International Journal of Innovative Computing, Information and Control
Volume **15**, Number **3**, June **2019**

ICIC International ©2019 ISSN 1349-4198
pp. 1079–1091

A NONLINEAR CONTROLLER FOR POWER SYSTEMS WITH STATCOM BASED ON BACKSTEPPING AND RAPID-CONVERGENT DIFFERENTIATOR

Adirak Kanchanaharuthai[1] and Ekkachai Mujjalinvimut[2]

[1]Department of Electrical Engineering
Rangsit University
52/347 Muang-Ake, Phaholyothin Rd., Lak-Hok, Muang, Patumthani 12000, Thailand
adirak@rsu.ac.th

[2]Department of Electrical Engineering
Faculty of Engineering
King Mongkut's University of Technology Thonburi
Pracha Uthit Road, Bang Mod, Bangkok 10140, Thailand
ekkachai.muj@kmutt.ac.th

Received August 2018; revised December 2018

Abstract. *This paper addresses a backstepping control design for power systems with STATCOM. With the help of a coordination of backstepping control strategy and rapid-convergent differentiator, the developed control law is designed to avoid the problem of "explosion of terms" arising in conventional backstepping method. The differentiator design is introduced to get the differential estimations of the virtual control function. The obtained differentials can be used instead of the derivative of the virtual control functions in each design step. Additionally, the derivative estimations of rapid-convergent differentiator have high precision and no chattering phenomenon. In order to show the effectiveness of the presented design, simulation results indicate that the presented control can effectively improve dynamic performances, rapidly suppress system oscillations of the overall closed-loop dynamics, and outperforms a conventional backstepping control technique.*
Keywords: Backstepping control, Rapid-convergent differentiator, STATCOM, Generator excitation

1. **Introduction.** It is well-known that modern power systems have the rapid increase of the size and complexity. When power system operation is confronted with unavoidable disturbances, maintaining power system stability is one of the most important problems. Therefore, this problem has attracted much attention from a number of researchers. Currently, there are three effective and promising methods that are used to improve system stability under unpredictable disturbances. The first method is a utilization of generator excitation control [1, 2, 3, 4, 5, 6, 7]. The second method is a combination of the excitation and energy storage system [8]. The third method is a coordination of the excitation and Flexible AC Transmission System (FACTS) devices [9, 10]. These schemes focus on improving power system stability and accomplishing the desired control objectives.

Since there are recently fast developments in power electronic devices, FACTS devices have been devoted to not only providing an opportunity to effectively tackle the existing transmission facilities, but also dealing with several constraints to build new transmission lines. In this paper, the Static Synchronous Compensator (STATCOM) [9, 10] of particular interest can be employed to increase the grid transfer capability through enhanced

DOI: 10.24507/ijicic.15.03.1079

voltage stability, significantly provide smooth and rapid reactive power compensation for voltage support, and enhance both damping oscillation and transient stability. So far, the generator excitation controller [1] and STATCOM controller [10] have separately been designed. However, in order to further enhance the power system stability of power systems, the combination of generator excitation and STATCOM is a promising and effective method and has attracted much attention in literature for years.

To the best of our knowledge, based on directly the nonlinear control strategy, there is little prior work that has been devoted to a coordination of generator excitation and STATCOM. In [11, 12], an adaptive coordinated generator excitation and STATCOM control strategy was designed via generalized Hamiltonian control for stability enhancement of large-scale power systems. With the help of the zero dynamic design and pole-assignment scheme, a coordinated controller [13] for the single-machine infinite bus system was investigated. A nonlinear coordinated controller [14] has been developed through a combination of the passivity design and backstepping technique. Kanchanaharuthai et al. [15] have developed an Interconnection and Damping Assignment-Passivity Based Control (IDA-PBC) strategy for a coordination of generator excitation and STATCOM/battery energy storage for transient stability and voltage regulation enhancement of multi-machine power systems. In [16], a coordinated Immersion and Invariance (I&I) control scheme has been developed for transient stability improvement and voltage regulation. Kanchanaharuthai [17] presented adaptive I&I control and adaptive backstepping scheme to enhance transient stability and voltage regulation for power systems with STATCOM in the presence of unknown parameters. Recently, based on Takagi-Sugeno (T-S) fuzzy scheme, a nonlinear stabilizer design [18] for power systems with random loads and STATCOM was presented and tested on both single and multi-machine power systems.

Motivated by the literature above, this paper continues this line of investigation and further extends the backsteppping design reported in [17]. In this paper, a control algorithm for designing a nonlinear controller for power systems with STATCOM via a backstepping control [19] and rapid-convergent differentiator [20] is developed. In accordance with the idea presented in [19], even if backstepping design is a powerful tool for control design and successfully applicable for several real systems, it has an important drawback. This drawback is the problem of "explosion of complexity" and often occurs in large-scale systems, thereby leading to a difficulty in finding out the derivative of the virtual control functions in each design step. In order to overcome this disadvantage, the differentiator is applied to estimating instead of computing the direct derivative of the virtual control functions. In recent years, there are several methods to compute differentiators such as a linear differentiator [21], Levant differentiator [22], a finite-time convergent differentiator based on singular perturbation techniques [23], an augmented nonlinear differentiator [24, 25], and a rapid-convergent nonlinear differentiator [20]. It can be obviously observed that differentiators reported in [21, 22, 23, 24, 25] are effective but quite complicated, because their design procedures and differentiator structures are too complex. In contrast, the recent-convergent differentiator used in this paper becomes simpler since it consists of two major terms: a nonlinear term (comprising of continuous power function) and a linear correction term. The first term is a continuous power function with a perturbation parameter. The second term is employed to restrain high-frequency noise and small bounded noises. Its structure of the designed differentiator is simple and has the short computation time. Moreover, this differentiator can avoid the chattering phenomenon, offers desired dynamical performances, and has simpler design process than other differentiators above.

Therefore, the major contributions of this paper can be outlined as follows: (i) a systematic strategy consisting of backstepping design and rapid-convergent differentiator to stabilize the power systems with STATCOM has not been investigated before; (ii) the derivative estimations of rapid-convergent differentiator developed for power systems with STATCOM have high precision and no chattering phenomenon; (iii) all trajectories of the overall closed-loop system are bounded and converge to a small neighborhood of the equilibrium point; and (iv) in comparison with a conventional backstepping control, the developed control law without an analytical differentiator offers better dynamic performances and can enhance power system stability.

The rest of this paper is organized as follows. Simplified synchronous generator and STATCOM models are briefly described and control problem formulation is given in Section 2. Control design is given in Section 3. Simulation results are given in Section 4. Conclusions are given in Section 5.

2. **Power System Model Description.**

Power system models with STATCOM. The complete dynamical model [16, 17] of the Synchronous Generator (SG) connected to an infinite bus with STATCOM dynamics can be expressed as follows:

$$\begin{cases} \dot{\delta} = \omega - \omega_s \\ \dot{\omega} = \dfrac{1}{M}\left(P_m - P_e - P_s - D(\omega - \omega_s)\right) \\ \dot{P}_e = (-a + \cot\delta(\omega - \omega_s))P_e + \dfrac{bV_\infty \sin 2\delta}{2(X_1 + X_2)} + \dfrac{V_\infty \sin\delta}{(X_1 + X_2)} \cdot \dfrac{u_f}{T_0'} \\ \dot{P}_s = \mathcal{N}(\delta, P_e, P_s)(-a + \cot\delta(\omega - \omega_s))P_e + \dfrac{\mathcal{N}(\delta, P_e, P_s)bV_\infty \sin 2\delta}{2(X_1 + X_2)} \\ \qquad + \dfrac{\mathcal{N}(\delta, P_e, P_s)V_\infty \sin\delta}{(X_1 + X_2)} \cdot \dfrac{u_f}{T_0'} + \dfrac{P_e X_1 X_2}{\Delta(\delta, P_e)} \cdot \dfrac{1}{T}\left(-\left(\dfrac{P_s \Delta(\delta, P_e)}{P_e X_1 X_2} - I_{Qe}\right) + u_q\right) \end{cases} \tag{1}$$

with

$$\Delta(\delta, P_e) = \sqrt{\left(\frac{P_e(X_1 + X_2)X_2}{V_\infty \sin\delta}\right)^2 + (V_\infty X_1)^2 + 2X_1X_2P_e(X_1 + X_2)\cot\delta} \tag{2}$$

$$\mathcal{N}(\delta, P_e, P_s) = \frac{P_s}{P_e} - \frac{P_s}{\Delta(\delta, P_e)^2}\left(X_1X_2\cot\delta(X_1 + X_2) + P_e\left(\frac{X_2(X_1 + X_2)}{V_\infty \sin\delta}\right)^2\right) \tag{3}$$

$$P_e = \frac{EV_\infty \sin\delta}{(X_1 + X_2)},\ P_s = \frac{P_e I_Q X_1 X_2}{\Delta(\delta, E)},\ I_Q = \frac{P_s \Delta(\delta, P_e)}{P_e X_1 X_2} \tag{4}$$

$$a = \frac{X_d + X_T + X_L}{(X_1 + X_2)T_0'},\ b = \frac{X_d - X_d'}{(X_1 + X_2)T_0'}V_\infty \tag{5}$$

where δ is the power angle of the generator, ω denotes the relative speed of the generator, $D \geq 0$ is a damping constant, P_m is the mechanical input power, E denotes the generator transient voltage source, $P_e = \frac{EV_\infty \sin\delta}{X_{d\Sigma'}}$ is the electrical power, without STATCOM, delivered by the generator to the voltage at the infinite bus V_∞, ω_s is the synchronous machine speed, $\omega_s = 2\pi f$, H represents the per unit inertial constant, f is the system frequency and $M = 2H/\omega_s$. X_d' denotes the direct axis transient reactance of SG and X_d denotes the direct axis reactance of SG. X_T is the reactance of the transformer, and X_L denotes the reactance of the transmission line. For simplicity, X_1 is the reactance consisting of the direct axis transient reactance of SG and the reactance of the transformer, and X_2 is the reactance of the transmission line. T_0' is the direct axis transient short-circuit time constant. u_f is the field voltage control input to be designed. I_Q denotes the injected or absorbed STATCOM currents as a controllable current source, I_{Qe} is an equilibrium point of STATCOM currents, u_q is the STATCOM control input to be designed, and T is a time constant of STATCOM models.

For convenience, let us introduce new state variables as follows:

$$x_1 = \delta - \delta_e, \; x_2 = \omega - \omega_s, \; x_3 = P_e, \; x_4 = P_s \tag{6}$$

Subsequently, after differentiating the state variables (6), we have the power system with STATCOM which can be written in the following form of an affine nonlinear system[1]:

$$\dot{x} = f(x) + g(x)u(x) \tag{7}$$

where

$$\left\{ \begin{array}{l} f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{1}{M}\left(P_m - x_3 - x_4 - Dx_2\right) \\ (-a + x_2 \cot x_1)x_3 + \dfrac{bV_\infty \sin 2x_1}{2(X_1 + X_2)} \\ \mathcal{N}(-a + x_2 \cot x_1)x_3 + \dfrac{\mathcal{N} bV_\infty \sin 2x_1}{2(X_1 + X_2)} \\ -\dfrac{x_3 X_1 X_2 \left(\frac{x_4 \Delta(x_1, x_3)}{x_3 X_1 X_2} - I_{qe}\right)}{\Delta(x_1, x_3)} \end{bmatrix} \\ g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ g_{31}(x) & 0 \\ g_{41}(x) & g_{42}(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{V_\infty \sin x_1}{(X_1 + X_2)} & 0 \\ \dfrac{\mathcal{N} V_\infty \sin x_1}{(X_1 + X_2)} & \dfrac{x_3 X_1 X_2}{\Delta(x_1, x_3)} \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \; u(x) = \begin{bmatrix} \dfrac{u_f}{T_0'} \\ \dfrac{u_q}{T} \end{bmatrix} \end{array} \right. \tag{8}$$

The region of operation is defined in the set $\mathcal{D} = \left\{x \in \mathcal{S} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \mid 0 < x_1 < \frac{\pi}{2}\right\}$. The open loop operating equilibrium is denoted by $x_e = [0, \; 0, \; x_{3e}, \; x_{4e}]^T = [0, \; 0, \; P_{ee}, \; P_{se}]^T = [0, \; 0, \; P_m, \; 0]^T$.

For the sake of simplicity, the power system considered (7) and (8) can be expressed as follows.

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{M}\left(P_m - x_3 - x_4 - Dx_2\right) \\ \dot{x}_3 = f_3(x) + g_{31}(x)\dfrac{u_f}{T_0'} \\ \dot{x}_4 = f_4(x) + g_{41}(x)\dfrac{u_f}{T_0'} + g_{42}(x)\dfrac{u_q}{T} \end{array} \right. \tag{9}$$

Control Problem Formulation: The main goal of this paper is to solve the control problem of the stabilization of the power systems with STATCOM (9). The control problem can be formulated as follows. For the system (9), with the help of the combination of a nonlinear differentiator technique [20] and backstepping design, find out, if possible, a nonlinear controller $u(x)$ such that all trajectories of the overall closed-loop system are bounded.

[1]It is assumed that all functions and mapping are smooth, i.e., $\mathbb{C}^\infty$, throughout this paper.

For the developed design procedure in the next section, the combination of the backstepping strategy and nonlinear differentiator design will be developed to obtain a feedback stabilizing nonlinear control without an analytical differentiator. In the following section, the developed control is designed step by step to achieve the desired performances.

3. Nonlinear Controller Design.

In this section, the control law for stabilizing power systems with STATCOM is developed. The main control development can be accomplished in the following three subsections. The first subsection is to introduce a rapid-convergent differentiator to obtain the differential estimations of the system signals. The second subsection is to develop a backstepping strategy combined with the rapid-convergent differentiator from the first subsection. The final subsection presents the overall closed-loop system stability analysis via Lyapunov stability arguments.

3.1. Rapid-convergent differentiator.

In this subsection, we concentrate on the differentiator estimation of the system signals, in particular differentiator of the virtual control law in the next subsection. So far, there exist several techniques to estimate the differentiators of arbitrary signals such as linear differentiators [21], a second-order (or high-order) sliding mode method [22], a finite-time convergent differentiator [23], and augmented nonlinear differentiators [24, 25]. However, the differentiator based on sliding mode algorithm has unavoidably the time-lagging phenomenon and the chattering problems. Furthermore, the finite-time differentiator has the complicated structure. As a result, a rapid-convergent differentiator, which has simpler structure, is introduced to estimate the differentiator of desired virtual signals. This differentiator has an ability to keep more rapidly convergence at all times. The rapid-convergent differentiator used in the control strategy is given in the following lemma.

Lemma 3.1. [20] *The rapid-convergent second-order differentiator is designed as*

$$\begin{cases} \dot{y}_1 = y_2 \\ \epsilon^2 \dot{y}_2 = -a_{10}(y_1 - v(t)) - a_{11}\mathrm{sig}(y_1 - v(t))^{\frac{\alpha}{(2-\alpha)}} - a_{20}\epsilon y_2 - a_{21}\mathrm{sig}(\epsilon y_2)^{\alpha} \\ y_{\mathrm{out}} = y_2 \end{cases} \tag{10}$$

where y_1, y_2, and y_{out} denote the system state and the output of the rapid-convergent differentiator, respectively, $\mathrm{sig}(y)^\alpha = |y|^\alpha \mathrm{sign}(y)$, $\alpha > 0$. *It is clear that $sig(y)^\alpha = y^\alpha$ only if $\alpha = q/p$ where p, q are positive odd numbers. For a continuous and piecewise two-order differentiator signal $v(t)$, there exists $\gamma > 0$ (where $\rho\gamma > 2$ and $\rho = \min\{\alpha, \alpha/(2-\alpha)\} = \alpha/(2-\alpha)$), such that*

$$y_i - v^{(i-1)}(t) = O\left(\epsilon^{\rho\gamma - i + 1}\right) \tag{11}$$

for $t \geq \epsilon\Gamma(\Xi(\epsilon)e(0))$, $i = 1$, and $t_j > t \geq t_{j-1} + \epsilon\Gamma(\Xi(\epsilon)e_+(t))$, $j = 1, 2, \ldots, k+1$, $i = 2$, respectively, with $y_2(t_j) - v'_-(t_j) = O\left(\epsilon^{\rho\gamma-1}\right)$, $j = 1, 2, \ldots, k$, where $\alpha \in (0, 1)$, $\epsilon > 0$ is the perturbation parameter and $O(\epsilon^{\rho\gamma-i+1})$ is the approximation of $\epsilon^{\rho\gamma-i+1}$ order [28] *between y_i and $v^{(i-1)(t)}$; $e_i = y_i - v^{(i-1)}(t)$, $i = 1, 2$, $e = [e_1, e_2]^T$, $e_+(t_{j-1}) = [e_1(t_{j-1}), e_2(t_{j-1})]^T$, $e_2^+(t_{j-1}) = y_2 - v'_+(t_{j-1})$, and $\Xi(\epsilon) = \mathrm{diag}\{1, \epsilon\}$.*

3.2. Rapid-convergent differentiator based-backstepping design.

In this subsection, the backstepping scheme combined with the rapid-convergent differentiator is used to find out the control law capable of achieving the desired control performances. Similar to the idea reported in [26], the proposed control procedure is developed step by step as follows.

Step 1: First, we focus on the first subsystem (9), and then let $z_1 = x_1$, $z_2 = x_2 - \alpha_1$ where α_1 denotes the virtual control that needs to be designed. The Lyapunov function candidate is chosen as

$$V_1 = \frac{1}{2} z_1^2 \tag{12}$$

Then the time derivative of V_1 along the system trajectories becomes

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 x_2 = z_1 (z_2 + \alpha_1) \tag{13}$$

Choose the virual control law α_1 as

$$\alpha_1 = -\left(c_1 + \frac{1}{2}\right) z_1 \tag{14}$$

where c_1 is a design parameter. Thus, after combining (13) with (14), we have

$$\dot{V}_1 = -\left(c_1 + \frac{1}{2}\right) z_1^2 + z_1 z_2 \leq -c_1 z_1^2 + \frac{1}{2} z_2^2 \tag{15}$$

Step 2: Let $z_2 = x_2 - \alpha_1$ and introduce the Lyapunov function candidate as:

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{16}$$

By calculating the derivative of (16), we have

$$\dot{V}_2 = \dot{V}_1 + z_2(\dot{x}_2 - \dot{\alpha}_1) = \dot{V}_1 + z_2 \left(\frac{1}{M} (P_m - x_3 - x_4 - Dx_2) - \dot{\alpha}_1 \right) \tag{17}$$

Define x_2 as the input of the rapid-convergent differentiator (10), i.e., $v_1(t) = x_2$ and define the differentiator output $y_{\text{out1}} = y_{x_2}$. Therefore, the rapid-convergent differentiator with the input x_2 is as follows.

$$\begin{cases} \dot{y}_{11} = y_{12} \\ \epsilon^2 \dot{y}_{12} = -a_{10}(y_{11} - v_1(t)) - a_{11} \text{sig}(y_{11} - v_1(t))^{\frac{\alpha}{(2-\alpha)}} - a_{20} \epsilon y_{12} - a_{21} \text{sig}(\epsilon y_{12})^{\alpha} \\ y_{\text{out1}} = y_{12} = y_{x_2} \end{cases} \tag{18}$$

Based on Lemma 3.1, we obtain

$$P_m - \frac{1}{2} D x_2 = \frac{M}{2} \left(y_{x_2} - O\left(\epsilon_{x_2}^{\rho\gamma - 3}\right) \right) - x_3 \tag{19}$$

$$-\frac{1}{2} D x_2 = \frac{M}{2} \left(y_{x_2} - O\left(\epsilon_{x_2}^{\rho\gamma - 3}\right) \right) - x_4 \tag{20}$$

where y_{x_2} can be directly computed from (18). Let us define $z_3 = x_3 - \alpha_2$ and $z_4 = x_4 - \alpha_3$ where α_2 and α_3 are the virtual control functions to be defined. After substituting (19) and (20) into (17), we have

$$\dot{V}_2 = \dot{V}_1 - \frac{1}{M} z_2 (z_3 + z_4) + z_2 \left(-\frac{1}{M} (\alpha_2 + \alpha_3 + x_3 + x_4) + y_{x_2} - \dot{\alpha}_1 - O\left(\epsilon_{x_2}^{\rho\gamma - 3}\right) \right) \tag{21}$$

It can be seen that $\dot{\alpha}_1$ can be directly computed using the analytical differentiator. In order to determine the virtual control functions α_2 and α_3 used in the next step, with the help of Young inequality [27], one has

$$-\frac{1}{M} z_2 z_3 \leq \frac{1}{4M^2 \eta_1^2} z_2^2 + \eta_1^2 z_3^2 \tag{22}$$

$$-\frac{1}{M} z_2 z_4 \leq \frac{1}{4M^2 \eta_2^2} z_2^2 + \eta_2^2 z_4^2 \tag{23}$$

$$-\frac{1}{M}z_2x_3 \le \frac{1}{4M^2\eta_3^2}z_2^2x_3^2+\eta_3^2 \tag{24}$$

$$-\frac{1}{M}z_2x_4 \le \frac{1}{4M^2\eta_4^2}z_2^2x_4^2+\eta_4^2 \tag{25}$$

$$z_2\left(y_{x_2}-\dot{\alpha}_1\right) \le \frac{1}{4\eta_5^2}z_2^2\left(y_{x_2}-\dot{\alpha}_1\right)^2+\eta_5^2 \tag{26}$$

$$z_2O\left(\epsilon_{x_2}^{\rho\gamma-3}\right) \le \frac{1}{2}z_2^2+O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right) \tag{27}$$

where $\eta_i > 0$, $i = 1, 2, \dots, 5$ are design parameters. By substituting (22)-(27) into (21), we have

$$\begin{aligned}\dot{V}_2 \le& -c_1z_1^2+\left(1+\frac{1}{4M^2\eta_1^2}+\frac{1}{4M^2\eta_2^2}+\frac{x_3^2}{4M^2\eta_3^2}+\frac{x_4^2}{4M^2\eta_4^2}+\frac{\left(y_{x_2}-\dot{\alpha}_1\right)^2}{4\eta_5^2}\right)z_2^2+\eta_1^2z_3^2\\&+\eta_4^2z_4^2+\eta_3^2+\eta_4^2+\eta_5^2+O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right)-\frac{1}{M}z_2(\alpha_2+\alpha_3)\end{aligned} \tag{28}$$

Design the virtual control functions α_2 and α_3 as

$$\begin{cases}\alpha_2=\dfrac{Mz_2}{2}\left(c_2+1+\dfrac{1}{4M^2\eta_1^2}+\dfrac{1}{4M^2\eta_2^2}+\dfrac{x_3^2}{4M^2\eta_3^2}+\dfrac{(y_{x_2}-\dot{\alpha}_1)^2}{4\eta_5^2}\right)\\ \alpha_3=\dfrac{Mz_2}{2}\left(c_2+1+\dfrac{1}{4M^2\eta_1^2}+\dfrac{1}{4M^2\eta_2^2}+\dfrac{x_4^2}{4M^2\eta_4^2}+\dfrac{(y_{x_2}-\dot{\alpha}_1)^2}{4\eta_5^2}\right)\end{cases} \tag{29}$$

where $c_2 > 0$ is a design parameter. By substituting (29) into (28), we obtain

$$\dot{V}_2 \le -c_1z_1^2-c_2z_2^2+\eta_1^2z_3^2+\eta_4^2z_4^2+\eta_3^2+\eta_4^2+\eta_5^2+O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right) \tag{30}$$

Step 3: Define the Lyapunov function from Step 2 as

$$V_3 = V_2+\frac{1}{2}z_3^2+\frac{1}{2}z_4^2 \tag{31}$$

Then the time derivative of V_3 along the system trajectories turns into as follows:

$$\dot{V}_3=\dot{V}_2+z_3\left(f_3(x)+g_{31}(x)\frac{u_f}{T_0'}-\dot{\alpha}_2\right)+z_4\left(f_4(x)+g_{41}(x)\frac{u_f}{T_0'}+g_{42}(x)\frac{u_q}{T}-\dot{\alpha}_3\right) \tag{32}$$

It can be observed from (29) and (32) that the direct computation in the derivative of α_2 and α_3 is quite difficult. Similar to Step 2, we define α_2 and α_3 as the input of the rapid-convergent differentiators (10), i.e., $v_2(t) = \alpha_2$ and $v_3(t) = \alpha_3$, respectively. Therefore, two rapid-convergent differentiators with the input α_2 and α_3, respectively, are as follows.

$$\begin{cases}\dot{y}_{21}=y_{22}\\ \epsilon^2\dot{y}_{22}=-a_{10}(y_{21}-v_2(t))-a_{11}\mathrm{sig}(y_{21}-v_2(t))^{\frac{\alpha}{(2-\alpha)}}-a_{20}\epsilon y_{22}-a_{21}\mathrm{sig}(\epsilon y_{22})^{\alpha}\\ y_{\text{out}2}=y_{22}=y_{\alpha_2}\\ \dot{y}_{31}=y_{32}\\ \epsilon^2\dot{y}_{32}=-a_{10}(y_{31}-v_3(t))-a_{11}\mathrm{sig}(y_{31}-v_3(t))^{\frac{\alpha}{(2-\alpha)}}-a_{20}\epsilon y_{32}-a_{21}\mathrm{sig}(\epsilon y_{32})^{\alpha}\\ y_{\text{out}3}=y_{32}=y_{\alpha_3}\end{cases} \tag{33}$$

Then, we define two differentiator outputs from (33) as y_{α_2} and y_{α_3}, respectively. With the help of Lemma 3.1, we get

$$\dot{\alpha}_2 = y_{\alpha_2}-O\left(\epsilon_{\alpha_2}^{\rho\gamma-3}\right) \tag{34}$$

$$\dot{\alpha}_3 = y_{\alpha_3}-O\left(\epsilon_{\alpha_3}^{\rho\gamma-3}\right) \tag{35}$$

where y_{α_2} and y_{α_3} can be directly computed from (33).

From (32)-(35), we obtain

$$\begin{aligned}
\dot{V}_3 &= \dot{V}_2 + z_3\left(f_3(x) + g_{31}(x)\frac{u_f}{T_0'} - y_{\alpha_2} + O\left(\epsilon_{\alpha_2}^{\rho\gamma-3}\right)\right) \\
&\quad + z_4\left(f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + g_{42}(x)\frac{u_q}{T} - y_{\alpha_3} + O\left(\epsilon_{\alpha_3}^{\rho\gamma-3}\right)\right) \\
&\le -c_1z_1^2 - c_2z_2^2 + z_3\underbrace{\left(f_3(x) + g_{31}(x)\frac{u_f}{T_0'} - y_{\alpha_2} + \left(\eta_1^2 + \frac{1}{2}\right)\right)}_{-c_3z_3} \\
&\quad + z_4\underbrace{\left(f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + g_{42}(x)\frac{u_q}{T} - y_{\alpha_3} + \left(\eta_2^2 + \frac{1}{2}\right)\right)}_{-c_4z_4} + \eta_3^2 + \eta_4^2 + \eta_5^2 \\
&\quad + O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_3}^{2\rho\gamma-6}\right)
\end{aligned} \tag{36}$$

From (36), in order to achieve the desired control performance, the proposed control law can be designed as

$$u_f(x) = -\frac{T_0'}{g_{31}(x)}\left[f_3(x) + \left(c_3 + \eta_1^2 + \frac{1}{2}\right)z_3 - y_{\alpha_2}\right] \tag{37}$$

$$u_q(x) = -\frac{T}{g_{42}(x)}\left[f_4(x) + g_{41}(x)\frac{u_f}{T_0'} + \left(c_4 + \eta_2^2 + \frac{1}{2}\right)z_4 - y_{\alpha_3}\right] \tag{38}$$

where c_3 and c_4 denote positive design parameters. The nonlinear control design is completed here.

3.3. **Closed-loop stability analysis.**

Theorem 3.1. *Consider the power systems with STATCOM (9). If the rapid-convergent differentiators are constructed as in (10), (18) and (33), then the desired control law $u(x)$ chosen as in (37) and (38) will guarantee that state trajectories z_k converge to the radius $\sqrt{2b/a}$, and all trajectories in the overall closed-loop system are bounded, where $a = \min_{1\le k\le 4}\{2c_k\}$, $b = \sum_{i=3}^{5}\eta_i^2 + O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_3}^{2\rho\gamma-6}\right)$ for $i = 3, 4, 5$.*

Proof: After substituting the proposed control law (37) and (38) into (36), we have

$$\begin{aligned}
\dot{V}_3 &\le -c_1z_1^2 - c_2z_2^2 - c_3z_3^2 - c_4z_4^2 + \eta_3^2 + \eta_4^2 + \eta_5^2 + O\left(\epsilon_{x_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_2}^{2\rho\gamma-6}\right) + O\left(\epsilon_{\alpha_3}^{2\rho\gamma-6}\right) \\
&\le -aV_3 + b
\end{aligned} \tag{39}$$

From (39), it is easy to show that

$$V_3(t) \le \left(V_3(0) - \frac{b}{a}\right)e^{-at} + \frac{b}{a} \tag{40}$$

The expression above can indicate that all trajectories in the closed-loop system are bounded. In particular, it is evident that

$$z_k^2 \le 2\left(V_3(0) - \frac{b}{a}\right)e^{-at} + \frac{2b}{a} \tag{41}$$

Therefore, it means that $\lim_{t\to+\infty} z_k \le \sqrt{\frac{2b}{a}}$. From the definition of the system state variables x_k, $(k = 1, 2, 3, 4)$ and α_i, $(i = 1, 2, 3)$, it is obvious that the x_k are also bounded. Thus, the developed strategy can guarantee that all signals of the overall closed-loop system are bounded, converge to a small neighborhood of the desired equilibrium point, and achieve the desired requirements. This completes the proof.

Remark 3.1. *It is easy to see that even if the developed control law does not need to employ the analytical differentiator in the design process and depends upon design parameters c_k like the conventional backstepping control, it has the additional design parameters (η_j) that are used to further enhance the desired control performances.*

Remark 3.2. *It can be observed that the proposed strategy is a systematic control strategy to design the desired control law by using three steps. The presented three steps are similar to the conventional backstepping strategy but replace the derivative of the virtual control functions $(\dot{\alpha}_i)$ with the output of rapid-convergent differentiator (y_{outj}) and the approximation of $\epsilon^{p\gamma-i-1}$ order between y_{outj} and $v_j(t)$.*

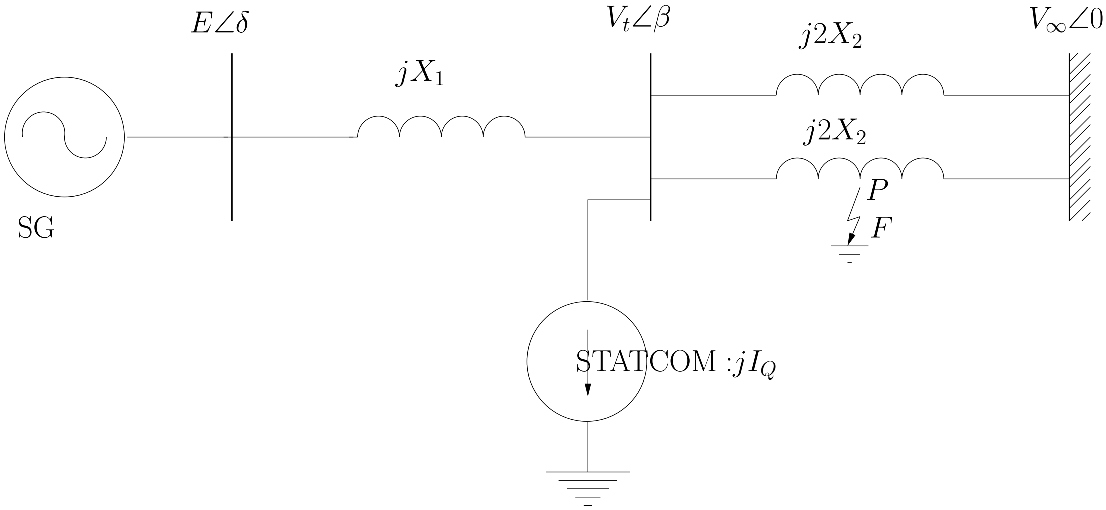

FIGURE 1. A single line diagram of SMIB model with STATCOM

4. **Simulation Results.** In this section, in order to verify the effectiveness of the presented approach, we consider a Single-Machine Infinite Bus (SMIB) power system with STATCOM as shown in Figure 1. The developed controller is evaluated through MATLAB environment, and the performance of the designed control is compared with that of the following nonlinear controller:

- A Conventional Backstepping Controller (CBC) [19].

$$u_f(x) = -\frac{T_0'}{g_{31}(x)}\left[c_3e_3 + f_3(x) - \frac{e_2}{M} - \dot{\alpha}_2\right] \tag{42}$$

$$u_q(x) = -\frac{T}{g_{42}(x)}\left[c_4e_4 + f_4(x) + g_{41}(x)\frac{u_f}{T_0'} - \frac{e_2}{M} - \dot{\alpha}_3\right] \tag{43}$$

where $e_1 = x_1$, $e_2 = x_2 - \alpha_1$, $e_3 = x_3 - \alpha_2$, $e_4 = x_4 - \alpha_4$, $\alpha_1 = -c_1x_1$, $\alpha_2 = P_m - \frac{D}{2}x_2 - \frac{M}{2}(c_2e_2 + e_1 - \dot{\alpha}_1)$, $\alpha_3 = -\frac{D}{2}x_2 - \frac{M}{2}(c_2e_2 + e_1 - \dot{\alpha}_1)$, $c_k = 20$, $(k = 1, 2, 3, 4)$.

The physical parameters (pu.), the controller parameters, and initial parameters used for this power system model are as follows:

- The parameters of synchronous generators, STATCOM, and transmission line: $\omega_s = 2\pi f$ rad/s, $D = 0.2$, $H = 5$, $f = 60$ Hz, $T_0' = 4$, $V_\infty = 1\angle 0°$, $X_d = 1.1$, $X_d' = 0.2$, $X_T = 0.1$, $T = 1$, $X_2 = X_L = 0.2$, $P_m = 1$,
- The tuning parameters of the proposed controller are $c_k = 20$, $(k = 1, 2, 3, 4)$, $\eta_j = 2$, $\epsilon = 1/300$, $\alpha = 1/3$.

- Initial parameters $\delta_e = 0.4964$ rad, $\omega_e = \omega_s$, $P_{ee} = 1$ pu., $P_{se} = 0$ pu., $y_{i1} = y_{i2} = 0$, $(i = 1, 2, 3)$. These initial parameters can be directly determined from setting all time derivatives of the complete dynamical model in [16] to zero and then directly solved from the resulting algebraic equations to obtain the equilibrium point $(\delta_e, \omega_e, E'_e, I_{qe})$. Afterwards, by substituting such equilibrium point into (4), we obtain the initial parameters above used throughout the simulations.

The SMIB power system consisting of generator excitation and STATCOM has been simulated using the the physical parameters and initial conditions above. In the simulations, in order to demonstrate the effectiveness of the developed strategy over the conventional backstepping control, we assume that there is a symmetrical three phase short circuit (temporary fault) occurring on one of the transmission lines as shown in Figure 1. Further, we assume that the system is in a pre-fault steady state, a fault occurs at $t = 0.5$ sec., the fault is isolated by opening the breaker of the faulted line at $t = 1.0$ sec., and the transmission line is recovered without the fault at $t = 2.5$ sec. Afterward the system is in a post-fault state.

The simulation results are presented in Figures 2-4 and discussed as follows. It can be seen that Figure 2 shows the time responses of power angle (δ), frequency $(\omega - \omega_s)$, transient voltage (E) and STATCOM current (I_Q), eventually settling down to the pre-fault state values, under two controllers. Figure 3 shows the active power $(P_e + P_s)$ and terminal voltage (V_t) under the proposed control and the CBC scheme.

It is evident from Figures 2 and 3 that the proposed scheme and the CBC scheme can successfully stabilize the system, and the developed scheme demonstrates a more good transient behavior compared with the CBC scheme. In particular, it is easy to see that time responses of the CBC method eventually return to the pre-fault values after the transient fault is clear, but its time responses have very slow rate of convergence and are quite

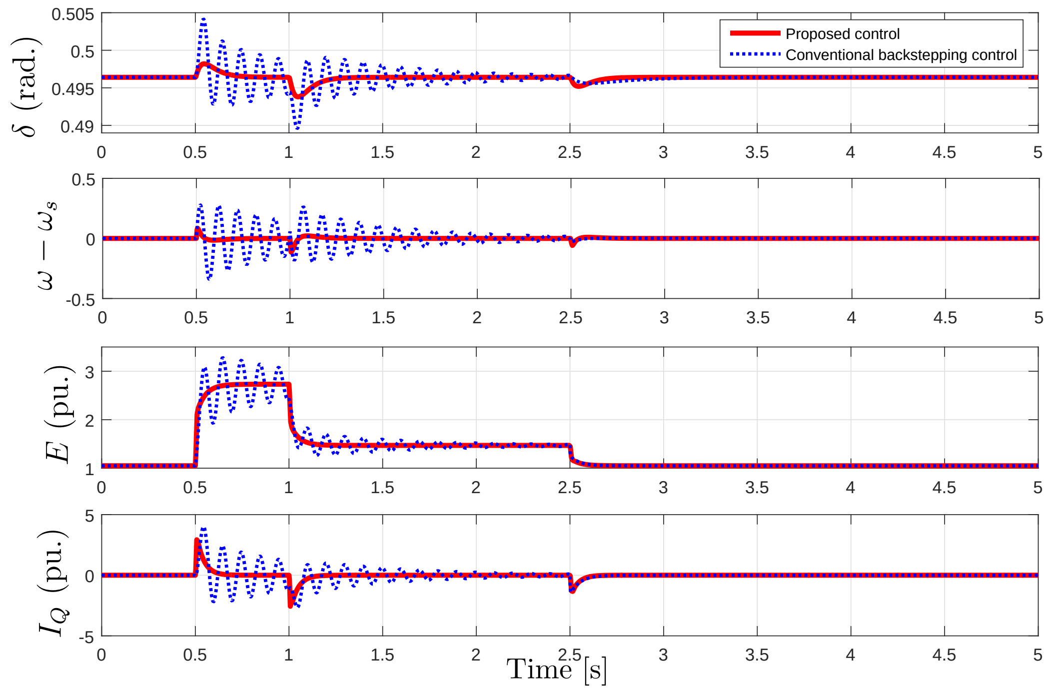

FIGURE 2. Controller performance – Power angles (δ) (rad.), frequency $(\omega - \omega_s)$ rad/s, transient voltage (E), and STATCOM current (I_Q) (Solid: Proposed control, Dashed: Conventional backstepping control)

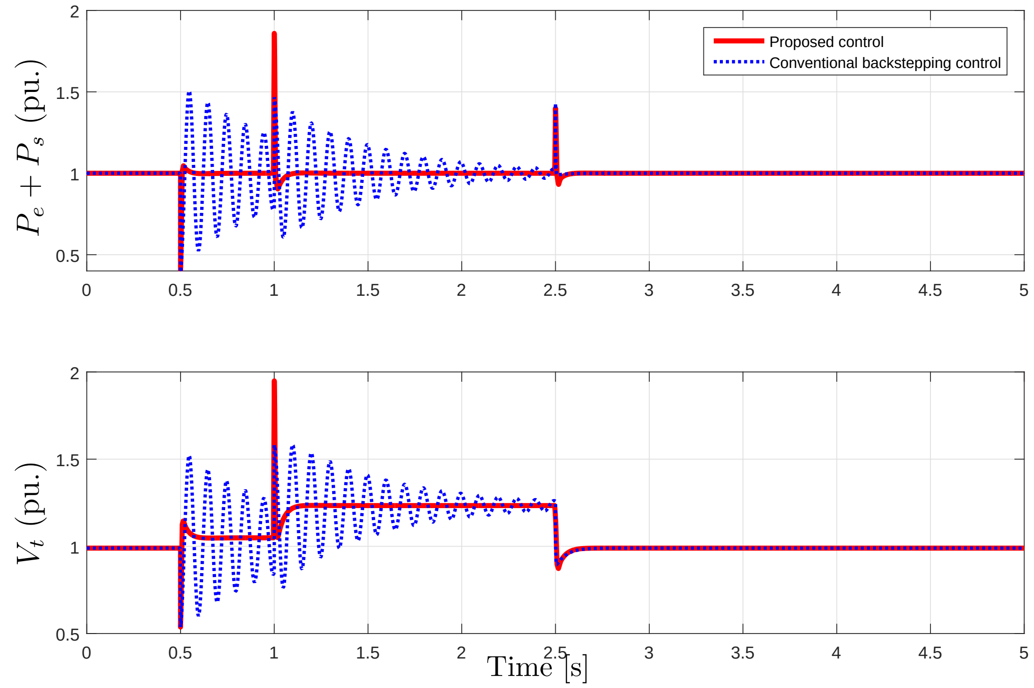

FIGURE 3. Controller performance – Active power $(P_e + P_s)$ (pu.) and the terminal voltage (V_t) (pu.) (Solid: Proposed control, Dashed: Conventional backstepping control)

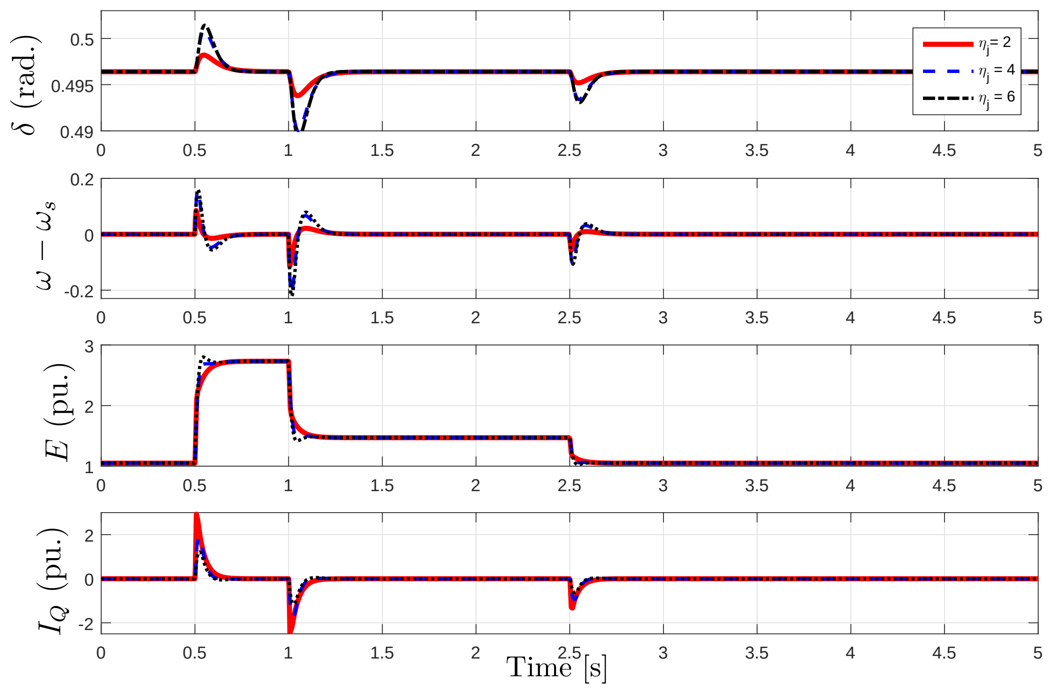

FIGURE 4. Time responses of power angle, frequency, transient voltage, and STATCOM current under different coefficient control $\eta_j = 2, 4, 6$

oscillatory as compared with the proposed controller under the same tuning parameters $c_k = 20$, $(k = 1, 2, 3, 4)$. It can be observed that the developed control law is designed by combining rapid-convergent differentiator with the backstepping scheme. Especially, it adds the additional degrees of freedom into the design procedure to further improve the system performances. Figure 4 shows the time trajectories of power angle, frequency, transient voltage, and STATCOM current of the proposed control under different control coefficients η_j, $j = 3, 4, 5$. This means that apart from the suitable selection of c_k like the CBC strategy, the dynamic performances of the presented strategy can be further improved by selecting appropriate control parameters η_j. Obviously, the small values of η_j lead to improvement of better transient performances with less regulation time.

As indicated in the simulation results above. It can be, overall, concluded that the proposed control law is effectively designed for transient stabilization and voltage regulation following short circuit. The developed control can stabilize the power system with STATCOM and steer the overall closed-loop system to a small neighborhood of the desired equilibrium point. Moreover, both active power and the terminal voltage can be quickly regulated to the reference active power and voltage values without oscillations as compared to the CBC method. Even though the presented method does not need to use the analytic differentiator, its control performances can be further improved by selecting the suitable parameters of η_j. In summary, the proposed method obviously outperforms the CBC one in terms of fast convergence speed, rapid reduction of oscillations, and shorter settling time.

5. **Conclusions.** In this paper, a nonlinear control strategy has been developed for power systems with STATCOM. In order to avoid the problem of "explosion of term" arising in the conventional backstepping method, the developed strategy has been designed by combining backstepping method with rapid-convergent differentiator. This combination is able to offer better dynamical performances as compared with the Conventional Backstepping Control (CBC) and avoid the chattering phenomenon arising in other kinds of differentiators. Based on Lyapunov control theory, the stability analysis of closed-loop system has been provided. Further, the presented control law is able to ensure that all trajectory states converge to a small neighborhood of the desired equilibrium point. The simulation results have confirmed that the proposed nonlinear control can improve obviously better dynamical performances than the CBLC method, although there is no analytical differentiator in the presented design process. Future study will be devoted to extension of this approach to a nonlinear controller for multimachine power systems with STATCOM or other kinds of FACTS devices.

REFERENCES

[1] P. Kundur, *Power System Stability and Control*, McGraw-Hill, 1994.
[2] A. S. Bazanella and C. L. Conceicao, Transient stability improvement through excitation control, *International Journal of Robust and Nonlinear Control*, vol.14, pp.891-910, 2004.
[3] W. Dib, G. Kenne and F. Lamnabhi-Lagarrigue, An application of immersion and invariance to transient stability and voltage regulation of power systems with unknown mechanical power, *Proc. of Joint 48th IEEE CDC and 28th CCC*, Shanghai, China, 2009.
[4] W. Dib, R. Ortega, A. Astolfi and D. Hill, Improving transient stability of multi-machine power systems: Synchronization via immersion and invariance, *Proc. of American Control Conference*, San Francisco, CA, USA, 2011.
[5] M. Galaz, R. Ortega, A. Bazanella and A. Stankovic, An energy-shaping approach to excitation control of synchronous generators, *Automatica*, vol.39, pp.111-119, 2003.
[6] R. Ortega, M. Galaz, A. Astolfi, Y. Sun and T. Shen, Transient stabilization of multi-machine power systems with nontrivial transfer conductances, *IEEE Trans. Automatic Control*, vol.50, pp.60-75, 2005.

[7] M. O. Paul and E. P. Gerardo, Output feedback excitation control of synchronous generators, *International Journal of Robust and Nonlinear Control*, vol.14, pp.879-890, 2004.

[8] A. Kanchanahanathai, V. Chankong and K. A. Loparo, Nonlinear generator excitation and superconducting magnetic energy storage control for transient stability enhancement via immersion and invariance, *Transactions of the Institute of Measurement and Control*, vol.37, no.10, pp.1217-1231, 2015.

[9] N. G. Hingorani and L. Gyugyi, *Understanding FACTS: Concepts and Technology of Flexible AC Transmission Systems*, IEEE Press, 1999.

[10] Y. H. Song and A. T. John, *Flexible AC Transmission Systems (FACTS)*, IEE Power and Energy Series 30, London, U.K., 1999.

[11] Q. J. Liu, Y. Z. Sun, T. L. Shen and Y. N. Song, Adaptive nonlinear co-ordinated excitation and STATCOM based on Hamiltonian structure for multimachine-power-system stability enhancement, *IEE Proc. of Control Theory and Applications*, vol.150, no.3, pp.285-294, 2003.

[12] K. Wang and M. L. Crow, Hamiltonian theory based coordinated nonlinear control of generator excitation and STATCOMs, *Proc. of North American Power Symposium*, 2010.

[13] L. Gu and J. Wang, Nonlinear coordinated control design of excitation and STATCOM of power systems, *Electric Power System Research*, vol.77, no.7, pp.788-796, 2007.

[14] B. Zou and J. Wang, Coordinated control for STATCOM and generator excitation based on passivity and backstepping technique, *Proc. of Electric Information and Control Engineering*, 2010.

[15] A. Kanchanaharuthai, V. Chankong and K. A. Loparo, Transient stability and voltage regulation in multi-machine power systems vis-à-vis STATCOM and battery energy storage, *IEEE Trans. Power Systems*, vol.30, no.5, pp.2404-2416, 2015.

[16] A. Kanchanaharathai, Immersion and invariance-based non-linear coordinated control for generator excitation and static synchronous compensator for power systems, *Electric Power Components and Systems*, vol.42, no.10, pp.1004-1015, 2014.

[17] A. Kanchanaharuthai, Nonlinear adaptive controller design for power systems with STATCOM via immersion and invariance, *ECTI Trans. Electrical Engineering, Electronics, and Communications*, vol.14, no.2, pp.35-46, 2016.

[18] A. H. Abolmasoumi and M. Moradi, Nonlinear T-S fuzzy stabilizer design for power systems including random loads and static synchronous compensator, *International Trans. Electrical Energy Systems*, vol.28, no.1, pp.1-19, 2018.

[19] M. Krstic, I. Kanellakopoulos and P. V. Kokotivic, *Nonlinear and Adaptive Control Design*, John Willey & Sons, 1995.

[20] X. Wang and B. Shirinzadeh, Rapid-convergent nonlinear differentiator, *Mechanical Systems and Signal Processing*, vol.28, pp.414-431, 2012.

[21] H. K. Khalil, Robust servomechanism output feedback controllers for feedback linearizable systems, *Automatica*, vol.30, no.10, pp.1587-1599, 1994.

[22] A. Levant, Higher-order sliding modes, differentiation and output feedback control, *International Journal of Control*, vol.76, no.9/10, pp.924-941, 2003.

[23] X. Wang, Z. Chen and G. Yang, Finite-time-convergent differentiator based on singular perturbation technique, *IEEE Trans. Automatic Control*, vol.52, no.9, pp.1731-1737, 2007.

[24] X. Shao, J. Liu, W. Yang, J. Tang and J. Li, Augmented nonlinear differentiator design, *Mechanical Systems and Signal Processing*, vol.90, pp.268-284, 2017.

[25] X. Shao, J. Liu, J. Li, H. Cao, C. Shen and X. Zhang, Augmented nonlinear differentiator design and application to nonlinear uncertain systems, *ISA Transactions*, vol.67, pp.30-46, 2017.

[26] X. Yang, X. Zheng and Y. Chen, Position tracking control law for an electro-hydraulic servo system based on backstepping and extended differentiator, *IEEE/ASME Trans. Mechatronics*, vol.23, no.1, pp.132-140, 2018.

[27] C. Qian and W. Lin, Non-smooth stabilizers for nonlinear systems with uncontrollable unstable linearization, *Proc. of the 39th IEEE Conference on Decision and Control*, Sydney, NSW, pp.1655-1660, 2000.

[28] H. K. Khalil, *Nonlinear Systems*, Prentice Hall, 2002.