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SECOND-ORDER STOCHASTIC DOMINANT PORTFOLIO REBALANCING

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Abstract. *The Second-order Stochastic Dominance (SSD) criterion evaluates if one return distribution dominates another, implying that a dominant one has higher probability of positive return and lower probability of negative return. The study aims to apply the SSD condition as a rebalancing rule and develop a new optimization model for portfolio rebalancing accordingly. The resulting portfolio will thus rebalance only when its current allocation is stochastically dominated by a new optimal allocation; otherwise the portfolio remains unchanged. Performance of the proposed rebalancing strategy is then compared with traditional strategies such as periodic and buy-and-hold rebalancing. Return and risk in the out-of-sample period shows that the proposed method outperforms the others.*

Keywords: Stochastic dominance, Portfolio optimization, Portfolio rebalancing

1. **Introduction.** Portfolio optimization problems typically aim to find an optimal allocation over a predetermined investment horizon by trading off expected return and risk. An investment horizon is basically a length of time an investor expects to hold a portfolio. Generally, investments in financial securities require some time to pay off depending on the class of asset. An investment horizon for bonds or equities, for instance, might take around one to three years while some other types of investment such as real estates may take much longer time. Thus, along the way of an investment term, it is important to assure that the portfolio can maintain its efficiency[1] until the end of the investment horizon. This results in maintaining the risk-reward profile of the investors and the ability of investors to capture buy-low/sell-high opportunities [2, 3, 4].

Portfolio rebalancing is an approach commonly employed to alleviate the effect of market fluctuation on portfolio efficiency. Rebalancing is generally performed to restore a portfolio that has drifted (due to financial market movements) from its target asset allocation (optimized allocation). Among several rebalancing strategies, periodic rebalancing (constant mix) is the most widely-used [5]. Conventionally, a portfolio is reallocated to its target allocation in every month assuming that market dynamics do not affect expected return and risk of the portfolio and hence the efficiency is attained. As opposed

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[1]A portfolio is deemed efficient if its expected return and risk is Pareto optimal. That is, no other portfolios can achieve higher return than an efficient portfolio without taking more risk [1].

to the recurring rebalancing, the buy-and-hold rebalancing strategy lets a portfolio allocation deviate along the way from its target without intervention. The advantage is saving transaction costs while the downside is that inflating the proportion of performing assets could cause severe loss when the market trend abruptly reverses. Also, academics have widely debated the issue of transaction costs and have pointed out that portfolio rebalancing results in higher operational costs and that portfolios with high transaction levels tend to underperform those with passive investment strategies [6, 7, 8, 9].

[10] categorizes rebalancing strategies in literature as (i) buy-and-hold, (ii) periodic rebalancing, (iii) threshold rebalancing which either reallocates a portfolio to original allocation or some predefined ranges of asset weights when some certain thresholds are breached[2]. Studies have also shown that the need for and outcome of rebalancing depend on the market environment [14]. In markets that are trending, portfolio rebalancing was found to yield a lower return as compared with less frequently rebalanced portfolios. On the other hand, in a mean-reverting market, periodic rebalancing helps realize profit from selling performing assets and buying undervalued ones.

To combine strengths of each rebalancing strategy described earlier, i.e., (i) saving transaction costs from buy-and-hold, (ii) de-risking from periodic reallocation and (iii) seeking new target allocation when markets change from threshold rebalancing, we therefore propose a new rebalancing strategy. The Second-order Stochastic Dominance criterion (SSD) is adopted as an efficiency measure (threshold) and is subsequently used to formulate a rebalancing model. Given two return distributions, one "second-order stochastically dominates" another if, on the average, that return distribution is more likely to have positive return and less likely to have negative return than another. The application of the SSD criterion on a rebalancing model thus finds a potentially more superior allocation than an existing one. The optimization model is formulated to find the number of units for buying or selling each asset relative to an existing allocation to achieve an SSD portfolio. In addition, the data used in the optimization problem is updated with new observations in every rebalancing period so that an SSD allocation is adapted to new market environment.

To our knowledge, the SSD conditions are generally used in portfolio construction, e.g., [15, 16]. The crucial benefits of constructing SSD portfolios are documented in [17, 18, 19] that downside risk is reduced and the upside potential is improved. One major requirement when establishing SSD portfolios is that a reference portfolio must be specified. In the context of portfolio construction, many have used bond or stock indices as benchmarks. This inspires us to select an existing portfolio as a benchmark when developing a rebalancing strategy.

Although many developments have been made in applying SSD in portfolio selection, only a little literature (see [20, 21] for example) has applied SSD in portfolio rebalancing and none of them takes account of transaction costs or uses SSD in finding new target allocation. The literature gaps we have seen are that: (i) none of rebalancing strategies trade off reallocation benefit with transaction cost and (ii) existing SSD rebalancing models do not include transaction costs. To bridge such gaps, the main contribution of our studies is that we introduce an optimization model providing an SSD optimal allocation a portfolio should rebalance to. The suggested allocation could be the allocation carried over from the previous period (no reallocation and hence save transaction costs) or a new allocation most efficient in SSD sense.

[2]The thresholds are based on, for example, portfolio volatility [11], portfolio beta [12] and allocation deviation [13].

In order to ascertain the merit of a new model, we compare performance of our proposed rebalancing strategy with the two popular methods that are generally used in the industry, i.e., buy-and-hold and periodic rebalancing. The experimental results show that the portfolio applying the proposed methodology generates more resilient returns than those following conventional rebalancing approaches.

The paper is structured as follows: Section 2, Background, elaborates relevant background knowledge used in formulating the proposed rebalancing model. Section 3, Methodology, shows how our rebalancing strategy is constructed as an optimization model. Descriptions on the objective function and constraints are also given. Section 4, Results and Discussions, provides experimental results and discussions on comparing performances of the proposed rebalancing method with buy-and-hold and periodic rebalancing strategies. Lastly, Section 5, Conclusions, summarizes the findings of this study.

2. **Background.** This section lays a foundation to the development of a new portfolio rebalancing model. First, common practices of portfolio rebalancing are exhibited. Then, the concept of stochastic dominance is examined and later employed as a criterion to evaluate if, at a given time, a portfolio needs to be reallocated. Lastly, we demonstrate how to formulate a portfolio optimization problem under a stochastic dominance criterion.

2.1. **Portfolio rebalancing.** Portfolio management generally involves asset allocation and rebalancing [22]. The process typically begins with setting an investment horizon determining a finite period of management until termination or liquidation, and then an optimization problem is set up to find an efficient portfolio (in terms of risk and return). An allocation obtained is expected to remain efficient until the end of investment horizon. However, market fluctuations usually lead to a deterioration of portfolio efficiency along the investment period and hence there arises a need for portfolio rebalancing to retain the efficiency [5].

Rebalancing is an action that brings a deviated allocation of a portfolio back to a target which is typically an efficient allocation derived from optimization. After an implementation of an initial portfolio, the allocation may drift from original proportion due to unequal return of each security when time passes. To restore the portfolio's original risk and return characteristics, we need a rebalancing strategy on portfolio allocation.

As part of the portfolio construction process, it is important for portfolio managers to develop a rebalancing strategy that formally addresses "how often, how far, and how much", that is:

1) How frequently the portfolio should be rebalanced (monthly, quarterly or yearly)?
2) How far an asset allocation can be allowed to deviate from its target before it is rebalanced?
3) Whether periodic rebalancing should restore a portfolio to its target or to a close approximation of the target?

Several rebalancing strategies are developed accordingly. In the study, the following two widely-used strategies [23] are the candidates in comparison with the new rebalancing strategy.

1) *Buy-and-Hold (BAH)* – this strategy leaves portfolio allocation drifts with market movements and changes nothing from original allocation. The example is given in Table 1.
2) *Periodic (PRD)* – also known as constant mix, this strategy pegs portfolio allocation to its original by selling off outperforming assets and buying in underperforming assets at a prespecified frequency such as monthly or yearly. The example is given in Table 2.

To demonstrate how the rebalancing strategies are implemented, consider a portfolio of two assets, namely A and B, as follows.

TABLE 1. An implementation of the buy-and-hold rebalancing strategy on a two-asset portfolio. Notice that no transaction is needed to execute this strategy and the units of assets A and B are constant over time while monthly allocations deviate from the original proportion due to market fluctuations.

		month-0	month-1	month-2	month-3
asset A	price	10.2	11.6	10.7	12.2
	unit	50	50	50	50
	allocation (%)	47.2	52.3	49.3	55.5
asset B	price	5.7	5.3	5.5	4.9
	unit	100	100	100	100
	allocation (%)	52.8	47.7	50.7	44.5
market value		1,080	1,110	1,085	1,100
portfolio return (%)			2.78	−2.25	1.38

TABLE 2. Rebalancing a two-asset portfolio under the periodic strategy to maintain constant allocation over time. In every month, a portfolio needs to sell the performing asset (A) off and buy in the non-performing asset (B) to keep the allocation equal to its original. Note that transaction costs are not included in this example.

		month-0	month-1	month-2	month-3
asset A	price	10.2	11.6	10.7	12.2
	unit	50	41	50	39
	allocation (%)	47.2	47.2	47.2	47.2
asset B	price	5.7	5.3	5.5	4.9
	unit	100	100	109	109
	allocation (%)	52.8	52.8	52.8	52.8
market value		1,080	1,003	1,135	1,011
portfolio return (%)			−7.15	13.14	−10.88

We can see from Table 1 that the buy-and-hold strategy lets the portfolio allocation float with financial markets. Thus, the strategy produces superior returns in markets with a prolonged upward (or downward) bias but at the expense of compounding portfolio risk (since the positions of performing assets are accumulated along the time and there could occur a large loss on a portfolio when those assets begin to fall). In contrast, the periodic rebalancing requires buying and selling assets in every period to maintain the original allocation. It, therefore, involves transaction costs (which are not shown in Table 2). This strategy could unintentionally reduce return by selling performing assets to buy non-performing ones, and it thus pays off well in stagnant as well as oscillating markets [24].

2.2. **Conditional Value-at-Risk (CVaR).** A good selection of an appropriate risk measure is vital to portfolio optimization problems. The Basel III regulatory framework employs percentiles of a loss distribution, i.e., the Value-at-Risk (VaR) as a standard risk measure. It represents a maximum loss under a specified probability (confidence level) over a certain time period. Usually, VaR is specified at 95% confidence level to portray intensity of extreme losses (that could only occur at 5% chance).

Conditional Value-at-Risk is an extension to VaR in which it gives information of an average of losses greater than VaR. Furthermore, CVaR contains desirable properties of coherent risk measure which VaR does not (see [25]). Our portfolio optimization problem is thus formulated upon CVaR risk measure.

By definition, CVaR is defined on portfolio losses, hence we need to define the loss function which represents negative returns of a portfolio, whereby the portfolio return is the summation of individual asset return ξ_i weighed by an allocation w_i,

$$f_L(w, \xi) = -\sum_i w_i \xi_i.$$

Accordingly, CVaR is then defined by [26] as

$$\text{CVaR}_\beta = \alpha + \frac{1}{(1-\beta)N} \sum_{s=1}^{N} e_s, \tag{1}$$

where

$$e_s \geq f_L(w, \xi_s) - \alpha, \tag{2}$$

$$e_s \geq 0. \tag{3}$$

That is, from Equation (1), CVaR at a confidence level β is equal to the average of losses greater than VaR (denoted as α), in which the losses greater than VaR is defined in Equation (2).

2.3. **Stochastic dominance.** Stochastic Dominance (SD) criteria are analytical tools for decision making under risk and uncertainty [21]. They are a form of stochastic ordering usually applied in situations where one investment (represented by a probability distribution) can be ranked as superior to another investment. In the context of stochastic dominance, an investment is a portfolio with known return distribution.

2.3.1. *First-order Stochastic Dominance (FSD).* Stochastic dominance criterion ranks random variables by a point-wise comparison subject to some general conditions for an investors risk preferences. Suppose that two random variables X_0 and X_1 represent returns of portfolios P_0 and P_1, respectively. Further, let $F_0(t)$ and $F_1(t)$ denote Cumulative return Distribution Functions (CDFs) corresponding to $\Pr[X_0 < t]$ and $\Pr[X_1 < t]$, respectively.

According to [27], the First-order Stochastic Dominance (FSD) criterion requires that P_1 dominates P_0 with the notation of $P_1 \succeq_{FSD} P_0$ if and only if

$$\begin{aligned} F_1(t) &\leq F_0(t); \quad \forall t \in \mathbb{R}, \\ F_1(t) &< F_0(t); \quad \exists t \in \mathbb{R}. \end{aligned} \tag{4}$$

The shortcoming of FSD is thus if there are two portfolios with the same expected return, we cannot distinguish them by the FSD condition. In such cases, we need some adjustments on the stochastic dominance condition.

2.3.2. *Second-order Stochastic Dominance (SSD).* The FSD theorem assumes that investors generally prefer more to less (of expected returns). It does not, however, take account of risk in choosing investments. To incorporate risk aversion into a condition, the concept of Second-order Stochastic Dominance (SSD) is therefore developed accordingly [27].

Rather than focusing on each $F(t)$ along the support of a return distribution, we consider the average value (up to t) of $F(t)$ instead. That is,

$$\begin{aligned} \mathbb{E}[F_1(t)] &\leq \mathbb{E}[F_0(t)]; \quad \forall t \in \mathbb{R}, \\ \mathbb{E}[F_1(t)] &< \mathbb{E}[F_0(t)]; \quad \exists t \in \mathbb{R}, \end{aligned} \tag{5}$$

where $\mathbb{E}[\cdot]$ is the expectation operator.

Unlike the FSD, the second-order stochastic dominance can still hold when two portfolios have the same mean return. Hence, we can use the SSD criterion to distinguish between investments or portfolios with the same mean.

In general, when comparing two investments X_0 and X_1 by FSD and SSD (altogether), [21] shows that possible answers will be

1) Nothing can be concluded.
2) X_1 is first- and second-order stochastic dominant to X_0 (or X_0 to X_1).
3) X_1 is second-order stochastic dominant to X_0 (or X_0 to X_1) but not first-order stochastic dominant.

2.4. **CVaR representation of second-order stochastic dominance.** The second-order stochastic dominance criterion can be expressed as a function of CVaR risk measure [28]. The CVaR representation of SSD criterion is useful in formulating a model to compare CVaRs of two portfolios. Details on how CVaR can represent the SSD condition are given below.

It is shown in [29] that

$$\begin{aligned} \text{CVaR}_\beta(X_1) &\leq \text{CVaR}_\beta(X_0); \quad \forall \beta \in (0,1), \\ \text{CVaR}_\beta(X_1) &< \text{CVaR}_\beta(X_0); \quad \exists \beta \in (0,1). \end{aligned} \tag{6}$$

Thus, the SSD condition can be verified by comparing CVaRs of two portfolios at every confidence level β [29]. From (5) and (6), we can also see that second-order stochastic dominance takes account of both expected return and risk (in terms of CVaR) which is suitable in comparing risk-return characteristics of portfolios at the same time. Thus, SSD is selected as a measure to rank portfolios in the study.

3. **Methodology.** This section introduces an approach to formulate a novel portfolio rebalancing strategy using the second-order stochastic dominance criterion. The performance of the proposed strategy is subsequently compared with the buy-and-hold and periodic rebalancing strategies.

3.1. **An SSD portfolio rebalancing model.** From (6), it is known that evaluating SSD criterion on two random variables can also be achieved by comparing their CVaRs at every confidence level. By its definition in (6), [29] formulates the SSD condition from CVaR as follows:

$$\sum_{\beta \in B} \text{CVaR}_\beta(X_1) \leq \sum_{\beta \in B} \text{CVaR}_\beta(X_0), \tag{7}$$

where B is a set containing the range of confidence levels. Note that $\beta = 0$ and $\beta = 1$ are unspecified, so, in what follows, we specify $B = \{0.01, 0.02, \ldots, 0.99\}$.

Suppose that a rebalancing routine is set to monthly from month-1 onwards and a portfolio is implemented in month-0, then from month-1 onwards the portfolio is subject to reallocation. It is known that the buy-and-hold and periodic rebalancing are appropriate for trending and oscillating markets respectively [24]. An alternative strategy is to combine their advantages through the SSD criterion.

For instance, suppose that a portfolio is set up in month-0 with allocation obtained from optimization, then in month-1, the decision is to let the allocation float as in buy-and-hold strategy or to reallocate the allocation to an SSD portfolio. Hence, there are two

TABLE 3. Notations for an SSD portfolio rebalancing model

Notation	Range	Description
Y_0		A set of portfolio losses for P_0. (Positive values are losses and negative values are profit)
Y_1		A set of portfolio losses for P_1. (Positive values are losses and negative values are profit)
A		A set of assets in a portfolio.
B		A set of confidence levels. In this study $B = \{0.01, 0.02, \ldots, 0.99\}$.
r	$\mathbb{N}$	An index of realizations in a probability distribution (discrete).
N_r		The number of realizations in a probability distribution.
i	$\mathbb{N}$	An index of assets, $i \in A$.
x_i^r	$\mathbb{R}$	A simulated return time series of r realizations of an asset i.
w_i^0, w_i^1	$[0,1]$	Allocations of portfolios P_0 and P_1, respectively.
u_i^0, u_i^1	$\mathbb{R}^+$	The number of units of asset i in a portfolio P_0 and P_1, respectively.
b_i	$\mathbb{R}^+$	The number of units an asset i is bought.
s_i	$\mathbb{R}^+$	The number of units an asset i is sold.
p_0^r, p_1^r	$[0,1]$	Probabilities associated with each realization r of Y_0 and Y_1, respectively.
e_0^r, e_1^r	$\mathbb{R}$	Auxiliary variables for calculating CVaRs of Y_0 and Y_1, respectively.
β	$(0,1)$	A confidence level for CVaR. $\beta \in B$.
Z_i	$\mathbb{R}^+$	A market price of an asset i.
π_i	$\mathbb{R}^+$	A fixed cost for buying an asset i.
ψ_i	$\mathbb{R}^+$	A fixed cost for selling an asset i.
ρ_i	$\mathbb{R}^+$	A variable cost for buying an asset i.
λ_i	$\mathbb{R}^+$	A variable cost for selling an asset i.

candidates, the current portfolio with drifted allocation $Port_0$ and the "newly optimized"[3] portfolio $Port_1$. Note that $Port_0$ is an efficient portfolio derived from optimization in month-0 and in month-1 its allocation has drifted due to market fluctuation. In calculating CVaRs of the two portfolios, asset returns and asset prices are updated from month-0 with new realizations, the in-sample dataset is thus continuously growing when time passes. With the notations given in Table 3, a deterministic optimization model is formulated accordingly.

$$\text{maximize} \quad \sum_{\beta \in B} \text{CVaR}_\beta(Y_0) - \sum_{\beta \in B} \text{CVaR}_\beta(Y_1) \tag{8}$$

subject to

$$\text{CVaR}_\beta(Y_0) = \text{VaR}_\beta(Y_0) + \frac{1}{1-\beta} \sum_{r=1}^{N_r} p_0^r e_0^r, \tag{9}$$

$$e_0^r \geq y_0^r - \text{VaR}_\beta(Y_0), \tag{10}$$

$$e_0^r \geq 0, \tag{11}$$

$$Y_0 = \left\{ y_0^1, y_0^2, \ldots, y_0^{N_r} \right\}, \tag{12}$$

$$y_0^r = -\sum_{i \in A} w_i^0 x_i^r, \tag{13}$$

[3]A portfolio is optimized again in an upcoming month with updated data.

$$\text{CVaR}_{\beta}(Y_1) = \text{VaR}_{\beta}(Y_1) + \frac{1}{1-\beta}\sum_{r=1}^{N_r} p_1^r e_1^r, \tag{14}$$

$$e_1^r \geq y_1^r - \text{VaR}_{\beta}(Y_1), \tag{15}$$

$$e_1^r \geq 0, \tag{16}$$

$$Y_1 = \left\{y_1^1, y_1^2, \ldots, y_1^{N_r}\right\}, \tag{17}$$

$$y_1^r = -\sum_{i \in A} w_i^1 x_i^r, \tag{18}$$

$$w_i^1 = \frac{u_i^1 Z_i}{\sum\limits_{i \in A}\left(u_i^1 Z_i - \pi_i - \psi_i - b_i\rho_i - s_i\lambda_i\right)}, \tag{19}$$

$$u_i^1 = u_i^0 + b_i - s_i, \tag{20}$$

$$b_i, s_i \in \mathbb{R}^+, \tag{21}$$

$$B = \{0.01, 0.02, \ldots, 0.99\}. \tag{22}$$

The objective function (8) aims to verify if $Port_1 \succeq_{SSD} Port_0$ by comparing their summations of CVaRs across all confidence levels. The decision variables of the problem are b_i and s_i indicating if there are any adjustments on the drifted allocation of the portfolio $Port_0$. Consequently, the possible solutions of the optimization problem are:

1) $b_i = 0$ and $s_i = 0$ for all i, this suggests that the floated allocation is still the most efficient in new market environment (month-1) and an efficient portfolio obtained from the optimization in month-1, $Port_1$, is the same as the drifted portfolio $Port_0$. Thus, there is no need to rebalance the portfolio in this case.
2) $b_i \neq 0$ and $s_i \neq 0$ for some i, this indicates that $Port_1 \succeq_{SSD} Port_0$ or the new allocation second-order stochastically dominates the existing one. Rebalancing is made by buying and selling assets according to the values of b_i and s_i.

In short, the portfolio rebalances to a new allocation only if its deviated allocation is not SSD (this allocation is dominated by some other allocations according to the second-order stochastic dominance criterion); otherwise, the allocation remains floated with market fluctuations. For constraints (9) to (22) the descriptions are given as follows.

- Constraint (9) denotes a definition of CVaR of $Port_0$ which is an average of losses greater than the VaR value. Particularly, constraints (10) and (11) define e_0^r as losses greater than VaR.
- Constraints (12) and (13) show that, for $Port_0$, a set of portfolio losses is created by multiplying portfolio returns with -1.
- Likewise, for $Port_1$, constraints (14) to (16) demonstrate how CVaR and VaR of the portfolio are constructed, and constraint (17) along with constraint (18) define how portfolio losses are calculated.
- Constraint (19) calculates a weight of each asset. That is, a market value of an asset divided by a market value of the portfolio net of transaction costs.
- Constraint (20) defines that a net number of unit comes from initial unit plus the net of buy-in and sell-out. Note that the number of units bought and sold are treated as real numbers (constraint (21)).
- Lastly, constraint (22) indicates the number of confidence levels constitutes in creating the SSD condition from CVaRs.

The optimization problem (8) is performed repeatedly along the investment horizon of the portfolio. The example in the optimization model given earlier demonstrates rebalancing decision occurring in month-1. Suppose that, in month-1, $Port_1 \succeq_{SSD} Port_0$ then the portfolio is carried over to month-2 and repeat the same optimization problem with updated data where $Port_1$ becomes the existing portfolio in the optimization model of month-2 and the routine goes on until the end of portfolio management horizon. The SSD rebalancing routine can be illustrated as in Figure 1.

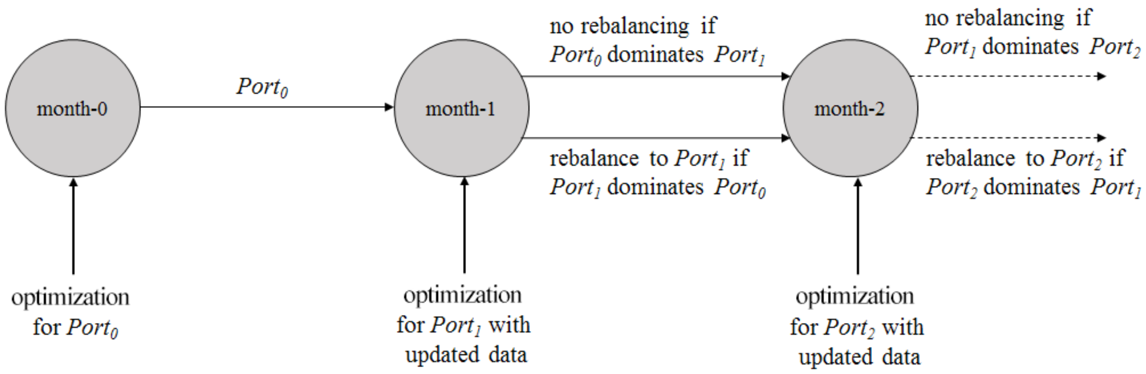

FIGURE 1. A diagram showing an application of second-order stochastic dominance in formulating a rebalancing routine. The rebalancing decisions occur from month-1 onwards deciding to let the allocation drifted further or to rebalance it to an allocation obtained from a new optimization.

The summation of confidence levels in (8) can be intuitively viewed as a robust version of CVaR minimization with respect to β. When there is no particular choice of the confidence level β, we take the average of them (or summing over a prespecified range of β's) to obtain a β-neutral minimum CVaR.

The SSD rebalancing strategy can be alternatively viewed as a comparison of a once-efficient portfolio in new environment $Port_0$ and a current-efficient portfolio $Port_1$. Although, based on updated market data, $Port_1$ is the most efficient in terms of risk and return but it could be dominated by the drifted portfolio using a second-order stochastic dominance point of view.

Note that the optimization problem (8) is not established to attain the risk-return profile[4] of a portfolio but to instigate a reallocation that takes consideration of new information and avoids redundant, too frequent, portfolio adjustments. The key objective of the proposed rebalancing strategy is that the reallocation frequency could be reduced more or less from adopting periodic rebalancing thereby decreasing unnecessary transactions. In addition, since the strategy accounts for new market data, it therefore allows a portfolio to adjust the allocation to avoid potential risk from holding too large portion of performing assets as in the buy-and-hold strategy.

4. **Results and Discussions.** This section elaborates the results of implementing the SSD rebalancing strategy on a portfolio. The portfolio performance is portrayed as out-of-sample returns which are compared with performances of portfolios employing other rebalancing policies, specifically, the Periodic Rebalancing (PRD) and the Buy-and-Hold (BAH).

[4]If the risk-return profile needs to be preserved, the constraints on specified values of expected return and CVaRs must be imposed.

The three rebalancing strategies possess different natures of allocation dynamics. The SSD ensures that its allocation is second-order stochastic dominant to any others at any rebalancing period, which either causes a reallocation or keeps the same allocation as in a preceding period (see Section 3.1 for details). The PRD always reallocates a portfolio to its inception allocation; hence there always occurs a transaction in a rebalancing routine.

4.1. **Data.** To compare performances of portfolios employing SSD, PRD and BAH rebalancing policies, we set up an optimal portfolio using in-sample data from Jan-02 to Dec-15 of four asset classes in the US, i.e., treasury bills, treasury bonds, corporate bonds and equities. Then we observe monthly returns of the three strategies over the out-of-sample data from Jan-16 to Sep-18. The risk-return profiles of asset classes in our analysis are displayed in Figure 2.

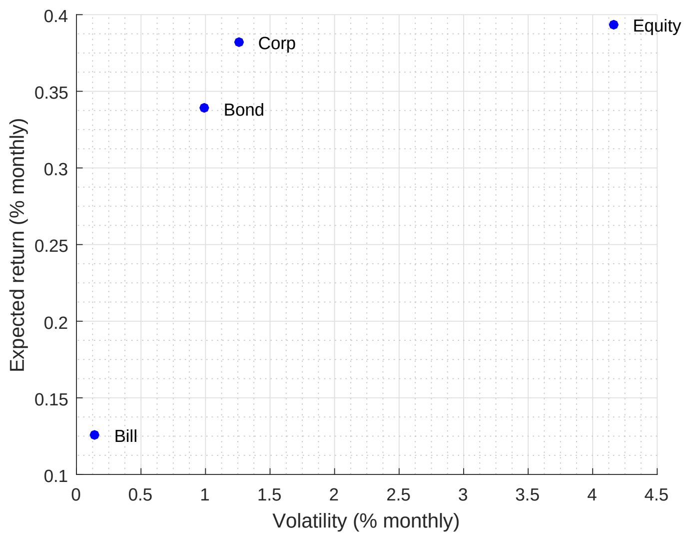

FIGURE 2. The risk-return profile of each asset class in the US market. Expected returns are estimated using historical mean and volatilities are standard deviations. The data length covers an in-sample period of Jan-02 to Dec-15.

Treasury bill data were retrieved from Bloomberg, using a generic 3-month US treasury bill as a proxy. US treasury bonds data were collected from JPMorgan US Government Bond Index (with 1-30 years maturity). Corporate bonds are from Barclays US Corporate Bond Index (with S&P credit ratings of A or over). Lastly, equities are represented with S&P 500 stock index.

Returns shown in Figure 2 are calculated using historical average while volatilities are from standard deviations. The safest asset in a portfolio is treasury bills, followed by treasury bonds that have slightly higher risk but significantly higher return. A-plus corporate bonds demonstrates slightly higher return with slightly higher risk than bonds. The riskiest asset is equities, giving marginally higher returns but with substantially higher volatility than corporate bonds.

4.2. **Results.** We run an out-of-sample test to measure monthly returns of portfolios practising different rebalancing strategies along the period. Not only focusing on monthly returns, their fluctuations are also taken into account in our evaluation. To assess a rebalancing policy that yields robust and stable returns, an average monthly return over return volatility along the course is taken as a performance measure.

The in-sample data provides a starting portfolio with the following allocation: Low, Medium and High-risk portfolios are taken from 10th, 50th and 90th percentiles of optimal portfolios on the efficient frontier. These allocations are also the target weights for the PRD portfolios, i.e., the portfolios must reallocate back to these proportions in the out-of-sample period starting from Jan-16 onwards.

TABLE 4. Optimal asset allocations derived from a mean-variance optimization with in-sample data

	Low-risk	Medium-risk	High-risk
Bills	87.28	42.93	0.00
Bonds	8.90	47.93	76.78
Corporate bonds	1.99	2.69	13.79
Equities	1.83	6.45	9.43
Total	100.00	100.00	100.00

During the out-of-sample period, portfolio allocations under BAH and SSD strategies may vary over time while the allocation of PRD remains the same as it always reallocates to the original one. Table 5 shows, at different risk appetite, allocations of PRD, BAH and SSD portfolios each month during the testing period. Note that PRD weights are constant because the portfolio always reallocates back to the original allocation. The varied allocation of BAH, although doing no transaction, is a consequence of changing market environment – more allocation in performing assets and vice versa. In contrast, the SSD portfolio adjusts its allocation to the one that is stochastically dominant in each period which can also be viewed as environmental adaptation. The risk levels (low, medium and high) influence portfolio composition in a way that more risky assets are preferred when a portfolio seeks for higher risk (which is supposed to give higher return also).

Figure 3 shows portfolio returns during the out-of-sample period and average return over volatility of the three portfolios. In terms of monthly returns, the SSD portfolio generally gives higher return than the PRD for all risk appetites. Notice that month-to-month returns for the SSD and the PRD are fairly smooth and remain positive along the way despite the downward trend. In contrast, the BAH demonstrates highly fluctuating return path comparing with the two. The BAH portfolio experiences both positive and negative returns along the course. Such characteristics can be better comprehended when observing their average return over volatility. It is obvious that the BAH portfolio is significantly penalized by its volatility while the SSD slightly outperforms the PRD thanks to its higher monthly returns.

Since allocations are the main driver to portfolio monthly return, we plot allocation evolution over the out-of-sample period of each asset class with the corresponding market return in Figures 4 to 9. This is to see if each rebalancing strategy develops a favourable alignment of allocation amidst fluctuating market environment.

A synchronicity between asset weight and asset return leads to good performance of the portfolios, e.g., more weight in assets with positive return and less weight in those with negative return. For SSD portfolios (Figures 4 to 6), asset weights vary more or less in the same direction with asset returns. Only corporate bonds investment that remain constant under low and medium risk appetites (Figures 4 and 5) which is presumably an optimal decision to keep the risk-return reward on target. Nonetheless, corporate bond allocation gradually increases in place of declining government bonds when the SSD portfolio seeks higher risk so as to achieve higher return (Figure 6).

TABLE 5. Asset allocations (in %) of all rebalancing policies along the out-of-sample period. Periodic rebalancing allocations are constant as it reallocates the portfolio back to an original allocation in every period.

Asset	Jan-16	Feb-16	Mar-16	Apr-16	May-16	Jun-16	Jul-16	Aug-16	Sep-16	Oct-16	Nov-16	Dec-16	Jan-17	Feb-17	Mar-17	Apr-17	May-17	Jun-17	Jul-17	Aug-17	Sep-17	Oct-17	Nov-17	Dec-17	Jan-18	Feb-18	Mar-18	Apr-18	May-18	Jun-18	Jul-18	Aug-18	Sep-18
Periodic (PRD)																																	
Low-risk																																	
Bills	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28	87.28
Bonds	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90	8.90
Corp bonds	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Equities	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
Medium-risk																																	
Bills	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93	42.93
Bonds	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93	47.93
Corp bonds	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69
Equities	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45	6.45
High-risk																																	
Bills	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bonds	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78	76.78
Corp bonds	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79
Equities	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43	9.43
Buy-and-Hold (BAH)																																	
Low-risk																																	
Bills	87.16	87.16	87.14	87.14	87.12	87.21	87.13	87.24	87.20	87.10	87.13	87.18	87.12	87.08	86.95	86.95	86.94	86.81	86.74	86.79	86.79	86.87	86.99	86.96	86.92	86.83	86.83	86.77	86.72	86.77	86.70	86.66	86.68
Bonds	9.04	8.95	9.00	8.99	8.99	8.96	8.99	9.01	9.07	9.03	9.00	8.99	9.13	9.16	9.16	9.15	9.14	9.25	9.24	9.20	9.21	9.17	9.03	9.02	9.02	9.03	9.03	9.07	9.09	9.04	9.05	9.09	9.03
Corp bonds	2.03	2.02	2.02	2.02	2.02	2.00	2.01	2.01	2.02	2.03	2.03	2.02	2.04	2.05	2.07	2.08	2.07	2.10	2.10	2.10	2.10	2.09	2.05	2.05	2.05	2.06	2.06	2.08	2.08	2.08	2.09	2.10	2.09
Equities	1.77	1.87	1.83	1.85	1.87	1.83	1.86	1.75	1.70	1.84	1.84	1.81	1.72	1.71	1.82	1.82	1.85	1.85	1.91	1.91	1.91	1.87	1.94	1.97	2.00	2.07	2.07	2.09	2.11	2.12	2.16	2.15	2.19
Medium-risk																																	
Bills	42.65	42.71	42.64	42.64	42.60	42.76	42.62	42.79	42.69	42.55	42.62	42.71	42.51	42.42	42.22	42.22	42.22	41.92	41.83	41.94	41.92	42.09	42.37	42.34	42.26	42.10	42.10	41.98	41.88	42.00	41.89	41.79	41.87
Bonds	48.42	48.02	48.21	48.16	48.14	48.11	48.15	48.36	48.61	48.29	48.20	48.21	48.75	48.87	48.69	48.67	48.58	48.90	48.78	48.66	48.70	48.63	48.13	48.05	48.02	47.94	47.95	48.02	48.06	47.88	47.87	47.99	47.76
Corp bonds	2.73	2.72	2.72	2.72	2.71	2.70	2.70	2.70	2.72	2.72	2.72	2.72	2.73	2.74	2.76	2.77	2.77	2.78	2.79	2.79	2.78	2.78	2.75	2.74	2.74	2.75	2.75	2.76	2.77	2.77	2.77	2.78	2.77
Equities	6.21	6.55	6.43	6.48	6.55	6.43	6.54	6.15	5.97	6.45	6.46	6.36	6.01	5.97	6.33	6.34	6.44	6.40	6.61	6.61	6.60	6.49	6.76	6.87	6.97	7.20	7.20	7.24	7.30	7.35	7.46	7.44	7.59
High-risk																																	
Bills	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bonds	77.07	76.59	76.78	76.70	76.65	76.83	76.72	77.22	77.43	76.79	76.73	76.87	77.40	77.45	76.91	76.85	76.73	76.83	76.54	76.48	76.53	76.64	76.29	76.14	76.01	75.71	75.73	75.67	75.60	75.47	75.31	75.35	75.13
Corp bonds	13.91	13.87	13.87	13.87	13.83	13.79	13.77	13.82	13.88	13.84	13.88	13.87	13.88	13.91	13.96	14.01	13.98	13.99	14.00	14.03	14.00	14.02	13.93	13.92	13.91	13.90	13.89	13.91	13.92	13.96	13.97	13.97	13.96
Equities	9.02	9.54	9.35	9.43	9.52	9.38	9.51	8.97	8.69	9.36	9.39	9.26	8.71	8.64	9.13	9.15	9.29	9.18	9.46	9.49	9.47	9.34	9.78	9.94	10.08	10.39	10.38	10.41	10.48	10.57	10.72	10.67	10.91
Second-order Stochastic Dominance (SSD)																																	
Low-risk																																	
Bills	88.44	88.88	88.54	88.69	88.66	88.32	89.02	88.88	88.67	89.18	88.57	88.52	88.93	88.58	88.41	88.57	88.56	88.36	88.82	88.71	88.59	88.89	89.57	87.74	88.93	88.64	87.89	86.37	88.78	88.80	88.35	88.77	87.38
Bonds	7.91	7.34	7.73	7.54	7.65	8.02	7.27	7.57	7.89	7.20	7.76	7.77	7.49	7.82	7.86	7.89	7.92	7.91	7.56	7.62	7.68	7.51	6.77	8.40	7.51	7.62	8.32	9.48	7.44	7.57	7.87	7.62	8.69
Corp bonds	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Equities	1.66	1.79	1.74	1.78	1.70	1.67	1.72	1.56	1.45	1.63	1.68	1.72	1.59	1.60	1.74	1.55	1.53	1.74	1.63	1.68	1.74	1.60	1.67	1.87	1.57	1.75	1.81	2.17	1.79	1.64	1.80	1.61	1.94
Medium-risk																																	
Bills	88.83	89.24	88.63	89.10	88.89	88.67	89.42	89.26	89.10	89.58	88.48	88.57	89.31	88.77	88.09	88.60	88.81	88.49	89.25	88.66	88.99	89.30	89.98	88.23	89.41	89.13	88.10	86.96	88.93	89.20	88.87	89.04	87.92
Bonds	6.91	6.38	7.00	6.52	6.81	7.06	6.30	6.59	6.86	6.20	7.18	7.09	6.52	7.00	7.51	7.23	7.06	7.15	6.54	7.02	6.66	6.50	5.79	7.28	6.43	6.54	7.51	8.32	6.68	6.57	6.78	6.73	7.53
Corp bonds	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69
Equities	1.58	1.69	1.68	1.69	1.61	1.58	1.59	1.45	1.35	1.53	1.65	1.66	1.49	1.54	1.70	1.48	1.44	1.67	1.52	1.63	1.67	1.51	1.54	1.80	1.46	1.64	1.70	2.03	1.71	1.54	1.66	1.53	1.85
High-risk																																	
Bills	90.64	91.09	90.55	91.48	90.80	91.31	91.13	91.30	91.23	91.32	91.05	91.74	91.14	90.89	90.33	90.90	91.24	90.66	91.23	91.07	90.89	91.92	92.38	90.67	91.88	91.75	91.52	90.59	91.38	91.95	92.05	92.08	91.55
Bonds	2.23	1.65	2.10	0.97	1.98	0.84	1.87	1.52	1.75	1.75	1.23	0.21	1.95	1.84	2.00	1.68	1.33	1.84	1.68	1.34	1.86	0.40	0.30	1.70	0.97	0.62	0.26	0.84	0.75	0.35	0.00	0.14	0.18
Corp bonds	6.02	6.09	6.15	6.43	6.10	6.90	5.86	6.22	6.17	5.84	6.76	7.07	5.92	6.28	6.48	6.46	6.49	6.38	6.03	6.58	6.06	6.80	6.37	6.44	6.29	6.64	7.26	7.34	6.73	6.79	6.98	6.94	7.17
Equities	1.11	1.18	1.20	1.13	1.12	0.95	1.14	0.96	0.85	1.10	0.95	0.98	0.98	0.99	1.20	0.96	0.94	1.12	1.07	1.01	1.19	0.88	0.95	1.19	0.87	1.00	0.97	1.23	1.13	0.91	0.97	0.85	1.11

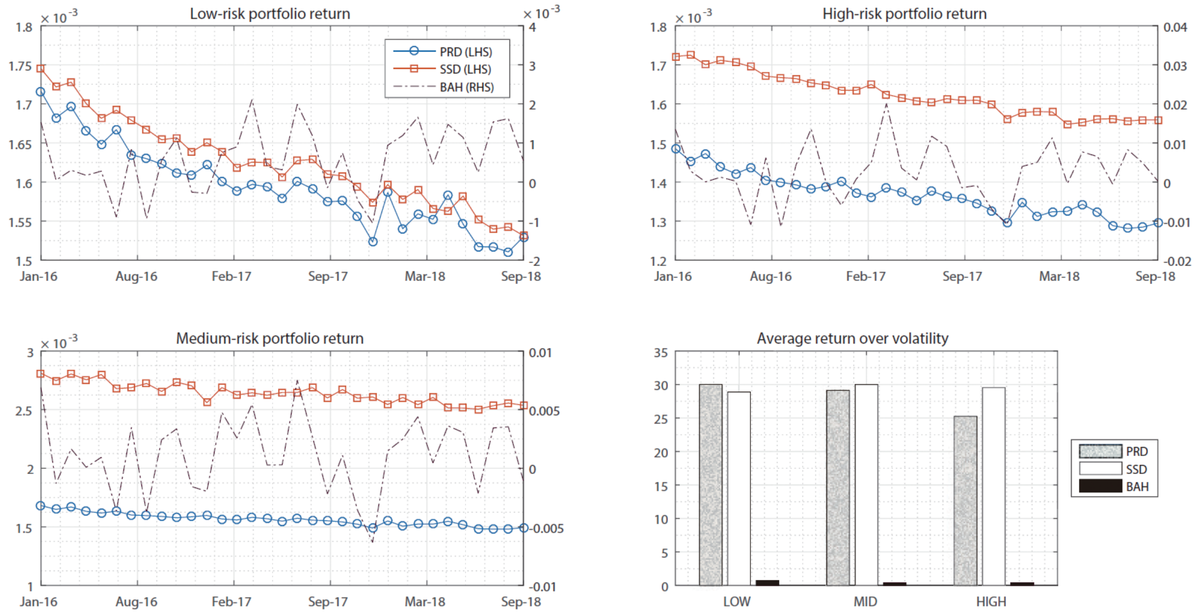

FIGURE 3. A comparison of out-of-sample portfolio returns under different rebalancing policies. Portfolios using SSD and PRD rebalancing strategies demonstrate similar return paths with SSD showing slightly higher return-s, regardless of risk appetite. A buy-and-hold policy, in contrast, exhibits fluctuating returns (hovering in both positive and negative return territo-ries) along the course. The comparison of return steadiness (average return over return volatility) during the out-of-sample period in the bottom-right panel shows that the SSD strategy gives highest return-to-risk ratio in all risk appetites, except for the lowest one.

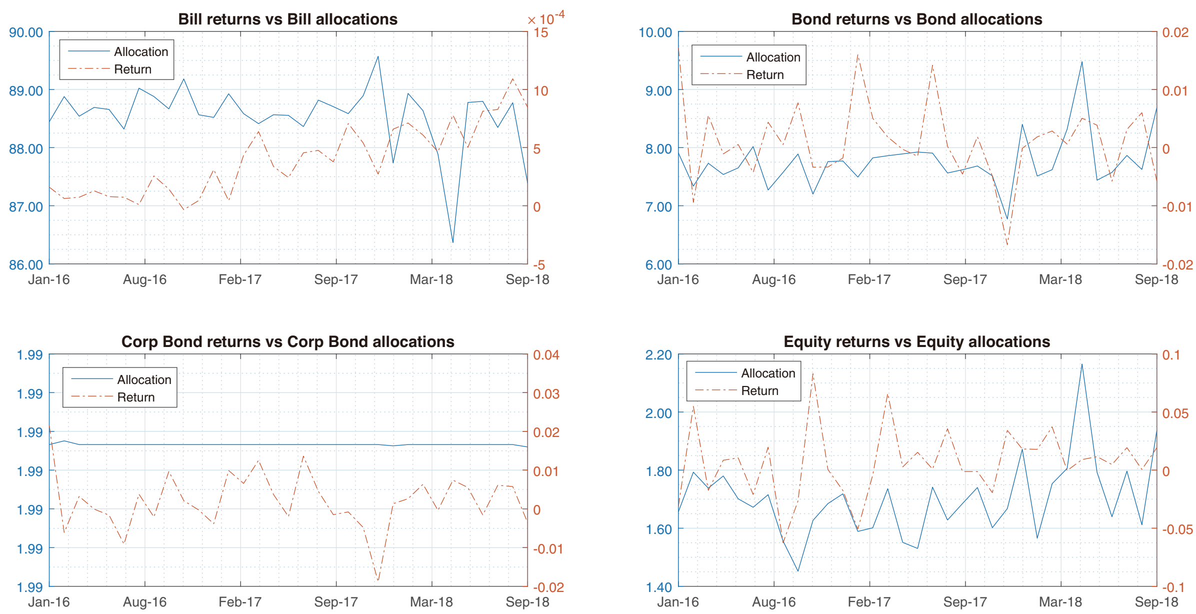

FIGURE 4. Asset allocation dynamics of a low-risk SSD portfolio

In contrast to the characteristics of SSD portfolios, the BAH strategy does no rebal-ancing and let portfolio allocations drift with a momentum of market returns – the most

FIGURE 5. Asset allocation dynamics of a medium-risk SSD portfolio

FIGURE 6. Asset allocation dynamics of a high-risk SSD portfolio

performing asset gradually gets its larger share in the portfolio and reduces the less performing ones. For the low and medium-risk portfolio (Figure 7), proportion of bills are taken away in order to invest in more performing asset classes – note that although bill returns are increasing, the magnitudes are relatively very small when comparing to the others. The fact that BAH portfolio allocations are driven by market momentum often makes portfolio positions lagged behind market trends.

For the PRD portfolios, since the target allocations are constant, we show no comparison between the allocations and asset returns in the out-of-sample period. The constant weights make the PRD portfolio allocations unchanged regardless of market movements. The optimal allocations rely only on information from the in-sample data to weather the storm in the out-of-sample period. Put it another way, the fixed allocation might perform

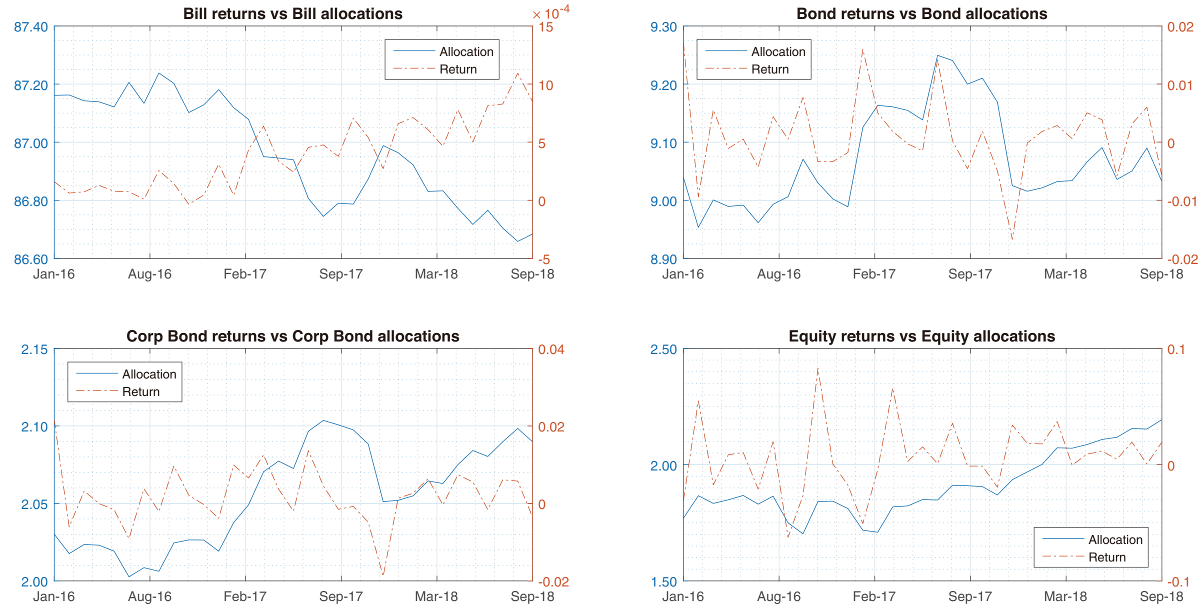

FIGURE 7. Asset allocation dynamics of a low-risk buy-and-hold portfolio

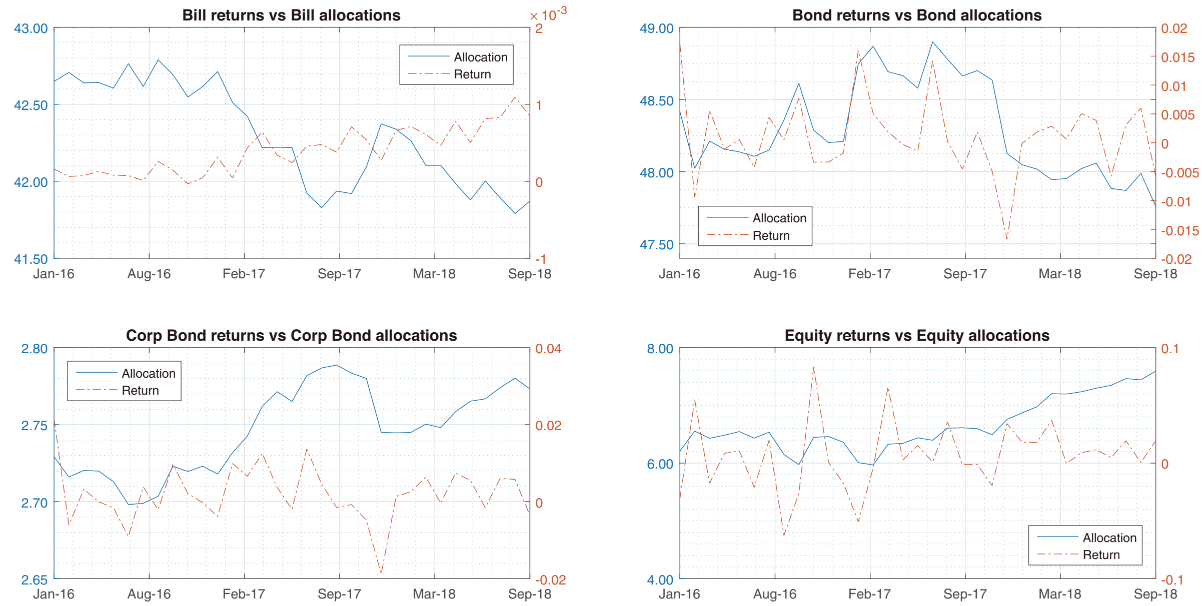

FIGURE 8. Asset allocation dynamics of a medium-risk buy-and-hold portfolio

very poorly if out-of-sample market conditions are so extreme that can be regarded as unforeseen events by available historical data.

One interesting fact when comparing different rebalancing strategies is that the PRD and BAH allocations are relatively similar, while the SSD's are significantly different from the rest (see Table 5) especially at medium and high risk appetites. Since the SSD portfolios harness new information to adjust their allocations any time of rebalancing (unlike the other rebalancing strategies), the new optimal allocations thus gear towards fresher market trends and correlations. This results in a more synchronous fashion of allocations and market returns, and also better performance in terms of average return per volatility.

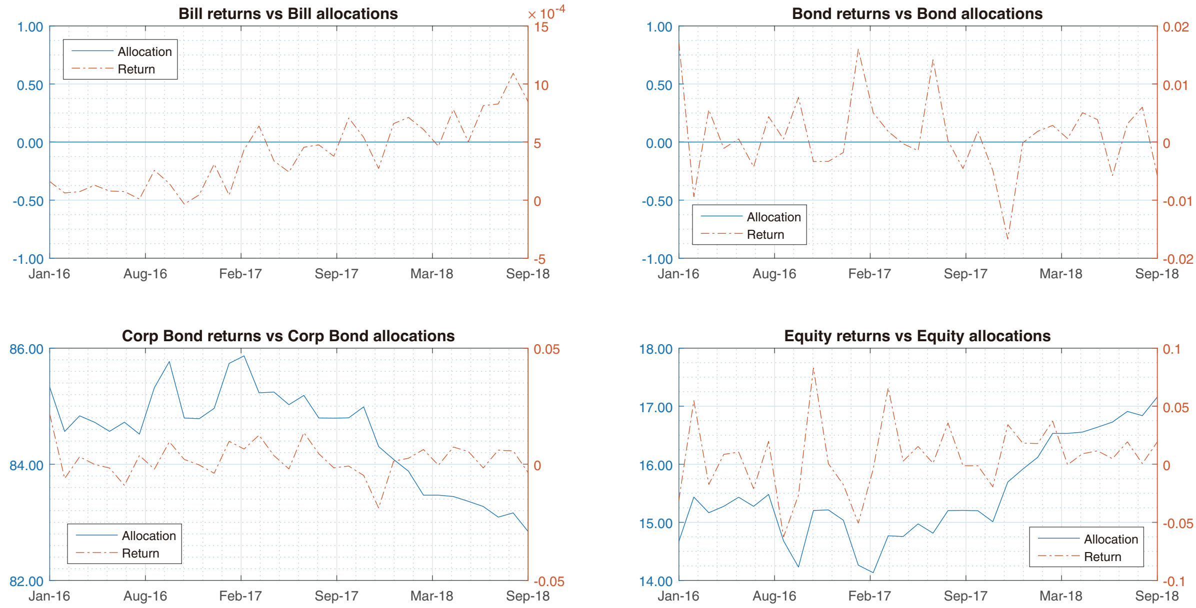

Figure 9. Asset allocation dynamics of a high-risk buy-and-hold portfolio

5. Conclusions.

This section provides a summary on findings of implementing the Second-order Stochastic Dominance (SSD) criterion as a portfolio rebalancing strategy. The SSD condition is used in comparing if one return distribution dominates another – a dominant distribution means higher probability of positive returns and vice versa. Consequently, we apply it to evaluating when a portfolio needs to be reblanced.

Since the SSD criterion can be expressed as a function of Conditional Value-at-Risk (CVaR), an SSD optimal allocation can be found from solving a problem similar to mean-CVaR portfolio optimization. The results give an allocation that dominates others in the SSD sense in which a portfolio will reallocate to in a rebalancing period. If a current allocation is already SSD, then the portfolio needs no rebalance, saving unnecessary transaction costs.

In order to investigate how well the proposed rebalancing strategy performs, a comparison with traditional rebalancing policies, i.e., Periodic (PRD) and Buy-and-Hold (BAH) is conducted. The performance is measured on average return over volatility of portfolio returns in the out-of-sample period. It is shown that, in most circumstances, the SSD rebalancing approach yields higher and more stable returns than other methods. When investigate further, it is observed that the SSD portfolios reshuffle their allocations in a more timely manner with asset returns than PRD and BAH strategies. That is most possibly due to the fact that a portfolio is adjusted to more information accumulated along course of an investment horizon.

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