PERFORMANCE ANALYSIS AND OPTIMIZATION 
OF AN ENERGY-SAVING STRATEGY WITH SLEEP MODE 
IN COGNITIVE RADIO NETWORKS

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ABSTRACT. In cognitive radio networks, a great deal of energy is wasted when the base station becomes idle. To solve this problem, we propose an energy-saving strategy with the sleep mode in this paper. Considering that primary user packets can interrupt the transmission of secondary user packets at any time and occupy the spectrum with pre-emptive priority, we establish a two-dimensional continuous-time Markov chain model to capture the stochastic behavior of the network. Moreover, we obtain the stable condition of the system, and then we derive the expressions for the performance measures in terms of energy-saving rate of system and average latency of secondary user packets. Numerical results with analysis as well as simulation show that the equilibrium arrival rate of SU packets is always greater than the socially optimal arrival rate of SU packets. Based on this observation, we present a pricing policy to optimize the system performance socially.

Keywords: Cognitive radio networks, Energy-saving strategy, Markov chain, Sleep mode, Social optimization

1. Introduction. With the rapid development of bandwidth-intensive applications and the explosive surge of mobile traffic on the base station (BS), it is foreseen that the access network will suffer from a dramatic increase in energy consumption in the next decade. Therefore, the energy savings in computer networks are paid more and more academic and industrial attention [1-3]. As a promising technology, green cognitive radio networks (CRNs) have been studied with the goal of improving energy efficiency.

Based on the proof that energy efficiency is a unimodal function, Li et al. optimized the sensing time by using the golden section search algorithm and then maximized the energy efficiency [4]. By mathematically formulating the achievable data rate of a cooperative CRN in terms of the energy consumption, and jointly designing the sensing time, sensing threshold and number of cooperative secondary users (SUs), Wu et al. proposed an energy utility function and achieved the maximum data rate with significant energy saving [5]. By developing a spectrum scheduling scheme based on particle swarm optimization (PSO) algorithm, Qu et al. obtained a globe solution to the joint problem of channel allocation and power control [6]. By using standard convex optimization techniques, Naeem et al. considered the optimization problem of finding the power allocation that maximizes the energy-efficiency of a CRN with outage constraint [7].

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On the other hand, the most straightforward and valid way to reduce energy consumption is to set the device to sleep state when the system is idle. In order to save the power consumption of BSs, Li et al. analyzed how and when to turn some BSs into sleeping mode based on the predicted traffic loads in radio access networks (RANs) [8]. In [9], Peng et al. studied the energy saving problem by switching off some macro BSs (MABSs) with the constraints of downlink coverage and uplink power. They also demonstrated that the sleep mode could reduce energy consumption significantly without violating the coverage performance. In [10], Chen et al. investigated the topology architecture: random cluster head and sub-cluster head with sleep mode (RCHSCHSM), and developed a sleep mode for sensor nodes based on correlations among sensor data within sub-clusters in RCHSCHSM. In [11], Wu et al. proposed a sleeping mode control for pico BS (PBS) in heterogeneous networks to save energy cost with adaption to the time-varying traffic load. In order to reduce energy consumption and achieve green cloud service, in [12], Jin et al. proposed a type of energy saving strategy in the cloud environment based on synchronous sleeping of the reservation virtual machines. Taking account of both the energy saving in BSs and the latency of the SU packets, in [13], Li et al. proposed an energy saving strategy with N-policy sleep mode.

Motivated by this observation, in this paper, we investigate an energy-saving strategy with an aperiodic sleep mode involved in green CRNs. The contributions of our paper are three fold. First, a novel energy-saving strategy is proposed. With this proposed strategy, energy consumption can be reduced; meanwhile, the response performance of primary user (PU) packets will not be affected. Second, a Markov chain composed of the total number of SU packets and the BS state is established to analyze the proposed strategy. Last, we provide a pricing policy for SU packets to coordinate the Nash equilibrium and the social optimization.

The rest of this paper is organized as follows. In Section 2, the proposed strategy is described and a two-dimensional continuous-time Markov chain model is built. In Section 3, after giving the stable condition of the system, the steady-state probability distribution is derived. In Section 4, system performance is evaluated with numerical results. In Section 5, system is optimized by using a pricing policy. Finally, conclusions are drawn in Section 6.


2.1. Energy-saving strategy with a sleep mode. In this paper, we consider only one licensed spectrum. A central controller can allocate the spectrum for PU packets and SU packets in CRNs. In conventional CRNs, BS keeps awake even though there are not any packets to be transmitted. When BS is idle, a lot of energy will be wasted. In order to save the energy, we introduce an energy-saving strategy with a sleep mode. In this sleep mode, BS will be switched among three states, namely, sleep state, awake state and listening state.

Sleep State: When the transmission of PU packets and SU packets in the system is completed and no new packets arrive at the system, BS will be switched to sleep state; at the same time, a sleep timer will be started. It is necessary to point out that the time length of the sleep timer is a random value given as a system parameter.

- As long as there is a PU packet arrival, sleep timer will be terminated immediately and BS will be switched to awake state. In other words, this sleep mode has no effect on PU packets. So this strategy ensures the performance of PU packets.
- If there are only SU packet arrivals, the sleep period will be terminated when the sleep timer expires, and BS will be switched to awake state. These arrived SU packets
queue in the buffer following the arrival order. It should be noticed that a buffer with infinite capacity is provided only for SU packets.

- If there is no packet arrival before the sleep timer expires, BS will be switched to listening state.

**Listening State:** Only from sleep state can BS be switched to listening state. When BS is in listening state, BS will listen to the licensed spectrum continuously. On the condition that one or more than one packet arrives at the system, BS will be switched to awake state immediately.

**Awake State:** BS can be switched to awake state from either sleep state or listening state. When BS is in awake state, the packets in the system will be transmitted. For awake state, activities of PU packets and SU packets are given as follows.

- When a PU packet arrives at the system, the activity of this newly arriving PU packet will be discussed for three cases. If the licensed spectrum is occupied by an SU packet, the newly arriving PU packet will interrupt the transmission of this SU packet and occupy the licensed spectrum immediately with pre-emptive priority; If the licensed spectrum is occupied by another PU packet, the newly arriving PU packet will be blocked since there is no buffer provided for PU packets in BS; If the licensed spectrum is not occupied by any other packets, i.e., the BS is idle, and the newly arriving PU packet will certainly occupy the licensed spectrum immediately.

- When one SU packet arrives at the system, it will queue in the buffer according to the first in first out (FIFO) principle. Once the licensed spectrum is available, the SU packet queueing at the head of the buffer will access the licensed spectrum immediately. If there is a PU packet arrival during the transmission procedure of an SU packet, the transmission of the SU packet will be interrupted by the arriving PU packet immediately. In order to improve the throughput performance of SU packets, we suppose that this interrupted SU packet will be discarded. Otherwise, the SU packet which is being transmitted will be transmitted continuously. If there is no packet to be transmitted, BS will be switched to sleep state from awake state.

2.2. **Markov chain model.** Based on the energy-saving strategy with a sleep mode mentioned above, we build a two-dimensional continuous-time Markov chain model.

Considering the continuous-time structure, we assume that the inter-arrival times and transmission times for both of SU packets and PU packets are independent, identically distributed (i.i.d) random variables following exponential distributions. Specifically speaking, the arriving intervals of PU packets and SU packets follow exponential distributions with parameters $\lambda_{pu}$ ($\lambda_{pu} > 0$) and $\lambda_{su}$ ($\lambda_{su} > 0$) respectively. The transmission times of a PU packet and an SU packet follow exponential distributions with parameters $\mu_{pu}$ ($\mu_{pu} > 0$) and $\mu_{su}$ ($\mu_{su} > 0$) respectively. In addition, we regard the time length of the sleep timer as a random variable $\xi$ and following exponential distribution with parameter $\theta$ ($\theta > 0$). Obviously, the average time length $E[\xi]$ of the sleep timer is $\frac{1}{\theta}$.

The total number of SU packets is defined as the system level, and the state of BS combining with the condition of the licensed spectrum is defined as the system stage. We use $X(t) = i$ ($i \geq 0$) to express the system level at the instant $t$, and use $Y(t) = j$ ($j = 0, 1, 2, 3$) to express the system stage at the instant $t$: $j = 0$ means BS is in sleep state; $j = 1$ means BS is in awake state and the licensed spectrum is being occupied by a PU packet; $j = 2$ means BS is in awake state and the licensed spectrum is being occupied by an SU packet; $j = 3$ means BS is in listening state, that is, BS is idle. $\{X(t), Y(t)\}$ constitutes a two-dimensional Markov chain. The state spaces of this Markov chain are


\[ \Omega = \{(i, j) : i \geq 0, j = 0, 1, 2, 3\} \]

We denote \( \pi_{i,j} \) as the steady-state distribution of the two-dimensional Markov chain. \( \pi_{i,j} \) can be given as follows: \( \pi_{i,j} = \lim_{t \to \infty} P\{X(t) = i, Y(t) = j\} \).

2.3. Stable condition of the system. In CRNs, there are two kinds of packets, namely SU and PU packets. In order to give the stable condition of the system, we investigate the traffic intensity \( \rho_{su} \) of SU packets and the traffic intensity \( \rho_{pu} \) of PU packets, respectively.

From the perspective of SU packets, \( \rho_{su} \) means the number of SU packets arriving at the system during the average actual transmission time of an SU packet. Since the priority of an SU packet is lower than that of a PU packet, an SU packet leaves the system either when the transmission of this SU packet is completed successfully or when the transmission of this SU packet is interrupted by a newly arriving PU packet. As a result, the average actual transmission time is \( \frac{1}{\mu_{su} + \lambda_{pu}} \). So the traffic intensity \( \rho_{su} \) of SU packets is given as follows:

\[ \rho_{su} = \frac{\lambda_{su}}{\mu_{su} + \lambda_{pu}} \]  

(1)

On the other hand, because of the higher priority of PU packets, neither SU packet nor system state has an effect on the activities of PU packets. Therefore, from the perspective of PU packets, the steady-state distribution for PU packets belongs to an ON/OFF Markov model. This model is composed of two alternating states: a busy state (ON), where a PU packet occupies the licensed spectrum, and no PU packets occupy the licensed spectrum in the idle state (OFF). With respect to [14], we can gain the probability that the licensed spectrum is occupied by a PU packet as \( \pi_{on} = \frac{\lambda_{pu}}{\mu_{pu} + \lambda_{pu}} \). We notice that \( \pi_{on} \) is just the traffic intensity \( \rho_{pu} \) of PU packets. Then \( \rho_{pu} \) is given as follows:

\[ \rho_{pu} = \frac{\lambda_{pu}}{\mu_{pu} + \lambda_{pu}} \]  

(2)

Combining the traffic intensity \( \rho_{su} \) of SU packets and the traffic intensity \( \rho_{pu} \) of PU packets, we obtain the traffic intensity \( \rho \) of the system, i.e., \( \rho = \rho_{su} + \rho_{pu} \). The necessary and sufficient constraint for the system to reach a steady-state is \( \rho < 1 \). So the stable condition of the system is given as follows:

\[ \frac{\lambda_{su}}{\mu_{su} + \lambda_{pu}} + \frac{\lambda_{pu}}{\mu_{pu} + \lambda_{pu}} < 1 \]  

(3)

We will analyze the system model under this stable condition.

3. System Analysis. The novelty of the system model studied in this paper lies in the combination of single-vacation and pre-emptive priority. To tackle this type of queueing model, one of the important steps is to construct the transition rate matrix.

Let \( Q \) be the one step state transition rate matrix of the \( \{(X(t), Y(t)), t \geq 0\} \), and \( Q(u, v) \) be the one step transition rate submatrix from the system level \( u \) to \( v \). Since there are four stages, namely 0, 1, 2, 3, in the system, each submatrix \( Q(u, v) \) is with order \( 4 \times 4 \).

According to different system levels, we discuss \( Q(u, v) \).

(1) If \( u = 0 \) and \( v = 0 \), it means there is no SU packet in the system. Letting \( Q(0, 0) = A_0 \), the one step transition rate matrix \( A_0 \) is given as follows:
\[
A_0 = \begin{pmatrix}
-\lambda_{su} - \lambda_{pu} - \theta & \lambda_{pu} & 0 & \theta \\
\mu_{pu} & -\lambda_{su} - \mu_{pu} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & \lambda_{pu} & 0 & -\lambda_{su} - \lambda_{pu}
\end{pmatrix}
\] (4)

The system stage being fixed at 0 means BS keeps in sleep state, i.e., there is no packet arrival and the sleep timer does not expire. The system stage changing to 1 from 0 means BS is switched to awake state from sleep state, and a newly arriving PU packet begins to occupy the licensed spectrum. The system stage changing to 3 from 0 means BS is switched to listening state from sleep state, that is, the sleep timer expires. The system stage changing to 0 from 1 means BS is switched to sleep state from awake state, and the licensed spectrum becomes idle, namely, the transmission of the PU packet is completed. The system stage being fixed at 1 means BS keeps in awake state and the licensed spectrum is still occupied by a PU packet, namely, the transmission of this PU packet is not completed yet and there is no SU packet arrival. The system stage changing to 1 from 3 means BS is switched to awake state from listening state, and a newly arriving PU packet begins to occupy the licensed spectrum. The system stage being fixed at 3 means BS is still in listening state, i.e., neither SU packet nor PU packet arrives at the system.

(2) If \( u = 0 \) and \( v = 1 \), it means there is an SU packet arrival in the system. Letting \( Q(0, 1) = C_0 \), the one step transition rate matrix \( C_0 \) is given as follows:

\[
C_0 = \lambda_{su} \begin{pmatrix}
1 & 1 \\
0 & 0 \\
1 & 0
\end{pmatrix}
\] (5)

The state transition may occur in the following three cases. The first case is the system stage being fixed at 0. That is, an SU packet could arrive at the system during sleep state. The second case is the system stage being fixed at 1. That is, an SU packet could arrive at the system when BS keeps in awake state and the licensed spectrum is occupied by a PU packet. The third case is the system stage changes to 2 from 3. That is, an SU packet could arrive at the system during listening state, then BS will be switched to awake state and this SU packet will occupy the licensed spectrum.

(3) If \( u = 1 \) and \( v = 0 \), it means that there is an SU packet departure from the system. Letting \( Q(1, 0) = B_1 \), the one step transition rate matrix \( B_1 \) is given as follows:

\[
B_1 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\mu_{su} & \lambda_{pu} & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (6)

The system stage changing to 0 from 2 means the transmission of an SU packet is completed. While, the system stage changing to 1 from 2 means the transmission of an SU packet is interrupted by a newly arriving PU packet.

(4) If \( u = i \) and \( v = i \) (\( i \geq 1 \)), it means that after a state transition, the number of SU packets in the system is fixed. Letting \( Q(i, i) = A \), the one step transition rate matrix \( A \) is given as follows:

\[
A = \begin{pmatrix}
-\lambda_{su} - \lambda_{pu} - \theta & \lambda_{pu} & \theta & 0 \\
0 & -\lambda_{su} - \mu_{pu} & \mu_{pu} & 0 \\
0 & 0 & -\lambda_{su} - \mu_{su} - \lambda_{pu} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (7)
The discussions on the system stage being fixed at 0 or 1, as well as the system stage changing to 1 from 0 are similar to Item (1).

The system stage changing to 2 from 0 means BS is switched to awake state from sleep state, that is, the sleep timer expires, and an SU packet begins to occupy the licensed spectrum. The system stage changing to 2 from 1 means BS keeps in awake state and an SU packet begins to occupy the licensed spectrum, namely, the transmission of the PU packet is completed. The system stage being fixed at 2 means BS keeps in awake state and the licensed spectrum is still occupied by an SU packet, i.e., the transmission of this SU packet is not completed yet and there is no any other packet arrival.

(5) If \( u = i \) (\( i \geq 1 \)) and \( v = i + 1 \), it means that after a state transition, the number of SU packets in the system increases by one. For the case \( i \geq 1 \), a new SU packet arrives and the system stages can only be fixed at 0, 1, or 2. That is, BS can only keep in sleep state or awake state. Letting \( Q(i, i + 1) = C \), the one step transition rate matrix \( C \) is given as follows:

\[
C = \lambda_{su} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}
\]  

(8)

(6) If \( u = i \) (\( i \geq 2 \)) and \( v = i - 1 \), it means after a state transition, the number of SU packets in the system decreases by one. The discussions on the system stage are similar to Item (3), so we will not cover those here. Letting \( Q(i, i - 1) = B \), the one step transition rate matrix \( B \) is given as follows:

\[
B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \lambda_{pu} & \mu_{su} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]  

(9)

Combining Equations (4)-(9), the one step state transition rate \( Q \) for the system is given by

\[
Q = \begin{pmatrix} A_0 & C_0 \\ B_1 & A & C \\ B & A & C \\ \cdots \cdots \cdots \end{pmatrix}
\]  

(10)

It is easy to see that the one step state transition rate \( Q \) is a three-diagonal matrix and the system state transition occurs only in adjacent levels. Therefore, the stochastic process \( \{X(t), Y(t)\} \) is a quasi-birth-and-death (QBD) process.

Let \( \Pi_i \) be the steady-state probability vector for the system being at level \( i \) (\( i \geq 0 \)). \( \Pi_i \) can be given as follows:

\[
\Pi_i = (\pi_{i,0}, \pi_{i,1}, \pi_{i,2}, \pi_{i,3})
\]  

(11)

We denote \( \Pi \) as:

\[
\Pi = (\Pi_0, \Pi_1, \Pi_2, \ldots)
\]  

(12)

Combining the steady-state equation and the normalization condition in the Markov chain, we have

\[
\begin{cases}
\Pi Q = 0 \\
\Pi e = 1
\end{cases}
\]  

(13)

where \( e \) is a one’s column vector with appropriate dimension.

For the QBD process, we can obtain the steady-state probability distribution of the system by using the matrix geometric solution method.
Let \( R \) be the minimum non-negative solution of the matrix equation \( R^2B + RA + C = 0 \). The condition for the QBD process to be positive recurrence is that the spectral radius \( SP(R) \) of \( R \) being less than 1 and the homogeneous linear equations

\[
(\Pi_0, \Pi_1)B[R] = 0
\]

have a positive solution.

\( B[R] \) is an 8-dimensional stochastic matrix given as follows:

\[
B[R] = \begin{bmatrix}
A_0 & C_0 \\
B_1 & A + RB
\end{bmatrix}
\]

When the QBD process is positive recurrent, its steady-state distribution satisfies

\[
\begin{cases}
\Pi_i = \Pi_1R^{i-1}, i \geq 1 \\
\Pi_0e + \Pi_1(I - R)^{-1}e = 1
\end{cases}
\]

From Equations (14)-(16), we can give the steady-state probability vector \( \Pi \).

4. Performance Measures and Numerical Results.

4.1. Performance measures. In this section, we derive some performance measures to evaluate the energy-saving strategy with a sleep mode in CRNs.

The energy-saving rate is one of the important indices for measuring the power consumption. In awake state, BS expends the same energy as that in conventional CRNs, in which the BS is always awake, so there is no energy conservation. In sleep state, some devices are turned off and energy is saved at utmost. In listening state, BS monitors whether there are packet arrivals or not, the energy consumption in listening state is less than that in awake state. Let \( f_1 \) and \( f_2 \) be the energy-saving parameters in sleep state and listening state, respectively. Obviously, \( f_1 > f_2 \). Using the steady-state distribution given in Section 3, the energy-saving rate \( \zeta \) can be given as follows:

\[
\zeta = f_1 \sum_{i=0}^{\infty} \pi_{i,0} + f_2 \sum_{i=0}^{\infty} \pi_{i,3}
\]

The average latency of SU packets is an important index for evaluating the experience of users. In the energy-saving strategy with a sleep mode, SU packets arriving in sleep state could not be transmitted until the sleep timer is terminated. As a result, the average number of SU packets waiting in the system buffer is greater than that in conventional CRNs, and the average latency of SU packets will be greater. Following Little’s law [15,16], the average latency \( \omega \) of SU packets is given as follows:

\[
\omega = \frac{\sum_{i=0}^{\infty} \sum_{j=0}^{3} i \pi_{i,j}}{\lambda_{su}}
\]

4.2. Numerical experiments and simulation experiments. In order to analyze the influences of the arrival rate \( \lambda_{su} \) of SU packets, the arrival rate \( \lambda_{pu} \) of PU packets and the average time length \( E[\xi] \) of the sleep timer on the system performance, we provide numerical experiments and simulation experiments to show the variation tendency of the system performance in terms of the system energy-saving rate and the average latency of SU packets.

As long as the system is stable, the trends and laws of the experiments results will not be affected by the settings of parameters. In numerical experiments and simulation experiments, as an example, we set parameters as follows: the energy-saving parameters in sleep state as \( f_1 = 0.98 \), the energy-saving parameters in listening state as \( f_2 = 0.67 \),
Figure 1 shows how the energy-saving rate changes with respect to arrival rates of PU packets and SU packets. In Figure 1, we find that the energy-saving rate increases as the average time length $E_\xi$ of the sleep timer increases. As the average time length of the sleep timer increases, more possible is that the BS is to be asleep. Since the energy is mainly saved during sleep state, the energy-saving rate will increase. Additionally, we observe that for the same arrival rate $\lambda_{pu}$ of PU packets, the higher the arrival rate $\lambda_{su}$ of SU packets is, the lower the energy-saving rate $\zeta$ will be. The more frequently the SU packets arrive at the system, the longer the system will be in awake state. Relatively speaking, the system is less likely to be in sleep state or listening state, so the smaller the energy-saving rate will be.

By setting the average time length as $E_\xi = 0$ in Figure 1, we can compare the results for energy-saving rate between the strategy without sleep mode and our proposed strategy with sleep mode. We find that our proposed strategy performs better in improving the energy efficiency.

We also notice that for the same arrival rate $\lambda_{su}$ of SU packets, the energy-saving rate $\zeta$ will decrease as the arrival rate $\lambda_{pu}$ of PU packet increases. There are two reasons leading to this tendency. One of the reasons is that the higher the arrival rate of PU packets is, the more possible is that the sleep timer will be terminated in advance, while the more possible is that the system will be awake, then the lower the energy-saving rate will be. The other reason is similar to the explanation for illustrating why the energy-saving rate decreases as the arrival rate of SU packets increases.

Figure 2 shows how the average latency $\omega$ of SU packets changes with the average time length $E_\xi$ of the sleep timer for different arrival rates of PU packets and SU packets.

From Figure 2, we observe that the average latency $\omega$ of SU packets will increase along with the increasing average time length $E_\xi$ of the sleep timer. It is because that with the increasing average time length of the sleep timer, the time for the system being in
sleep state will be longer, the SU packets arriving in sleep state will queue in the buffer until the sleep timer is terminated. The queue length of the buffer will be longer, the transmission of SU packets queueing in the buffer will be delayed, so the average latency of SU packets will be greater. On the other hand, for the same arrival rate $\lambda_{pu}$ of PU packet, the bigger the arrival rate $\lambda_{su}$ of SU packet is, the higher the average latency of SU packets will be. The reason is that the more frequently the SU packets arrive at the system, the larger the average number of SU packets in the buffer is, so the greater the average latency of SU packets will be.

By setting the average time length as $E[\xi] = 0$ in Figure 2, we compare the results for the average latency of SU packets between the strategy without sleep mode and our proposed strategy with sleep mode. We find that our proposed strategy has to make sacrifices on response performance.

Moreover, in Figure 2, we see that for the same arrival rate $\lambda_{su}$ of SU packets, when the average time length $E[\xi]$ of the sleep timer is shorter, the greater the arrival rate $\lambda_{pu}$ of PU packets is, the greater the average latency $\omega$ of SU packets will be; however, when the average time length $E[\xi]$ of the sleep timer is larger, the consequence will be opposite. When the average time length of the sleep timer is shorter, the more the PU packets arrive at the system, the greater the possible is that the licensed spectrum is to be occupied by PU packets, and the longer the SU packets have to wait in the system buffer, and the higher the average latency of SU packets will be. On the other hand, when the average time length of the sleep timer becomes larger, the more frequently the PU packets arrive at the system, the more likely is that the sleep timer is terminated ahead of time due to the arrival of a PU packet, the longer the system will be in awake state. During awake state, all the packets in the system could be transmitted continuously, so the lower the average latency of SU packets will be.

Combining Figures 1 and 2, we find that the numerical experiments match well with simulation results, and we also find that both of the energy-saving rate and the average latency of SU packets increase as the average time length of sleep timer increases.
off the effect of energy conservation and the quality of experience for SU packets, we suggest that the sleeping parameter should be moderate. In practice, when the users are strict with real time, the sleeping parameter should be set higher to decrease the average latency. On the other hand, when the user requirement for real time is lower, the sleeping parameter should be set lower to improve the energy efficiency.

5. Nash Equilibrium and Social Optimization. In the proposed energy-saving strategy with a sleep mode, when an SU packet arrives at the system, this SU packet may queue in the buffer, and the transmission will be delayed. Even this SU packet is being transmitted, the transmission may be interrupted by a newly arriving PU packet. That is to say, the transmission of an SU packet is not guaranteed. So, from the perspective of SU packets, we make optimization for the actions of SU packets to maximize their benefits. Firstly, we analyze the Nash equilibrium behavior and social optimization problem. Then, we provide a pricing policy for SU packets to coordinate the Nash equilibrium and the social optimization.

5.1. Nash equilibrium behavior and social optimization problem. Each SU packet can be seen as an individual with the target of maximizing its net benefit. Based on the system model built in Section 2, we analyze the Nash equilibrium behavior. First of all, we give some hypotheses as follows.

- The reward for an SU packet to be transmitted successfully is $R$.
- The cost for an SU packet is $C$ per unit time sojourn in the system.
- The individual net benefit function for each SU packet is the same, and the net benefits for all SU packets can be added together.

We denote the net benefit function of an SU packet by

$$U_i(\lambda_{su}) = RS - C\omega$$  \hspace{1cm} (19)

where $S$ is the probability that an SU packet is transmitted successfully, $S = \frac{1}{\lambda_{su}} \sum_{i=0}^{\infty} \pi_{i,2}\mu_{su}$, $\omega$ is the average delay of SU packets given in Equation (18).

By gathering the individual net benefits obtained of all SU packets arrived at the system, the social net benefit $U_s(\lambda_{su})$ can be calculated as follows:

$$U_s(\lambda_{su}) = \lambda_{su}U_i(\lambda_{su})$$  \hspace{1cm} (20)

Maximizing the social net benefit $U_s(\lambda_{su})$ in Equation (20), we can gain the socially optimal arrival rate $\lambda^*$ of SU packets as follows:

$$\lambda^* = \arg \max_{0<\lambda_{su}} \{\lambda_{su}U_i(\lambda_{su})\}$$  \hspace{1cm} (21)

In CRNs, all SU packets want to occupy the licensed spectrum to gain benefit. However, with the increasing number of SU packets in the system, the average delay of SU packets will increase too, and then the individual net benefit of an SU packet will be reduced. So, there is an equilibrium behavior between different SU packets. In order to further analyze the functions for the individual net benefit $U_i(\lambda_{su})$ and the social net benefit $U_s(\lambda_{su})$ of SU packets, we explore the monotonic property of $U_i(\lambda_{su})$ in Equation (19) and $U_s(\lambda_{su})$ in Equation (20) with numerical experiments.

Applying the same parameters as used in Section 4.2 and setting $R = 5$ and $C = 1$ as an example, we show the change trends for the individual net benefit $U_i(\lambda_{su})$ of an SU packet and the social net benefit $U_s(\lambda_{su})$ of SU packets along with the arrival rate of SU packets in Figures 3 and 4, respectively.

In Figure 3, we find that the individual net benefit $U_i(\lambda_{su})$ of an SU packet will decrease with respect to the increasing arrival rate $\lambda_{su}$ of SU packets. What is more, there exists
a value of $\lambda_{su}$ for $U_i(\lambda_{su}) = 0$. In other words, there is a Nash equilibrium behavior for the arrival rate $\lambda_e$ of SU packets, and $\lambda_e$ is unique. Letting $U_i(\lambda_e) = 0$, we can obtain the equilibrium arrival rate $\lambda_e$ of SU packets.

In Figure 4, we observe that the social net benefit $U_s(\lambda_{su})$ firstly shows a rising trend and then shows a downward trend along with the arrival rate $\lambda_{su}$ of SU packets increases. Generally speaking, there exists a maximum social benefit $U^*_s$ of SU packets and a socially optimal arrival rate $\lambda^*_su$ of SU packets.

We make a comparison between the equilibrium arrival rate $\lambda_e$ in Figure 3 and the socially optimal arrival rate $\lambda^*_su$ in Figure 4, and then we observe that for the same
sleeping parameter $\theta$ and arrival rate $\lambda_{pu}$ of PU packets, the equilibrium arrival rate $\lambda_e$ of SU packets is always greater than the socially optimal arrival rate $\lambda^*_{su}$ of SU packets. That is, the Nash equilibrium does not lead to the social optimization, and the number of SU packets joining the system buffer under the Nash equilibrium is more than that under the social optimization.

5.2. Pricing policy for SU packets. In order to oblige each SU packet to adopt the social optimization strategy, we impose an admission fee $P$ as a cost for an SU packet to access the licensed spectrum. Therefore, we modify the net benefit $U_{ip}(\lambda_{su})$ of an SU packet as follows:

$$U_{ip}(\lambda_{su}) = RS - C\omega - P$$  \hspace{1cm} (22)

By setting $\lambda_{su} = \lambda^*_{su}$ in Equation (22), we can get the individual net benefit $U_{ip}(\lambda^*_{su})$ of an SU packet as follows:

$$U_{ip}(\lambda^*_{su}) = RS - C\omega - P$$  \hspace{1cm} (23)

Letting $U_{ip}(\lambda^*_{su}) = 0$, we can solve the admission fee $P$ as follows:

$$P = RS - C\omega$$  \hspace{1cm} (24)

With the same parameters as used in Figures 1-4, we provide numerical results of the admission fee $P$ in Table 1.

**Table 1. Numerical results of the admission fee $P$**

<table>
<thead>
<tr>
<th>Sleeping parameter $\theta$</th>
<th>Arrival rate $\lambda_{pu}$ of PU packets</th>
<th>Admission fee $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.15</td>
<td>1.4756</td>
</tr>
<tr>
<td>0.55</td>
<td>0.15</td>
<td>1.3897</td>
</tr>
<tr>
<td>0.55</td>
<td>0.25</td>
<td>0.9799</td>
</tr>
</tbody>
</table>

From Table 1, we observe that for the same sleep parameter $\theta$, the greater the arrival rate $\lambda_{pu}$ of PU packets is, the lower the admission fee $P$ will be. The more frequently the PU packets arrive at the system, the greater the probability is that the transmission of an SU packet will be interrupted by a newly arriving PU packet, which will reduce the enthusiasm of SU packets to join the system. To not lose SU packets in such a situation, the admission fee should be set lower to attract more SU packets.

In addition, we find that for the same arrival rate $\lambda_{pu}$ of PU packets, the greater the sleeping parameter $\theta$ is, the higher the admission fee $P$ will be. A greater sleeping parameter means a shorter time length of a sleep timer, that is to say, the less likely is that the system being in sleep state, the shorter the time for an SU packet waiting in the buffer for transmission, which will lead more SU packets to join the system. In order to prevent overpopulation of SU packets and to ensure the service quality of CRNs, the admission fee should be set higher to restrain the coming of SU packets.

6. Conclusions. In this paper, for the purpose of saving energy consumption in CRNs, we proposed an energy-saving strategy with a sleep mode. We built a two-dimensional continuous-time Markov chain model to capture the stochastic behaviour of SU and PU packets. With the aid of matrix geometric solution method, we gave the steady-state probability distribution and evaluated the system performance mathematically. Moreover, we carried out theoretical calculation and statistical simulation to validate the effectiveness of the proposed energy-saving strategy with a sleep mode. By investigating the Nash equilibrium behaviour and resolving the social optimization problem, we presented a pricing policy to SU packets and optimized the system socially.
The research in this paper has a potential application in achieving green communication in CRNs. As a future direction, we will investigate the energy-saving strategy based on N-policy to improve the response performance of SU packets.

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REFERENCES


