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INTEGRATED FAULT ESTIMATION AND FAULT-TOLERANT CONTROL FOR RIGID SPACECRAFT ATTITUDE SYSTEM WITH MULTIPLE ACTUATOR FAULTS

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Abstract. *This paper proposes an integrated fault estimation and fault-tolerant control (FE/FTC) method for a rigid spacecraft attitude system in the presence of multiple actuator faults. The proposed scheme solves a problem of fault-tolerant controller design in the presence of actuator loss of effectiveness (LOE) fault, bias fault, uncertainties and external disturbances. Firstly, an adaptive sliding mode fault estimation observer is designed for the faulty rigid spacecraft attitude system to get the estimated values of actuator faults under the existence of unknown inertia matrix uncertainties and external disturbances. Then, an integrated FE/FTC attitude control strategy is developed by using both fractional-order nonsingular terminal sliding mode (FONTSM) method and backstepping technology, and it could guarantee that the attitude error variables of faulty closed-loop attitude system are stable within a finite time. Finally, simulation results are presented to demonstrate the effectiveness of the proposed technique.*
Keywords: Fault estimation, Fault-tolerant control, Sliding mode, Rigid spacecraft

1. Introduction.

With the rapid development of space technology, space flight missions such as satellite surveillance, space station docking and installation, spacecraft formation flying has higher attitude control requirements. The attitude control for spacecraft is an important and practical problem and some results have been reported in the literature recently [1,2]. The attitude dynamics of the spacecraft is highly nonlinear and time-varying in the presence of actuator dynamics, actuator saturation, reaction wheel friction, space environmental disturbances, the moment of inertia uncertainty and even unknown actuator faults [3]. These uncertainties and disturbances will degrade the performance of the attitude control, and even cause mission failure and catastrophic accidents, especially in cases of actuator faults [4]. Therefore, the robust and fault-tolerant control technology needs to be taken into consideration in the design of modern spacecraft attitude systems.

Fault-tolerant control theory has been greatly developed in recent years, some of which have been applied to many fields [5-7]. Especially in the aerospace industry, in order to ensure the stability of the spacecraft attitude system and the acceptable control performance of attitude tracking in cases of actuator or sensor faults, the design of fault-tolerant controllers has attracted more and more attention in recent years. An active FTC scheme

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for flexible spacecraft attitude control system with sensor faults was designed in [8], which requires a fault-detection and isolation (FDI) mechanism. FDI can potentially help avoid the development of serious faults, and the detailed fault information generated during the fault diagnosis process is very valuable to design fault-tolerant controller. However, the overall performance of such FTC methods is directly influenced by the time delay between fault occurrence and fault isolation as well as the accuracy of the FDI algorithm. In contrast, passive FTC without a fault diagnosis mechanism can potentially respond more quickly to the occurrence of faults and automatically adjust the controller signal to compensate for the effects of multiple simultaneous faults in [9]. Although passive FTC strategy improves the response speed to faults, there is no accurate fault information that reduces the performance of fault-tolerant controller. In order to avoid the disadvantages of active FTC strategy and passive FTC strategy, an integrated FE/FTC strategy is proposed in some literature [10-12]. For example, an integrated FE/FTC approach is presented for a class of linear discrete time systems with additive or multiplicative faults in [10], and in this approach, the reconstructed or estimated fault signals are used directly in the control system to compensate for the effects of the faults. [11] focused on the FTC approach using FE and fault compensation for a class of linear systems with system state uncertainty. In [12], a decoupling approach to the integrated design of FE/FTC is proposed for linear systems in the presence of unknown bounded actuator faults and perturbations. The direct use of FE for topics designed for FTC systems without the need for a reconfigurable mechanism brings significant convenience and application potential.

However, the integrated FE/FTC design of the attitude system of rigid spacecraft has the big challenge since the fault estimation observer and fault-tolerant controller reconfiguration roles have a bi-directional uncertainty, and it is more complex when compared with passive FTC within a closed-loop system, i.e., without an FE function. The complexity arises from the joint multi-objectives of robust closed-loop stability, robust residual performance, and robust fault tolerance with stable reconfiguration, generally operating in the presence of disturbances and uncertainties. In addition, the available literature for the integrated design of FE/FTC for the attitude system of rigid spacecraft in actuator faulty case is very limited. Therefore, the above statement motivates us to do this study.

It is worth noting that adaptive sliding mode control technology is considered as an effective method to deal with external disturbances and model uncertainties in nonlinear systems, and has been widely used in fault-tolerant control systems (FTCS) [13,14]. In [13], a sliding mode observer was designed to reconstruct the lumped fault including actuator fault and external disturbances, and a velocity-free attitude controller was synthesized to asymptotically stabilize the attitude. In [14], an adaptive sliding mode controller is derived by using on-line updating law to estimate the bound of actuator fault such that any information of the fault is not required. The previous fault-tolerant attitude control scheme can only produce asymptotic convergence of attitude errors, while the second-order sliding mode and NTSMC methods are used for [15,16], respectively, to achieve finite-time fault-tolerant attitude control. For complex lumped perturbations, fractional-order (FO) controllers using FO calculus have been widely validated to be more efficient than integer-order (IO) counterparts [17]. The FO controller has been applied to robot manipulators in [18-20]. Due to the FONTSM surface and the fast-TSM-type reaching law, the proposed control design ensures fast convergence and high tracking accuracy under uncertainty in [18].

This paper investigates the integrated design of FE/FTC for attitude tracking systems of rigid spacecraft in the presence of the LOE fault, the bias fault, unknown uncertainties and external disturbances. The main contributions in this paper are as follows.

• Compared with the fault estimation observer design approach in [13], the sliding mode fault estimation observer developed in this paper does not require the known upper bound information on the uncertainties and external disturbances, which could effectively estimate the amplitude of unknown actuator fault effects.

• The FONTSM manifold proposed in this paper could ensure better dynamic performance in the sliding mode phase than some existing nonsingular terminal sliding mode (NTSM) manifolds, such as [16,21]. In the case of multiple actuator faults, it is proven that the FONTSM attitude controller can ensure fast convergence and high tracking accuracy of the faulty closed-loop attitude system in finite time.

• Compared with passive FTC [6,9], which could not acquire the unknown fault estimated values, the integrated FE/FTC scheme designed in this paper has better fault-tolerant control effectiveness. This strategy relies on a fault estimation observer to obtain the accurate fault estimation, which will be used in the design of fault-tolerant attitude controller, such that the better fault accommodation performance could be achieved.

The rest of the paper is organized as follows. In Section 2, the rigid spacecraft attitude mathematical model and the control problem are described. A sliding mode fault estimation observer is designed for the faulty attitude system of rigid spacecraft to get the estimated values of the actuator LOE fault and bias fault in Section 3. In Section 4, an integrated FE/FTC controller is designed to eliminate the actuator faults. Simulation results of Section 5 are presented to demonstrate the effectiveness of the proposed method. Conclusions are given in Section 6.

2. **Attitude Kinematics and Dynamics.**

In this section, the Euler-angles moment equations are applied to describing the attitude system of rigid spacecraft, the equations of motion in terms of kinematics are given by [3]

$$\begin{cases} \dot{\nu} = \omega_2 \sin\gamma + \omega_3 \cos\gamma \\ \dot{\psi} = (\omega_2 \cos\gamma - \omega_3 \sin\gamma)/\cos\nu \\ \dot{\gamma} = \omega_1 - \tan\nu(\omega_2 \cos\gamma - \omega_3 \sin\gamma) \end{cases} \tag{1}$$

where ν, ψ, γ are pitch angle, yaw angle and roll angle, respectively. ω_1, ω_2 and ω_3 denote the angular velocities with respect to the body-fixed frame.

For the simplicity, the kinematic Equation (1) could be rewritten as

$$\dot{\sigma} = G(\sigma)\omega \tag{2}$$

where $\sigma = [\nu, \psi, \gamma]^T$, $\omega = [\omega_1, \omega_2, \omega_3]^T$ and

$$G(\sigma) = \begin{bmatrix} 0 & \sin\gamma & \cos\gamma \\ 0 & \cos\gamma/\cos\nu & -\sin\gamma\cos\nu \\ 1 & -\tan\nu\cos\gamma & \tan\nu\sin\gamma \end{bmatrix}.$$

The dynamic equation of the rigid spacecraft with respect to the uncertainties of inertia matrix is described as [21]

$$(J + \triangle J)\dot{\omega} = -\omega^{\times}(J + \triangle J)\omega + Mu + T_d \tag{3}$$

where $J \in R^{3\times3}$ is the symmetric inertia matrix of rigid spacecraft, $\triangle J$ represents the uncertainties of inertia matrix, $u = [u_1, u_2, \ldots, u_N]^T$ is the control torque, $T_d = [T_{d1}, T_{d2}, T_{d3}]^T$ represents the external disturbance torque, and $\omega^{\times}$ is a skew-symmetric matrix. $M \in R^{3\times N}$ is the actuator distribution matrix, and N is the number of actuators.

After some manipulations, Equation (3) can be transformed into the following form

$$\dot{\omega} = -J^{-1}\omega^{\times}J\omega + J^{-1}Mu + J^{-1}d \tag{4}$$

where $d = T_d - \omega^\times \triangle J\omega - \triangle J\dot{\omega}$ and it can be viewed as a kind of generalized perturbation for spacecraft dynamics equation.

In this paper, different types of actuator faults are taken into account. The actuator fault model is described as [21]

$$u = Eu_c + u_f \tag{5}$$

where $u_c = [u_{c1}, u_{c2}, \ldots, u_{cN}]^T$ is command control torque; $E = \text{diag}\{e_1, e_2, \ldots, e_N\}$ is the reaction wheel effectiveness matrix, and e_i satisfying $0 \le e_i \le 1$ $(i = 1, 2, \ldots, N)$. If $e_i = 1$, it means that the ith actuator works normally, and $0 < e_i < 1$ $(i = 1, 2, \ldots, N)$ indicates that the ith actuator is producing a reduced effectiveness. The value $e_i = 0$ implies that the ith actuator undergoes a complete failure. $u_f = [u_{f1}, u_{f2}, \ldots, u_{fN}]^T$ is the additive bias fault.

Substituting the actuator fault model (5) into the kinetics (4), the faulty attitude dynamics can be described as

$$\dot{\omega} = -J^{-1}\omega^\times J\omega + J^{-1}MEu_c + J^{-1}Mu_f + J^{-1}d \tag{6}$$

Next, the following assumptions and lemmas are given in order.

Assumption 2.1. *It is a reasonable assumption that the nonlinear function $\omega^\times J\omega$ is locally Lipschitz bounded with a Lipschitz constant ε, which can be formulated in the following [25]*

$$\|\omega^\times J\omega - \hat{\omega}^\times J\hat{\omega}\| \le \varepsilon\|\omega - \hat{\omega}\| = \varepsilon\|\tilde{\omega}\| \tag{7}$$

Assumption 2.2. *Using the calculation in [3], the disturbance can be probably estimated as to be bounded. It is reasonable to assume that there always exists an unknown constant d_M such that $\|d\| \le d_M$.*

Assumption 2.3. *In this study, fault-tolerant control of actuator faults is implemented by installing redundant actuators, i.e., the number of actuators required is $N > 3$ and the number of total failure actuators is no more than $N - 3$ [1].*

Lemma 2.1. [20]: *The fractional integrator I^a_{b+} and I^a_{c-} using power a, $\Re(a) > 0$ are bounded in $L_p(b, c)$, $1 < p < \infty$.*

$$\|I^a_{b+}y\|_p \le W\|y\|_p, \quad \|I^a_{c-}y\|_p \le W\|y\|_p, \quad \left(W = \frac{\left(c - b^{\Re(a)}\right)}{\Re(a)\|\Gamma(a)\|}\right).$$

Lemma 2.2. [21]: *The extended Lyapunov description of finite-time stability with faster finite time convergence is given as:*

$$\dot{V}(x) + r_1V(x) + r_2V^{r_0}(x) \le 0 \tag{8}$$

and the convergence time is given as:

$$T_k \le \frac{1}{r_1(1 - r_0)} \ln \frac{r_1V^{1-r_0}(x_0) + r_2}{r_2} \tag{9}$$

where $r_1 > 0$, $r_2 > 0$ and $0 < r_0 < 1$.

Remark 2.1. *Assumption 2.1 is reasonable because system state $\omega(t)$ could be directly measured by the gyros and it is continuously differentiable. In Assumption 2.2, the generalized disturbance torques include the inertia uncertainty, the gravity gradient torque, the magnetic disturbance torque, the aerodynamic torque, the solar radiation torque, internal disturbance torque and other environmental torques. Assumption 2.3 means that, although the N actuators ($N > 3$) may suffer from partial loss of actuator effectiveness or even complete failure, the number of totally failed actuators is no more than $N - 3$*

such that ME^3M^T remains positive definite. If more than $N-3$ actuators have totally failed, the matrix ME^3M^T becomes singular and the system is underactuated. In this situation, other types of actuators have to be employed [26]. The underactuated system is not considered further in this paper.

In this study, the LOE fault, bias fault, uncertainties and external disturbances are considered in the system (6). Figure 1 shows the integrated FE/FTC design structure, which includes the design of the fault estimation observer and the fault-tolerant controller. To deal with the multiple actuator faults of system (6), an adaptive sliding mode fault estimation observer and a fault-tolerant controller will be designed separately in Section 3 and Section 4.

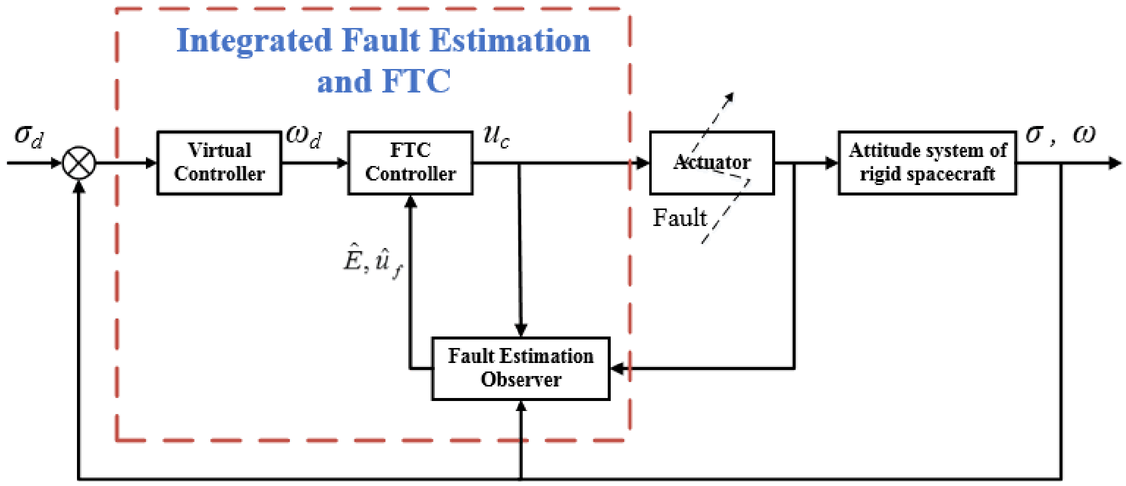

FIGURE 1. The structure of the integration of FE/FTC developed in this paper

3. **Fault Estimation Observer Design.** In this position, a sliding mode fault estimation observer is presented to achieve the estimation of LOE fault and bias fault.

As E is a diagonal matrix, the term Eu_c in (6) can be rearranged as

$$Eu_c = Ue \tag{10}$$

where $U = \text{diag}\{u_{c1}, u_{c2}, \ldots, u_{cN}\}$ and $e = [e_1, e_2, \ldots, e_N]^T$.

The faulty rigid spacecraft Equation (6) can be transformed into

$$\dot{\omega} = -J^{-1}\omega^\times J\omega + J^{-1}MUe + J^{-1}Mu_f + J^{-1}d \tag{11}$$

An adaptive sliding mode fault estimation observer is designed as

$$\dot{\hat{\omega}} = \Lambda(\omega - \hat{\omega}) - J^{-1}\hat{\omega}^\times J\hat{\omega} + J^{-1}MU\hat{e} + J^{-1}M\hat{u}_f + J^{-1}K(t)\,\text{sign}\,(\tilde{\omega}) \tag{12}$$

where $K(t)$ is a positive parameter, and Λ is a diagonal matrix determined in advance.

Let $\tilde{e} = e - \hat{e}$, subtracting (12) from (11), we can obtain the following equation

$$\dot{\tilde{\omega}} = -\Lambda\tilde{\omega} - J^{-1}(\omega^\times J\omega - \hat{\omega}^\times J\hat{\omega}) + J^{-1}MU\tilde{e} + J^{-1}M\tilde{u}_f + J^{-1}d - J^{-1}K(t)\,\text{sign}\,(\tilde{\omega}) \tag{13}$$

where $\tilde{\omega} = \omega - \hat{\omega}$ and $\tilde{u}_f = u_f - \hat{u}_f$.

The adaptive laws for estimation of e and u_f are given as follows

$$\dot{\hat{e}} = \lambda_1 U^T M^T J^{-T}\tilde{\omega}, \quad \dot{\hat{u}}_f = \lambda_2 M^T J^{-T}\tilde{\omega} \tag{14}$$

where λ_1 and λ_2 are positive constants.

The adaptive control gain $K(t)$ is given

$$\dot{K}(t) = \begin{cases} \lambda_3\|J^{-1}\|\|\tilde{\omega}\|\text{sign}(\|\tilde{\omega}\| - \epsilon), & \text{if } K > \mu \\ \mu, & \text{if } K \leq \mu \end{cases} \tag{15}$$

where $K(0) > 0$, $\lambda_3 > 0$, $\epsilon > 0$, and $\mu > 0$ is very small.

Lemma 3.1. [22]: *For the sliding mode fault estimation observer (12), the adaptive gain K, defined in (15), has an upper bound K^* for all $t > 0$ with $K^* > d_M$.*

Theorem 3.1. *For the faulty spacecraft attitude systems (2) and (6), suppose that Assumptions 2.1-2.3 hold, and the following conditions are satisfied $\lambda_{\min}(\Lambda) - \varepsilon\|J^{-1}\| \geq 0$, then the proposed fault estimation observer (12) can provide an accurate estimation of the actuator loss of effectiveness fault and the bias fault.*

Proof: In this position, the Lyapunov candidate is selected as follows

$$V_0 = \frac{1}{2}\tilde{\omega}^T\tilde{\omega} + \frac{1}{2\lambda_1}\tilde{e}^T\tilde{e} + \frac{1}{2\lambda_2}\tilde{u}_f^T\tilde{u}_f + \frac{1}{2\lambda_3}(K - K^*)^2 \tag{16}$$

The time derivative of V_0 is given as

$$\begin{aligned} \dot{V}_0 &= \tilde{\omega}^T\dot{\tilde{\omega}} + \frac{1}{\lambda_1}\tilde{e}^T\dot{\tilde{e}} + \frac{1}{\lambda_2}\tilde{u}_f^T\dot{\tilde{u}}_f + \frac{1}{\lambda_3}(K - K^*)\dot{K} \\ &= \tilde{\omega}^T\big[-\Lambda\tilde{\omega} - J^{-1}(\omega^\times J\omega - \hat{\omega}^\times J\hat{\omega}) + J^{-1}MU\tilde{e} + J^{-1}M\tilde{u}_f \\ &\quad + J^{-1}d - J^{-1}K\,\text{sign}\,(\tilde{\omega})\big] - \frac{1}{\lambda_1}\tilde{e}^T\dot{\hat{e}} - \frac{1}{\lambda_2}\tilde{u}_f^T\dot{\hat{u}}_f + \frac{1}{\lambda_3}(K - K^*)\dot{K} \end{aligned} \tag{17}$$

Introduce parameter $\|\tilde{\omega}^T\|\|J^{-1}\|K^*$ as

$$\begin{aligned} \dot{V}_0 \leq &-(\lambda_{\min}(\Lambda) - \varepsilon\|J^{-1}\|)\|\tilde{\omega}\|^2 + \|\tilde{\omega}^T\|\|J^{-1}\|(d_M - K) \\ &- \|\tilde{\omega}^T\|\|J^{-1}\|K^* + (K - K^*)\|J^{-1}\|\|\tilde{\omega}\|\,\text{sign}\,(\|\tilde{\omega}\| - \epsilon) \\ &- \frac{1}{\lambda_2}\tilde{u}_f^T\dot{\hat{u}}_f + \frac{1}{\lambda_3}(K - K^*)\dot{K} \end{aligned} \tag{18}$$

Substituting first equation of (15) into (18), we can obtain

$$\begin{aligned} \dot{V}_0 \leq &-\left(\lambda_{\min}(\Lambda) - \varepsilon\|J^{-1}\|\right)\|\tilde{\omega}\|^2 + \|\tilde{\omega}^T\|\|J^{-1}\|(d_M - K^*) \\ &- (K - K^*)\underbrace{[\|\tilde{\omega}^T\|\|J^{-1}\| - \|J^{-1}\|\|\tilde{\omega}\|\,\text{sign}\,(\|\tilde{\omega}\| - \epsilon)]}_{\beta} \end{aligned} \tag{19}$$

From Lemma 3.1, one can obtain

$$\dot{V}_0 \leq -\left(\lambda_{\min}(\Lambda) - \varepsilon\|J^{-1}\|\right)\|\tilde{\omega}\|^2 - \beta(K - K^*) \tag{20}$$

According to (15), the two cases are analyzed as follows.

- **Case 1:** Suppose that $\|\tilde{\omega}\| < \epsilon$, if $\lambda_{\min}(\Lambda) - \varepsilon\|J^{-1}\| \geq 0$, one gets $\dot{V}_0 \leq 0$. It is always possible to choose Λ such that the previous inequality fulfils. Therefore, convergence to a domain $\|\tilde{\omega}\| < \epsilon$ is guaranteed from any initial condition $\|\tilde{\omega}\| > \epsilon$.
- **Case 2:** Suppose that $\|\tilde{\omega}\| < \epsilon$, function β in (20) can be negative. It means that $\dot{V}_0$ would be an indefinite sign and it is impossible to draw conclusions about the stability of the attitude system. Therefore, $\|\tilde{\omega}\|$ can increase over ϵ. As soon as $\|\tilde{\omega}\|$ becomes greater than ϵ, V_0 starts decreasing. Apparently, decrease of V_0 can be achieved via increase of $K(t)$ allowing $\|\tilde{\omega}\|$ to increase before it starts decreasing down to $\|\tilde{\omega}\| < \epsilon$. The proof is similar as in [22] and is not presented here.

From the above, it can be concluded that the estimated error converges near the origin. Thus, we complete the proof.

Remark 3.1. *In Equation (15), the μ parameter only guarantees a positive value for $K(t)$. The sequel, without loss of generality, only considers the case of $K(t) > \mu$ for discussion and proof.*

Remark 3.2. *Compared with the observer design approach developed in [13], the priori knowledge of uncertainties/disturbances is required. In addition, only one type of faults is estimated in [13]. However, the adaptive sliding mode observer proposed in this study does not require the prior knowledge of uncertainties/disturbances and multiple actuator faults are estimated simultaneously. Therefore, it is one of the main contributions of the sliding mode fault estimation observer designed in this paper.*

Remark 3.3. *Although the sliding mode fault estimation observer is proposed under unknown uncertainties/disturbances in [24], the sliding mode fault estimation observer must select too high switching term parameters to suppress external disturbances. In this study, the switching term gain of the sliding mode fault estimation observer is designed by using an adaptive method, which greatly relaxes the strict conditions of the fault estimation observer design in [24]. When the sliding mode variable $\tilde{\omega}(t)$ enters the boundary layer of the sliding mode surface, the proposed gain-adaptation law (15) allows the gain parameter K is adaptive to uncertainties/disturbances and declining to the minimum degree of stability of the sliding mode variable $\tilde{\omega}(t)$. The gain-adaptation law does not overestimate uncertainties/disturbances magnitude and the chattering of the sliding mode variable $\tilde{\omega}(t)$ is reduced in the sliding mode surface such that fault information obtained from the sliding mode fault estimation observer is more accurate.*

In the above, the designed fault estimation observer obtained accurate fault information. Then, we will design a fault-tolerant controller based on integrated FE/FTC strategy to complete the attitude tracking of the faulty closed-loop attitude system.

4. Fault-Tolerant Controller Design.

In this section, based on adaptive sliding mode fault estimation observer, an integrated FE/FTC controller by using backstepping and the sliding mode method is designed to compensate for the actuator faults.

4.1. Preliminaries of fractional calculus.

Fractional calculus is a generalization of integration and differentiation to fractional order fundamental operation. In this paper, the notation D^α indicates the Reimann-Liouville (RL) fractional derivative [17].

Definition 4.1. *The αth-order RL fractional derivative of function $f(t)$ is defined*

$$ {}_{t_0}D_t^\alpha f(t) = \frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha} = \frac{1}{\Gamma(m-\alpha)} \frac{\mathrm{d}^m}{\mathrm{d}t^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} \mathrm{d}\tau \tag{21} $$

where $\Gamma(\alpha)$ is the gamma function, t_0 is the initial time, and $m-1 < \alpha \le m$, $m \in N$.

Property 4.1. *The RL fractional derivative operator ${}_{t_0}D_t^\alpha$ commutes with $\mathrm{d}^n/\mathrm{d}t^n$*

$$ \frac{\mathrm{d}^n \left({}_{t_0}D_t^\alpha f(t)\right)}{\mathrm{d}t^n} = {}_{t_0}D_t^\alpha \left(\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}\right) = {}_{t_0}D_t^{\alpha+n} f(t) \tag{22} $$

4.2. FONTSM fault-tolerant tracking controller design.

Combined with backstepping technology and sliding mode control method, the design steps of FONTSM fault-tolerant controller are as follows.

Step 1: Two new error variables are defined

$$ z_1 = \sigma - \sigma_d, \quad z_2 = \omega - \omega_d \tag{23} $$

where σ_d is desired attitude angle commands, and ω_d is desired angular velocity commands.

The virtual control law is selected as follows with respect to the outer loop error z_1

$$\dot{z}_1 = G(\sigma)(\omega_d + z_2) - \dot{\sigma}_d \tag{24}$$

Consider a Lyapunov function

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{25}$$

The time derivative of V_1 is

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T [G(\sigma)(\omega_d + z_2) - \dot{\sigma}_d] \tag{26}$$

Design the virtual control ω_d as

$$\omega_d = -cz_1 + G^{-1}(\sigma)\dot{\sigma}_d \tag{27}$$

where c is a design parameter.

From Equation (27), one can obtain

$$\dot{V}_1 \leq -c\|z_1\|^2 + z_1^T G(\sigma) z_2 \tag{28}$$

Step 2: The time derivative of z_2 is

$$\dot{z}_2 = \dot{\omega} - \dot{\omega}_d = -J^{-1}\omega^{\times} J\omega + J^{-1}MEu_c + J^{-1}Mu_f + J^{-1}d - \dot{\omega}_d \tag{29}$$

According to the attitude errors and the angular velocity errors, a sliding mode surface is selected as follows [20]:

$$s = z_2 + aD^{\alpha_1}\left[\mathrm{sig}(z_1)^{\beta_1}\right] + bD^{\alpha_2 - 1}\left[\mathrm{sig}(z_1)^{\beta_2}\right] \tag{30}$$

where $a = \mathrm{diag}\{a_1, a_2, a_3\}$, $b = \mathrm{diag}\{b_1, b_2, b_3\}$, α_1, α_2 and β_1, β_2 are control parameters. For any vector $x \in R^3$, there is $\mathrm{sig}(x)^\alpha = [|x_1|^\alpha \mathrm{sign}(x_1), |x_2|^\alpha \mathrm{sign}(x_2), |x_3|^\alpha \mathrm{sign}(x_3)]^T$.

According to the sliding mode surface (30), the spacecraft dynamics Equation (6) and using Definition 4.1 and Property 4.1, it can obtain

$$\dot{s} = -J^{-1}\omega^{\times} J\omega + J^{-1}MEu_c + J^{-1}Mu_f + J^{-1}d + aD^{\alpha_1+1}\left[\mathrm{sig}(z_1)^{\beta_1}\right] + bD^{\alpha_2}\left[\mathrm{sig}(z_1)^{\beta_2}\right] \tag{31}$$

To guarantee fast convergence and stability with actuator faults and the existence of complex time-varying disturbances, the fast-TSM-type reaching law is adopted as [22]

$$\dot{s} = -\varepsilon_1 \mathrm{sign}(s) - \varepsilon_2 \mathrm{sig}(s)^r \tag{32}$$

where $\varepsilon_i = \mathrm{diag}\{\varepsilon_{i1}, \ldots, \varepsilon_{in}\}$, $i = 1, 2$ are diagonal parameter matrices and $0 < r < 1$.

Define $\Upsilon = M\tilde{u}_f + d$. Then, the proposed control schemes can be given as (33)-(36) based on the newly proposed FONTSM (30) and fast-TSM-type reaching law (32)

$$u_c = \hat{E}^2 M^T \left(J^{-1} M \hat{E}^3 M^T\right)^{-1} (u_{cn} + u_{cm}) \tag{33}$$

$$u_{cn} = J^{-1}\omega^{\times} J\omega - J^{-1}M\hat{u}_f - J^{-1}\hat{\Upsilon} \tag{34}$$

$$u_{cm} = -aD^{\alpha_1+1}\left[\mathrm{sig}(z_1)^{\beta_1}\right] - bD^{\alpha_2}\left[\mathrm{sig}(z_1)^{\beta_2}\right] - \varepsilon_1 \mathrm{sign}(s) - \varepsilon_2 \mathrm{sig}(s)^r \tag{35}$$

$$\dot{\hat{\Upsilon}} = \rho J^{-T} s \tag{36}$$

where $\hat{\Upsilon}$ is an estimation of Υ. ρ, ε_1 and ε_2 are positive control parameters.

According to the integrated FE/FTC scheme designed in (33)-(36), a significant result is given as follows.

Theorem 4.1. *For the rigid spacecraft attitude systems (2) and (6) in actuator faulty case, suppose that Assumptions 2.1-2.3 are satisfied, if a FONTSM manifold is chosen as (30), the virtual controller (27) and the fault-tolerant controller (33) designed in this study guarantee that the attitude error variables z_1 and z_2 are stable within a finite time.*

Proof: Define the following Lyapunov function:

$$V_2 = V_0 + V_1 + \frac{1}{2}s^T s + \frac{1}{2\rho}\tilde{\Upsilon}^T\tilde{\Upsilon} \tag{37}$$

The time derivative of V_2 is given as

$$\begin{aligned}\dot{V}_2 &= \dot{V}_0 + \dot{V}_1 + s^T\dot{s} - \frac{1}{\rho}\tilde{\Upsilon}^T\dot{\hat{\Upsilon}} \\ &= \dot{V}_0 + \dot{V}_1 + s^T\big[-J^{-1}\omega^{\times}J\omega + J^{-1}MEu_c + J^{-1}Mu_f + J^{-1}d\big] \\ &\quad + aD^{\alpha_1+1}\left[\mathrm{sig}(z_1)^{\beta_1}\right] + bD^{\alpha_2}\left[\mathrm{sig}(z_1)^{\beta_2}\right] - \frac{1}{\rho}\tilde{\Upsilon}^T\dot{\hat{\Upsilon}}\end{aligned} \tag{38}$$

From Theorem 3.1, it is easily derived that $\dot{V}_0 < 0$. By selecting the appropriate c and substituting (33)-(36) into (38), we can obtain

$$\begin{aligned}\dot{V}_2 &\le -s^T\left[\varepsilon_1\mathrm{sign}(s) + \varepsilon_2\mathrm{sig}(s)^r\right] + s^TJ^{-1}\left(M\tilde{u}_f + d - \hat{\Upsilon}\right) - \tilde{\Upsilon}^TJ^{-T}s \\ &\le -s^T\left[\varepsilon_1\mathrm{sign}(s) + \varepsilon_2\mathrm{sig}(s)^r\right] \\ &\le -2\lambda_{\min}(\varepsilon_1)\left(1 - \frac{\Psi}{V_2}\right)V_2 - 2^{\frac{(1+r)}{2}}\lambda_{\min}(\varepsilon_2)\left(1 - \frac{\Psi}{V_2}\right)^{\frac{(1+r)}{2}}V_2^{\frac{(1+r)}{2}}\end{aligned} \tag{39}$$

where $\Psi = V_0 + V_1 + \frac{1}{2\rho}\tilde{\Upsilon}^T\tilde{\Upsilon}$, $\frac{\Psi}{V_2} < 1$ and $\left(\frac{\Psi}{V_2}\right)^{\frac{(1+r)}{2}} < 1$.

Using Lemma 2.2 to (39), the settling time can be calculated as

$$\begin{aligned}T_k \le &\frac{1}{2\lambda_{\min}(\varepsilon_1)\left(1 - \frac{\Psi}{V_2}\right)\left[1 - \frac{(1+r)}{2}\right]} \\ &\times \ln\frac{2\lambda_{\min}(\varepsilon_1)\left(1 - \frac{\Psi}{V_2}\right)V_2^{\frac{(1+r)}{2}} + 2^{\frac{(1+r)}{2}}\lambda_{\min}(\varepsilon_2)\left(1 - \frac{\Psi}{V_2}\right)^{\frac{(1+r)}{2}}V_2^{\frac{(1+r)}{2}}(0)}{2^{\frac{(1+r)}{2}}\lambda_{\min}(\varepsilon_2)\left(1 - \frac{\Psi}{V_2}\right)^{\frac{(1+r)}{2}}V_2^{\frac{(1+r)}{2}}}\end{aligned} \tag{40}$$

Therefore, the system states will asymptotically converge to the designed fractional order nonsingular terminal sliding mode surface given in (30) under the condition $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$. Then, it could be easily deduced that the system trajectory will converge into a neighborhood of the origin in finite time.

In the following, the attitude system dynamic performance in sliding mode phase will be analyzed. It is assumed that Φ is the boundary value of the neighborhood near the origin, when $0 < |s_i| < \Phi$, $(i = 1, 2, 3)$, it can be obtained

$$s_i = \dot{z}_{2i} + a_iD^{\alpha_{1i}}\left[\mathrm{sig}(z_{1i})^{\beta_{1i}}\right] + b_iD^{\alpha_{2i}-1}\left[\mathrm{sig}(z_{1i})^{\beta_{2i}}\right] \tag{41}$$

Afterwards, similar analysis tricks from above procedure are adopted here. Equation (41) can be transformed into following two forms as

$$\dot{z}_{2i} + a_iD^{\alpha_{1i}}\left[\mathrm{sig}(z_{1i})^{\beta_{1i}}\right] + \left(b_i - s_iD^{\alpha_{2i}-1}\left[\mathrm{sig}(z_{1i})^{\beta_{2i}}\right]^{-1}\right)D^{\alpha_{2i}-1}\left[\mathrm{sig}(z_{1i})^{\beta_{2i}}\right] = 0 \tag{42}$$

$$\dot{z}_{2i} + \left(a_i - s_iD^{\alpha_{1i}}\left[\mathrm{sig}(z_{1i})^{\beta_{1i}}\right]^{-1}\right)D^{\alpha_{1i}}\left[\mathrm{sig}(z_{1i})^{\beta_{1i}}\right] + b_iD^{\alpha_{2i}-1}\left[\mathrm{sig}(z_{1i})^{\beta_{2i}}\right] = 0 \tag{43}$$

For Equation (42), it will still remain the newly proposed FONTSM form given in (30) when following inequality holds:

$$b_i - s_iD^{\alpha_{2i}-1}\left[\mathrm{sig}(z_{1i})^{\beta_{2i}}\right]^{-1} > 0 \tag{44}$$

According to Lemma 2.1, it is easily known that

$$\left|D^{\alpha_{2i}-1}\left[\operatorname{sig}(z_{1i})^{\beta_{2i}}\right]\right| \le W_i |z_{1i}|_{\max}^{\beta_{2i}} \tag{45}$$

where W_i stands for the ith element of W given in Lemma 2.1.

Therefore, the theoretical control errors will be bounded and can be given as

$$|z_{1i}| \le |z_{1i}|_{\max} \le \left(W_i^{-1} b_i^{-1} \Phi\right)^{\beta_{2i}} \tag{46}$$

Applying the same analysis procedure to (44), we can get $\left|D^{\alpha_{1i}}\left[\operatorname{sig}(z_{1i})^{\beta_{1i}}\right]\right| \le a_i^{-1}\Phi$. Then, we have $|z_{2i}| \le |s_i| + \left|D^{\alpha_{1i}}\left[\operatorname{sig}(z_{1i})^{\beta_{1i}}\right]\right| + \left|D^{\alpha_{2i}-1}\left[\operatorname{sig}(z_{1i})^{\beta_{2i}}\right]\right| \le 3\Phi$. Therefore, the stability of the attitude system is ensured and the attitude tracking errors will be bounded within a finite time. This completes the proof of Theorem 4.1.

Remark 4.1. *The FONTSM given in [18,19] can be written as*

$$\bar{s} = z_2 + bD^{\alpha_2-1}\left[\operatorname{sig}(z_1)^{\beta_2}\right] \tag{47}$$

where the parameters are exactly the same as (30).

When system trajectory reaches the sliding mode plane, we have $s = 0$. *Then, following equality will hold considering* $0 < \alpha_2 < 1$, $z_2 = -bD^{\alpha_2-1}\left[\operatorname{sig}(z_1)^{\beta_2}\right] = -bI^{1-\alpha_2}\left[\operatorname{sig}(z_1)^{\beta_2}\right]$. *Therefore, the right-hand term is actually an FO-integral term. This design may result in deterioration of the overall control performance. Meanwhile, due to the additional FO difference term* $aD^{\alpha_1}\left[\operatorname{sig}(z_1)^{\beta_1}\right]$, *our proposed FONTSM (30) does not have this problem.*

Remark 4.2. *Compared with passive FTC [6,9,23] without accurate fault information, the integrated FE/FTC controller designed by FONTSM technology and backstepping method uses accurate information from the fault estimation observer to effectively compensate for actuator faults. Thus, the integrated FE/FTC scheme improves dynamic convergence speed and tracking accuracy of the faulty closed-loop systems in finite time.*

5. **Simulation Results.**

To verify the effectiveness and performance of the proposed attitude control scheme, numerical simulations have been performed using the rigid spacecraft systems (2) and (6). The physical parameters of the rigid body of the spacecraft are selected from [21].

$$J = \begin{bmatrix} 30 & 5.3 & 6.4 \\ 5.3 & 27 & 10 \\ 6.4 & 10 & 19 \end{bmatrix} \ \text{kg}\cdot\text{m}^2, \quad M = \begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}$$

The inertial uncertainty $\triangle J$ is proposed as $\triangle J = \operatorname{diag}\{\sin(0.1t), 2\sin(0.2t), \sin(0.3t)\}$ and the external disturbance vector is chosen as $T_d = 0.001[\sin(t), 3\sin(t), 5\sin(t)]^T$ N·m. The initial attitude of the rigid spacecraft is $\sigma(0) = [-0.6, 0.5, 0.5]^T$, with an initial angular velocity of $\omega_0 = [0.9, 0.6, -0.4]^T$. For the simplicity of the simulation, the expected reference attitude angle command σ_d is assumed to be zero.

The first actuator occurs a bias fault $u_{f1} = 0.04$ N·m after $t = 20$ s, and the second actuator decreases 40% control effectiveness after $t = 20$ s, the third actuator work normal. The observer gains are ultimately chosen as $\Lambda = 5\operatorname{diag}\{1, 1, 1\}$, $\lambda_1 = 0.5$, $\lambda_2 = 0.6$ and $\lambda_3 = 0.8$. As for FTC controller part, the parameters in the FTC controller are selected as $a = b = \operatorname{diag}\{1, 1, 1\}$, $\alpha_1 = \operatorname{diag}\{0.01, 0.01, 0.01\}$, $\alpha_2 = \operatorname{diag}\{0.99, 0.99, 0.99\}$, $\beta_1 = \operatorname{diag}\{0.9, 0.9, 0.9\}$, $\beta_2 = \operatorname{diag}\{0.8, 0.8, 0.8\}$, $r = 5/7$, $\varepsilon_1 = 1.5\operatorname{diag}\{1, 1, 1\}$ and $\varepsilon_2 = 2.8\operatorname{diag}\{1, 1, 1\}$. In addition, the adaptive parameters are selected as $\rho = 0.3$.

To show the effectiveness of the proposed integrated FE/FTC scheme for rigid spacecraft attitude system with actuator faults, the necessary simulation comparison results are given in this section. Firstly, a closed-loop spacecraft attitude control system is established with a passive fault tolerant attitude control approach developed in [21]. Figures

2-4 show the attitude angle output curves, the angular velocity curves and the control input torque curves in actuator fault case. It is easily known that the closed-loop attitude system under the passive FTC approach proposed in [21] has the unaccepted dynamic performance, and could not compensate for the effects of unknown actuator faults to the closed loop attitude system. By utilizing the integrated FT/FTC scheme developed in this study, the estimated unknown actuator faults could be obtained by the designed fault estimation observer, which is depicted in Figure 5. Meanwhile, the attitude angle curves, the angular velocity curves and the control input torque curves using fault-tolerant attitude control scheme developed in this study are depicted in Figures 6-8, and it is obviously shown that the designed integrated FE/FTC approach could guarantee that the faulty closed-loop attitude system have satisfied fault-tolerant capability. To further emphasize the superiority of the proposed method, the control performance comparisons using different FTC schemes are also given in Table 1, which includes attitude tracking accuracy, angular velocity accuracy and stability time. Summarizing the above simulation comparison results, it can be easily seen that the proposed integrated FE/FTC scheme has better fault-tolerant effectiveness than the passive FTC scheme developed in [21] under general external disturbances and actuator faults.

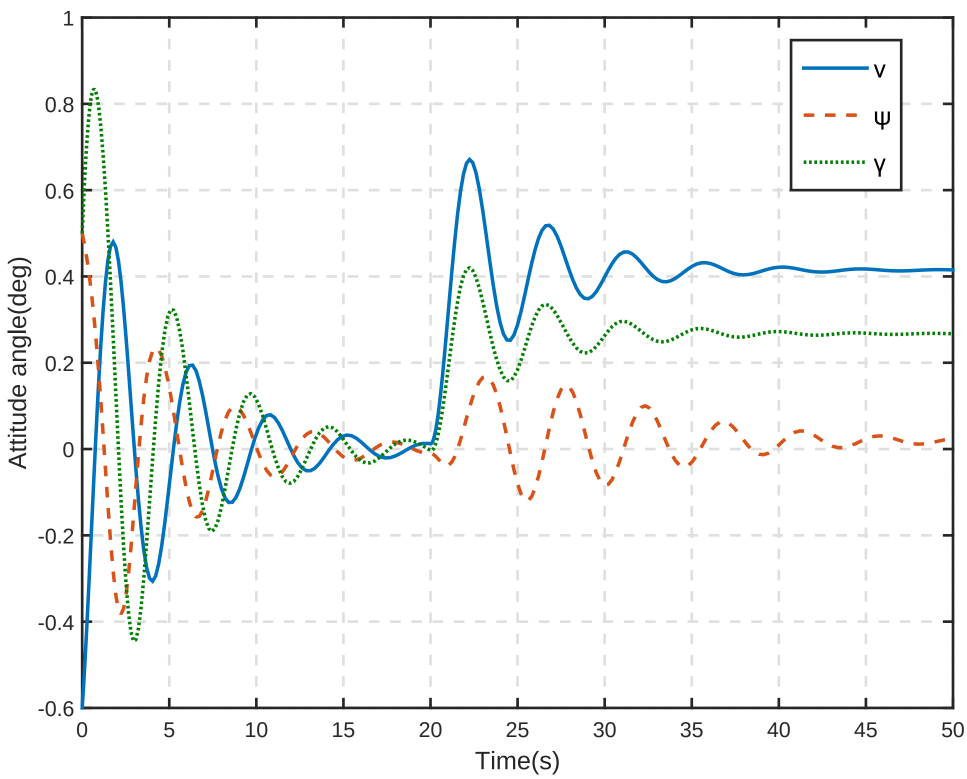

FIGURE 2. The attitude angle in actuator faulty case using passive FTC strategy in [21]

6. **Conclusions.** In this paper, an integrated FE/FTC method is proposed for the rigid spacecraft attitude system under the actuator loss of effectiveness fault and the bias fault case. Different from the existing literature, the fault estimation observer achieves precise estimation of actuator faults without any prior information of uncertainties and disturbances. According to the fault estimation information, a FONTSM-based fault-tolerant controller combined with backstepping method is designed, and it could guarantee

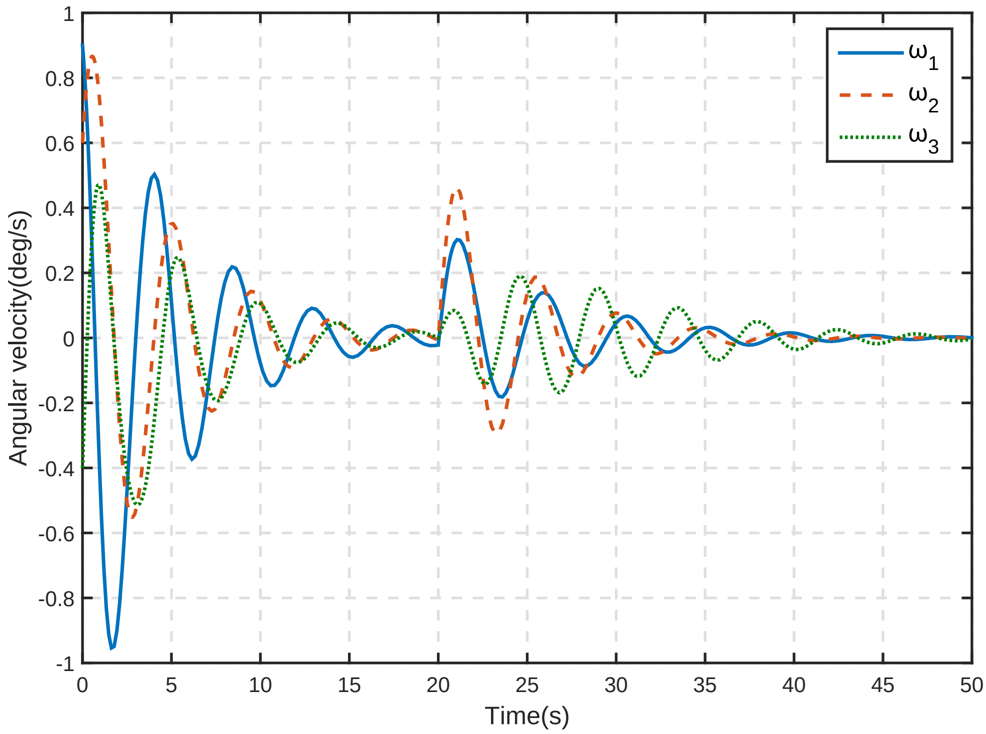

Figure 3. The angular velocity in actuator faulty case using passive FTC strategy in [21]

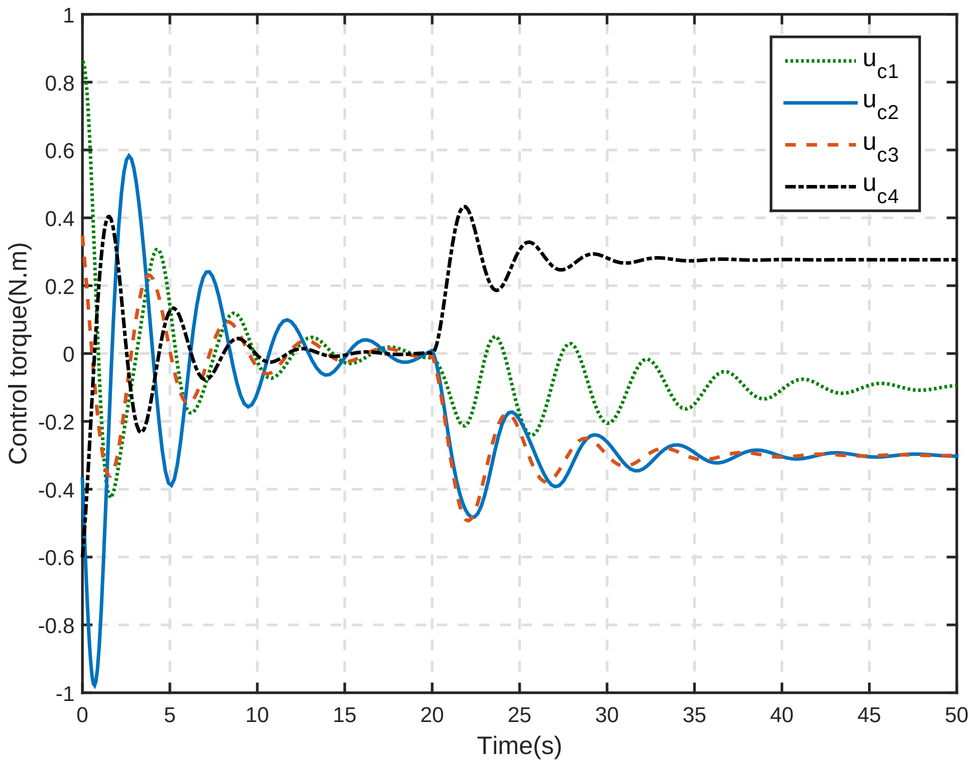

Figure 4. The control torque in actuator faulty case using passive FTC strategy in [21]

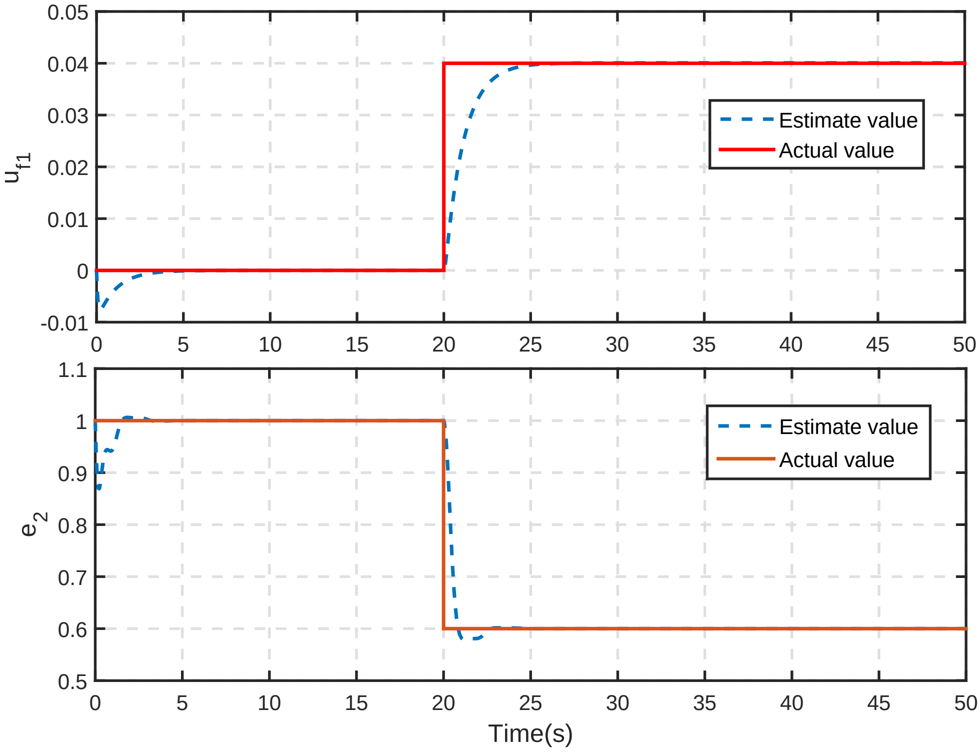

Figure 5. The actual actuator faults and its estimated value

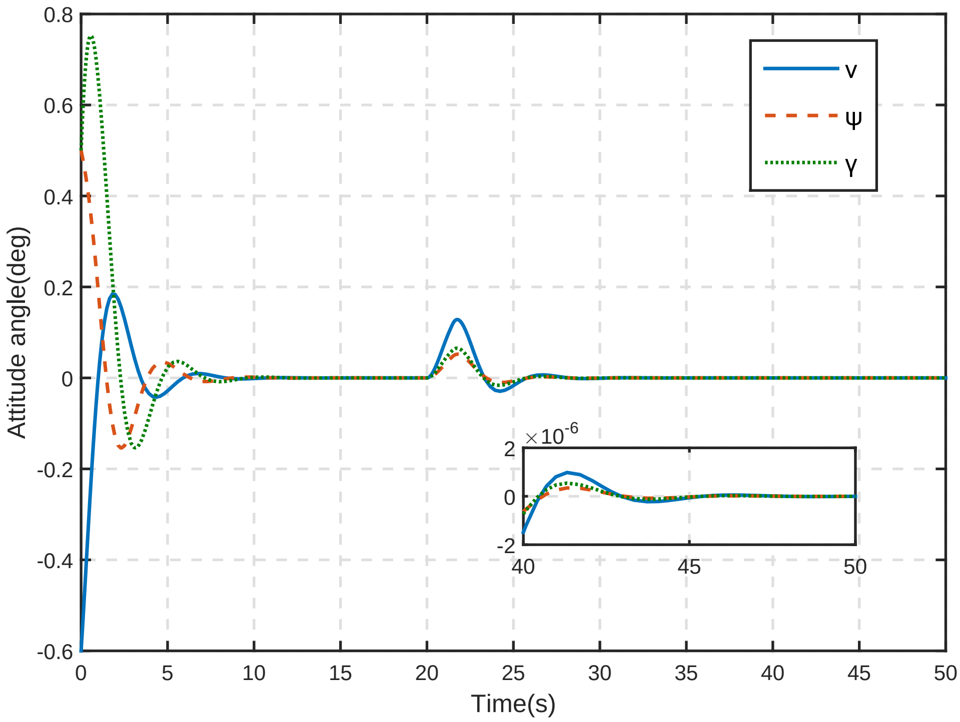

Figure 6. The attitude angle in actuator faulty case using integrated FE/FTC strategy in this paper

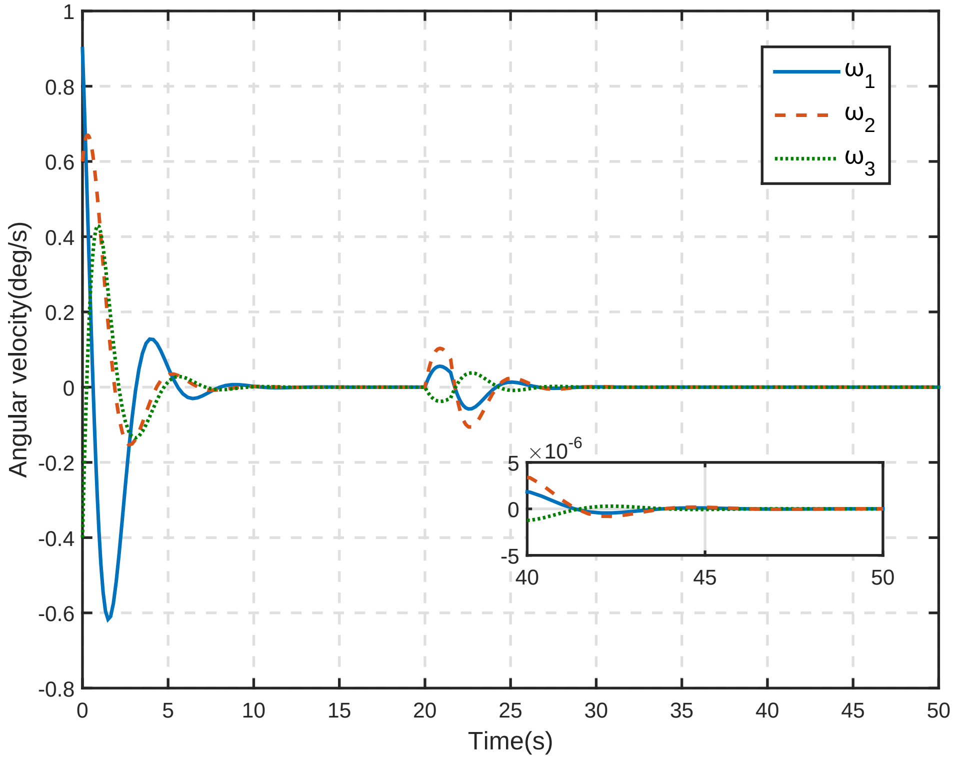

FIGURE 7. The angular velocity in actuator faulty case using integrated FE/FTC strategy in this paper

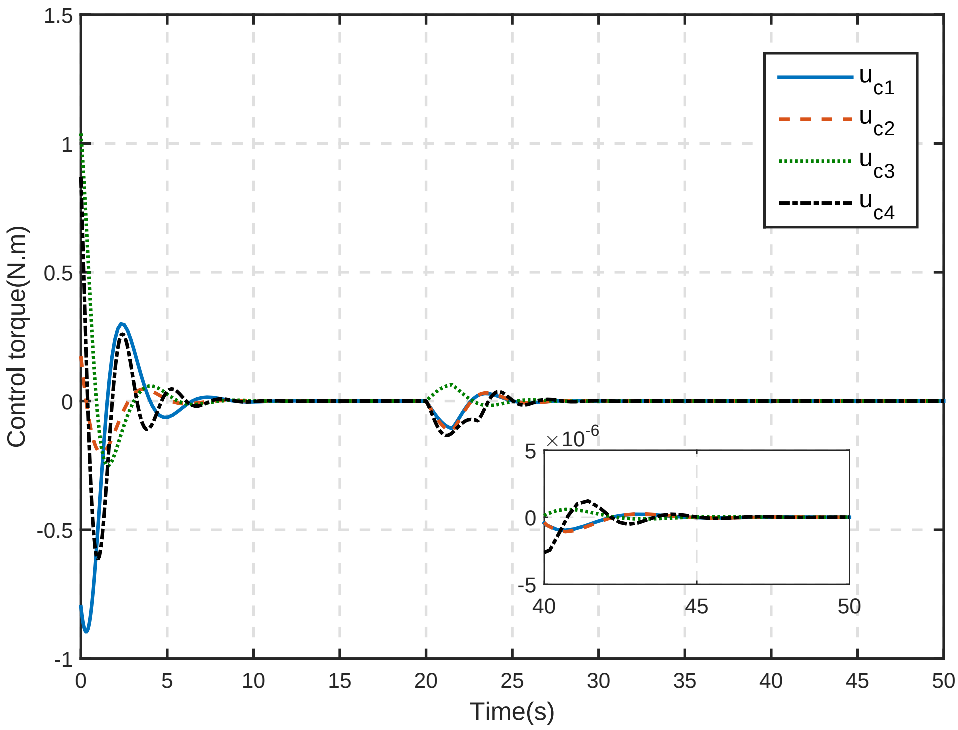

FIGURE 8. The control torque in actuator faulty case using integrated FE/FTC strategy in this paper

Table 1. The control performance comparisons under different control schemes

FTC scheme		FTC in [21]	FTC in this paper
Attitude tracking accuracy (deg)	ν	± 0.412	$\pm 2.6 \times 10^{-6}$
	ψ	± 0.008	$\pm 2.3 \times 10^{-6}$
	γ	± 0.264	$\pm 1.2 \times 10^{-6}$
Angular velocity accuracy (deg/s)	ω_1	± 0.055	$\pm 4.4 \times 10^{-6}$
	ω_2	± 0.067	$\pm 2.0 \times 10^{-6}$
	ω_3	± 0.089	$\pm 3.5 \times 10^{-6}$
Stabilization time (s)		Infinity	28 s

that the attitude error variables z_1 and z_2 of faulty closed-loop attitude system is stable within a finite time. Simulation results are given to demonstrate the effectiveness of the proposed fault-tolerant control scheme. Although the integrated FE/FTC design of this paper effectively solves the multiple actuator faults problem, it does not consider the fault-tolerant control in the existence of actuator saturation and actuator faults simultaneously. It is a very practical problem in the engineering applications and will be our future research work.

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