8), $\left\{\left\|\tilde{\theta}_i(t)\right\|^2\right\}$ is a non increasing sequence bounded below by zero, which results in Property 3.1.

Property 3.1.

$$\left\|\hat{\theta}_i(t) - \theta_0\right\| \le \left\|\hat{\theta}_i(t-1) - \theta_0\right\| \le \left\|\hat{\theta}_i(0) - \theta_0\right\|, \quad t \ge 1 \tag{9}$$

Combining Equations (3) and (7), we note that

$$\hat{\theta}_i(t) - \hat{\theta}_i(t-1) = \frac{-a_i(t)\varphi_i(t)}{c_i + \varphi_i(t)^T\varphi_i(t)}w_i(t) \tag{10}$$

Hence, square on both sides of (10)

$$\left\|\hat{\theta}_i(t) - \hat{\theta}_i(t-1)\right\|^2 = \frac{a_i(t)^2\varphi_i(t)^T\varphi_i(t)}{\left[c_i + \varphi_i(t)^T\varphi_i(t)\right]^2}w_i(t)^2 \tag{11}$$

Using the following inequalities

$$\limsup_{t\to\infty} \frac{a_i(t)w_i(t)^2}{c_i + \varphi_i(t)^T\varphi_i(t)} \le \frac{4\Delta^2}{c_i} \tag{12}$$

or

$$\limsup_{t\to\infty} \frac{a_i(t)\left[c_i + \varphi_i(t)^T\varphi_i(t)\right]w_i(t)^2}{\left[c_i + \varphi_i(t)^T\varphi_i(t)\right]^2} \le \frac{4\Delta^2}{c_i} \tag{13}$$

then

$$\limsup_{t\to\infty} \frac{a_i(t)\varphi_i(t)^T\varphi_i(t)w_i(t)^2}{\left[c_i + \varphi_i(t)^T\varphi_i(t)\right]^2} \le \frac{4\Delta^2}{c_i} \tag{14}$$

Substituting (14) into (11) and squaring on both sides, then Property 3.2 is obtained.

Property 3.2.

$$\limsup_{t\to\infty} \left\|\hat{\theta}_i(t) - \hat{\theta}_i(t-1)\right\| \le \frac{2\Delta}{\sqrt{c_i}} \tag{15}$$

From these two properties, parameter estimator will converge to its true value with the number of iterations increasing, and the approximate error between two adjacent parameter estimators is dependent of that bounded value corresponding to uncertain regions Δ.

It is straightforward to extend the projection algorithm with dead zone to the case of the least squares algorithm with dead zone.

$$\begin{aligned}\hat{\theta}_i(t) &= \hat{\theta}_i(t-1) + \frac{a_i(t)P_i(t-2)\varphi_i(t)}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)}\left[y_i(t)-\varphi_i(t)^T\hat{\theta}_i(t-1)\right] \\ P_i(t-1) &= P_i(t-2) - \frac{a_i(t)P_i(t-2)\varphi_i(t)^T\varphi_i(t)P_i(t-2)}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)}\end{aligned} \tag{16}$$

where initial iterative value $\hat{\theta}_i(0)$ is given, $P_i(-1) = P_0 > 0$, and

$$a_i(t) = \begin{cases} 1 & \dfrac{\left|y_i(t)-\varphi_i(t)^T\hat{\theta}_i(t-1)\right|^2}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)} > \Delta^2 > 0 \\ 0 & \text{otherwise}\end{cases} \tag{17}$$

The analysis of the least squares algorithm with dead zone (16) is similar to the projection algorithm with dead zone (3), and we only give one property of (16).

Property 3.3.

$$\lim_{t\to\infty}\sup\frac{w_i(t)^2}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)} \le \Delta^2 \tag{18}$$

Proof: We define

$$\begin{aligned}w_i(t) &= y_i(t)-\varphi_i(t)^T\hat{\theta}_i(t-1) = -\varphi_i(t)^T\tilde{\theta}_i(t-1)+e_i(t) \\ \tilde{\theta}_i(t-1) &= \hat{\theta}_i(t-1)-\theta_o\end{aligned} \tag{19}$$

Subtracting θ_0 from both sides of (16), we obtain

$$\hat{\theta}_i(t)-\theta_0 = \hat{\theta}_i(t-1)-\theta_0 - \frac{a_i(t)P_i(t-2)\varphi_i(t)}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)}\varphi_i(t)^T\tilde{\theta}_i(t-1) \tag{20}$$

Using the definition of (19)

$$\tilde{\theta}_i(t) = \tilde{\theta}_i(t-1) - \frac{a_i(t)P_i(t-2)\varphi_i(t)\varphi_i(t)^T\tilde{\theta}_i(t-1)}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)} \tag{21}$$

Using the recursive relation of $P_i(t-2)$ and the matrix inversion lemma, we have

$$\begin{aligned}\tilde{\theta}_i(t) &= \tilde{\theta}_i(t-1)\left[1-\frac{a_i(t)P_i(t-2)\varphi_i(t)\varphi_i(t)^T}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)}\right] \\ &= \left[P_i(t-2)-\frac{a_i(t)P_i(t-2)\varphi_i(t)\varphi_i(t)^TP_i(t-2)}{1+a_i(t)\varphi_i(t)^TP_i(t-2)\varphi_i(t)}\right]\times P_i(t-2)^{-1}\tilde{\theta}_i(t-1)\end{aligned} \tag{22}$$

Applying the recursive relation

$$\begin{aligned}P_i(t-1)P_i(t-2)^{-1} &= \left[P_i(t-2)^{-1}+a_i(t)\varphi_i(t)\varphi_i(t)^T\right]^{-1}P_i(t-2)^{-1} \\ &= \left(P_i(t-2)\left[P_i(t-2)^{-1}+a_i(t)\varphi_i(t)\varphi_i(t)^T\right]\right)^{-1} \\ &= \left[1+P_i(t-2)a_i(t)\varphi_i(t)\varphi_i(t)^T\right]^{-1}\end{aligned} \tag{23}$$

Substituting (23) and the definition of $P_i(t-1)$ into (22), we get

$$\tilde{\theta}_i(t) = P_i(t-1)P_i(t-2)^{-1}\tilde{\theta}_i(t-1) \tag{24}$$

Now defining $V(t) = \tilde{\theta}_i(t)P_i(t-1)^{-1}\tilde{\theta}_i(t)$, we have

$$\begin{aligned} V(t) - V(t-1) &= \left[\tilde{\theta}_i(t) - \tilde{\theta}_i(t-1)\right]^T P_i(t-2)^{-1}\tilde{\theta}_i(t-1) \\ &= -\frac{\tilde{\theta}_i(t-1)^T a_i(t)P_i(t-2)\varphi_i(t)\varphi_i(t)^T P_i(t-2)^{-1}\tilde{\theta}_i(t-1)}{1 + a_i(t)\varphi_i(t)^T P_i(t-2)\varphi_i(t)} \\ &= -\frac{\tilde{\theta}_i(t-1)^T a_i(t)\varphi_i(t)\varphi_i(t)^T\tilde{\theta}_i(t-1)}{1 + a_i(t)\varphi_i(t)^T P_i(t-2)\varphi_i(t)} \end{aligned} \tag{25}$$

Then $V(t)$ is a non-negative, non-increasing function.

$$\tilde{\theta}_i(t)^T P_i(t-1)\tilde{\theta}_i(t) \leq \tilde{\theta}_i(0)^T P_i(-1)\tilde{\theta}_i(0) \tag{26}$$

Using the matrix inversion lemma again

$$P_i(t)^{-1} = P_i(t-1)^{-1} + a_i(t)\varphi_i(t)\varphi_i(t)^T \tag{27}$$

we have that

$$\lambda_{\min}\left[P_i(t)^{-1}\right] \geq \lambda_{\min}\left[P_i(t-1)^{-1}\right] \geq \lambda_{\min}\left[P_i(-1)^{-1}\right] \tag{28}$$

Equation (28) implies that

$$\begin{aligned} &\lambda_{\min}\left[P_i(-1)^{-1}\right]\left\|\tilde{\theta}_i(t)\right\|^2 \leq \lambda_{\min}\left[P_i(t-1)^{-1}\right]\left\|\tilde{\theta}_i(t)\right\|^2 \\ &\leq \tilde{\theta}_i(0)^T P_i(-1)^{-1}\tilde{\theta}_i(0) \leq \lambda_{\min}\left[P_i(-1)^{-1}\right]\left\|\tilde{\theta}_i(0)\right\|^2 \end{aligned} \tag{29}$$

From Equation (29), we see

$$\left\|\tilde{\theta}_i(t)\right\|^2 \leq \frac{\lambda_{\max}\left[P_i(-1)^{-1}\right]}{\lambda_{\min}\left[P_i(-1)^{-1}\right]}\left\|\tilde{\theta}_i(0)\right\|^2$$

It means that

$$\left\|\hat{\theta}_i(t) - \theta_0\right\|^2 \leq \frac{\lambda_{\max}\left[P_i(-1)^{-1}\right]}{\lambda_{\min}\left[P_i(-1)^{-1}\right]}\left\|\hat{\theta}_i(0) - \theta_0\right\|^2 \tag{30}$$

Summing from 1 to M on Equation (25)

$$\begin{aligned} V(M) &= V(0) - \sum_{t=1}^{M}\frac{\tilde{\theta}_i(t-1)^T a_i(t)\varphi_i(t)\varphi_i(t)^T\tilde{\theta}_i(t-1)}{1 + a_i(t)\varphi_i(t)^T P_i(t-2)\varphi_i(t)} \\ &= V(0) - \sum_{t=1}^{M}\frac{a_i(t)w_i(t)^2}{1 + a_i(t)\varphi_i(t)^T P_i(t-2)\varphi_i(t)} \end{aligned} \tag{31}$$

Since $V(M)$ is nonnegative, we have

$$\lim_{M\to\infty}\sum_{t=1}^{M}\frac{a_i(t)w_i(t)^2}{1 + a_i(t)\varphi_i(t)^T P_i(t-2)\varphi_i(t)} < \infty \tag{32}$$

Combine Equations (28) and (32) to obtain Property 3.3 in (18).

In summary, this section proposes the projection algorithm with dead zone and least squares algorithm with dead zone to identify the unknown parameters θ_i, $i = 1, 2, \ldots, N$ in the presence of bounded noise. Three properties of these two algorithms are analyzed and applied in other adaptive control theories, too. After identifying these N parameters, the anomaly detection problem can be determined by comparing or clustering these N parameters. From above description, we see that the problem of anomaly detection for multi UAVs formation is to identify unknown parameters. However, how to measure the accuracy of these identified parameters, finite sample properties of these identified parameters can be used to achieve this goal. However, finite sample properties need strong probabilistic analysis. So in order to relax this requirement on probabilistic analysis, dynamic programming techniques are used to measure the accuracy of the above anomaly detection step.

4. **Dynamic Programming Techniques in Anomaly Detection.** Dynamic programming deals with situations where decisions are made in stages. The outcome of each decision may not be fully predictable but can be anticipated to some extent before the next decision is made. A key aspect of such situations is that decisions cannot be viewed in isolation since one must balance the desire for lower present cost with the undesirability of high future cost. The dynamic programming technique captures this tradeoff. At each stage, it ranks decisions based on the sum of the present cost and the expected future cost, assuming optimal decision making for subsequent stage. Here in this section, dynamic programming technique is introduced in the anomaly detection process in multi UAVs formation.

In author's paper [10], a new anomaly detector is summarized as a hypothesis testing problem. Given the residual $\varepsilon_i(t)$ at time instant t, that new anomaly detector is defined as

$$D_0(\varepsilon_i(t)) = \begin{cases} 1 & \|\varepsilon_i(t)\|^2 \geq T_0 \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

where 1 corresponds to the abnormal state, 0 is the normal state, and T_0 is the chosen threshold. However, in above anomaly detector D_0, new observations are neglected and a decision may pay a cost or risk to accept one of these two hypotheses. So there exists a compromise between the cost of new observations and the higher probability of accepting the wrong hypothesis.

Let $y_i(1), y_i(2), \ldots, y_i(M)$ be the sequence of observations. Based on (1) and (2), the mean value corresponding to each parameter θ_i is nominated value θ_0 or its identified value $\hat{\theta}_i$. Then the probability distribution of observation $y_i(t)$ is either f_0 or f_1, and we are trying to decide on one of these.

$$\begin{cases} f_0 = \dfrac{1}{\sqrt{2\pi\sigma}} \exp\left[-\dfrac{\left(y_i(t) - \varphi_i(t)^T\theta_0\right)^2}{2\sigma} \right] \\ f_1 = \dfrac{1}{\sqrt{2\pi\sigma}} \exp\left[-\dfrac{\left(y_i(t) - \varphi_i(t)^T\hat{\theta}_i\right)^2}{2\sigma} \right] \end{cases} \tag{34}$$

where σ is the variance of noise $e_i(t)$, here for the sake of simplicity, we give a statistical description on noise $e_i(t)$ not its bounded characterization. In case of bounded noise, dynamic programming technique in anomaly detection is the topic of our next paper. For any observation $y_i(t)$, f_0 and f_1 denote the probabilities of $y_i(t)$, when f_0 and f_1 are true distributions, respectively. At time instant t, using observations $y_i(1), y_i(2), \ldots, y_i(t)$, we may either stop collecting new observation and accept either f_0 or f_1, or we may collect new observations at cost $C > 0$. If we stop collecting and make a choice, then we obtain zero cost when our choice is correct, and cost L_0 and L_1 if we choose incorrectly f_0 and f_1. Assume a priori probability that the true distribution is f_0 is p.

Then we encounter one hypothesis testing problem with two states

$$\begin{cases} x_0 & \text{true distribution is } f_0 \\ x_1 & \text{true distribution is } f_1 \end{cases} \tag{35}$$

One conditional probability from dynamic programming algorithm is given as

$$p_t = P(x_t = x_0 | y_i(1), y_i(2), \ldots, y_i(t)) \tag{36}$$

From probability theory [12], that conditional probability p_t can be generated recursively as

$$\begin{aligned} p_0 &= \frac{p f_0(y_i(1))}{p f_0(y_i(1)) + (1-p) f_1(y_i(1))} \\ p_{t+1} &= \frac{p_t f_0(y_i(t+1))}{p_t f_0(y_i(t+1)) + (1-p_t) f_1(y_i(t+1))} \\ t &= 1, 2, \ldots\ M-1 \end{aligned} \tag{37}$$

Then the optimal expected cost for the last observation $y_i(M)$ is

$$J_M(p_M) = \min[(1-p_M)L_0, p_M L_1] \tag{38}$$

where $(1-p_M)L_0$ is the expected cost for accepting f_0, and $p_M L_1$ is the expected cost for accepting f_1. Using Equation (37), the optimal expected cost for time instant t is given as

$$\begin{aligned} J_t(p_t) = \min \Bigg[& (1-p_t)L_0, p_t L_1, \\ & C + E_{y_i(t+1)} \left\{ J_{t+1} \left(\frac{p_t f_0(y_i(t+1))}{p_t f_0(y_i(t+1)) + (1-p_t) f_1(y_i(t+1))} \right) \right\} \Bigg] \end{aligned} \tag{39}$$

where expectation operation over $y_i(t+1)$ is taken with respect to the probability distribution.

$$p(y_i(t+1)) = p_t f_0(y_i(t+1)) + (1-p_t) f_1(y_i(t+1))$$

For $t = 1, 2, \ldots, M$, we have

$$\begin{cases} J_t(p_t) = \min\left[(1-p_t)L_0, p_t L_1, C + A_t(p_t)\right] \\ A_t(p_t) = E_{y_i(t+1)} \left\{ J_{t+1} \left(\dfrac{p_t f_0(y_i(t+1))}{p_t f_0(y_i(t+1)) + (1-p_t) f_1(y_i(t+1))} \right) \right\} \end{cases} \tag{40}$$

An optimal policy for the last time instant is obtained from the minimization.

$$\begin{cases} \text{accept } f_0 \text{ if } p_M \geq \gamma \\ \text{accept } f_1 \text{ if } p_M < \gamma \end{cases}$$

where γ is determined as the relation

$$\gamma = \frac{L_0}{L_0 + L_1}$$

From convex optimization theory [13], function $A_t(p_t)$ is concave, and satisfying for all t, and $p \in [0, 1]$.

$$\begin{cases} A_t(0) = A_t(1) = 0 \\ A_{t-1}(p) \leq A_t(p) \end{cases} \tag{41}$$

Using the concave property of function $A_t(p_t)$, for all $p \in [0, 1]$, we have

$$J_{M-1}(p) \leq \min[(1-p)L_0, pL_1] = J_M(p)$$

Applying the stationary of the system and the monotonicity property [14], we obtain

$$J_t(p) \leq J_{t+1}(p)\ \forall\ t \text{ and } p \in [0, 1]$$

Using Equation (40), for all t, and $p \in [0, 1]$, we have

$$A_{t+1}(p) \leq A_t(p)$$

From Equation (41), we obtain that if

$$C + A_{M-1}\left(\frac{L_0}{L_0+L_1}\right) < \frac{L_0L_1}{L_0+L_1}$$

then an optimal policy for each time instant t is given as

$$\begin{cases} \text{accept } f_0 \text{ if } p_t \geq \alpha_t \\ \text{accept } f_1 \text{ if } p_M < \beta_t \\ \text{continue new observations if } \beta_t < p_t < \alpha_t \end{cases}$$

where scalars α_t, β_t are determined as

$$\begin{cases} \beta_t L_1 = C + A_t(\beta_t) \\ (1-\alpha_t)L_0 = C + A_t(\alpha_t) \end{cases}$$

We have monotonicity inequalities

$$\cdots \leq \alpha_{t+1} \leq \alpha_t \leq \alpha_{t-1} \leq \cdots \leq 1 - \frac{C}{L_0}$$
$$\cdots \geq \beta_{t+1} \geq \beta_t \geq \beta_{t-1} \geq \cdots \geq \frac{C}{L_1}$$

Then as $M \to \infty$, the sequences $\{\alpha_t\}$, $\{\beta_t\}$ converge to their limit values $\overline{\alpha}$, $\overline{\beta}$ respectively, and the optimal policy is approximated by the stationary policy.

$$\begin{cases} \text{accept } f_0 \text{ if } p_t \geq \overline{\alpha} \\ \text{accept } f_1 \text{ if } p_t < \overline{\beta} \\ \text{continue new observations if } \overline{\beta} < p_t < \overline{\alpha} \end{cases}$$

Now the conditional probability p_t is obtained as

$$\begin{aligned} p_t &= \frac{pf_0(y_i(1))f_0(y_i(2))\cdots f_0(y_i(t))}{*} \\ * &= pf_0(y_i(1))f_0(y_i(2))\cdots f_0(y_i(t)) + (1-p)f_1(y_i(1))f_1(y_i(2))\cdots f_1(y_i(t)) \end{aligned} \tag{42}$$

From Equation (42), the stationary policy can be reformulated as

$$\begin{cases} \text{accept } f_0 \text{ if } R_t \geq A \\ \text{accept } f_1 \text{ if } R_t < B \\ \text{continue new observations if } B < R_t < A \end{cases} \tag{43}$$

where

$$A = \frac{(1-p)\overline{\alpha}}{p(1-\overline{\alpha})}, \quad B = \frac{(1-p)\overline{\beta}}{p\left(1-\overline{\beta}\right)}, \quad R_t = \frac{f_0(y_i(1))f_0(y_i(2))\cdots f_0(y_i(t))}{f_1(y_i(1))f_1(y_i(2))\cdots f_1(y_i(t))}$$

And R_t can be generated by means of its recursive form.

$$R_{t+1} = \frac{f_0(y_i(t+1))}{f_1(y_i(t+1))}R_t$$

The optimal policy (43) is known as the sequential probability ratio test and used to replace that anomaly detector in [10]. The advantage of our stationary policy is that here the cost of collecting new observations and the higher probability of accepting the wrong hypothesis are all taken into account simultaneously.

5. **Simulation Example.** Consider the multi UAVs formation flight process and the flight performance data corresponding to each UAV can be collected in ground control station to construct database. One linear unknown relation can be set up by using flight data. The whole flow of information with respect to the ground control station is seen in Figure 2.

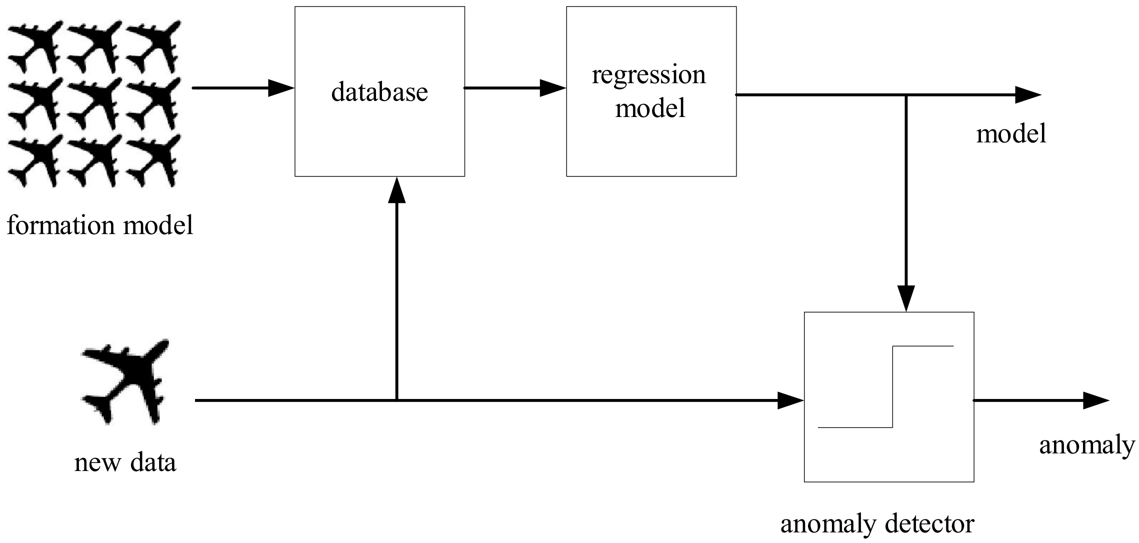

FIGURE 2. The information flow in formation flight

In Figure 2, the formation flight data are stored in database, and then one linear regression model is constructed through the database. This model uses different unknown parameters to represent different UAVs. Based on this linear regression model with respect to unknown parameters, we apply the projection algorithm with dead zone to solving all unknown parameters.

Assume that there are nine UAVs, where three UAVs are known as abnormal in prior. The number of collected measurements is 500, i.e., in Equation (1), it holds that

$$N = 9,\ k = 3,\ M = 500$$

According to the ith UAV at time instant t, $y_i(t)$ denotes the angle of attack, $\varphi_i(t)$ is one regression vector consisted by velocity, dynamic pressure, elevator deflection and the stabilizer deflection angle, and measurement noise $e_i(t)$ is a bounded noise, where the upper bound is 0.5, i.e.,

$$|e_i(t)| \leq 0.5$$

Further let the mean and variance corresponding to nominated parameter value θ_0 in normal formation flight be that

$$\overline{\theta_0} = \begin{bmatrix} 0.8 \\ -3 \\ -0.6 \\ 0.5 \end{bmatrix} . \Sigma_0 = \begin{bmatrix} 0.04 & 0.12 & 0.02 & 0.02 \\ 0.12 & 0.08 & 0.08 & 0.01 \\ 0.02 & 0.09 & 0.03 & 0.1 \\ 0.02 & 0.2 & 0.1 & 0.05 \end{bmatrix}$$

Then the nominated parameter θ_0 is a stochastic variable

$$\theta_0 \sim N\left(\overline{\theta_0}, \Sigma_0\right)$$

That regression vector $\varphi_i(t)$ is also a stochastic variable

$$\varphi_i(t) \sim N\left(\overline{\varphi}, \Sigma_\varphi\right)$$

where

$$\overline{\varphi} = \begin{bmatrix} 0.9 \\ -1.2 \\ -2.8 \\ 0.7 \end{bmatrix} . \Sigma_{\varphi} = \begin{bmatrix} 0.25 & -0.02 & 0.12 & -0.04 \\ -0.02 & 0.45 & 0.03 & -0.05 \\ 0.12 & 0.03 & 0.9 & -1.2 \\ -0.04 & -0.52 & -1.2 & 3 \end{bmatrix}$$

Substituting above data information into Equations (1) and (3) and applying our projection algorithm with dead zone to solving these nine unknown parameters, the parameter estimation errors among these nine parameter estimators $\{\theta_i\}_{i=1}^{N}$ and their nominated value $\overline{\theta_0}$ is plotted in Figure 3. From Figure 3, we see that the parameter error between the first parameter θ_1 and nominated value $\overline{\theta_0}$ converges to zero, and this result also holds for other parameters such as θ_3, θ_4, θ_5, θ_7, θ_8. Then those six UAVs corresponding to these six parameters are normal. Meanwhile, those three parameters $(\theta_2, \theta_6, \theta_9)$ deviate from nominated value $\overline{\theta_0}$, and it means that these three UAVs are regarded as abnormal state. In Figure 4, the parameter estimations corresponding to the first UAV are given. From the iterative curves in Figure 4, we see that these identified parameters will all converge to their own true values, i.e., the estimation errors between parameter estimators and their nominated value will be zero. Then we can conclude that the first UAV is normal. The similar analysis can be expanded for other UAVs in the whole formation flight.

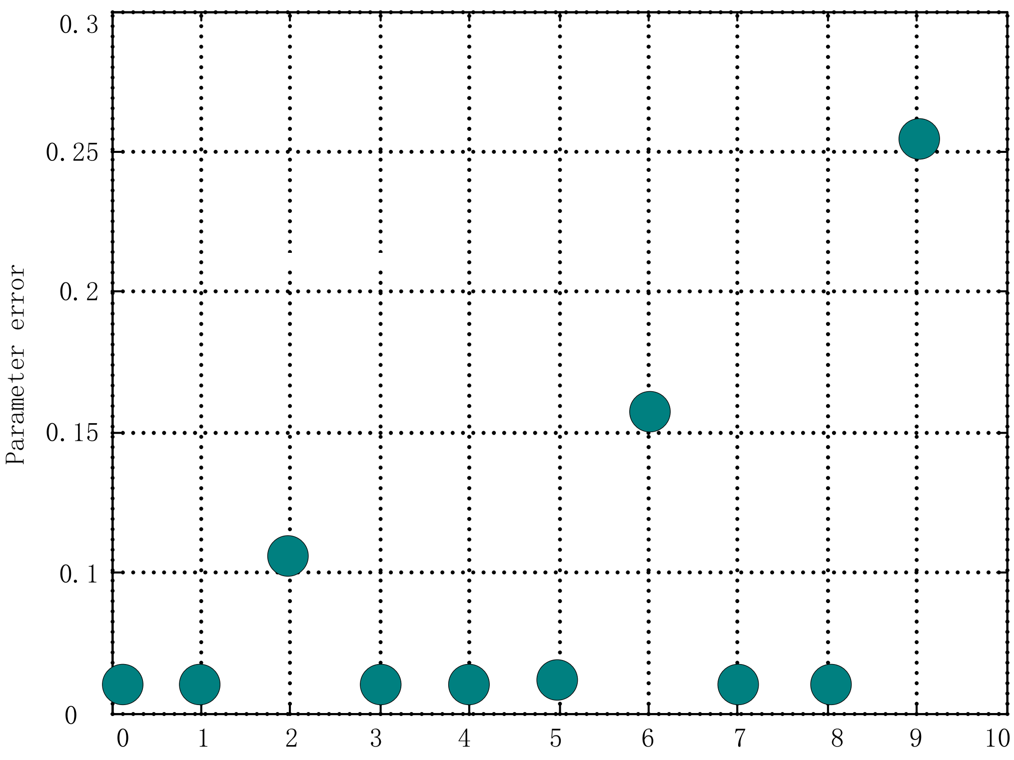

Figure 3. Parameter estimation errors among parameter estimators and their nominated values

6. **Conclusion.** This paper transforms an anomaly detection problem of multi UAVs formation into a problem of linear parameter identification. To relax strict statistical assumption on measurement noise in identifying unknown parameters, here the measurement noise is assumed to be unknown but bounded description. Then one projection algorithm with dead zone and its modified form are proposed to identify those unknown

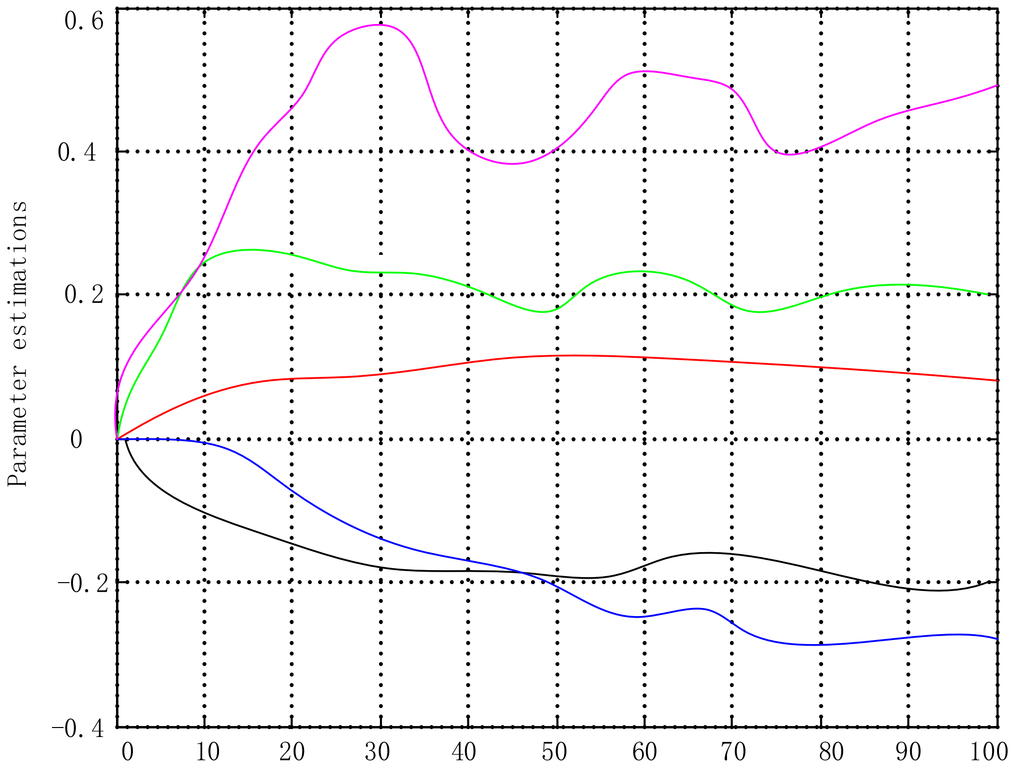

FIGURE 4. Parameter estimations

parameters. To further study the anomaly detection problem, and take account of the cost of collecting new observations and the higher probability of accepting the wrong hypothesis, the dynamic programming technique is introduced to balance the desire for lower present cost with the undesirability of high future cost. However, dynamic programming technique is applied in anomaly detection of multi UAVs formation in the presence of a statistical description of measurement noise, so that how to use dynamic programming technique in case of bounded noise is our subject for the next paper.

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