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NONFRAGILE MEMORY-BASED OUTPUT FEEDBACK CONTROL FOR FUZZY MARKOV JUMP GENERALIZED NEURAL NETWORKS WITH REACTION-DIFFUSION TERMS

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Abstract. *This paper investigates the stabilization issue of T-S fuzzy Markov jump generalized neural networks (GNNs) with reaction-diffusion terms. A nonfragile memory-based control strategy that contains a constant signal transmission delay is proposed. Additionally, the controller gain optimization method and the principle for the number of selected variables in the derived process are also analyzed in this paper. Firstly, based on the original T-S fuzzy Markov jump GNNs, a full-order observer with designed controller is established. Then the stable criteria of the considered error system are proposed and two relevant corollaries are also derived. Finally, three numerical examples are given to demonstrate the validity of the related results and the superiority of the designed controller.*

Keywords: GNNs, Markov jump, Nonfragile memory-based control, Reaction-diffusion terms, T-S fuzzy model

1. **Introduction.** In the past few decades, a lot of scholars have concentrated on the study of dynamical neural networks (NNs) owing to their immense implementation in numerous fields, including pattern recognition, image processing, combinatorial optimization, associate memory design, speed detection of moving objects and other areas [1, 2, 3, 4]. In addition, the NNs can be classified into local field NNs (LFNNs) [5] and static NNs (SNNs) [6] by the utilization of local field states or neural states of neurons. Scholars have to study LFNNs and SNNs respectively in earlier years, because their models are often different [7]. However, recently, the new unified NNs called generalized NNs (GNNs) have been proposed to combine the LFNNs and SNNs [8, 9]. As a result, analyzing the dynamic behaviors of GNNs instead of LFNNs and SNNs separately is of great importance [10].

On another research front, Markov jump parameters have been taken into account in GNNs, because there are often random situations in practical application which can be suitably described by Markov jump models. Thus, many research results about Markov jump GNNs (MJGNNs) have been reported. For example, the authors in [10, 11] analyzed stability for a class of MJGNNs, the dissipativity of MJGNNs was studied in [12], and [13] investigated exponential stability of semi-MJGNNs. Moreover, in engineering fields, diffusion effects cannot be neglected in NNs when electrons are moving in asymmetric

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electromagnetic fields [14, 15]. Therefore, it is necessary to consider reaction-diffusion effects in the research of MJGNNs, which implies that the whole dynamics and structure of GNNs depend not only on the evolution time of each variable, but also on its position status [16]. However, due to the inherent complex characteristics of nonlinear systems, the control effects are not always satisfactory [17]. Fortunately, T-S fuzzy model can provide an effective method to represent complex nonlinear systems by local linear systems with their linguistic description, so that the inherent complex characteristics of nonlinear systems can be averted [18, 19, 20].

Uncertainties widely exist in controller, which may generate instability of systems. Therefore, many scholars have studied the nonfragile control problems, which can take the uncertainties of the controller into consideration [21, 22, 23, 24]. Moreover, the response of the practical controller is not only affected by the current input signal, but also often affected by the previous ones; thus, it is necessary to take signal transmission delay into consideration. Memory control strategy, also named proportional retarded control scheme, is proposed in [25, 26], where the updating signal successfully transmitted from the sampler to controller and zero-order holder at the sampling instant has experienced a signal transmission delay. However, to the best of our knowledge, the nonfragile memory-based controller has not been considered for T-S fuzzy MJGNNs with reaction-diffusion terms, which is the most important motivation of this paper.

As is known to us all, the core of stabilization issue is to design a suitable controller such that the considered system is stable. However, there are often the phenomena that obtained controller gains are too large to implement. Inspired by [27], the controller gain optimization method is analyzed in this paper. What is more, the selection of variables can greatly influent the conservatism of the main results. In this paper, to better show the importance of the selected variables' number in the process of deriving the main results, a comparative example is proposed. The analysis of the above two problems is another motivation of this paper.

Based on discussions mentioned above, the main objective of this paper is to establish a stable criterion for T-S fuzzy MJGNNs with reaction-diffusion terms via nonfragile memory-based control scheme. The main contributions of this work are summarized as below.

(1) Compared with some existing results such as [15, 16, 28], this paper innovatively integrates reaction-diffusion terms and Markov jump parameters with T-S fuzzy GNNs, so that the influence of spatial position and the parameters variation are fully considered, which makes system model established in this paper more general and more suitable for practical engineering.

(2) Considering the change of controller's parameters in the actual project and the influence of the previous input signal on the controller, this paper makes the first attempt to design a nonfragile memory-based control scheme for T-S fuzzy MJGNNs with reaction-diffusion terms, so that the controller designed in this paper has stronger anti-interference ability and memory function.

(3) We apply the controller gain optimization method to T-S fuzzy MJGNNs with reaction-diffusion terms, as a result, the controller gains may be more suitable in practical engineering application. At the same time, the influence of selected variables' number in Lyapunov function was analyzed, and a conclusion is reached through a corresponding numerical example.

The rest of this paper is organized as follows. The system model and some preliminaries are introduced in Section 2. In Section 3, the main results of this paper are obtained, including the controller design method and two related corollaries. Three numerical examples are given in Section 4. Finally, conclusions are drawn in Section 5.

Notations: Throughout this paper, $\mathbb{R}^n$ represents n-dimensional Dirichlet space and $\mathbb{R}^{n\times n}$ denotes $n \times n$ real matrix set; $\Omega = \{s \,||s_k| \le l_k, k = 1, 2, \dots, q\}$ denotes a compact set with smooth boundary $\partial\Omega$; υ_i, $i = 1, 2, \dots, n$, represents the n-dimensional column vector with the ith element equal to 1 and 0 elsewhere; the notation M^T represents the transpose of the matrix M; A^{-1} represents the inverse matrix of matrix A; for symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \ge Y$) means that the matrix $X - Y$ is positive definite (respectively, positive semidefinite); the shorthand $diag\{\cdots\}$ denotes a diagonal or block diagonal matrix; $*$ represents the elements below the main diagonal of the symmetric block matrix; $col\{\cdots\}$ denotes expressing the elements in $\{\cdots\}$ as column vectors; $Sym\{A\} = A + A^T$; $C^n(\Omega)$ represents the family of continuously n-times differentiable real-valued functions defined on Ω; I_n is the n-dimensional unit matrix; $\{\delta_t\}$ is a continuous-time Markovian process with right continuous trajectories and takes the values in a finite set $S = (1, 2, \dots, \mathcal{S})$ with transition probability matrix $\Psi \triangleq \{\varphi_{\alpha\beta}\}$ given by

$$\Pr\{\delta_{t+\Delta} = \beta \,|\delta_t = \alpha\} = \begin{cases} \varphi_{\alpha\beta}\Delta + o(\Delta) & \alpha \ne \beta \\ 1 + \varphi_{\alpha\alpha}\Delta + o(\Delta) & \alpha = \beta \end{cases}$$

where $\Delta > 0$, $\lim\limits_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$, and $\varphi_{\alpha\beta} \ge 0$ for $\alpha \ne \beta$ is the transition rate from mode α at time t to mode β at time $t+\Delta$ and $\varphi_{\alpha\alpha} = -\sum_{\beta\in S,\alpha\ne\beta} \varphi_{\alpha\beta}$, $\Pr\{\delta_{t+\Delta} = \beta \,|\delta_t = \alpha\}$ denotes that under the condition of $\delta_t = \alpha$, the probability of occurrence of $\delta_{t+\Delta} = \beta$; matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. **Problem Statement.**

Fix a probability space (Ω, F, P), and consider the following Markov jump GNNs with reaction-diffusion terms depicted by a T-S fuzzy model with r rules:

Plant Rule *i*: IF $\vartheta_1(s, t)$ is F_{i1} and ... and $\vartheta_p(s, t)$ is F_{ip}, THEN

$$\begin{aligned} \frac{\partial u_\rho(s,t)}{\partial t} &= \sum_{k=1}^{q} \frac{\partial}{\partial s_k}\left(a_{\rho k}\frac{\partial u_\rho(s,t)}{\partial s_k}\right) - b_{\rho i}(\delta_t)u_\rho(s,t) + \sum_{l=1}^{n} c_{\rho l i}(\delta_t)g_l(w_{\rho l}(\delta_t)u_l(s,t)) \\ &\quad + \sum_{l=1}^{n} d_{\rho l i}(\delta_t)g_l(w_{\rho l}(\delta_t)u_l(s,t-h(t))), \\ \Im_\rho^u(s,t) &= \hbar_{\rho i}(\delta_t)u_\rho(s,t), \quad \rho = 1, 2, \dots, n \end{aligned} \tag{1}$$

where $\vartheta_1(s, t), \dots, \vartheta_p(s, t)$ are premise variables; $F_{i\omega}$ ($i = 1, 2, \dots, r$, $\omega = 1, 2, \dots, p$) is fuzzy set characterized by membership function; $s = (s_1, s_2, \dots, s_q)^T \in \Omega \subset \mathbb{R}^q$, with $\Omega = \{s \,||s_k| \le l_k, k = 1, 2, \dots, q\}$, and l_k is a positive constant; $u_\rho(s, t)$ is the state of the ρ^{th} neuron at time t and space $s = (s_1, s_2, \dots, s_q)^T$; $\Im_\rho^u(s, t)$ is the measurement output of the ρ^{th} neuron at time t; $a_{\rho k} > 0$ ($\rho = 1, 2, \dots, n$; $k = 1, 2, \dots, q$) represents the transmission diffusion operator along the ρ^{th} neuron; $b_{\rho i}(\delta_t) > 0$ represents the rate with which the ρ^{th} unit will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $c_{\rho l i}(\delta_t)$ and $d_{\rho l i}(\delta_t)$ are the connection weights coefficients of the neurons; $\hbar_{\rho i}(\delta_t)$ is feedback connection weight of the neurons; $g_l(\cdot)$ stands for the neuron activation functions; $h(t)$ is the time-varying delay and satisfies:

$$0 < h_1 < h(t) < h_2, \quad \frac{dh(t)}{dt} \le h,$$

where h_1, h_2 and h are given constants. $w_{\rho l}$ is the value of the synaptic connectivity from neuron l to neuron ρ.

Besides, the boundary conditions and initial conditions of (1) are supposed as:

$$u_\rho(s,t)=0,\quad (s,t)\in\partial\Omega\times[-h_2,+\infty)$$
$$u_\rho(s,\ell)=\phi_\rho(s,\ell),\quad (s,\ell)\in\Omega\times[-h_2,0]$$

for $\rho=1,2,\ldots,n$, where $\phi_\rho(s,\ell)$ is a vector-valued continuous and bounded function defined on $\Omega\times[-h_2,0]$.

Remark 2.1. *The GNNs (1) consists of some familiar neural networks as its particular cases. If $c_{\rho li}(\delta_t)=d_{\rho li}(\delta_t)=1$, the system model (1) reduced to Markov jump delayed static neural networks with reaction-diffusion terms. And if $w_{\rho l}(\delta_t)=1$, it falls into a class of Markov jump delayed local field neural networks with reaction-diffusion terms. As a result, compared with some existing results such as* [6, 14, 28]*, the system model considered in this paper is more general and may fulfill a wider range of production needs.*

For simplicity, the system (1) can be rewritten as the following compact form:

$$\begin{aligned}\frac{\partial u(s,t)}{\partial t}&=\sum_{k=1}^{q}\frac{\partial}{\partial s_k}\left(\mathcal{A}_k\frac{\partial u(s,t)}{\partial s_k}\right)+\sum_{i=1}^{r}\theta_i(\vartheta(s,t))\big[-\mathcal{B}_i(\delta_t)u(s,t)\\&\quad+\mathcal{C}_i(\delta_t)g(W(\delta_t)u(s,t))+\mathcal{D}_i(\delta_t)g(W(\delta_t)u(s,t-h(t)))\big]\\ \Im^u(s,t)&=\sum_{i=1}^{r}\theta_i(\vartheta(s,t))\mathcal{H}_i(\delta_t)u(s,t)\end{aligned}\tag{2}$$

where $u(s,t)=(u_1(s,t),u_2(s,t),\ldots,u_n(s,t))^T$, $\mathcal{A}_k=diag\{a_{1k},a_{2k},\ldots,a_{nk}\}$, $\mathcal{B}_i(\delta_t)=diag\{b_{1i}(\delta_t),b_{2i}(\delta_t),\ldots,b_{ni}(\delta_t)\}$, $\mathcal{C}_i(\delta_t)=(c_{\rho li}(\delta_t))_{n\times n}$, $\mathcal{D}_i(\delta_t)=(d_{\rho li}(\delta_t))_{n\times n}$, $\mathcal{H}_i(\delta_t)=diag\{\hbar_{1i}(\delta_t),\hbar_{2i}(\delta_t),\ldots,\hbar_{ni}(\delta_t)\}$, $g(W(\delta_t)u(s,t))=(g(W(\delta_t)u_1(s,t)),g(W(\delta_t)u_2(s,t)),\ldots,g(W(\delta_t)u_n(s,t)))^T$, and $\theta_i(\vartheta(s,t))$ denotes the normalized membership function of the inferred fuzzy set $\varpi_i(\vartheta(s,t))$ satisfying

$$\theta_i(\vartheta(s,t))=\frac{\varpi_i(\vartheta(s,t))}{\sum_{i=1}^{r}\varpi_i(\vartheta(s,t))},\quad \varpi_i(\vartheta(s,t))=\prod_{\omega=1}^{p}F_{i\omega}(\vartheta(s,t)),$$

in which $F_{i\omega}(\vartheta(s,t))$ is the grade membership function of $\vartheta(s,t)$ in $F_{i\omega}$. It is assumed that

$$\varpi_i(\vartheta(s,t))\ge 0,\quad i=1,2,\ldots,r,\quad \sum_{i=1}^{r}\varpi_i(\vartheta(s,t))>0,\quad \forall t\ge 0.$$

Hence, $\theta_i(\vartheta(s,t))$ satisfies $\theta_i(\vartheta(s,t))\ge 0$, $i=1,2,\ldots,r$, $\sum_{i=1}^{r}\theta_i(\vartheta(s,t))=1$ for any $\vartheta(s,t)$.

Assumption 2.1. [29] *The activation function g_l is continuously bounded, and there exist some real constants v_l^- and v_l^+ such that*

$$v_l^-\le\frac{g_l(\sigma_1)-g_l(\sigma_2)}{\sigma_1-\sigma_2}\le v_l^+,$$

where v_l^- and v_l^+ may be positive, zero or negative.

Inspired by [28], the following full-order observer can be modeled as:

$$\begin{aligned}\frac{\partial v_\mu(s,t)}{\partial t}&=\sum_{k=1}^{q}\frac{\partial}{\partial s_k}\left(\mathcal{A}_k\frac{\partial v_\mu(s,t)}{\partial s_k}\right)+\sum_{i=1}^{r}\theta_i(\vartheta(s,t))[-\mathcal{B}_i(\delta_t)v_\mu(s,t)\\&\quad+\mathcal{C}_i(\delta_t)g(W(\delta_t)v_\mu(s,t))+\mathcal{D}_i(\delta_t)g(W(\delta_t)v_\mu(s,t-h(t)))+w_\mu(s,t)],\end{aligned}$$

$$\Im_\mu^v(s,t) = \sum_{i=1}^{r} \theta_i(\vartheta(s,t))\mathcal{H}_i(\delta_t)v_\mu(s,t), \quad \mu = 1,2,\dots,n \tag{3}$$

where $v_\mu(s,t) = (v_{\mu 1}(s,t), v_{\mu 2}(s,t), \dots, v_{\mu n}(s,t)) \in \mathbb{R}^n$ is the estimation of the neuron state, and $w_\mu(s,t) \in \mathbb{R}^n$ is the nonfragile fuzzy memory-based controller to be designed.

Besides, the boundary and initial conditions of (3) are defined as:

$$\begin{aligned} v_\mu(s,t) &= 0, \quad (s,t) \in \partial\Omega \times [-h_2, +\infty) \\ v_\mu(s,\ell) &= \sigma_\mu(s,\ell), \quad (s,\ell) \in \Omega \times [-h_2, 0] \end{aligned}$$

for $\mu = 1,2,\dots,n$, where $\sigma_\mu(s,\ell)$ are vector-valued continuous and bounded functions defined on $\Omega \times [-h_2, 0]$.

Inspired by [26, 30, 31], the stabilization problem is investigated under the parallel distributed compensation scheme, which means that the memory-based controller for rule j can be designed as:

Controller Rule *j*: IF $\vartheta_1(s,t)$ is F_{j1} and ... and $\vartheta_p(s,t)$ is F_{jp}, THEN

$$\begin{aligned} w_\mu(s,t) = {} & [K_{1j} + \Delta K_{1j}(t)] \left[\Im_\mu^v(s,t) - \Im^u(s,t)\right] \\ & + [K_{2j} + \Delta K_{2j}(t)] \left[\Im_\mu^v(s,t-\eta) - \Im^u(s,t-\eta)\right], \quad j = 1,2,\dots,r \end{aligned} \tag{4}$$

where η is a constant delay, and K_{1j}, K_{2j} are appropriate dimensional controller gain matrices.

The uncertainties $\Delta K_{1j}(t)$ and $\Delta K_{2j}(t)$ represent the possible controller gain fluctuations. It is assumed that $\Delta K_{1j}(t)$ and $\Delta K_{2j}(t)$ have the following form:

$$[\Delta K_{1j}(t), \Delta K_{2j}(t)] = Q_j Y_j(t) [\mathcal{N}_{1j}, \mathcal{N}_{2j}]$$

where Q_j, $\mathcal{N}_{1j}$ and $\mathcal{N}_{2j}$ are known constant matrices with appropriate dimensions, and $Y_j(t)$ is an unknown matrices function satisfying $Y_j^T(t)Y_j(t) \le I$.

Let $y_\mu(s,t) = v_\mu(s,t) - u(s,t)$ be the error vector, and then the error system represented by a compact form is obtained:

$$\begin{aligned} \frac{\partial y_\mu(s,t)}{\partial t} = {} & \sum_{k=1}^{q} \frac{\partial}{\partial s_k}\left(\mathcal{A}_k \frac{\partial y_\mu(s,t)}{\partial s_k}\right) + \sum_{i=1}^{r}\theta_i(\vartheta(s,t))\sum_{j=1}^{r}\theta_j(\vartheta(s,t))\big[-\mathcal{B}_{i\alpha}y_\mu(s,t) \\ & + \mathcal{C}_{i\alpha}\tilde{g}\left(W_\alpha y_\mu(s,t)\right) + \mathcal{D}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t-h(t))) \\ & + [K_{1j} + \Delta K_{1j}(t)]\,\mathcal{H}_{i\alpha}y_\mu(s,t) + [K_{2j} + \Delta K_{2j}(t)]\,\mathcal{H}_{i\alpha}y_\mu(s,t-\eta)\big], \\ & \mu = 1,2,\dots,n \end{aligned} \tag{5}$$

where $\alpha = 1,2,\dots,\mathcal{S}$, and $\tilde{g}(W_\alpha y_\mu(s,t)) = g(W_\alpha v_\mu(s,t)) - g(W_\alpha u(s,t))$.

From Assumption 2.1, the neuron activation function satisfies

$$v_l^- \le \frac{\tilde{g}_l(\sigma)}{\sigma} \le v_l^+, \quad l = 1,2,\dots,n.$$

Define $V_1 = diag\{v_1^-, v_2^-, \dots, v_n^-\}$ and $V_2 = diag\{v_1^+, v_2^+, \dots, v_n^+\}$ are constant matrixes.

In this paper, we shall use the following definition and lemmas, which play crucial roles in the proof of the main results.

Definition 2.1. [32]*: Let $V(y_\mu(s,t),t)$ be the stochastic Lyapunov function of the system (5), and define its weak infinitesimal operator as*

$$\begin{aligned} \mathcal{L}V(y_\mu(s,t),\delta_t,t) = {} & \frac{\partial}{\partial t}V(y_\mu(s,t),\delta_t,t) + \left[\frac{\partial}{\partial y_\mu(s,t)}V(y_\mu(s,t),\delta_t,t)\right]\frac{\partial y_\mu(s,t)}{\partial t} \\ & + \sum_{\beta=1}^{\mathcal{S}}\varphi_{\alpha\beta}V(y_\mu(s,t),\beta,t) \end{aligned}$$

Definition 2.2. [32]*: System (5) is said to be globally asymptotically stable in mean square if* $\lim_{t\to\infty}\left\{\|y_\mu(s,t)\|^2\right\}=0$ *for any initial conditions.*

Lemma 2.1. [33]*: For any matrix* $M\in\mathbb{R}^{n\times n}$, $M=M^T>0$ *and* $a\le x\le b$, *the following inequation holds:*

$$-\int_a^b \omega^T(x)M\omega(x)dx\le-\frac{1}{b-a}\left(\int_a^b\omega(x)dx\right)^T M\int_a^b\omega(x)dx$$

where a *and* b *are given scalars.*

Lemma 2.2. [34]*: Let* $g_1,g_2,\dots,g_N:\mathbb{R}^m\to\mathbb{R}^1$ *have positive values in an open subset* E *of* $\mathbb{R}^m$. *Then the reciprocally convex combination of* g_i *over* E *satisfies*

$$\left\{\nu_i\,\middle|\,\nu_i>0,\sum_i\nu_i=1\right\}\sum_i\frac{1}{\nu_i}g_i(t)=\sum_i g_i(t)+\max_{f_{i,j}(t)}\sum_{i\neq j}f_{i,j}(t)$$

subject to

$$\left\{f_{i,j}:\mathbb{R}^m\to\mathbb{R}^1,f_{i,j}(t)=f_{j,i}(t),\begin{bmatrix}g_i(t)&f_{i,j}(t)\\f_{j,i}(t)&g_j(t)\end{bmatrix}\ge0\right\}$$

Lemma 2.3. [35]*: Given real matrices* A, B *and* D *that with appropriate dimensions and a scalar* $\varepsilon>0$, *moreover,* $D^TD\le I$, *for any vectors* $x,y\in\mathbb{R}^n$, *the following inequation holds:*

$$2x^TADBy\le\varepsilon^{-1}x^TAA^Tx+\varepsilon y^TB^TBy$$

Lemma 2.4. [36]*: Let* Ω *be a cube* $|x_k|<\tilde{l}_k$ $(k=1,2,\dots,m)$, *and let* $\nu(x)$ *be a real-valued function belonging to* $C^1(\Omega)$ *which satisfies* $\nu(x)\left|_{\partial\Omega}\right.=0$. *Then*

$$\int_\Omega\nu(x)dx\le\tilde{l}_k^2\int_\Omega\left|\frac{\partial\nu(x)}{\partial x_k}\right|dx$$

Our target is to design a controller (4), such that the response system (5) is globally asymptotically stable in mean square. To achieve this aim, the main results will be proposed in the next section.

3. Main Results.

In this section, by constructing Lyapunov functional and utilizing multiple integration method, new criteria to ensure the stability of the considered T-S fuzzy MJGNNs with reaction-diffusion terms and the corresponding controller design scheme will be proposed.

For the purpose of simplicity, some vector notations are denoted as follows:

$$\chi_\mu^T(s,x)=\left[y_\mu^T(s,t),\tilde{g}^T(W_\alpha y_\mu(s,t))\right]$$

$$\begin{aligned}\nabla(s,t)=col\Bigg\{&y_\mu(s,t),\dot{y}_\mu(s,t),y_\mu(s,t-h_1),y_\mu(s,t-h(t)),y_\mu(s,t-h_2),y_\mu(s,t-\eta),\\&\tilde{g}(W_\alpha y_\mu(s,t)),\tilde{g}(W_\alpha y_\mu(s,t-h_1)),\tilde{g}(W_\alpha y_\mu(s,t-h(t))),\tilde{g}(W_\alpha y_\mu(s,t-h_2)),\\&\int_{t-h_2}^{t-h(t)}y_\mu(s,x)dx,\int_{t-h(t)}^{t-h_1}y_\mu(s,x)dx,\int_{t-\eta}^{t}\dot{y}_\mu(s,x)dx\Bigg\}\end{aligned}$$

Theorem 3.1. *For given scalars* $\varepsilon>0$, $\eta>0$, $h_2>h_1>0$, *the error system (5) is asymptotically stable in mean square if there exist* $\mathcal{P}_\alpha>0$, $(\alpha=1,2,\dots,\mathcal{S})$, $\Gamma>0$, $\mathcal{Q}_e=\begin{bmatrix}\mathcal{Q}_{e11}&\mathcal{Q}_{e12}\\ *&\mathcal{Q}_{e22}\end{bmatrix}>0$, $(e=1,2,3)$, $\mathcal{U}_1,\mathcal{U}_2>0$, $\mathcal{Z}>0$, *positive definite diagonal* *matrix* Θ_o $(o = 1, 2, 3)$ *and appropriate dimensional arbitrary matrices* G_ϖ $(\varpi = 1, 2, 3, 4)$, $\tilde{\mathcal{Z}}$, $\hat{K}_{1j}$ *and* $\hat{K}_{2j}$ $(j = 1, 2, \ldots, r)$ *such that the following LMIs hold*

$$\begin{bmatrix} \Sigma_{ij} & \left(\gamma_1 \upsilon_1^T \Gamma + \gamma_2 \upsilon_2^T \Gamma\right) Q_j \\ * & -\varepsilon I_n \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} \mathcal{Z} & \tilde{\mathcal{Z}} \\ * & \mathcal{Z} \end{bmatrix} > 0 \tag{7}$$

where

$$\begin{aligned}
\Sigma_{ij} &= \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 + \Xi_5 + \Xi_6 \\
\Xi_1 &= Sym\left\{\upsilon_1^T(\mathcal{P}_\alpha - \gamma_1\Gamma)\upsilon_2\right\} - \upsilon_1^T\left[2\gamma_1\Gamma\tilde{\mathcal{A}} + 2\gamma_1\Gamma\mathcal{B}_{i\alpha} - \sum_{\beta\in S}\varphi_{\alpha\beta}\mathcal{P}_\beta - 2\gamma_1\hat{K}_{1j}\mathcal{H}_{i\alpha}\right]\upsilon_1 \\
&\quad + Sym\left\{\upsilon_1^T\gamma_1\Gamma\left[\mathcal{C}_{i\alpha}\upsilon_7 + \mathcal{D}_{i\alpha}\upsilon_9\right]\right\} + Sym\left\{\upsilon_1^T\gamma_1\hat{K}_{2j}\mathcal{H}_{i\alpha}\upsilon_6\right\} - 2\upsilon_2^T\gamma_2\Gamma\upsilon_2 \\
&\quad + Sym\Big\{\upsilon_2^T\gamma_2\Gamma\left[-\mathcal{B}_{i\alpha}\upsilon_1 + \mathcal{C}_{i\alpha}\upsilon_7 + \mathcal{D}_{i\alpha}\upsilon_9\right] + \upsilon_2^T\gamma_2\hat{K}_{1j}\mathcal{H}_{i\alpha}\upsilon_1 + \upsilon_2^T\gamma_2\hat{K}_{2j} \\
&\quad \times \mathcal{H}_{i\alpha}\upsilon_6\Big\} + \varepsilon(\mathcal{N}_{1j}\mathcal{H}_{i\alpha}\upsilon_1 + \mathcal{N}_{2j}\mathcal{H}_{i\alpha}\upsilon_6)^T(\mathcal{N}_{1j}\mathcal{H}_{i\alpha}\upsilon_1 + \mathcal{N}_{2j}\mathcal{H}_{i\alpha}\upsilon_6) \\
\Xi_2 &= \left(\upsilon_1^T, \upsilon_7^T\right)(\mathcal{Q}_1 + h_1\mathcal{Q}_2 + h_2\mathcal{Q}_3)\left(\upsilon_1^T, \upsilon_7^T\right)^T - (1-h)\left(\upsilon_4^T, \upsilon_9^T\right)\mathcal{Q}_1\left(\upsilon_4^T, \upsilon_9^T\right)^T \\
&\quad - \left(\upsilon_3^T, \upsilon_8^T\right)h_1\mathcal{Q}_2\left(\upsilon_3^T, \upsilon_8^T\right)^T - \left(\upsilon_5^T, \upsilon_{10}^T\right)h_2\mathcal{Q}_3\left(\upsilon_5^T, \upsilon_{10}^T\right)^T \\
\Xi_3 &= \eta\left(\upsilon_1^T\mathcal{U}_1\upsilon_1 - \upsilon_6^T\mathcal{U}_1\upsilon_6\right) + (h_2 - h_1)\left(\upsilon_3^T\mathcal{U}_2\upsilon_3 - \upsilon_5^T\mathcal{U}_2\upsilon_5\right) \\
\Xi_4 &= (h_2 - h_1)^2\upsilon_1^T\mathcal{Z}\upsilon_1 - \left(\upsilon_{11}^T, \upsilon_{12}^T\right)\begin{bmatrix} \mathcal{Z} & \tilde{\mathcal{Z}} \\ * & \mathcal{Z} \end{bmatrix}\left(\upsilon_{11}^T, \upsilon_{12}^T\right)^T \\
\Xi_5 &= Sym\left\{\left[\upsilon_7^T - \upsilon_1^T W_\alpha^T V_1^T\right]\Theta_1\left[V_2 W_\alpha\upsilon_1 - \upsilon_7\right]\right\} \\
&\quad + Sym\left\{\left[\upsilon_9^T - \upsilon_4^T W_\alpha^T V_1^T\right]\Theta_2\left[V_2 W_\alpha\upsilon_4 - \upsilon_9\right]\right\} \\
&\quad + Sym\left\{\left[\upsilon_7^T - \upsilon_9^T - \upsilon_1^T W_\alpha^T V_1^T + \upsilon_4^T W_\alpha^T V_1^T\right]\Theta_3\left[V_2 W_\alpha\upsilon_1 - V_2 W_\alpha\upsilon_4 - \upsilon_7 + \upsilon_9\right]\right\} \\
\Xi_6 &= Sym\left\{\left(\upsilon_4^T G_1 + \upsilon_7^T G_2 + \upsilon_9^T G_3 + \upsilon_{13}^T G_4\right) \times (\upsilon_1 - \upsilon_6 - \upsilon_{13})\right\}
\end{aligned}$$

In addition, $K_{1j} = \Gamma^{-1}\hat{K}_{1j}$, $K_{2j} = \Gamma^{-1}\hat{K}_{2j}$.

Proof: To derive Theorem 3.1, Lyapunov functional method will be adopted here. That is, Lyapunov functional $V(y_\mu(s,t),t)$ should be constructed such that

$$\mathcal{L}V(y_\mu(s,t),t) \le \int_\Omega \sum_{\mu=1}^{n}\left\{\sum_{i=1}^{r}\theta_i(\vartheta(s,t))\sum_{j=1}^{r}\theta_j(\vartheta(s,t))\nabla^T(s,t)\bar{\Sigma}_{ij}\nabla(s,t)\right\}ds < 0$$

where $\bar{\Sigma}_{ij} = \Sigma_{ij} + \varepsilon^{-1}\left(\gamma_1\upsilon_1^T\Gamma + \gamma_2\upsilon_2^T\Gamma\right)Q_jQ_j^T\left(\gamma_1\upsilon_1^T\Gamma + \gamma_2\upsilon_2^T\Gamma\right)^T$ and Σ_{ij} has been defined in (6). As a result, the error system (5) is asymptotically stable in mean square.

To achieve this objective, we choose the following Lyapunov functional candidates:

$$V(y_\mu(s,t),t) = \sum_{\varrho=1}^{4} V_\varrho(y_\mu(s,t),t) \tag{8}$$

where

$$V_1(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n}\left\{y_\mu^T(s,t)\mathcal{P}_\alpha y_\mu(s,t) + \sum_{k=1}^{q}\mathcal{A}_k\left(\frac{\partial y_\mu(s,t)}{\partial s_k}\right)^T\Gamma\left(\frac{\partial y_\mu(s,t)}{\partial s_k}\right)\right\}ds$$

$$V_2(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ \int_{t-h(t)}^{t} \chi_\mu^T(s,x)\mathcal{Q}_1\chi_\mu(s,x)dx + h_1 \int_{t-h1}^{t} \chi_\mu^T(s,x)\mathcal{Q}_2\chi_\mu(s,x)dx \right.$$
$$\left. + h_2 \int_{t-h2}^{t} \chi_\mu^T(s,x)\mathcal{Q}_3\chi_\mu(s,x)dx \right\} ds$$
$$V_3(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ \eta \int_{t-\eta}^{t} y_\mu^T(s,x)\mathcal{U}_1 y_\mu(s,x)dx \right\} ds$$
$$+ \int_\Omega \sum_{\mu=1}^{n} \left\{ (h_2 - h_1) \int_{t-h_2}^{t-h_1} y_\mu^T(s,x)\mathcal{U}_2 y_\mu(s,x)dx \right\} ds$$
$$V_4(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} y_\mu^T(s,x)\mathcal{Z} y_\mu(s,x)dxd\theta \right\} ds$$

Then, from Definition 2.1, it can be deduced that for each $\alpha \in \mathcal{S}$,

$$\mathcal{L}V(y_\mu(s,t),t) = \sum_{\varrho=1}^{4} \mathcal{L}V_\varrho(y_\mu(s,t),t) \tag{9}$$

where

$$\mathcal{L}V_1(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ 2y_\mu^T(s,t)\mathcal{P}_\alpha \dot{y}_\mu(s,t) + \sum_{\beta\in S} y_\mu^T(s,t)\varphi_{\alpha\beta}\mathcal{P}_\beta y_\mu(s,t) \right.$$
$$\left. + 2\sum_{k=1}^{q} \mathcal{A}_k \left(\frac{\partial y_\mu(s,t)}{\partial s_k} \right)^T \Gamma \left(\frac{\partial \dot{y}_\mu(s,t)}{\partial s_k} \right) \right\} ds$$

According to the error system (5), one can have

$$0 = 2\int_\Omega \sum_{i=1}^{r} \left\{ \theta_i(\vartheta(s,t)) \sum_{j=1}^{r} \theta_j(\vartheta(s,t)) \left[\gamma_1 y_\mu^T(s,t)\Gamma + \gamma_2 \dot{y}_\mu^T(s,t)\Gamma\right] \left[-\dot{y}_\mu(s,t) \right.\right.$$
$$+ \sum_{k=1}^{q} \frac{\partial}{\partial s_k} \left(\mathcal{A}_k \frac{\partial y_\mu(s,t)}{\partial s_k} \right) - \mathcal{B}_{i\alpha} y_\mu(s,t) + \mathcal{C}_{i\alpha}\tilde{g}\left(W_\alpha y_\mu(s,t)\right) + \mathcal{D}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t-h(t)))$$
$$\left.\left. + \left[K_{1j} + \Delta K_{1j}(t)\right]\mathcal{H}_{i\alpha} y_\mu(s,t) + \left[K_{2j} + \Delta K_{2j}(t)\right]\mathcal{H}_{i\alpha} y_\mu(s,t-\eta) \right] \right\} ds \tag{10}$$

Then, using Lemma 2.3

$$\begin{aligned} &2\left[\gamma_1 y_\mu^T(s,t)\Gamma + \gamma_2 \dot{y}_\mu^T(s,t)\Gamma\right]\left[\Delta K_{1j}(t)\mathcal{H}_{i\alpha} y_\mu(s,t) + \Delta K_{2j}(t)\mathcal{H}_{i\alpha} y_\mu(s,t-\eta)\right] \\ &= 2\left[\gamma_1 y_\mu^T(s,t)\Gamma + \gamma_2 \dot{y}_\mu^T(s,t)\Gamma\right] Q_j Y_j(t) \left[\mathcal{N}_{1j}\mathcal{H}_{i\alpha} y_\mu(s,t) + \mathcal{N}_{2j}\mathcal{H}_{i\alpha} y_\mu(s,t-\eta)\right] \\ &\leq \varepsilon^{-1}\left[\gamma_1 y_\mu^T(s,t)\Gamma + \gamma_2 \dot{y}_\mu^T(s,t)\Gamma\right] Q_j Q_j^T \left[\gamma_1 y_\mu^T(s,t)\Gamma + \gamma_2 \dot{y}_\mu^T(s,t)\Gamma\right]^T \\ &\quad + \varepsilon\left[\mathcal{N}_{1j}\mathcal{H}_{i\alpha} y_\mu(s,t) + \mathcal{N}_{2j}\mathcal{H}_{i\alpha} y_\mu(s,t-\eta)\right]^T\left[\mathcal{N}_{1j}\mathcal{H}_{i\alpha} y_\mu(s,t) + \mathcal{N}_{2j}\mathcal{H}_{i\alpha} y_\mu(s,t-\eta)\right] \end{aligned} \tag{11}$$

Combining (10), (11) and Lemma 2.4, one can easily derive that

$$\mathcal{L}V_1(y_\mu(s,t),t)$$
$$\leq 2\int_\Omega \sum_{\mu=1}^{n} \Upsilon_\mu ds + \int_\Omega \sum_{\mu=1}^{n} \left\{ \sum_{\beta\in S} y_\mu^T(s,t)\varphi_{\alpha\beta}\mathcal{P}_\beta y_\mu(s,t) \right\} ds$$

$$+\int_{\Omega}\sum_{\mu=1}^{n}\Bigg\{\sum_{i=1}^{r}\theta_i(\vartheta(s,t))\sum_{j=1}^{r}\theta_j(\vartheta(s,t))\left[\varepsilon^{-1}\left(\gamma_1 y_\mu^T(s,t)\Gamma\right.\right.$$
$$\left.+\gamma_2\dot{y}_\mu^T(s,t)\Gamma\right)Q_jQ_j^T\left(\gamma_1 y_\mu^T(s,t)\Gamma+\gamma_2\dot{y}_\mu^T(s,t)\Gamma\right)^T+\varepsilon\left(\mathcal{N}_{1j}\mathcal{H}_{i\alpha}y_\mu(s,t)\right.$$
$$\left.\left.+\mathcal{N}_{2j}\mathcal{H}_{i\alpha}y_\mu(s,t-\eta)\right)^T\left(\mathcal{N}_{1j}\mathcal{H}_{i\alpha}y_\mu(s,t)+\mathcal{N}_{2j}\mathcal{H}_{i\alpha}y_\mu(s,t-\eta)\right)\right]\Bigg\}ds \tag{12}$$

where

$$\begin{aligned}\Upsilon_\mu=&\sum_{i=1}^{r}\theta_i(\vartheta(s,t))\sum_{j=1}^{r}\theta_j(\vartheta(s,t))\left[y_\mu^T(s,t)\mathcal{P}_\alpha\dot{y}_\mu(s,t)-\gamma_1 y_\mu^T(s,t)\Gamma\dot{y}_\mu(s,t)\right.\\
&-\gamma_1 y_\mu^T(s,t)\Gamma\tilde{\mathcal{A}}y_\mu(s,t)-\gamma_1 y_\mu^T(s,t)\Gamma\mathcal{B}_{i\alpha}y_\mu(s,t)+\gamma_1 y_\mu^T(s,t)\Gamma\mathcal{C}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t))\\
&+\gamma_1 y_\mu^T(s,t)\Gamma\mathcal{D}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t-h(t)))+\gamma_1 y_\mu^T(s,t)\Gamma K_{1j}\mathcal{H}_{i\alpha}y_\mu(s,t)\\
&+\gamma_1 y_\mu^T(s,t)\Gamma K_{2j}\mathcal{H}_{i\alpha}y_\mu(s,t-\eta)-\gamma_2\dot{y}_\mu^T(s,t)\Gamma\dot{y}_\mu(s,t)+\gamma_2\dot{y}_\mu^T(s,t)\Gamma(-\mathcal{B}_{i\alpha}y_\mu(s,t)\\
&+\mathcal{C}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t))+\mathcal{D}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t-h(t))))+\gamma_2\dot{y}_\mu^T(s,t)\Gamma K_{1j}\mathcal{H}_{i\alpha}y_\mu(s,t)\\
&\left.+\gamma_2\dot{y}_\mu^T(s,t)\Gamma K_{2j}\mathcal{H}_{i\alpha}y_\mu(s,t-\eta)\right]\end{aligned}$$

$\tilde{\mathcal{A}}=diag\left\{\sum_{k=1}^{q}\frac{a_{1k}}{l_k^2},\sum_{k=1}^{q}\frac{a_{2k}}{l_k^2},\dots,\sum_{k=1}^{q}\frac{a_{nk}}{l_k^2}\right\}$ and $l_k>0$ are given scalars.

In addition,

$$\begin{aligned}\mathcal{L}V_2(y_\mu(s,t),t)\le&\int_{\Omega}\sum_{\mu=1}^{n}\left\{\chi_\mu^T(s,t)(\mathcal{Q}_1+h_1\mathcal{Q}_2+h_2\mathcal{Q}_3)\chi_\mu(s,t)\right\}ds\\
&-(1-h)\int_{\Omega}\sum_{\mu=1}^{n}\left\{\chi_\mu^T(s,t-h(t))\mathcal{Q}_1\chi_\mu(s,t-h(t))\right\}ds\\
&-h_1\int_{\Omega}\sum_{\mu=1}^{n}\left\{\chi_\mu^T(s,t-h_1)\mathcal{Q}_2\chi_\mu(s,t-h_1)\right\}ds\\
&-h_2\int_{\Omega}\sum_{\mu=1}^{n}\left\{\chi_\mu^T(s,t-h_2)\mathcal{Q}_3\chi_\mu(s,t-h_2)\right\}ds\end{aligned}\tag{13}$$

$$\begin{aligned}\mathcal{L}V_3(y_\mu(s,t),t)=&\,\eta\int_{\Omega}\sum_{\mu=1}^{n}\left\{y_\mu^T(s,t)\mathcal{U}_1y_\mu(s,t)-y_\mu^T(s,t-\eta)\mathcal{U}_1y_\mu(s,t-\eta)\right\}ds\\
&+(h_2-h_1)\int_{\Omega}\sum_{\mu=1}^{n}\left\{y_\mu^T(s,t-h_1)\mathcal{U}_2y_\mu(s,t-h_1)\right.\\
&\left.-y_\mu^T(s,t-h_2)\mathcal{U}_2y_\mu(s,t-h_2)\right\}ds\end{aligned}\tag{14}$$

$$\begin{aligned}\mathcal{L}V_4(y_\mu(s,t),t)=&\int_{\Omega}\sum_{\mu=1}^{n}\Bigg\{(h_2-h_1)^2y_\mu^T(s,t)\mathcal{Z}y_\mu(s,t)\\
&-(h_2-h_1)\int_{t-h_2}^{t-h_1}y_\mu^T(s,x)\mathcal{Z}y_\mu(s,x)dx\Bigg\}ds\end{aligned}\tag{15}$$

For $h_1\le h(t)\le h_2$, the following inequalities can be deduced by Lemmas 2.1 and 2.2:

$$-(h_2-h_1)\int_{t-h_2}^{t-h_1}y_\mu^T(s,x)\mathcal{Z}y_\mu(s,x)dx$$

$$\leq -\frac{h_2-h_1}{h_2-h(t)}\left(\int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx\right)^T \mathcal{Z}\left(\int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx\right)$$
$$-\frac{h_2-h_1}{h(t)-h_1}\left(\int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx\right)^T \mathcal{Z}\left(\int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx\right)$$
$$\leq -\begin{bmatrix} \int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx \\ \int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx \end{bmatrix}^T \begin{bmatrix} \mathcal{Z} & \tilde{\mathcal{Z}} \\ * & \mathcal{Z} \end{bmatrix} \begin{bmatrix} \int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx \\ \int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx \end{bmatrix}$$

and

$$\begin{bmatrix} \mathcal{Z} & \tilde{\mathcal{Z}} \\ * & \mathcal{Z} \end{bmatrix} > 0$$

From Assumption 2.1, we can obtain the following inequations for n-dimensional positive definite diagonal matrix Θ_o $(o = 1, 2, 3)$:

$$0 \leq 2[\tilde{g}(W_\alpha y_\mu(s,t)) - V_1 W_\alpha y_\mu(s,t)]^T \Theta_1 [V_2 W_\alpha y_\mu(s,t) - \tilde{g}(W_\alpha y_\mu(s,t))] \tag{16}$$

$$0 \leq 2[\tilde{g}(W_\alpha y_\mu(s,t-h(t))) - V_1 W_\alpha y_\mu(s,t-h(t))]^T \Theta_2 [V_2 W_\alpha y_\mu(s,t-h(t)) - \tilde{g}(W_\alpha y_\mu(s,t-h(t)))] \tag{17}$$

$$0 \leq 2\left\{\tilde{g}(W_\alpha y_\mu(s,t)) - \tilde{g}(W_\alpha y_\mu(s,t-h(t))) - V_1\left[W_\alpha y_\mu(s,t) - W_\alpha y_\mu(s,t-h(t))\right]\right\}^T \Theta_3 \left\{V_2[W_\alpha y_\mu(s,t) - W_\alpha y_\mu(s,t-h(t))] - \tilde{g}(W_\alpha y_\mu(s,t)) + \tilde{g}(W_\alpha y_\mu(s,t-h(t)))\right\} \tag{18}$$

By the Leibnitz-Newton formula, it is clear that the following equality is true for arbitrary matrices G_ϖ $(\varpi = 1, 2, 3, 4)$ with appropriate dimensions:

$$0 = 2\left[y_\mu^T(s,t-h(t))G_1 + \tilde{g}^T(y_\mu(s,t))G_2 + \tilde{g}^T(y_\mu(s,t-h(t)))G_3 + \int_{t-\eta}^{t} \dot{y}_\mu^T(s,x)dx \times G_4\right] \times \left[y_\mu(s,t) - y_\mu(s,t-\eta) - \int_{t-\eta}^{t} \dot{y}_\mu^T(s,x)dx\right] \tag{19}$$

Let $\hat{K}_{1j} = \Gamma K_{1j}$, $\hat{K}_{2j} = \Gamma K_{2j}$, then, invoking (8)-(19) and calculating the mathematical expectation of $\mathcal{L}V(y_\mu(s,t),t)$, for each $\alpha \in S$, it can be derived that:

$$\mathbb{E}\{\mathcal{L}V(y_\mu(s,t),t)\} \leq \nabla^T(s,t)\bar{\Sigma}_{ij}\nabla(s,t)$$

where $\bar{\Sigma}_{ij} = \Sigma_{ij} + \varepsilon^{-1}\left(\gamma_1 \upsilon_1^T\Gamma + \gamma_2\upsilon_2^T\Gamma\right) Q_j Q_j^T \left(\gamma_1\upsilon_1^T\Gamma + \gamma_2\upsilon_2^T\Gamma\right)^T$, and Σ_{ij} is defined in Theorem 3.1. It clearly shows that $\mathbb{E}\{\mathcal{L}V(y_\mu(s,t),t)\} < 0$ if and only if (6) and (7) hold. From Lyapunov stability theory, one can obtain that the closed-loop error system (5) is globally asymptotically stable in mean square. Additionally, $K_{1j} = \Gamma^{-1}\hat{K}_{1j}$, $K_{2j} = \Gamma^{-1}\hat{K}_{2j}$. This completes the proof. □

Remark 3.1. *In this paper, some accessional variables are involved in $\nabla(s,t)$ such as $\int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx$, $\int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx$ and $\int_{t-\eta}^{t} \dot{y}_\mu(s,x)dx$, and these new additional variables can strengthen the combination of the terms $y_\mu(s,t)$, $y_\mu(s,t-h(t))$, $y_\mu(s,t-\eta)$, $y_\mu(s,t-h_1)$ and $y_\mu(s,t-h_2)$, which help to derive modified stability criteria and less conservative results of the proposed systems. Nevertheless, if the number of variables is too large, it will increase the amount of calculations. The relevant proof and simulation will be demonstrated in Corollary 3.2 and numerical Example 4.2.*

Remark 3.2. *Two zero terms (10) and (19) are adopted to derive main results, which differently operate on the criterion. The first zero term (10) is mainly aimed at presenting the matrix* Γ *to combine with* K_{1j} *and* K_{2j}*, so that the controller gains can be directly deduced by the utilization of Theorem 3.1. As a result, the calculations are greatly reduced compared with* [28, 37]*. Moreover, to decrease the zero units in the main matrix of (6), which can generate conservatism of the main results and instability of the control system, the second zero term (19) that can strengthen the combination of the variables* $y_\mu(s,t)$*,* $y_\mu(s,t-\eta)$ *and the other proposed terms. As a result, improved stability criteria are obtained.*

Remark 3.3. *It is significant to note that the control gains derived from Theorem 3.1 may be much larger, which is difficult to come true so that the control cost would be increased. To further reduce the conservativeness and improve the feasibility of control scheme, one can impose restrictions on the magnitude of the control gains* K_{1j} *and* K_{2j} [38]*. By restricting* $\|\Gamma^{-1}\| < \tau_1$*,* $\left\|\hat{K}_{1j}\right\| < \tau_2$*,* $\left\|\hat{K}_{1j}\right\| < \tau_3$*, where* τ_1*,* τ_2 *and* τ_3 *are given positive constants, such that*

$$\begin{pmatrix} -\tau_1\Gamma & I \\ I & -\tau_1\Gamma \end{pmatrix} < 0 \tag{20}$$

$$\begin{pmatrix} -\tau_2 I & \hat{K}_{1j}^T \\ \hat{K}_{1j} & -\tau_2 I \end{pmatrix} < 0 \tag{21}$$

$$\begin{pmatrix} -\tau_3 I & \hat{K}_{2j}^T \\ \hat{K}_{2j} & -\tau_3 I \end{pmatrix} < 0 \tag{22}$$

Invoking (6), (7), and (20), (21), (22), one can obtain optimized control gains K_{1j} *and* K_{2j}*.*

Remark 3.4. *Considering* $K_{2j} + \Delta K_{2j}(t) = 0$*, that is, the memory function is missing in considered controller, then the error system (5) can be described as:*

$$\begin{aligned}\frac{\partial y_\mu(s,t)}{\partial t} = & \sum_{k=1}^{q} \frac{\partial}{\partial s_k}\left(\mathcal{A}_k \frac{\partial y_\mu(s,t)}{\partial s_k}\right) + \sum_{i=1}^{r} \theta_i(\vartheta(s,t)) \sum_{j=1}^{r} \theta_j(\vartheta(s,t))\big[-\mathcal{B}_{i\alpha} y_\mu(s,t) \\ & + \mathcal{C}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t)) + \mathcal{D}_{i\alpha}\tilde{g}(W_\alpha y_\mu(s,t-h(t))) \\ & + \big[K_j + \Delta K_j(t)\big]\mathcal{H}_{i\alpha} y_\mu(s,t)\big], \quad \mu = 1,2,\ldots,n \end{aligned} \tag{23}$$

where $\Delta K_j(t) = Q_j Y_j(t) \mathcal{N}_j$*.*

Then Corollary 3.1 about the stability problem of error system (23) can be deduced. Before given Corollary 3.1, we define:

$$\begin{aligned}\bar{\nabla}(s,t) = col\Big\{ & y_\mu(s,t), \dot{y}_\mu(s,t), y_\mu(s,t-h_1), y_\mu(s,t-h(t)), y_\mu(s,t-h_2), \tilde{g}(W_\alpha y_\mu(s,t)), \\ & \tilde{g}(W_\alpha y_\mu(s,t-h_1)), \tilde{g}(W_\alpha y_\mu(s,t-h(t))), \tilde{g}(W_\alpha y_\mu(s,t-h_2)), \\ & \int_{t-h_2}^{t-h(t)} y_\mu(s,x)dx, \int_{t-h(t)}^{t-h_1} y_\mu(s,x)dx \Big\}\end{aligned}$$

Corollary 3.1. *For given scalars* $\varepsilon > 0$*,* $\eta > 0$*,* $h_2 > h_1 > 0$*, the T-S fuzzy MJGNNs with reaction-diffusion terms (23) are asymptotically stable in mean square if there exist* $\bar{\mathcal{P}}_\alpha > 0$*,* $(\alpha = 1,2,\ldots,\mathcal{S})$*,* $\bar{\Gamma} > 0$*,* $\bar{\mathcal{Q}}_e > 0$*,* $(e = 1,2,3)$*,* $\bar{\mathcal{U}} > 0$*,* $\bar{\mathcal{Z}} > 0$*, positive definite diagonal matrix* $\bar{\Theta}_o$ $(o = 1,2,3)$ *and appropriate dimensional arbitrary matrices* $\hat{\mathcal{Z}}$ *and* $\hat{K}_j$

$(j = 1, 2, \dots, r)$ *such that the following LMIs hold:*

$$\begin{bmatrix} \bar{\Sigma}_{ij} & \left(\gamma_1 \upsilon_1^T \bar{\Gamma} + \gamma_2 \upsilon_2^T \bar{\Gamma}\right) \bar{Q}_j \\ * & -\varepsilon I_n \end{bmatrix} < 0$$

$$\begin{bmatrix} \bar{\mathcal{Z}} & \hat{\mathcal{Z}} \\ * & \bar{\mathcal{Z}} \end{bmatrix} > 0$$

where

$$\bar{\Sigma}_{ij} = \bar{\Xi}_1 + \bar{\Xi}_2 + \bar{\Xi}_3 + \bar{\Xi}_4 + \bar{\Xi}_5$$

$$\begin{aligned} \bar{\Xi}_1 = {} & Sym\left\{\upsilon_1^T\left(\bar{\mathcal{P}}_\alpha - \gamma_1 \bar{\Gamma}\right)\upsilon_2\right\} - \upsilon_1^T\left[2\gamma_1 \bar{\Gamma}\tilde{\mathcal{A}} + 2\gamma_1 \bar{\Gamma}\mathcal{B}_{i\alpha} - \sum_{\beta \in S} \varphi_{\alpha\beta} \bar{\mathcal{P}}_\beta - 2\gamma_1 \hat{K}_j \mathcal{H}_{i\alpha}\right]\upsilon_1 \\ & + Sym\left\{\upsilon_1^T \gamma_1 \bar{\Gamma}\left[\mathcal{C}_{i\alpha}\upsilon_6 + \mathcal{D}_{i\alpha}\upsilon_8\right]\right\} - 2\upsilon_2^T \gamma_2 \bar{\Gamma}\upsilon_2 \\ & + Sym\left\{\upsilon_2^T \gamma_2 \bar{\Gamma}\left[-\mathcal{B}_{i\alpha}\upsilon_1 + \mathcal{C}_{i\alpha}\upsilon_6 + \mathcal{D}_{i\alpha}\upsilon_8\right] + \upsilon_2^T \gamma_2 \hat{K}_j \mathcal{H}_{i\alpha}\upsilon_1\right\} \\ & + \varepsilon(\mathcal{N}_j \mathcal{H}_{i\alpha}\upsilon_1)^T \left(\mathcal{N}_j \mathcal{H}_{i\alpha}\upsilon_1\right) \end{aligned}$$

$$\begin{aligned} \bar{\Xi}_2 = {} & \left(\upsilon_1^T, \upsilon_6^T\right)\left(\bar{\mathcal{Q}}_1 + h_1 \bar{\mathcal{Q}}_2 + h_2 \bar{\mathcal{Q}}_3\right)\left(\upsilon_1^T, \upsilon_6^T\right)^T - (1-h)\left(\upsilon_4^T, \upsilon_8^T\right)\bar{\mathcal{Q}}_1\left(\upsilon_4^T, \upsilon_8^T\right)^T \\ & - \left(\upsilon_3^T, \upsilon_7^T\right) h_1 \bar{\mathcal{Q}}_2 \left(\upsilon_3^T, \upsilon_7^T\right)^T - \left(\upsilon_5^T, \upsilon_9^T\right) h_2 \bar{\mathcal{Q}}_3 \left(\upsilon_5^T, \upsilon_9^T\right)^T \end{aligned}$$

$$\bar{\Xi}_3 = (h_2 - h_1)\left(\upsilon_3^T \bar{\mathcal{U}}\upsilon_3 - \upsilon_5^T \bar{\mathcal{U}}\upsilon_5\right)$$

$$\bar{\Xi}_4 = (h_2 - h_1)^2 \upsilon_1^T \bar{\mathcal{Z}}\upsilon_1 - \left(\upsilon_{10}^T, \upsilon_{11}^T\right)\begin{bmatrix} \bar{\mathcal{Z}} & \hat{\mathcal{Z}} \\ * & \bar{\mathcal{Z}} \end{bmatrix}\left(\upsilon_{10}^T, \upsilon_{11}^T\right)^T$$

$$\begin{aligned} \bar{\Xi}_5 = {} & Sym\left\{\left[\upsilon_6^T - \upsilon_1^T W_\alpha^T V_1^T\right]\bar{\Theta}_1\left[V_2 W_\alpha \upsilon_1 - \upsilon_6\right]\right\} \\ & + Sym\left\{\left[\upsilon_8^T - \upsilon_4^T W_\alpha^T V_1^T\right]\bar{\Theta}_2\left[V_2 W_\alpha \upsilon_4 - \upsilon_8\right]\right\} \\ & + Sym\left\{\left[\upsilon_6^T - \upsilon_8^T - \upsilon_1^T W_\alpha^T V_1^T + \upsilon_4^T W_\alpha^T V_1^T\right]\bar{\Theta}_3\left[V_2 W_\alpha \upsilon_1 - V_2 W_\alpha \upsilon_4 - \upsilon_6 + \upsilon_7\right]\right\} \end{aligned}$$

In addition, $K_j = \bar{\Gamma}^{-1}\hat{K}_j$.

Proof: Firstly, define

$$\chi_\mu^T(s,t) = \left[y_\mu^T(s,t), \tilde{g}^T(W_\alpha y_\mu(s,t))\right]$$

Then, choose the following Lyapunov functional candidates:

$$\bar{V}(y_\mu(s,t),t) = \sum_{\varrho=1}^{4} \bar{V}_\varrho(y_\mu(s,t),t)$$

where

$$\bar{V}_1(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n}\left\{y_\mu^T(s,t)\bar{\mathcal{P}}_\alpha y_\mu(s,t) + \sum_{k=1}^{q}\mathcal{A}_k\left(\frac{\partial y_\mu(s,t)}{\partial s_k}\right)^T \bar{\Gamma}\left(\frac{\partial y_\mu(s,t)}{\partial s_k}\right)\right\}ds$$

$$\begin{aligned} \bar{V}_2(y_\mu(s,t),t) = {} & \int_\Omega \sum_{\mu=1}^{n}\left\{\int_{t-h(t)}^{t}\chi_\mu^T(s,x)\bar{\mathcal{Q}}_1\chi_\mu(s,x)dx + h_1\int_{t-h1}^{t}\chi_\mu^T(s,x)\bar{\mathcal{Q}}_2\chi_\mu(s,x)dx\right. \\ & \left. + h_2\int_{t-h2}^{t}\chi_\mu^T(s,x)\bar{\mathcal{Q}}_3\chi_\mu(s,x)dx\right\}ds \end{aligned}$$

$$\bar{V}_3(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n}\left\{(h_2 - h_1)\int_{t-h_2}^{t-h_1} y_\mu^T(s,x)\bar{\mathcal{U}}y_\mu(s,x)dx\right\}ds$$

$$\bar{V}_4(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} y_\mu^T(s,x) \bar{Z} y_\mu(s,x) dx d\theta \right\} ds$$

Because the next proof is similar to the proof of Theorem 3.1, it is omitted here. This completes the proof. □

If we only consider the necessary variables in $\nabla(s,t)$, that is,

$$\begin{aligned} \tilde{\nabla}(s,t) = col\,\{ & y_\mu(s,t), \dot{y}_\mu(s,t), y_\mu(s,t-h_1), y_\mu(s,t-h(t)), y_\mu(s,t-h_2), \\ & y_\mu(s,t-\eta), \tilde{g}(W_\alpha y_\mu(s,t)), \tilde{g}(W_\alpha y_\mu(s,t-h(t)))\} \end{aligned}$$

Then Corollary 3.2 is deduced as the following.

Corollary 3.2. *For given scalars $\varepsilon > 0$, $\eta > 0$, $h_2 > h_1 > 0$, the T-S fuzzy MJGNNs with reaction-diffusion terms (5) are asymptotically stable in mean square if there exist $\tilde{\mathcal{P}}_\alpha > 0$, $(\alpha = 1, 2, \ldots, \mathcal{S})$, $\tilde{\Gamma} > 0$, $\tilde{\mathcal{Q}}_e > 0$, $(e = 1, 2, 3)$, $\tilde{\mathcal{M}} > 0$, $\tilde{\mathcal{U}} > 0$, positive definite diagonal matrix $\tilde{\Theta}_o$ $(o = 1, 2, 3)$, and appropriate dimensional arbitrary matrices $\hat{K}_{1j}$, $\hat{K}_{2j}$ $(j = 1, 2, \ldots, r)$, such that the following LMI holds*

$$\begin{bmatrix} \tilde{\Sigma}_{ij} & \left(\gamma_1 \upsilon_1^T \tilde{\Gamma} + \gamma_2 \upsilon_2^T \tilde{\Gamma}\right) \tilde{Q}_j \\ * & -\varepsilon I_n \end{bmatrix} < 0$$

where

$$\begin{aligned}
\tilde{\Sigma}_{ij} &= \tilde{\Xi}_1 + \tilde{\Xi}_2 + \tilde{\Xi}_3 + \tilde{\Xi}_4 + \tilde{\Xi}_5 \\
\tilde{\Xi}_1 &= Sym\left\{ \upsilon_1^T \left(\tilde{\mathcal{P}}_\alpha - \gamma_1 \tilde{\Gamma} \right) \upsilon_2 \right\} - \upsilon_1^T \left[2\gamma_1 \tilde{\Gamma} \tilde{\mathcal{A}} + 2\gamma_1 \tilde{\Gamma} \mathcal{B}_{i\alpha} - \sum_{\beta \in S} \varphi_{\alpha\beta} \tilde{\mathcal{P}}_\beta - 2\gamma_1 \hat{K}_{1j} \mathcal{H}_{i\alpha} \right] \upsilon_1 \\
&\quad + Sym\left\{ \upsilon_1^T \gamma_1 \tilde{\Gamma} \left[\mathcal{C}_{i\alpha} \upsilon_7 + \mathcal{D}_{i\alpha} \upsilon_8 \right] \right\} + Sym\left\{ \upsilon_1^T \gamma_1 \hat{K}_{2j} \mathcal{H}_{i\alpha} \upsilon_6 \right\} - 2\upsilon_2^T \gamma_2 \tilde{\Gamma} \upsilon_2 \\
&\quad + Sym\left\{ \upsilon_2^T \gamma_2 \tilde{\Gamma} \left[-\mathcal{B}_{i\alpha} \upsilon_1 + \mathcal{C}_{i\alpha} \upsilon_7 + \mathcal{D}_{i\alpha} \upsilon_8 \right] + \upsilon_2^T \gamma_2 \hat{K}_{1j} \mathcal{H}_{i\alpha} \upsilon_1 + \upsilon_2^T \gamma_2 \hat{K}_{2j} \times \mathcal{H}_{i\alpha} \upsilon_6 \right\} \\
&\quad + \varepsilon (\mathcal{N}_{1j} \mathcal{H}_{i\alpha} \upsilon_1 + \mathcal{N}_{2j} \mathcal{H}_{i\alpha} \upsilon_6)^T (\mathcal{N}_{1j} \mathcal{H}_{i\alpha} \upsilon_1 + \mathcal{N}_{2j} \mathcal{H}_{i\alpha} \upsilon_6) \\
\tilde{\Xi}_2 &= \left(\upsilon_1^T\right) \left(\tilde{\mathcal{Q}}_1 + h_1 \tilde{\mathcal{Q}}_2 + h_2 \tilde{\mathcal{Q}}_3 \right) \left(\upsilon_1^T\right)^T - (1-h) \left(\upsilon_4^T\right) \tilde{\mathcal{Q}}_1 \left(\upsilon_4^T\right)^T - \left(\upsilon_3^T\right) h_1 \tilde{\mathcal{Q}}_2 \left(\upsilon_3^T\right)^T \\
&\quad - \left(\upsilon_5^T\right) h_2 \tilde{\mathcal{Q}}_3 \left(\upsilon_5^T\right)^T \\
\tilde{\Xi}_3 &= \upsilon_7^T \tilde{\mathcal{M}} \upsilon_7 - \upsilon_8^T \tilde{\mathcal{M}} \upsilon_8 \\
\tilde{\Xi}_4 &= \eta \left(\upsilon_1^T \tilde{\mathcal{U}} \upsilon_1 - \upsilon_6^T \tilde{\mathcal{U}} \upsilon_6 \right) \\
\tilde{\Xi}_5 &= Sym\left\{ \left[\upsilon_7^T - \upsilon_1^T W_\alpha^T V_1^T \right] \tilde{\Theta}_1 \left[V_2 W_\alpha \upsilon_1 - \upsilon_7 \right] \right\} \\
&\quad + Sym\left\{ \left[\upsilon_8^T - \upsilon_4^T W_\alpha^T V_1^T \right] \tilde{\Theta}_2 \left[V_2 W_\alpha \upsilon_4 - \upsilon_8 \right] \right\} \\
&\quad + Sym\left\{ \left[\upsilon_7^T - \upsilon_8^T - \upsilon_1^T W_\alpha^T V_1^T + \upsilon_4^T W_\alpha^T V_1^T \right] \tilde{\Theta}_3 \left[V_2 W_\alpha \upsilon_1 - V_2 W_\alpha \upsilon_4 - \upsilon_7 + \upsilon_8 \right] \right\}
\end{aligned}$$

In addition, $K_{1j} = \tilde{\Gamma}^{-1} \hat{K}_{1j}$, $K_{2j} = \tilde{\Gamma}^{-1} \hat{K}_{2j}$.

Proof: Choose the following Lyapunov functional candidates:

$$\tilde{V}(y_\mu(s,t),t) = \sum_{\kappa=1}^{4} \tilde{V}_\kappa(y_\mu(s,t),t)$$

where

$$\tilde{V}_1(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ y_\mu^T(s,t)\tilde{\mathscr{P}}_\alpha y_\mu(s,t) + \sum_{k=1}^{q} \mathscr{A}_k \left(\frac{\partial y_\mu(s,t)}{\partial s_k} \right)^T \tilde{\Gamma} \left(\frac{\partial y_\mu(s,t)}{\partial s_k} \right) \right\} ds$$

$$\tilde{V}_2(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ \int_{t-h(t)}^{t} y_\mu^T(s,x)\tilde{\mathscr{Q}}_1 y_\mu(s,x)dx + h_1 \int_{t-h_1}^{t} y_\mu^T(s,x)\tilde{\mathscr{Q}}_2 y_\mu(s,x)dx \right.$$
$$\left. + h_2 \int_{t-h_2}^{t} y_\mu^T(s,x)\tilde{\mathscr{Q}}_3 y_\mu(s,x)dx \right\} ds$$

$$\tilde{V}_3(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ \int_{t-h(t)}^{t} \tilde{g}^T(y_\mu(s,x))\tilde{\mathscr{M}}\tilde{g}(y_\mu(s,x))dx \right\} ds$$

$$\tilde{V}_4(y_\mu(s,t),t) = \int_\Omega \sum_{\mu=1}^{n} \left\{ \eta \int_{t-\eta}^{t} y_\mu^T(s,x)\tilde{\mathscr{U}} y_\mu(s,x)dx \right\} ds$$

Then the following proof is similar to the proof of Theorem 3.1. Hence, it is omitted here. This completes the proof. □

4. **Numerical Examples.** This section provides three numerical examples to illustrate the effectiveness and advantages of the proposed theoretical results.

Consider the following MJGNNs with reaction-diffusion terms under two T-S fuzzy rules:

$$\frac{\partial u(s,t)}{\partial t} = \sum_{k=1}^{2} \frac{\partial}{\partial s_k} \left(\mathscr{A}_k \frac{\partial u(s,t)}{\partial s_k} \right) + \sum_{i=1}^{2} \theta_i(\vartheta(s,t)) \left[-\mathscr{B}_{i\alpha} u(s,t) \right.$$
$$\left. + \mathscr{C}_{i\alpha} \tanh(W_\alpha u(s,t)) + \mathscr{D}_{i\alpha} \tanh(W_\alpha u(s,t-h(t))) \right]$$
$$\Im^u(s,t) = \sum_{i=1}^{2} \theta_i(\vartheta(s,t))\mathscr{H}_{i\alpha} u(s,t) \tag{24}$$

and the corresponding full-order observer

$$\frac{\partial v_\mu(s,t)}{\partial t} = \sum_{k=1}^{2} \frac{\partial}{\partial s_k} \left(\mathscr{A}_k \frac{\partial v_\mu(s,t)}{\partial s_k} \right) + \sum_{i=1}^{2} \theta_i(\vartheta(s,t)) \big[-\mathscr{B}_{i\alpha} v_\mu(s,t)$$
$$+ \mathscr{C}_{i\alpha} \tanh\big(W_\alpha v_\mu(s,t)\big) + \mathscr{D}_{i\alpha} \tanh\big(W_\alpha v_\mu(s,t-h(t))\big) + w_\mu(s,t) \big]$$
$$\Im_\mu^v(s,t) = \sum_{i=1}^{2} \theta_i(\vartheta(s,t))\mathscr{H}_{i\alpha} v_\mu(s,t), \quad \mu = 1,2 \tag{25}$$

We design the following nonfragile memory-based output feedback controller:

$$w_\mu(s,t) = \sum_{j=1}^{2} \theta_j(\vartheta(s,t)) \big\{ [K_{1j} + \Delta K_{1j}(t)] \big[\Im_\mu^v(s,t) - \Im^u(s,t) \big]$$
$$+ [K_{2j} + \Delta K_{2j}(t)] \big[\Im_\mu^v(s,t-\eta) - \Im^u(s,t-\eta) \big] \big\} \tag{26}$$

Then, the error system can be derived as:

$$\frac{\partial y_\mu(s,t)}{\partial t} = \sum_{k=1}^{q} \frac{\partial}{\partial s_k} \left(\mathscr{A}_k \frac{\partial y_\mu(s,t)}{\partial s_k} \right) + \sum_{i=1}^{2} \theta_i(\vartheta(s,t)) \sum_{j=1}^{2} \theta_j(\vartheta(s,t)) \big[-\mathscr{B}_{i\alpha} y_\mu(s,t)$$
$$+ \mathscr{C}_{i\alpha} \tanh\left(W_\alpha y_\mu(s,t)\right) + \mathscr{D}_{i\alpha} \tanh(W_\alpha y_\mu(s,t-h(t)))$$

$$+ [K_{1j} + \Delta K_{1j}(t)]\, \mathcal{H}_{i\alpha} y_\mu(s,t) + [K_{2j} + \Delta K_{2j}(t)]\, \mathcal{H}_{i\alpha} y_\mu(s, t-\eta)] \tag{27}$$

Example 4.1. *Firstly, we choose the following parameters for open-loop system (24):*

$$\Psi = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix},\ \mathcal{A}_k = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix},\ \mathcal{B}_{11} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},\ \mathcal{B}_{12} = \begin{bmatrix} 0.3 & 0 \\ 0 & 1.4 \end{bmatrix},$$

$$\mathcal{B}_{21} = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.5 \end{bmatrix},\ \mathcal{B}_{22} = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix},\ \mathcal{C}_{11} = \begin{bmatrix} 0.01 & -0.08 \\ 0.06 & 0.02 \end{bmatrix},\ \mathcal{C}_{12} = \begin{bmatrix} 1.01 & -2.08 \\ 0.06 & 1.02 \end{bmatrix},$$

$$\mathcal{C}_{21} = \begin{bmatrix} 0.15 & -1.08 \\ 0.75 & 2.02 \end{bmatrix},\ \mathcal{C}_{22} = \begin{bmatrix} 1.15 & -1.08 \\ 0.85 & 1.02 \end{bmatrix},\ \mathcal{D}_{11} = \begin{bmatrix} 0.16 & 0.2 \\ -0.4 & 0.9 \end{bmatrix},$$

$$\mathcal{D}_{12} = \begin{bmatrix} 0.12 & -0.2 \\ 0.3 & 0.1 \end{bmatrix},\ \mathcal{D}_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.9 \end{bmatrix},\ \mathcal{D}_{22} = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.6 \end{bmatrix},\ W_1 = \begin{bmatrix} 1.2 & 1 \\ 1 & 1.1 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 1.1 & 1 \\ 1 & 1.2 \end{bmatrix},\ V_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},\ V_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ \mathcal{H}_{11} = \begin{bmatrix} 0.3 & 0.12 \\ 0.1 & 0.22 \end{bmatrix},$$

$$\mathcal{H}_{12} = \begin{bmatrix} 0.23 & 0.1 \\ 0.1 & 0.32 \end{bmatrix},\ \mathcal{H}_{21} = \begin{bmatrix} 0.25 & 0.15 \\ 0.08 & 0.32 \end{bmatrix},\ \mathcal{H}_{22} = \begin{bmatrix} 0.35 & 0.1 \\ 0.11 & 0.42 \end{bmatrix},$$

and $h(t) = 1.5 + 0.5\sin(t)$*. Moreover, the boundary conditions and initial conditions are respectively set as*

$$u_1(s,t) = u_2(s,t) = 0, \quad (s,t) \in \partial\Omega \times [-h_2, +\infty)$$
$$u_1(s,\ell) = 0.05\sin(\pi s), \quad u_2(s,\ell) = 0.02\sin(\pi s), \quad (s,\ell) \in \Omega \times [-h_2, 0]$$

Then the dynamical behaviors of the open-loop system (24) is presented as Figure 1.

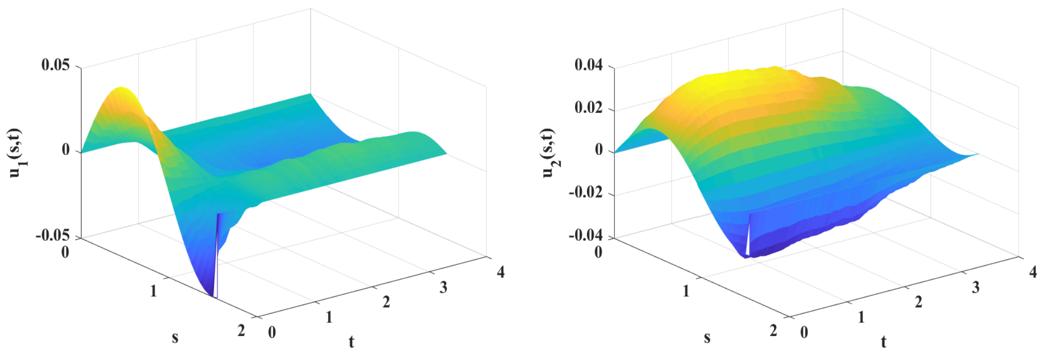

FIGURE 1. The dynamic behaviors of $u_1(s,t)$ and $u_2(s,t)$

It is obvious that the open-loop system (24) is emanative. Therefore, it is significant to design a corresponding controller to form a closed-loop system so as to make the system asymptotically stable. As a result, the observer (25) with controller (26) is developed for the open-loop system (24) to form the error system (27). Besides the parameters used above, the additional ones are employed here:

$$Q_1 = \begin{bmatrix} 1.78 & 0.5 \\ 0.51 & 2 \end{bmatrix},\ Q_2 = \begin{bmatrix} 1.2 & 0.6 \\ 0.41 & 1.4 \end{bmatrix},\ \mathcal{N}_{11} = \begin{bmatrix} 0.2 & 0.01 \\ 0.02 & 0.3 \end{bmatrix},\ \mathcal{N}_{12} = \begin{bmatrix} 0.5 & 0.01 \\ 0.01 & 0.4 \end{bmatrix}$$

$$\mathcal{N}_{21} = \begin{bmatrix} 0.6 & 0.02 \\ 0.01 & 0.7 \end{bmatrix},\ \mathcal{N}_{22} = \begin{bmatrix} 0.7 & 0.01 \\ 0.01 & 0.75 \end{bmatrix},\ \gamma_1 = 1.8,\ \gamma_2 = 1,\ \eta = 0.1,\ \varepsilon = 3.$$

By applying Theorem 3.1 with the LMI Toolbox in MATLAB, the controller gain matrices are obtained as:

$$K_{11} = \begin{bmatrix} -1144.89 & -95.29 \\ -884.77 & -594.67 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 28.85 & 159.93 \\ -81.29 & -39.99 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -10.42 & -6.94 \\ -10.34 & -8.95 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} -10.82 & 6.76 \\ 9.77 & 8.12 \end{bmatrix}.$$

Moreover, the corresponding dynamic behaviors of error system (27) are demonstrated as Figure 2, from which we can see that the considered error system (27) is asymptotically stable with the memory-based controller (26).

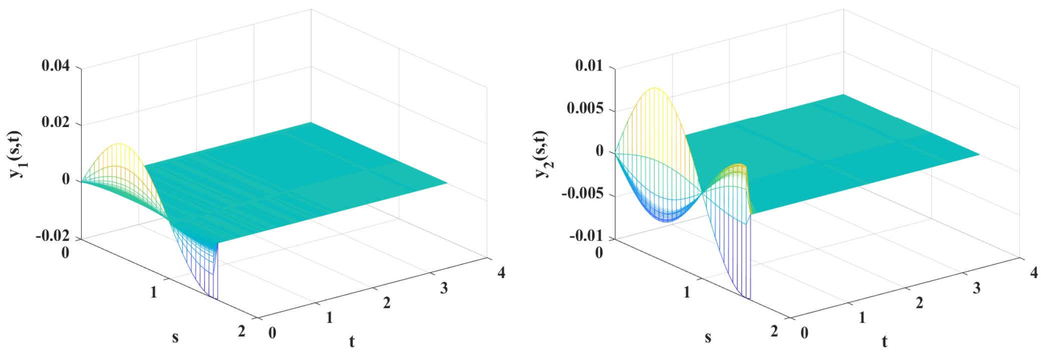

FIGURE 2. The dynamic behaviors of $y_1(s,t)$ and $y_2(s,t)$

It is essential to note that the elements of the aforementioned controller gains K_{11} *and* K_{12} *are so large that the control cost could be increased and even the controller cannot be implemented in some practical applications. Thus, the controller gains optimization strategy proposed in Remark 3.3 is utilized with* $\tau_1 = 19.4$, $\tau_2 = 5.1$. *Then by solving the LMIs (6), (7), (20), and (21), one can derive the feasible solution with*

$$K_{11} = \begin{bmatrix} 18.09 & 8.06 \\ 7.93 & 23.28 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 30.14 & 13.43 \\ 13.22 & 38.80 \end{bmatrix}.$$

It is obvious that the units of the nonfragile controller gains are much smaller than corresponding ones obtained only by applying Theorem 3.1, which means that the actual system is easier to achieve stability by the controller with the modified controller gains.

Example 4.2. *In order to reveal the superiority of Theorem 3.1 in control effect compared with Corollary 3.2, we show the following two images Figure 3 and Figure 4 under the same parameter conditions. Furthermore, to show Theorem 3.1 less conservative than Corollary 3.2, the maximum allowable upper bounds of time-varying delay* $h(t)$ *for different lower bounds are demonstrated in Table 1.*

It can be clearly found from Figures 3 and 4 that the control effect of Theorem 3.1 is more efficient than Corollary 3.2, and from Table 1, we can obtain that Theorem 3.1 can be applied to situations with large time-delay, while Corollary 3.2 may not be suitable. That is, the additional variables of Theorem 3.1 are essential for the stabilization problem of the considered system. They can strengthen the combination of the system's variables, so that the improved stable criteria and less conservative results of the proposed systems are derived.

Example 4.3. *To further highlight the superiority of the controller designed in this paper, a comparison between* [39] *and this paper is presented in this example. First of all, for the*

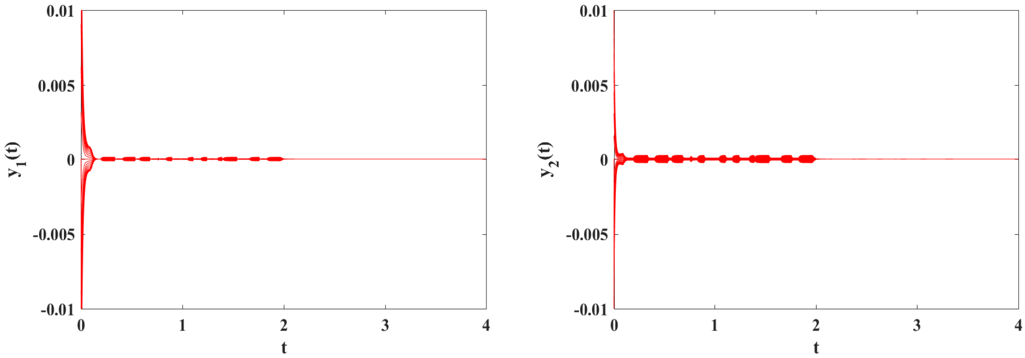

FIGURE 3. The dynamic behaviors of $y_1(t)$ and $y_2(t)$ under Theorem 3.1

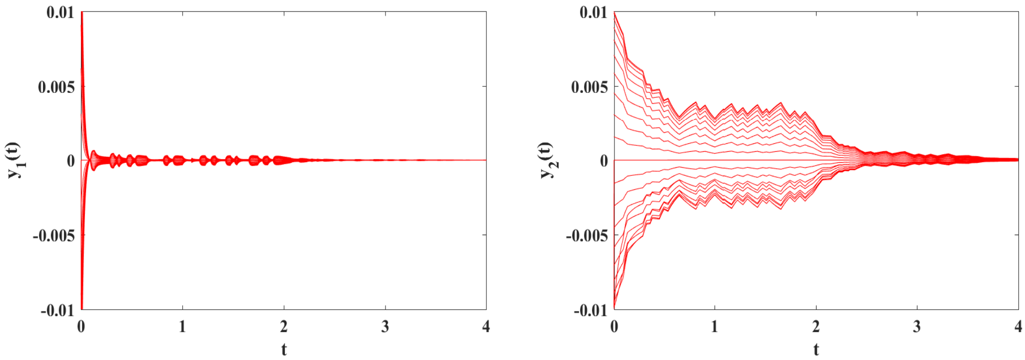

FIGURE 4. The dynamic behaviors of $y_1(t)$ and $y_2(t)$ under Corollary 3.2

TABLE 1. The maximum allowable upper bounds of time-varying delay $h(t)$ for different lower bounds

h_1	0.2	0.5	0.8	1.0
$h_{2\max}$ of Theorem 3.1	0.882	5.932	4.511	2.294
$h_{2\max}$ of Corollary 3.2	0.847	5.601	4.389	1.896

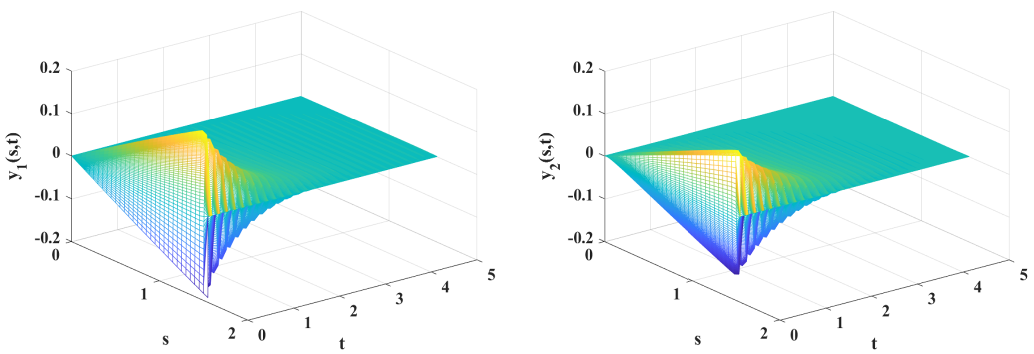

FIGURE 5. The state response of error system $y_1(s,t)$ and $y_2(s,t)$ with the controller proposed in this paper

convenience of comparison, we choose the same system parameters as [39] *except for the controller's. Then the trajectory of the error system with the controller designed in this paper is depicted in Figure 5. From where we can see that the stabilization time of the* *error system is about 2 seconds, while in* [39]*, the stabilization time of the error system is about 5 seconds (see* [39]*). That is, the control strategy proposed in this paper is more efficient and may have wider applicability.*

5. **Conclusion.** The nonfragile memory-based output feedback control problem for T-S fuzzy MJGNNs with reaction-diffusion terms has been addressed in this paper. Combining the Lyapunov functional method and the integral inequalities technique, we firstly present a new stable criterion for T-S fuzzy MJGNNs with reaction-diffusion terms and the design scheme of corresponding fuzzy nonfragile memory-based controller. Then we propose two related corollaries. Finally, the feasibility and effectiveness of the proposed method is verified. It is worth noting that the influence of the number of Lyapunov function variables has been analyzed, and the controller gain optimization method is also used in this paper. Compared with some existing results, the system model considered in this paper is more general and the processing method of some issue is more effective. In future work, based on [26, 30], the nonfragile memory-based sampled-data control strategy will be applied to the T-S fuzzy GNNs with Markovian jumping parameters and reaction-diffusion terms.

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