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FUZZY LEAST ABSOLUTE REGRESSION ANALYSIS BASED ON MELLIN TRANSFORMS

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Abstract. *Most of existing papers that have been published on the fuzzy regression analysis have used the least squares method with the distance between fuzzy numbers to construct the fuzzy regression model. The distance between fuzzy numbers is an important research topic in fuzzy regression analysis. In order to increase the explanatory performance of fuzzy regression model, the least absolute deviation Mellin transform (LADMT) method is proposed to determine regression coefficients. In this paper, we consider the fuzzy linear regression model with fuzzy input, fuzzy output and crisp parameters and introduce a new distance based on the Mellin transform of triangular fuzzy number, merging least absolute deviation method with the new Mellin transform distance. Finally, two examples are given to illustrate the effectiveness and feasibility of the method. Comparisons with existing methods show that based on the total estimation error using the absolute error and the same distance criterion, the performance of the LADMT method is satisfactory, and the calculation is relatively simple.*
Keywords: Fuzzy regression model, Mellin transform, Least absolute deviation

1. **Introduction.** Regression analysis is an important and common method in forecasting, evaluation and decision making, but traditional regression often depends on exact statistical value. In social and economic activities, the data sometimes cannot be recorded or collected precisely. The probability distributions for the observations either cannot be found or can be done so only with great difficulty. For such data, the traditional statistical regression model is not considered an appropriate method to describe the linguistic terms. In the real world, people often use natural language values to express qualitative concepts, for example, "very good", "temperature is not high", and "quite small" [1]. This is exactly the important basis for people to identify, analyze and even make decisions. In view of the fuzziness of language values in real world, Japanese scholar Tanaka et al. [2] proposed the fuzzy linear regression model for the first time, which is mainly used to reflect the fuzzy relation between independent variables and dependent variables. If the functional relationship is known, the model is called a parametric fuzzy regression model; otherwise, it is called a nonparametric fuzzy regression model. Several approaches to fuzzy regression analysis have been developed, starting from the pioneering works by Tanaka et al. [2], Celminš [3] and Diamond [4], based respectively on linear programming and least squares principles. Chang and Ayyub [5] performed a literature review and summarized

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the two main methods to solve the fuzzy regression problem. First, a fuzzy regression method is based on minimizing fuzziness for model-fitting by linear programming [6-12]. In these improved methods, the minimum fuzziness is used as the fitting criterion, and linear programming is kept as the problem-solving main tool. The main shortcoming of the minimum fuzziness criterion method is that the concept of least-squares is not utilized; therefore, fuzzy regression using least squares of errors as a criterion becomes the second method [3,4,13-22]. The existing least-squares estimate parameter methods were mainly through the α cut sets Euclidean distance of fuzzy numbers. In this paper we present a new least absolute deviation distance method from the viewpoint of Mellin transform to estimate parameter.

In this paper, we propose a new method by computing a simple linear programming to determine the regression parameter. We apply the least absolute deviation principle to constructing the fuzzy linear regression model with fuzzy inputs, fuzzy output and crisp parameters, introduce the Mellin transform distance between triangular fuzzy numbers, and use the Mellin transform distance measure of triangular fuzzy numbers to evaluate the fitting of the observed and estimated values. Two examples show that the proposed method is better than the existing fuzzy regression methods studied by some authors using the least squares method.

The rest of the paper is organized as follows. In Section 2, some basic concepts of fuzzy number, Mellin transform of triangular fuzzy numbers, Mellin transform distance measure are described. In Section 3, we propose fuzzy least absolute deviation Mellin transform distance method and put forward its calculation method. In Section 4, two examples are used to illustrate the application of the proposed method, whose performances are compared with those of existing methods. Section 5 summarizes the main results and draws conclusions.

2. **Preliminaries.**

In this section, we will briefly present triangular membership functions that are used in the model formulation. Also we introduce the concept of Mellin transform using proportional probability density function associated with the membership function of fuzzy numbers [23,24]. Finally, we define a new distance based on the Mellin transform of fuzzy numbers.

Definition 2.1. *A fuzzy number $A = (a_1, a_2, a_3)$ is called a normal triangular with a piecewise linear membership function $\mu_A(x)$ defined as*

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & else \end{cases} \tag{1}$$

Based on the extension principle by Zadeh [25], if $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then we have the algebraic operations of triangular fuzzy numbers in the following.

(i) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
(ii) $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
(iii) $k \cdot A = (ka_1, ka_2, ka_3)$, $k > 0$ or $k \cdot A = (ka_3, ka_2, ka_1)$, $k < 0$

Definition 2.2. *The Mellin transform $M_x(s)$ of a probability density function (PDF) $f(x)$ is defined as*

$$M_x(s) = \int_0^\infty x^{s-1} f(x)dx, \quad 0 < x < \infty \tag{2}$$

The Mellin transform has a unique one-to-one correspondence, i.e., $f(x) \leftrightarrow M_x(s)$. The moments of a distribution represent the expected values of the power of a random variable with distribution function $f(x)$. In general, the kth moment of a random variable X about a real number c is defined as

$$M_k(x) = E\left[(X-c)^k\right] = \int_x (x-c)^k f(x)dx \tag{3}$$

The moments of interest in economic analyses are those about the origin ($c = 0$) and those about the mean ($c = \mu$), typically for $k = 1$, 2, and 3. The kth moments about the origin and the mean are denoted, respectively, by $E[X^k]$ and μ_k

$$\mu_k = E\left[(X-\mu)^k\right] = \int_x (x-\mu)^k f(x)dx \tag{4}$$

The first moment about the origin represents the mean of the distribution, $\mu_1 = E[X]$, while the second moment about the mean represents the variance, $\mu_2 = \sigma^2$, of the distribution. A comparison of Equation (2) with Equation (3) shows that $M_x(s)$ is a special case of $M_k(x)$, where $c = 0$ and $k = s-1$. In other words, the Mellin transform $M_x(s) = E\left[X^{s-1}\right]$ provides an alternative method to establish a series of moments of a distribution if $f(x)$ is viewed as a PDF. Comparing the first two moments of a distribution with the Mellin transform, it allows the mean and variance to be expressed as the equation

$$\mu_1 = E[X] = M_x(2) \qquad \mu_2 = \sigma^2 = M_x(3) - [M_x(2)]^2 \tag{5}$$

The close correlation between the Mellin transform and the expected values makes it simple to establish some important operating properties involving products, quotients, and powers of random variables. The important Mellin transform operations summarized in Table 1 adapted from Park [26].

Table 1. Properties of Mellin transform

Property	PDF	Random variable	Mellin transform $M_x(s)$
Standard	$f(x)$	x	$M_x(s)$
Scaling	$f(ax)$	x	$a^{-s}M_x(s)$
Linear	$af(x)$	x	$aM_x(s)$
Translation	$x^a f(x)$	x	$M_x(a+s)$
Exponentiation	$f(x^a)$	x	$a^{-1}M_x(s/a)$

The PDF $f(x)$ corresponding to membership function $\mu(x)$ of triangular fuzzy number $A = (a_1, a_2, a_3)$ is given as

$$f_A(x) = c\mu_A(x) \tag{6}$$

Then, the proportional probability density function corresponding to triangular fuzzy number $A = (a_1, a_2, a_3)$ is given by

$$f_A(x) = \begin{cases} \dfrac{2(x-a_1)}{(a_3-a_1)(a_2-a_1)}, & a_1 \le x \le a_2 \\ \dfrac{2(a_3-x)}{(a_3-a_1)(a_3-a_2)}, & a_2 \le x \le a_3 \\ 0, & \text{else} \end{cases} \tag{7}$$

Graphically it is shown in Figure 1.

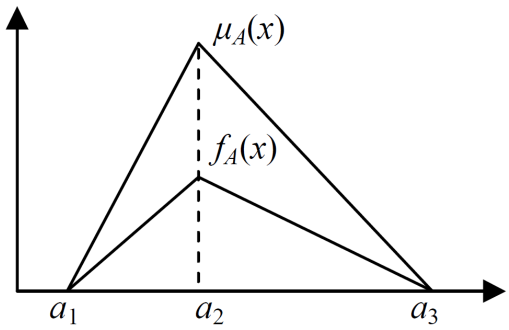

FIGURE 1. PDF of triangular fuzzy number

Further, using the Mellin transform, we obtain the Mellin transform of triangular fuzzy number is

$$M_x(s) = \frac{2}{(a_3 - a_1)s(s+1)} \left(\frac{a_3\left(a_3^s - a_2^s\right)}{a_3 - a_2} - \frac{a_1\left(a_2^s - a_1^s\right)}{a_2 - a_1} \right) \tag{8}$$

Let $s = 2, 3$, and then we substitute Formula (8) into Formula (5) and obtain the mean, variance of a triangular density function. Obviously, the normal triangular fuzzy numbers can be compared or ranked directly in terms of their Mellin transform. Therefore, we have the following definitions.

Definition 2.3. *Suppose $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the fuzzy number $A = B$ if and only if $(M_x(s))_A = (M_x(s))_B$, for all $s \in R$.*

The distance measure of fuzzy sets is an important topic in the fuzzy set theory. In many applications such as decision theory, signal processing, speech recognition, pattern recognition, correlation coefficient, and risk analysis, distance measure plays an important role. Consider two fuzzy numbers A and B, a real function $D(A, B)$ is called the distance measure between A and B, which satisfies the following axioms [27].

(i) $D(A, B) \geq 0$ and $D(A, B) = 0$ if and only if $A = B$, for any two fuzzy numbers A and B.

(ii) $D(A, B) = D(B, A)$, for any two fuzzy numbers A and B.

(iii) $D(A, B) + D(B, C) \geq D(A, C)$, for any three fuzzy numbers A, B and C.

We propose a new distance using Mellin transform between two triangular fuzzy numbers. The Mellin transform distance between two triangular fuzzy numbers A and B is as follows.

Definition 2.4. *Suppose A and B are two fuzzy numbers, the Mellin transform absolute deviation distance between A and B can be defined as*

$$D_M(A, B) = \sum_s \left|(M_x(s))_A - (M_x(s))_B\right| \tag{9}$$

Obviously, $D_M(A, B)$ is the distance measure, which satisfies three conditions of axioms. In this paper, we will use the least absolute deviation Mellin transform (LADMT) method of normal triangular fuzzy number to evaluate the error degree between observed and estimated values of fuzzy regression model.

In fact, normal triangular fuzzy number $A = (a_1, a_2, a_3)$ can be determined by two digital characteristics, namely, the mean and variance. Therefore, the Mellin transform absolute deviation distance measure based on the mean and standard deviation of two fuzzy numbers can be defined as

$$D_M(A, B) = |\mu_1(A) - \mu_1(B)| + |\sigma(A) - \sigma(B)| \tag{10}$$

where μ_1 and σ are shown in Equation (5).

3. **Fuzzy Linear Regression Model and Parameter Estimation.** The fuzzy linear regression model may be roughly classified by the conditions of independent and dependent variables into three categories by Choi and Buckley [28] as the following.

(i) Input independent variables are non-fuzzy, but output dependent variables and regression parameters are fuzzy.

(ii) Input independent variables and output dependent variables are both fuzzy, but regression parameters are non-fuzzy.

(iii) Input independent variables, output dependent variables and regression parameters are all fuzzy.

In this paper, we will use the fuzzy linear regression model for the second category. The fuzzy linear regression model in which input data and output data are both fuzzy is expressed as follows:

$$y_i = b_0 x_{0i} + b_1 x_{1i} + b_2 x_{2i} + \cdots + b_p x_{pi} \tag{11}$$

where y_i and x_{ji} are fuzzy numbers, regression parameter b_j is real number, $i = 1, \ldots, n$, $j = 0, 1, \ldots, p$.

Without loss of generality, we take account of the cases in which fuzzy regression input and output variables are normal triangular fuzzy numbers. Let $x_{ji} = (a_{ji}, \alpha_{ji}, \beta_{ji})$ ($j = 0, 1, 2, \ldots, p$; $i = 1, 2, \ldots, n$), with left width α_{ji} and right width β_{ji}, then the fuzzy regression model (11) can be rewritten as:

$$y_i = b_0(a_{0i}, \alpha_{0i}, \beta_{0i}) + b_1(a_{1i}, \alpha_{1i}, \beta_{1i}) + b_2(a_{2i}, \alpha_{2i}, \beta_{2i}) + \cdots + b_p(a_{pi}, \alpha_{pi}, \beta_{pi}) \tag{12}$$

From Definition 2.3, the left and right sides of the above Equation (12) are equal if and only if their Mellin transforms are equal in model (12). Because of the fuzziness of the regression model itself, model (12) is equivalent to the following model (13) based on Mellin transform.

$$M_x(y_i)_s = b_0 M_x(x_{0i})_s + b_1 M_x(x_{1i})_s + b_2 M_x(x_{2i})_s + \cdots + b_p M_x(x_{pi})_s \tag{13}$$

In fact, normal triangular fuzzy number can be determined by the mean and variance. So for the sake of simplicity, the model (13) can be simplified as the following regression models

$$\mu_1(y_i) = b_0 \mu_1(x_{0i}) + b_1 \mu_1(x_{1i}) + b_2 \mu_1(x_{2i}) + \cdots + b_p \mu_1(x_{pi}) \tag{14}$$

and

$$\sigma(y_i) = b_0 \sigma(x_{0i}) + b_1 \sigma(x_{1i}) + b_2 \sigma(x_{2i}) + \cdots + b_p \sigma(x_{pi}) \tag{15}$$

Prior to applying the least absolute deviation Mellin transform distance method, each estimate b_i should be determined as being either positive or negative for formulating the lower and upper bounds of the estimated fuzzy response in (12) based on fuzzy arithmetic. if $b_i < 0$ ($i = 0, 1, \ldots, p$), then according to the algebraic operation (iii), $b_i x_i$ is swapped, as the operation is in fuzzy arithmetic, when calculating the estimated fuzzy response using the fuzzy regression model. In order to address this, we first perform correlation analysis between the fuzzy response variable and each fuzzy explanatory variable using their defuzzified values, since only the trend between them is needed. Let $x_i = (x_{1i}, x_{2i}, \ldots, x_{ni})$ ($i = 1, 2, \ldots, p$) and $y = (y_1, y_2, \ldots, y_n)$, respectively. The correlation coefficient between the fuzzy response variable and the ith fuzzy explanatory variable is defined as

$$\rho(x_i, y) = \frac{\sum_k \left(E(x_{ki}) - \bar{E}(x_i)\right)\left(E(y_k) - \bar{E}(y)\right)}{\sqrt{\sum_k \left(E(x_{ki}) - \bar{E}(x_k)\right)^2 \left(E(y_k) - \bar{E}(y)\right)^2}} \tag{16}$$

where $E(*)$ is the possibilistic mean value of fuzzy number by Carlsson and Fullér [29], and $\bar{E}(x_k) = \frac{1}{n}\sum_{k=1}^{n} E(x_{ki})$, $\bar{E}(y) = \frac{1}{n}\sum_{k=1}^{n} E(y_k)$.

In order to estimate the model parameters, the least absolute deviation method based on the concept of distance is applied to estimating the regression parameters. Let E_i denote the absolute deviation between the estimated and observed fuzzy responses for the ith case by Mellin transform, which is formulated as

$$E = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} D_M(y_i, \tilde{y}_i) = \sum_{i=1}^{n} |\mu_1(y_i) - \mu_1(\tilde{y}_i)| + \sum_{i=1}^{n} |\sigma(y_i) - \sigma(\tilde{y}_i)| \tag{17}$$

According to Equations (14) and (15), the total error can be formulated as

$$E = \sum_{i=1}^{n} D_M(y_i, \tilde{y}_i) = \sum_{i=1}^{n} \left|\mu_1(y_i) - \sum_{j=0}^{p} b_j \mu_1(x_{ji})\right| + \sum_{i=1}^{n} \left|\sigma(y_i) - \sum_{j=0}^{p} b_j \sigma(x_{ji})\right| \tag{18}$$

Let $\lambda_1 = \max\limits_{i=1,\ldots,n} \left\{\left|\mu_1(y_i) - \sum_{j=0}^{p} b_j \mu_1(x_{ji})\right|\right\}$, $\lambda_2 = \max\limits_{i=1,\ldots,n} \left\{\left|\mu_2(y_i) - \sum_{j=0}^{p} b_j \mu_2(x_{ji})\right|\right\}$, then Equation (18) can be converted to a minimax deviation model, i.e.,

$$\begin{aligned} &\min(\lambda_1 + \lambda_2) \\ &\begin{cases} \left|\mu_1(y_i) - \sum\limits_{j=0}^{p} b_j \mu_1(x_{ji})\right| \le \lambda_1, \quad i = 1, \ldots, n \\ \left|\mu_2(y_i) - \sum\limits_{j=0}^{p} b_j \mu_2(x_{ji})\right| \le \lambda_2, \quad i = 1, \ldots, n \end{cases} \end{aligned} \tag{19}$$

Therefore, we have the linear programming model as follows

$$\begin{aligned} &\min(\lambda_1 + \lambda_2) \\ &\begin{cases} \mu_1(y_i) - \sum\limits_{j=0}^{p} b_j \mu_1(x_{ji}) \le \lambda_1 \\ \mu_1(y_i) - \sum\limits_{j=0}^{p} b_j \mu_1(x_{ji}) \ge -\lambda_1 \\ \mu_2(y_i) - \sum\limits_{j=0}^{p} b_j \mu_2(x_{ji}) \le \lambda_2 \\ \mu_2(y_i) - \sum\limits_{j=0}^{p} b_j \mu_2(x_{ji}) \ge -\lambda_2 \end{cases} \end{aligned} \tag{20}$$

By solving the linear programming model (20) it can determine the optimal parameters for achieving the minimum total error $\min\{E\}$. Therefore, the least absolute deviation method is derived to find the optimal coefficient estimates to minimize the total estimation error (18) of the fuzzy linear regression model.

In order to compare several methods suggested in the fuzzy regression, we use an error measure proposed by Kim and Bishu [30], and Kao and Chyu [31]. Since an estimated fuzzy number is expected to have membership function close to the observed fuzzy membership function, the error of the fitting of the membership functions can be defined by the ratio of the difference of membership values to the observed membership values. That is,

$$E = \frac{\int_{S_y \cup S_{\hat{y}}} |\mu_y(x) - \mu_{\hat{y}}(x)|\, dx}{\int_{S_y} \mu_y(x) dx} \tag{21}$$

If the difference of membership values between two membership functions becomes zero, the error of fit, E, comes close to zero (Figure 2). That is, the less the total sums of error

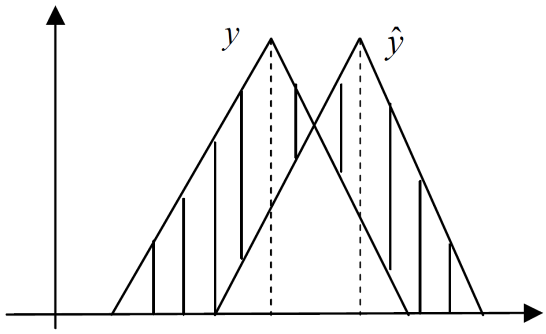

FIGURE 2. Difference of the membership functions between two fuzzy numbers

which is the difference between the membership functions of the predicted output and the observed output, the better the performance of the regression model.

4. **Illustrative Examples.** In this section, we will use two examples including two or several independent variables to illustrate our proposed fuzzy least absolute linear regression method and make some comparisons with some existing fuzzy regression methods.

Example 4.1. *Consider the data in Table 2, in which LR fuzzy numbers are converted to triangular fuzzy numbers, in the example given by Diamond and Körner, the data given by Choi and Buckley [28].*

According to Formula (20), we use matlab tool to calculate the estimate of regression coefficients. The KC method [31] is a two-stage methodology to obtain the regression coefficients. The FLAD method [28] is a method using least absolute deviation estimators to obtain fuzzy regression coefficients. The total sum of errors for the LADMT method is smaller than the total error for KC method and FLAD method using the same error measure Formula (21) in Table 2.

TABLE 2. Numerical data and the estimation errors in Example 4.1

n	x_1	x_2	y	Errors		
				LADMT	KC	FLAD
1	$(6.0, 0.3, 0.9)$	$(6.3, 0.9, 0.8)$	$(61.6, 6.2, 3.1)$	1.395	1.382	1.799
2	$(4.4, 0.4, 0.7)$	$(5.5, 0.8, 0.3)$	$(53.2, 2.7, 5.3)$	1.889	2.018	1.993
3	$(9.1, 0.5, 0.7)$	$(3.6, 0.2, 0.4)$	$(65.5, 9.8, 9.8)$	0.812	0.840	0.841
4	$(8.1, 1.2, 1.2)$	$(5.8, 0.8, 0.9)$	$(64.9, 3.2, 9.8)$	1.013	1.120	1.007
5	$(9.4, 0.7, 1.8)$	$(6.8, 0.3, 0.3)$	$(72.7, 3.6, 7.3)$	0.423	0.623	0.336
6	$(4.8, 0.2, 0.7)$	$(7.9, 1.2, 0.8)$	$(52.2, 2.6, 5.2)$	0.867	1.160	0.386
7	$(7.6, 0.4, 1.1)$	$(4.2, 0.2, 0.6)$	$(50.2, 2.5, 5.0)$	1.871	2.103	2.059
8	$(4.4, 0.2, 0.4)$	$(6.0, 0.6, 0.3)$	$(44.0, 2.2, 4.4)$	0.201	1.518	0.209
9	$(9.1, 0.9, 0.9)$	$(2.8, 0.1, 0.4)$	$(53.8, 8.1, 8.1)$	0.906	0.914	0.946
10	$(6.7, 0.7, 0.7)$	$(6.7, 1.0, 1.0)$	$(53.5, 8.1, 5.4)$	1.402	1.890	1.343
Total error				10.779	13.568	10.919

Example 4.2. *Performance evaluation is an important content of human resource management in the enterprise, because the work directly related to the employee's personal interest and position promotion, transfer and dismissal, and the result of assessment will affect the overall performance of human resources management function. One of the difficulties of employee performance evaluation is the subjective fuzziness of evaluation index,*

and the lack of objective scale. This leads to a lack of objective assessment results, and is only a subjective impression or feeling. According to the theory of human resource management, four subjective factors that influence work performance are considered. Four fuzzy explanatory variables and one fuzzy response are used to subjectively evaluate employees' work performance, work quality (x_1), inability to endure job stress (x_2), frequency of delays (x_3), and communication and coordination ability (x_4), respectively by Chen and Hsueh [15]. Intuitively, higher values for variables x_1 and x_4 and lower values for the variables x_2 and x_3 result in better performance. The dataset is composed of 30 fuzzy observations, as listed in Table 3. All the variables are asymmetrical triangular fuzzy numbers. Fuzzy triangular numbers, which are defined as a fuzzy subset of [0, 100], are produced for each fuzzy explanatory variable and the fuzzy response in the evaluation process [15,21].

Firstly, according to Formula (16), we use matlab to calculate the correlation coefficient between the fuzzy response variable and the ith fuzzy explanatory variable. The result is $\rho(x_1, y) = 0.9399$, $\rho(x_2, y) = -0.2697$, $\rho(x_3, y) = -0.4590$, $\rho(x_4, y) = 0.0563$. From the

TABLE 3. Performance evaluation sample

n	x_1	x_2	x_3	x_4	y	Predicted output
1	(50, 8, 8)	(98, 6, 2)	(71, 9, 11)	(70, 11, 13)	(30, 11, 9)	(31.467, 10.106, 10.992)
2	(29, 8, 8)	(76, 6, 2)	(61, 9, 11)	(46, 11, 13)	(20, 13, 10)	(17.084, 10.106, 10.992)
3	(41, 8, 8)	(88, 6, 2)	(73, 9, 11)	(58, 11, 13)	(25, 11, 12)	(24.370, 10.106, 10.992)
4	(60, 9, 7)	(62, 9, 10)	(79, 9, 6)	(66, 8, 9)	(45, 12, 10)	(46.684, 11.752, 10.346)
5	(49, 9, 7)	(50, 9, 10)	(79, 9, 6)	(54, 8, 9)	(38, 12, 8)	(38.613, 11.752, 10.346)
6	(59, 9, 7)	(60, 9, 10)	(85, 9, 6)	(64, 8, 9)	(43, 11, 9)	(45.230, 11.752, 10.346)
7	(61, 9, 11)	(77, 8, 6)	(85, 5, 8)	(18, 7, 13)	(40, 17, 11)	(37.504, 10.987, 13.463)
8	(58, 9, 11)	(75, 8, 6)	(82, 5, 8)	(16, 7, 13)	(38, 11, 12)	(35.573, 10.987, 13.463)
9	(55, 9, 11)	(72, 8, 6)	(79, 5, 8)	(13, 7, 13)	(37, 12, 12)	(33.752, 10.987, 13.463)
10	(66, 8, 7)	(59, 17, 11)	(39, 8, 9)	(83, 14, 11)	(60, 11, 12)	(59.994, 12.259, 12.286)
11	(69, 8, 7)	(63, 17, 11)	(49, 8, 9)	(87, 14, 11)	(59, 10, 9)	(60.751, 12.259, 12.286)
12	(59, 8, 7)	(53, 17, 11)	(39, 8, 9)	(77, 14, 11)	(54, 11, 8)	(54.680, 12.259, 12.286)
13	(74, 4, 6)	(89, 11, 5)	(70, 12, 13)	(82, 14, 10)	(61, 14, 3)	(55.581, 8.017, 10.482)
14	(41, 4, 6)	(57, 11, 5)	(58, 12, 13)	(50, 14, 10)	(34, 10, 8)	(32.569, 8.017, 10.482)
15	(49, 4, 6)	(65, 11, 5)	(66, 12, 13)	(58, 14, 10)	(38, 9, 9)	(37.426, 8.017, 10.482)
16	(76, 8, 7)	(75, 10, 8)	(37, 8, 11)	(75, 5, 10)	(64, 16, 9)	(64.163, 10.762, 10.560)
17	(57, 8, 7)	(56, 10, 8)	(18, 8, 11)	(56, 5, 10)	(56, 13, 7)	(52.627, 10.762, 10.560)
18	(72, 8, 7)	(71, 10, 8)	(33, 8, 11)	(71, 5, 10)	(63, 11, 9)	(61.734, 10.762, 10.560)
19	(78, 7, 8)	(65, 6, 6)	(82, 11, 11)	(64, 8, 12)	(66, 16, 5)	(60.697, 9.811, 11.145)
20	(58, 7, 8)	(45, 6, 6)	(62, 11, 11)	(44, 8, 12)	(49, 12, 9)	(48.555, 9.811, 11.145)
21	(72, 7, 8)	(59, 6, 6)	(76, 11, 11)	(58, 8, 12)	(55, 10, 12)	(57.055, 9.811, 11.145)
22	(90, 8, 5)	(95, 13, 3)	(80, 11, 8)	(72, 7, 13)	(67, 11, 14)	(65.282, 9.445, 10.313)
23	(68, 8, 5)	(73, 13, 3)	(58, 11, 8)	(50, 7, 13)	(53, 10, 9)	(51.926, 9.445, 10.313)
24	(71, 8, 5)	(76, 13, 3)	(61, 11, 8)	(53, 7, 13)	(54, 9, 10)	(53.747, 9.445, 10.313)
25	(92, 8, 6)	(76, 6, 9)	(78, 10, 6)	(27, 9, 15)	(70, 13, 7)	(66.211, 10.790, 9.663)
26	(94, 8, 6)	(78, 6, 9)	(80, 10, 6)	(29, 9, 15)	(68, 9, 10)	(67.425, 10.790, 9.663)
27	(87, 8, 6)	(71, 6, 9)	(73, 10, 6)	(22, 9, 15)	(65, 10, 9)	(63.175, 10.790, 9.663)
28	(94, 6, 5)	(51, 9, 8)	(30, 9, 11)	(29, 9, 16)	(75, 5, 14)	(80.447, 9.537, 9.482)
29	(95, 6, 5)	(52, 9, 8)	(31, 9, 11)	(30, 9, 16)	(84, 10, 7)	(81.054, 9.537, 9.482)
30	(86, 6, 5)	(43, 9, 8)	(22, 9, 11)	(21, 9, 16)	(80, 12, 6)	(75.590, 9.537, 9.482)
Total estimation error $\sum E_i$						1495.2

correlation coefficient, we find that work quality and communication and coordination ability is positively related to employees' work performance, while inability to endure job stress and frequency of delays are negatively related. This is consistent with the actual situation.

Then we use the least absolute deviation Mellin transform distance method to estimate the membership function of fuzzy output for the sample given in Table 3. By substituting the data in Table 3 into the linear programming model (20), then the optimal solution based on the LADMT method is obtained as follows:

$$y_M = 12.5876(1, 0, 0) + 0.8529x_1 - 0.2295x_2 - 0.1365x_3 + 0.1202x_4$$

The result is shown in Table 3 and Figure 3. Known from Figure 3, we can conclude that the center regression approximates pass all observed value, the left and right spread also approximate pass all observed value. It shows that our LADMT method is effective for fuzzy regression analysis about employee performance evaluation.

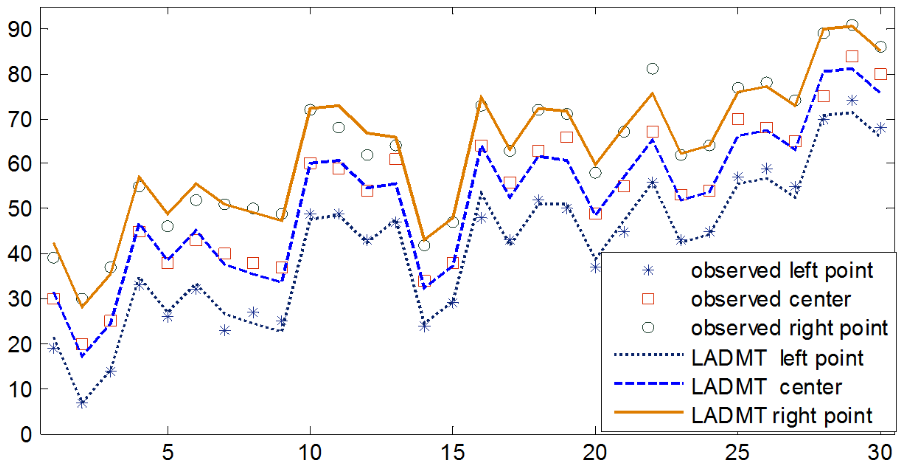

FIGURE 3. The result of fuzzy regression model

We will also use the data in Table 3 into Tanaka et al.'s method [2], Diamond's method [4] and Chen and Hsueh's method [15], and the regression equations are obtained as follows:

$$y_T = 17.1889(1, 0, 0) + 0.6527x_1 - 0.3978x_2 - 0.0020x_3 + 0.4138x_4$$
$$y_D = 10.6155(1, 0, 0) + 0.8764x_1 - 0.1815x_2 - 0.1489x_3 + 0.0824x_4$$
$$y_C = (12.093, 1.299, 0.039) + 0.859x_1 - 0.207x_2 - 0.134x_3 + 0.108x_4$$

When fuzzy regression model is adopted, the following task is to consider the difference between the membership values of the observed fuzzy number y_i and the estimated fuzzy number $\tilde{y}_i$. In order to minimize the total difference between estimated and observed response variables, many distances have been proposed to measure the total difference between y_i and $\tilde{y}_i$. Kim and Bishu [30] used an integration of the membership functions to compare the accuracy of the developed fuzzy regression model. However, this measure of performance has a weakness when y_i and $\tilde{y}_i$ are not overlapping. The value of the measure of performance will be the same regardless of how much they are overlapping [32]. D'Urso [33] proposed the Euclidean distance between two fuzzy numbers $y = (y_m, \alpha_y, \beta_y)$ and $\tilde{y} = (\tilde{y}_m, \alpha_{\tilde{y}}, \beta_{\tilde{y}})$, which is defined as

$$\Phi_E(y, \tilde{y}) = \sqrt{(y_m - \tilde{y}_m)^2\pi_c + ((y_m - \alpha_y) - (\tilde{y}_m - \alpha_{\tilde{y}}))^2 \pi_\alpha + ((y_m + \beta_y) - (\tilde{y}_m + \beta_{\tilde{y}}))^2 \pi_\beta} \qquad (22)$$

where π_c, π_α and π_β are arbitrary positive weights.

Roh et al. [34] defined a distance between $y = (y_m, \alpha_y, \beta_y)$ and $\tilde{y} = (\tilde{y}_m, \alpha_{\tilde{y}}, \beta_{\tilde{y}})$ to be

$$\Phi_R(y, \tilde{y}) = \sqrt{\int_0^1 (y^L(\gamma) - \tilde{y}^L(\gamma))^2 d\gamma + \int_0^1 (y^U(\gamma) - \tilde{y}^U(\gamma))^2 d\gamma} \tag{23}$$

where $y(\gamma) = \left[y^L(\gamma), y^U(\gamma)\right]$ and $\tilde{y}(\gamma) = \left[\tilde{y}^L(\gamma), \tilde{y}^U(\gamma)\right]$ are γ-cuts set of two fuzzy numbers.

To demonstrate the feasibility of the proposed LADMT method, its performance is compared with those of other models in Example 4.2. The above two distance measures Φ_E, Φ_R are employed to compare the efficiency of the fuzzy regression model estimated by the LADMT distance method.

Using the error estimate Equations (22) and (23), the error estimate values Φ_E, Φ_R for observed fuzzy number y_i and the estimated fuzzy number $\tilde{y}_i$ are calculated as

$$\Phi_E(y_T, y) = 456.9550,\ \Phi_E(y_D, y) = 87.5736,\ \Phi_E(y_C, y) = 99.8691,\ \Phi_E(y_M, y) = 107.668$$
$$\Phi_R(y_T, y) = 357.3663,\ \Phi_R(y_D, y) = 64.2912,\ \Phi_R(y_C, y) = 71.2459,\ \Phi_R(y_M, y) = 79.362$$

By comparison, it is found that the total error of LADMT method is greater than Diamond and Chen and Hsueh's method, but smaller than Tanaka et al.'s method. On the surface, LADMT method is not as good as Diamond and Chen and Hsueh's method. In fact, if from a single component error point of view, we find that the errors only on the individual points are larger, while the other points of the error are not too large. The error of individual points magnifies the overall error. This can be seen from Figure 4 and Figure 5. On the whole, the advantage of LADMT method is to obtain some accurate estimators for parameters. It does not need to perform more complex computation, but only need to calculate simple linear programming.

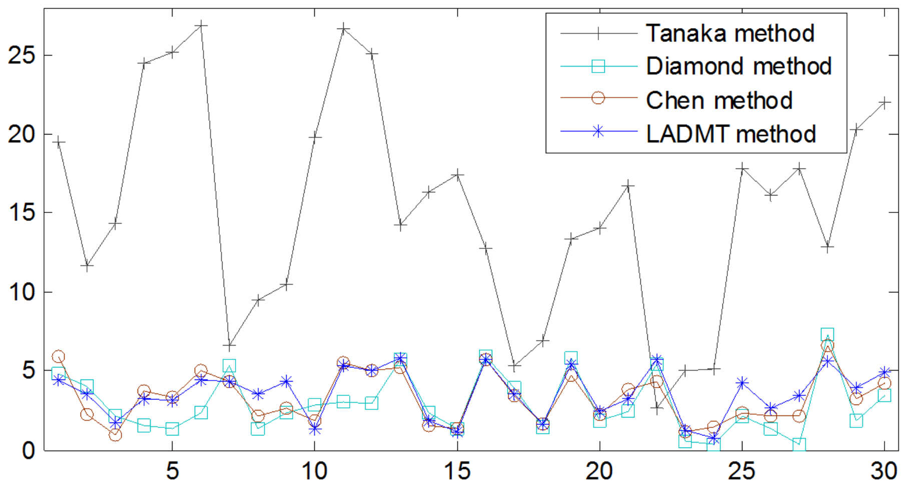

FIGURE 4. The error based on the distance measure Φ_E

5. Conclusion.

In this paper, we introduce the Mellin transform distance between triangular fuzzy numbers, and apply least absolute deviation Mellin transform distance method to constructing the fuzzy linear regression model with fuzzy inputs, fuzzy output and crisp parameters. It is easy to be calculated and has got the better results for evaluating the fitting affect of the observed and estimated values by minimax deviation sum of many different squared distances. Two examples also show that the method has good fitting effect and simple calculation.

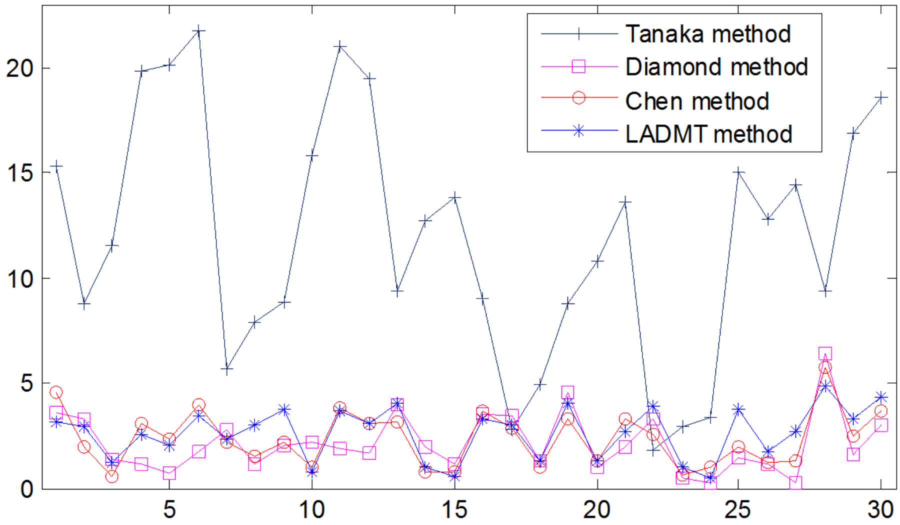

FIGURE 5. The error based on the distance measure Φ_R

It needs to point out that the least absolute deviation Mellin transform distance method can be extended to the fuzzy linear regression model with fuzzy input and fuzzy output, where fuzzy input/output is represented by generalized triangular or trapezoidal LR-fuzzy numbers.

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