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A SELF-REPAIRING FUNCTION EXPLOITING RESONANCE FOR HIGH-GAIN ADAPTIVE CONTROL WITH FAULTY SENSORS

Masanori Takahashi

Department of Electrical Engineering and Computer Science
Tokai University
9-1-1 Toroku, Higashi-ku, Kumamoto 862-8652, Japan
masataka@ktmail.tokai-u.jp

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Abstract. *For the high-gain adaptive feedback control (HGAFC), this paper presents a new self-repairing scheme against sensor failures. The retrofitted HGAFC system can automatically replace the failed sensor with the healthy backup if the failure occurs. The main difference from the conventional self-repairing control is that the method exploits an unbounded resonance of a linear detection filter in order to find failures. One can easily design the detection filter with a linear element of the second order, and this is compatible with the linear shunt compensators for constructing the stable HGAFCSs for plants with any relative degrees.*

Keywords: Self-repairing, Adaptive and robust control, Sensor failure, Resonance

1. **Introduction.**

High-gain adaptive feedback control (HGAFC) has been recognized as one of the most practical adaptive controls [1, 2]. Although the HGAFC systems (HGAFCSs) have the simple structures which do not depend on the mathematical models of the plants, the systems have high robustness with respect to disturbances, uncertainties and nonlinearities [3, 4, 5].

Apparently, the HGAFCSs work well under the assumption that the feedback signals measured by sensors are always correct. However, if the sensor fails, then the control stability and performance cannot be guaranteed theoretically. Unfortunately, in existing researches on the HGAFC, the sensor failure problem has never been discussed.

As a remedy, this paper presents a concrete modification of the HGAFCS to attain self-repairing control (SRC) against sensor failures, which can detect failures by self-testing, and replace the failed sensor with the healthy backup so as to maintain the stability. Just additionally installing the detection filter to the HGAFCS, makes it possible to achieve the SRC. The main difference from the existing SRCSs [6, 7, 8], is that the failure detection is based on a resonance of the detection filter (oscillator) in faulty situation. The detection filter is intentionally designed so that the unstable resonance is caused by the feedback signal measured by failed sensor. Hence, by monitoring the filtered signal, failure can be successfully found. Furthermore, the detection filter can be easily constructed of a linear element of the second order.

Fundamentally, the HGAFC could be applied to only a class of the minimum-phase plants with the relative degree one [1, 2]. Hence, for plants with the high relative degree of more than two, the compensators (e.g., pre and/or parallel compensators) [3, 4] should be introduced in order to construct the stable HGAFCSs. Fortunately, the above-mentioned detection filter has a high affinity to those compensators because of its linearity. Then, in this paper, the concrete design method for the parallel compensator is shown to make

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the augmented plants of the relative degree one. By using this compensator, the stable HGAFCS with the detection filter can be built up for plants with any relative degrees, and so an applicable class of the plants can be extremely expanded.

The remaining sections are organized as follows. Section 2 provides the brief review of the original version of the HGAFCS, and Section 3 states the problem on SRC against sensor failures. In Section 4, the proposed design method for the HGAFCS with the SRC function is presented, and the theoretical analysis of the SRC performance and the concrete design method for the parallel compensator are shown. Section 5 explores numerical simulations to confirm the effectiveness of the proposed HGAFCS. Section 6 concludes this paper.

2. **High-Gain Adaptive Feedback Control.** First of all, this section briefly reviews the conventional HGAFCSs.

Consider a linear time invariant plant Σ_P with the order $n \in \mathbb{I}$. Here, suppose that the plant Σ_P is a minimum-phase system of the relative degree one. Then, by appropriate linear transformation, the state space model of the plant Σ_P can be represented as follows [1]:

$$\Sigma_P : \quad \begin{aligned} \dot{y} &= ay + bu + \boldsymbol{h}^T \boldsymbol{z} \\ \dot{\boldsymbol{z}} &= \boldsymbol{F}\boldsymbol{z} + \boldsymbol{g}y \end{aligned} \tag{1}$$

where $y \in \mathbb{R}$ is the actual output, $u : \mathbb{R}^+ \to \mathbb{R}$ is the control input, and $\boldsymbol{z} \in \mathbb{R}^{n-1}$ is the state of the plant. Because of the minimum-phase characteristic, $\boldsymbol{F} \in \mathbb{R}^{(n-1)\times(n-1)}$ is the stable matrix, that is, all eigenvalues of $\boldsymbol{F}$ lie in the left-half plane of $\mathbb{C}$. Without loss of generality, the sign of high-frequency gain b is assumed to be positive.

For stabilization of the above-mentioned plant Σ_P, the adaptive high-gain feedback controller is given as follows.

$$u = -py \tag{2}$$

where $p : \mathbb{R}^+ \to \mathbb{R}^+$ is the feedback gain adaptively tuned by

$$\dot{p} = \gamma y^2 - \sigma p, \quad p(0) = p_0 > 0 \tag{3}$$

with any positive constants $\gamma > 0$ and $\sigma > 0$.

Then, we have the following lemma.

Lemma 2.1. *The HGAFCS constructed by (1)-(3), has the following properties.*
(P1) All the signals, y, $\boldsymbol{z}$ and p are bounded over the control time $[0, \infty)$.
(P2) For arbitrarily small λ, there exist γ and σ such that

$$\limsup_{t\to\infty} |y(t)| \le \lambda \tag{4}$$

Proof: Because $\boldsymbol{F}$ is the stable matrix, for arbitrarily given, positive definite, $\boldsymbol{Q} \in \mathbb{R}^{(n+1)\times(n+1)}$, there exists a positive definite $\boldsymbol{P} \in \mathbb{R}^{(n+1)\times(n+1)}$ such that

$$\boldsymbol{F}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{F} = -2\boldsymbol{Q} \tag{5}$$

Then, consider the following positive definite function $S : \mathbb{R}^+ \to \mathbb{R}^+$:

$$S = \frac{1}{2}\left\{y^2 + \boldsymbol{z}^T\boldsymbol{P}\boldsymbol{z} + \frac{b}{\gamma}\Delta^2\right\}, \quad \Delta \triangleq p^* - p \tag{6}$$

where $p^* > 0$ is an ideal feedback gain to be estimated by p.

From (2), (3), (5) and (6), the time derivative of S can be expressed as follows.

$$\dot{S} \le -\frac{1}{2}\left\{2bp^* - 2a - \|\boldsymbol{h}\|^2 - \|\boldsymbol{P}\boldsymbol{g}\|^2\right\} y^2$$

$$-\frac{1}{2}\left(2\lambda_{\min}[\boldsymbol{Q}]-3\right)\|\boldsymbol{z}\|^2-\frac{b\sigma}{2\gamma}\Delta^2+\underbrace{\frac{b\sigma}{2\gamma}(p^*)^2}_{\frac{1}{2}\beta} \tag{7}$$

Choose p^* and $\boldsymbol{Q}$ satisfying

$$2bp^*-2a-\|\boldsymbol{h}\|^2-\|\boldsymbol{Pg}\|^2>\min\left\{\frac{2\lambda_{\min}[\boldsymbol{Q}]-3}{\lambda_{\max}[\boldsymbol{P}]},\ \sigma\right\}\triangleq\alpha \tag{8}$$

Then, we have $\dot{S}\le-\alpha S+\beta/2$, which implies

$$S\le S(0)\exp\left(-\alpha t\right)+\frac{\beta}{2\alpha} \tag{9}$$

Hence, all the signals, y, v_1, v_2, $\boldsymbol{z}$ and p are bounded over the control time $[0,\ \infty)$. Thus, (P1) is true.

From (6) and (9), it can be shown that $|y|\le 2S$. Therefore, we have

$$\limsup_{t\to\infty}|y(t)|\le\frac{\beta}{\alpha} \tag{10}$$

Clearly, from the definition of α and β in (7) and (8), it can be shown that for given λ, there exist sufficiently large γ and sufficiently small σ such that $\beta/\alpha\le\lambda$. This means that $\limsup_{t\to\infty}|y(t)|\le\lambda$. Hence, (P2) is true. ■

Apparently, if the sensor fails, then the preceding lemma holds no longer because the inaccurate output signal might be fed back to the controller. In the following sections, the problem on the self-repairing against sensor failures will be discussed.

3. Problem on Self-Repairing Control against Sensor Failures.

For measurement of the actual output y, the two sensors ♯1 (primary) and ♯2 (backup) are exploited. Based on the dynamic redundancy [8], the measured signal is given by

$$y_S(t)=\begin{cases}y_1(t) & (t\le t_D)\\ y_2(t) & (t>t_D)\end{cases} \tag{11}$$

where $t_D\in\mathbb{R}^+$ is a failure detection time, which will be defined later. Each $y_i\in\mathbb{R}$, $i\in\{1,\ 2\}$ is the output of the sensor $\sharp i$. If the sensors are healthy, then we have $y_i=y$. When the failure of the primary sensor ♯1 is detected, the backup ♯2 is activated.

The failure scenario to be considered here, is given as follows.

$$y_1(t)=\varphi,\quad t\ge t_F \tag{12}$$

where $t_F\in\mathbb{R}^+$ is a failure time, and $\varphi\in\mathbb{R}$ is the stuck value of the faulty sensor. Of course, both t_F and φ are unknown.

The SRC problem is to replace the failed sensor with the backup automatically to maintain the control system stability.

4. A Self-Repairing Function for High-Gain Adaptive Feedback Control.

To solve the above-mentioned problem, this section shows the concrete modification method for the HGAFCS to have the self-repairing function against sensor failures.

4.1. **Introduction of the detection filter.** Introduce the linear detection filter Σ_D of the second order to the HGAFCS as follows.

$$\Sigma_D : \quad \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tau + y_S \end{bmatrix} \tag{13}$$

where $\omega \in \mathbb{R}^+$ is an arbitrary constant, and $\tau : \mathbb{R}^+ \to \mathbb{R}$ is an auxiliary signal for detection.

In this method, a resonance phenomenon of the above-mentioned filter Σ_D in the faulty situation (12) is utilized to find the sensor failure. Clearly, the detection filter Σ_D has a natural angular frequency ω in the case of $y_S(t) = \varphi,\ t \geq t_F$. Hence, the auxiliary signal which induces resonance, is given by

$$\tau = \tilde{\tau} \sin(\omega t) \tag{14}$$

where $\tilde{\tau} > 0$ is an any small amplitude of τ, whose detail will be discussed later.

Furthermore, the adaptive controller is modified as follows.

$$\Sigma_C : \quad u = -\tilde{p}\underbrace{(y_S + \delta v_2)}_{\tilde{y}_S} = -\tilde{p}\tilde{y}_S \tag{15}$$

$$\dot{\tilde{p}} = \gamma \tilde{y}_S^2 - \sigma \tilde{p} \tag{16}$$

where $\delta > 0$ is an any positive constant.

Then, we can obtain the following lemma on the system stability.

Lemma 4.1. *Consider the HGAFCS retrofitted by (13)-(16). If no failure occurs, then the following properties hold.*
(P3) All the signals, y, $\boldsymbol{z}$, v_1, v_2 and $\tilde{p}$ are bounded over the control time $[0,\ \infty)$.
(P4) For arbitrarily small λ, there exist γ, σ and $\tilde{\tau}$ such that the inequality (4) holds.

Proof: If the sensor is healthy, then we have $\tilde{y}_S = y + \delta v_2 \triangleq \tilde{y}$. From (1), (13) and (15), it follows that

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{\boldsymbol{z}} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & \boldsymbol{0} \\ -\omega^2 & -\delta & \boldsymbol{0} \\ \boldsymbol{0} & -\delta \boldsymbol{g} & \boldsymbol{F} \end{bmatrix}}_{\tilde{\boldsymbol{F}}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \boldsymbol{z} \end{bmatrix}}_{\tilde{\boldsymbol{z}}} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ \boldsymbol{g} \end{bmatrix}}_{\tilde{\boldsymbol{g}}} \tilde{y} + \begin{bmatrix} 0 \\ 1 \\ \boldsymbol{0} \end{bmatrix} \tau \tag{17}$$

$$\dot{\tilde{y}} = (a + \delta)\tilde{y} + bu + \tilde{\boldsymbol{h}}^T \tilde{\boldsymbol{z}} + \delta \tau \tag{18}$$

where $\tilde{\boldsymbol{h}} = \left[-\delta\omega^2,\ -(a+\delta)\delta,\ \boldsymbol{h}^T\right]^T$. In the above (17), $\tilde{\boldsymbol{F}} \in \mathbb{R}^{(n+1)\times(n+1)}$ is also the stable matrix because $\boldsymbol{F}$ is the stable matrix and δ is positive. Hence, for arbitrarily given, positive definite, $\tilde{\boldsymbol{Q}} \in \mathbb{R}^{(n+1)\times(n+1)}$, there exists a positive definite $\tilde{\boldsymbol{P}} \in \mathbb{R}^{(n+1)\times(n+1)}$ such that

$$\tilde{\boldsymbol{F}}^T \tilde{\boldsymbol{P}} + \tilde{\boldsymbol{P}}\tilde{\boldsymbol{F}} = -2\tilde{\boldsymbol{Q}} \tag{19}$$

Here, consider the following positive definite function $\tilde{S} : \mathbb{R}^+ \to \mathbb{R}^+$:

$$\tilde{S} = \frac{1}{2}\left\{\tilde{y}^2 + \tilde{\boldsymbol{z}}^T \tilde{\boldsymbol{P}} \tilde{\boldsymbol{z}} + \frac{b}{\gamma}\tilde{\Delta}^2\right\}, \quad \tilde{\Delta} \triangleq \tilde{p}^* - \tilde{p} \tag{20}$$

where $\tilde{p}^* > 0$ is an ideal feedback gain to be estimated by p.

From (15), (16), (17) and (19), the time derivative of S can be expressed as

$$\begin{aligned} \dot{\tilde{S}} \leq & -\frac{1}{2}\left\{2b\tilde{p}^* - 2(a+\delta) - \|\tilde{\boldsymbol{h}}\|^2 - \delta^2 - \|\tilde{\boldsymbol{P}}\tilde{\boldsymbol{g}}\|^2\right\}\tilde{y}^2 \\ & -\frac{1}{2}\left(2\lambda_{\min}[\tilde{\boldsymbol{Q}}] - 3\right)\|\tilde{\boldsymbol{z}}\|^2 - \frac{b\sigma}{2\gamma}\tilde{\Delta}^2 + \underbrace{\frac{b\sigma}{2\gamma}(\tilde{p}^*)^2 + \frac{1}{2}\tilde{\tau}^2\left(1 + \|\tilde{\boldsymbol{P}}\|^2\right)}_{\frac{1}{2}\tilde{\beta}} \end{aligned} \tag{21}$$

Now, we choose $\tilde{p}^*$ and $\tilde{\boldsymbol{Q}}$ satisfying

$$2b\tilde{p}^* - 2(a+\delta) - \|\tilde{\boldsymbol{h}}\|^2 - \delta^2 - \|\tilde{\boldsymbol{P}}\tilde{\boldsymbol{g}}\|^2 > \min\left\{\frac{2\lambda_{\min}[\tilde{\boldsymbol{Q}}] - 3}{\lambda_{\max}[\tilde{\boldsymbol{P}}]},\ \sigma\right\} \triangleq \tilde{\alpha} \tag{22}$$

Then, it follows that $\dot{\tilde{S}} \le -\tilde{\alpha}\tilde{S} + \tilde{\beta}/2$. This implies

$$\tilde{S} \le \tilde{S}(0)\exp(-\tilde{\alpha}t) + \frac{\tilde{\beta}}{2\tilde{\alpha}} \tag{23}$$

Hence, it is verified that all the signals, $y_S = y$, v_1, v_2, $\boldsymbol{z}$ and p are bounded over the control time $[0,\ \infty)$. Thus, (P3) is true.

From (15) and (20), it can be shown that

$$|y| = |\tilde{y} + \delta v_2| \le |\tilde{y}| + \delta\|\tilde{\boldsymbol{z}}\| \le 2\left(1 + \frac{\delta}{\lambda_{\min}[\tilde{\boldsymbol{P}}]}\right)\tilde{S} \tag{24}$$

Therefore, we have

$$\limsup_{t\to\infty}|y(t)| \le \frac{\tilde{\beta}}{\tilde{\alpha}}\left(1 + \frac{\delta}{\lambda_{\min}[\tilde{\boldsymbol{P}}]}\right) \tag{25}$$

Clearly, from the definition of $\tilde{\beta}$ in (21), it can be seen that for given λ, there exist sufficiently small σ, $\tilde{\tau}$ and sufficiently large γ such that $\tilde{\beta}/\tilde{\alpha}\left(1 + \delta/\lambda_{\min}[\tilde{\boldsymbol{P}}]\right) \le \lambda$. This means that $\limsup_{t\to\infty}|y(t)| \le \lambda$. Hence, (P4) holds. ■

The block diagram of the modified HGAFCS is illustrated in Figure 1. The part by the dashed line is additionally installed to the original HGAFCS, and contains the detection filter Σ_D and the auxiliary signal τ.

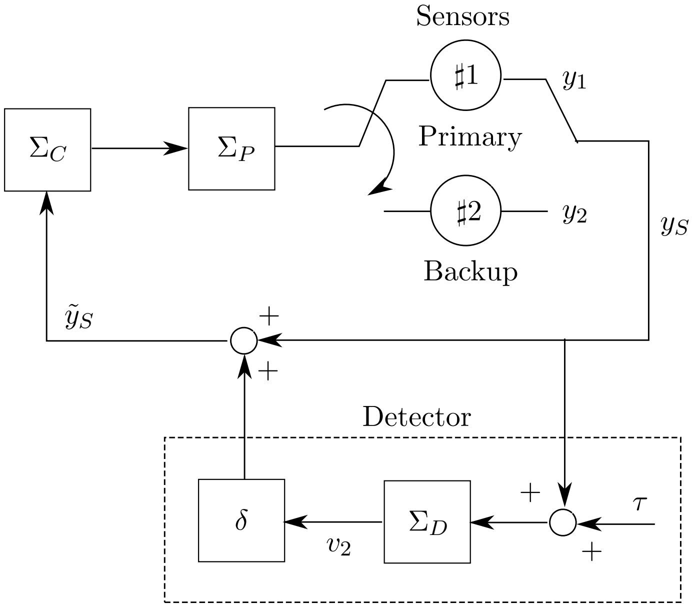

FIGURE 1. The HGAFCS modified with a detection filter, Σ_D

4.2. **Failure detection.** From Lemma 4.1, because of boundedness of v_2, there is a finite constant $\Gamma \in \mathbb{R}^+$ so that

$$|v_2(t)| < \Gamma, \quad t \in [0, \ t_F) \tag{26}$$

However, if the sensor $\sharp 1$ fails, then the detection filter Σ_D can be expressed as

$$\Sigma_D : \quad \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tau + \varphi \end{bmatrix} \tag{27}$$

By using the Laplace transformation, it follows that

$$v_2(t) = \mathcal{L}^{-1} \left[\frac{v_2(t_F)s + \varphi - \omega^2 v_1(t_F)}{s^2 + \omega^2} \right] + \mathcal{L}^{-1} \left[\frac{\tilde{\tau}\omega s}{(s^2 + \omega^2)^2} \right] \tag{28}$$

The first term in the R.H.S. above is bounded. However, the second term is calculated as $\tilde{\tau} t \sin(\omega t)/2$. That is, an unbounded resonance is induced by the auxiliary signal τ if the sensor fails. The inequality (26) holds no longer. Then, taking this unstable behavior of v_2 into consideration, we define the detection time t_D as follows.

$$t_D \triangleq \min \{ t \mid |v_2(t)| \geq \Gamma \} \tag{29}$$

For finite t_F, the detection time t_D is finite because the amplitude of v_2 diverges with $\mathcal{O}(t)$. Since the plant, the detection filter and the adaptive tuner do not have finite escape time, all the signals in the HGAFCS are bounded on the time $[t_F, \ t_D)$. After the detection time (the sensor is replaced), the HGAFCS can recover with the healthy backup sensor, and thus all the signals are bounded over the control time $[0, \ \infty)$.

The control performances can be summarized in the following theorem.

Theorem 4.1. *Consider the HGAFCS retrofitted by (13)-(16) and (29). The SRC can be achieved, that is,*
(P5) If the sensor $\sharp 1$ *fails, then the finite detection time* t_D *exists, and the system has the properties (P3) and (P4).*

Theorem 4.1 is apparent from the above discussions, so the proof is omitted here.

4.3. **Introduction of the compensators.** As mentioned in Section 2, the HGAFC can be applied to only a class of minimum-phase systems of the relative degree one. However, this might be a severe restriction in practical situations. To alleviate the condition on the relative degrees, some kinds of compensators have been proposed. According to [3, 4], the linear compensator is connected with the plant in parallel, and then the augmented plant always becomes a minimum-phase system of the relative degree one. This section shows that, for plants with any relative degrees, the HGAFC can be applied by introducing the parallel compensator.

Now, consider the HGAFCS with a parallel compensator Σ_H as shown in Figure 2. Let $G(s)$, $H(s)$ and $D(s)$ be the transfer functions of the plant Σ_P, the compensator Σ_H and the detection filter Σ_D respectively. Then, the transfer function from the input u to the output $\tilde{y}_{AS}$ of the augmented plant can be represented as

$$G(s)\left(1 + \delta D(s)\right) + H(s) = \underbrace{G(s) \left(\frac{s^2 + \delta s + \omega^2}{s^2 + \omega^2} \right)}_{\bar{G}(s)} + H(s) \tag{30}$$

Hence, by using the existing design method for the compensator $H(s)$ [3, 4], the augmented plant, $\bar{G}(s) + H(s)$ can become a minimum-phase system of relative degree one, and thus it can be stabilized by the controller, Σ_C.

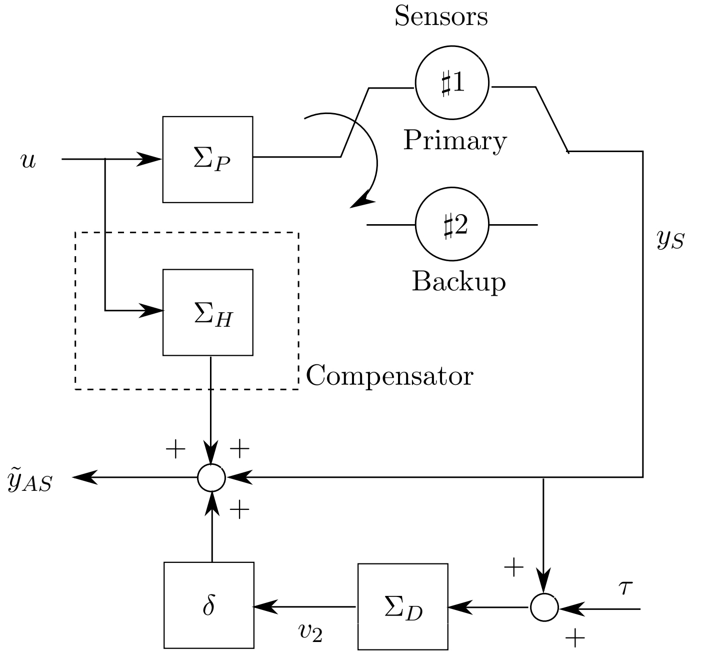

FIGURE 2. Introduction of the parallel compensator, Σ_H

5. **Numerical Example.**

This section shows the two numerical examples to confirm the effectiveness of the proposed HGAFC with the SRC function.

5.1. **Example 1.**

First, consider the following unstable system of the relative degree one:

$$\begin{aligned} \Sigma_P : \quad & \dot{y} = 0.1y + u + z, \quad y(0) = 0.5 \\ & \dot{z} = -z + y, \ \ z(0) = -0.5 \end{aligned} \tag{31}$$

Suppose that the failure time t_F is set as

$$t_F = 50 \ [\mathrm{s}], \quad \varphi = y_1(t_F) \tag{32}$$

Of course, the above values are assumed to be unknown in the controller design.

The detection filter Σ_D is designed with

$$\omega = 2, \quad \delta = 0.1, \quad \tilde{\tau} = 0.1 \tag{33}$$

and the design parameters for the adaptive gain, $\tilde{p}$ are given by

$$\gamma = 1, \quad \sigma = 0.001, \quad \tilde{p}(0) = 3$$

These parameters are selected by trial and error so that the output, y remains in the small ball of radius $\lambda = 0.25$. The threshold, Γ for detection is given by

$$\Gamma = 1.4$$

The simulation results are shown in Figures 3 and 4. In Figure 3, the actual output y and the measured output y_S are shown. Figure 4 shows the time response of the adaptive gain $\tilde{p}$ (top) and the absolute value of the filtered signal v_2 (bottom). From these results, it can be shown that the unstable plant (31) can be stabilized before and after the failure time. Also, the failed sensor can be successfully replaced after the failure. The detection time is $t_D \simeq 52$ [s].

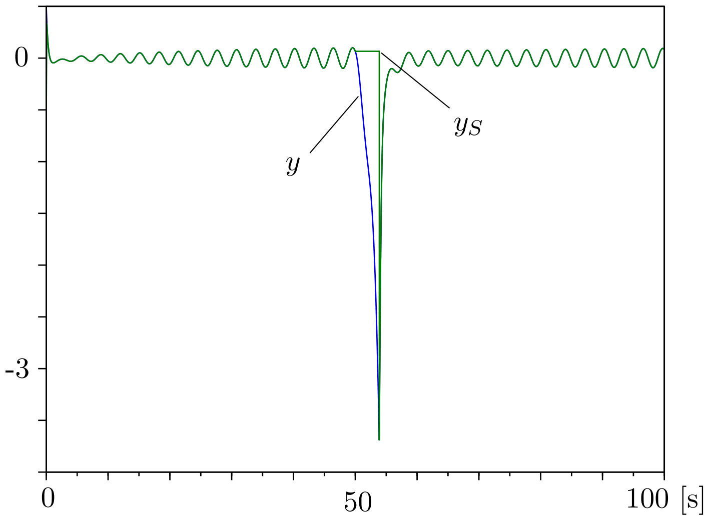

FIGURE 3. Simulation results: outputs, y and y_S

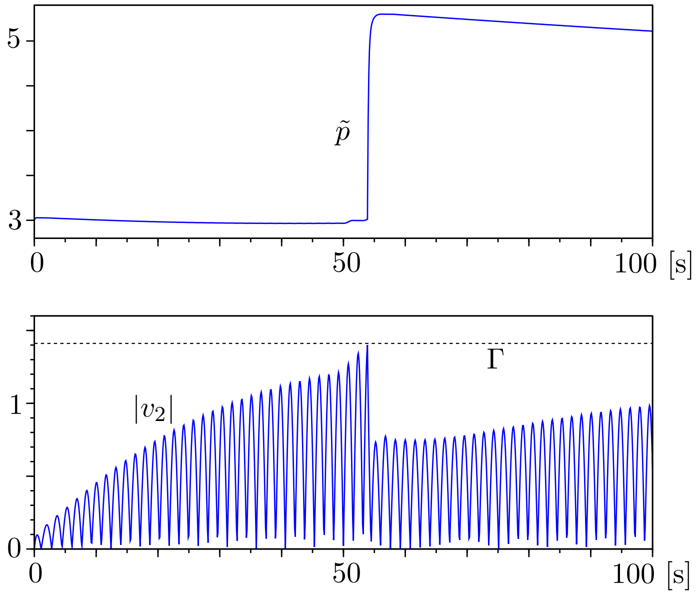

FIGURE 4. Simulation results: adaptive gain, $\tilde{p}$ (top) and absolute value of filtered signal, $|v_2|$ (bottom)

5.2. **Example 2.** Next, consider the following stable second order system:

$$\begin{aligned} \Sigma_P: \quad & \dot{x}_1 = -5x_1 + x_2, \quad x_1(0) = 0.5 \\ & \dot{x}_2 = -4x_2 + u, \quad x_2(0) = 0 \\ & y = x_1 \end{aligned} \tag{34}$$

The failure scenario is supposed as same as (32).

For the plant (34), the detection filter Σ_D is designed with the same parameters as (33). Clearly, $\bar{G}(s)$ given in (30) has the relative degree two,

$$\bar{G}(s) = \frac{1}{s^2 + 9s + 20} \left(\frac{s^2 + 0.1s + 4}{s^2 + 4} \right) \tag{35}$$

Hence, the following parallel compensator Σ_H of the first order is introduced [3].

$$H(s) = \frac{0.001}{s+3} \tag{36}$$

Then, the zeros of the augmented plant, $\bar{G}(s) + H(s)$ are

$$z = -1005.8986, -3.002041, -0.0496832 \pm 1.999466j \tag{37}$$

Thus, the augmented plant becomes a minimum-phase system of the relative degree one, and so the HGAFC, Σ_C can stabilize the augmented plant.

The design parameters for the adaptive gain, $\tilde{p}$ are given by

$$\gamma = 1, \quad \sigma = 0.001, \quad \tilde{p}(0) = 50$$

These parameters are selected by trial and error so that the output, y remains in the small ball of radius $\lambda = 0.25$. The threshold, Γ for detection is given by

$$\Gamma = 1.5$$

The simulation results are shown in Figures 5 and 6. In Figure 5, the actual output y and the measured output y_S are shown. Figure 6 shows the time response of the adaptive gain $\tilde{p}$ (top) and the absolute value of the filtered signal v_2 (bottom). Although the relative degree of the plant (34) is two, the plant can be stabilized by introducing the compensator (36). In addition, the output y converges to the small region asymptotically. After the failure, the failed sensor can be successfully replaced with the backup to maintain the stability. The detection time is $t_D \simeq 56$ [s].

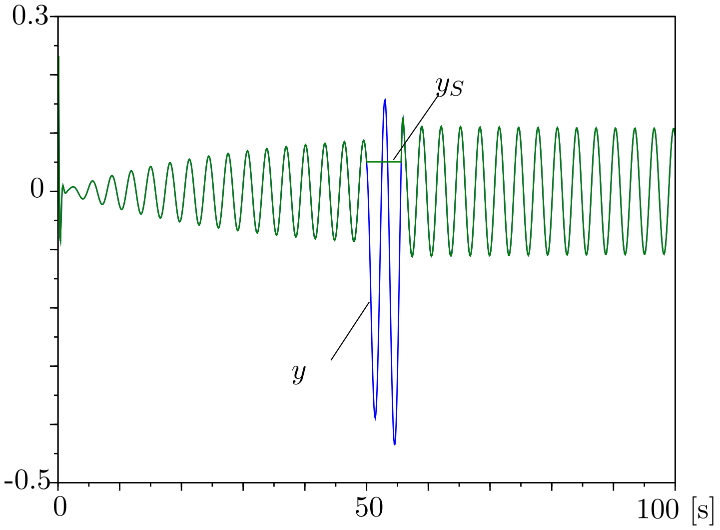

Figure 5. Simulation results: outputs, y and y_S

6. Concluding Remarks.

This paper has presented the concrete modification method for the HGAFCS to have the SRC function against sensor failures. Compared with SRCS developed in the previous works, the feature is that the failure detector can reliably perform despite its simple structure. Furthermore, the use of the linear detection filter Σ_D makes it possible to easily introduce the linear compensator Σ_H to the HGAFCS in order to construct the stable adaptive system for plants with any relative degree. Thus, the proposed HGAFCS with the SRC, can be applicable to more wider class of plants than conventional SRC systems.

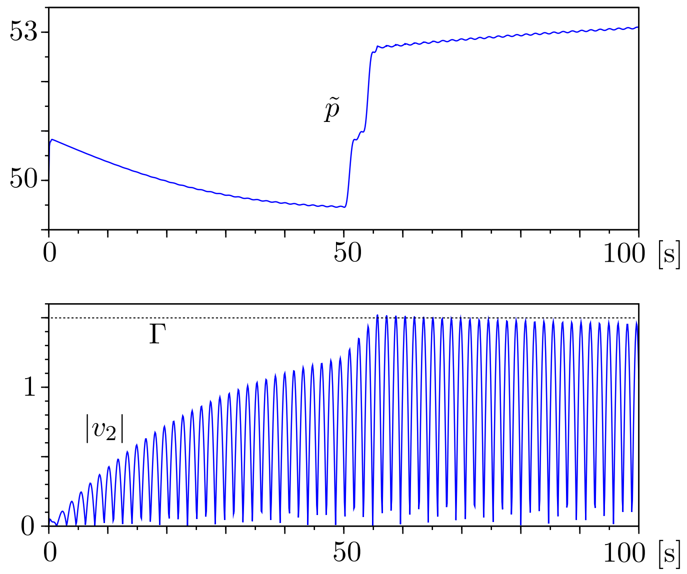

Figure 6. Simulation results: adaptive gain, $\tilde{p}$ (top) and absolute value of filtered signal, $|v_2|$ (bottom)

However, regarding the detection time t_D defined by (29), because the amplitude of v_2 is determined by $\tilde{\tau}$ in the faulty situation, the detection time can be shortened faster by larger $\tilde{\tau}$. Unfortunately, from the analysis of the convergence of y in Lemma 4.1, there might be a tradeoff between the control performance and the detection time, because a choice of large $\tilde{\tau}$ would widen the residual region of y large.

In this paper, only the stabilization has been considered, and the tracking control has not been discussed yet. Also, the method can be applicable for neither nonlinear systems nor MIMO systems. Various control problems are still left in future works.

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