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FAULT-TOLERANT TRACKING CONTROL FOR UNMANNED HELICOPTER ALTITUDE AND ATTITUDE SYSTEM BASED ON PRESCRIBED PERFORMANCE METHOD

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Abstract. *In this paper, an adaptive neural fault-tolerant tracking control method is proposed for the nonlinear unmanned helicopter (UH) altitude and attitude system using prescribed performance technique. The neural network (NN) and the disturbance observer are utilized to tackle the system uncertainties, unknown disturbance and actuator gain faults, respectively. Furthermore, the prescribed performance function technology is introduced to deal with output constraints of helicopter altitude and attitude system. In addition, the boundness of tracking errors of closed-loop system is ensured by Lyapunov stability analysis. The simulation results show that the designed adaptive neural fault-tolerant tracking control schemes is effective, and the tracking performance is satisfactory.*

Keywords: Unmanned helicopter, Fault-tolerant control, Prescribed performance, Adaptive control, Neural network, Disturbance observer

1. **Introduction.** In recent years, the UH system has been attracted increasing attention since it can be widely spread to various practical fields [1]. However, with the increasing requirements of maneuverability and autonomous flight, the traditional flight controller is no longer unable to satisfy the requirement of complex work environment [2]. This raised the need for an accurate nonlinear robust control design approach of the UH systems [3]. For tackling attitude angle tracking control problem, there are some efficient research works for nonlinear UH systems [4]. In [5], the adaptive NN optimal control method was studied for the regulation and tracking control of a UH. A class of disturbance observer based backstepping control approaches were studied for the small-scale UH systems in [6]. In [7], a robust adaptive constrained tracking control design method was proposed for small UH system with constrained flapping dynamics. A sliding mode control was designed for the UH systems in [8]. However, these results rarely involve the research on the control problem of UH under fault and tracking performance constraint [9]. Specially, the state dependent nonlinear actuator fault needs to be further considered for the UH. In this paper, based on the design technique of nonlinear adaptive controller, the prescribed performance technique will be employed to ensure the output performance of UH satisfies the tracking requirement.

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In any practical control engineering problem, there is always a deviation between the practical control system and the mathematical model which is used to design the system controller [10]. The disturbances and uncertainties of system will seriously affect the control accuracy, or even make the system unstable [11]. As a result, a large number of robust control studies have emerged which were used to overcome this problem. Among them, one efficient controller design technology is so-called disturbance observer based control method [12]. With the development of nonlinear disturbance observer, a series of control methods based on disturbance observer, including NN disturbance observer [13], sliding mode disturbance observer [14, 15] and fuzzy disturbance observer [16, 17], have been applied in the controller design process. In this paper, the nonlinear NN disturbance observer will be employed to deal with the uncertainties, disturbances and the state dependent faults of altitude and attitude system in the UH.

Another important issue associated with the UH system is transient performance and steady-state performance. As we all know, the prescribed performance function based adaptive control approach can not only guarantee the stability of the system, but also guarantee the transient performance of the closed-loop system [18]. Since prescribed performance function method was proposed, it has been widely developed for practical control systems [19]. In [20], the adaptive fuzzy output feedback control was proposed for an uncertain multi-input and multi-output (MIMO) nonlinear system by using the dynamic surface method. In [21], the adaptive control approach was studied for the linear mechanisms system using the prescribed performance technique. In [22], the prescribed performance function method was employed to the switched nonlinear system in order to solve the output tracking control problem for a class of non-strict feedback systems. However, the disturbance and fault have seldom been taken into account simultaneously in the existing research results. Owing to the long-running of transmission mechanism and the influence of external environment, the helicopter system may show actuator fault. Thus, the fault tolerant control should be considered in the control of helicopter system. The reliable output control based on disturbance observer was developed for time-delay systems with actuator faults in [23]. In [24], a fault-tolerant control scheme with post-fault transient improvement was proposed for aircraft control. Robust fault-tolerant control was studied for a fuzzy delay system with unavailable premise variables based on uncertain system method in [25]. In this paper, we will develop one fault-tolerant controller by using prescribed performance method for the altitude and attitude system of a UH.

Motivated by the need to tackle actuator fault, system uncertainty and external disturbance, a nonlinear adaptive control procedure will be proposed for altitude and attitude control system of the UH in this paper. And the paper will be organized as follows. Section 2 describes nonlinear UH mathematical model and related knowledge about NN. Section 3 provides a detailed description on the prescribed performance function based adaptive fault-tolerant tracking control design procedure. Section 4 gives simulation results and the analysis. Finally, some conclusions are presented in Section 5.

2. **Problem Statement and Preliminaries.** In this section, we will review some preliminary knowledge about nonlinear helicopter altitude and attitude system, and necessary assumptions and lemmas are given in the following contents. The attitude and altitude dynamics of UH system can be described as [8]

$$
\begin{aligned}
\dot{z} &= v \\
\dot{v} &= g - \frac{1}{m} g \cos\theta \cos\phi Z_{col} u_{col} + \Delta v + d_v \\
\dot{\phi} &= p + \frac{\sin\phi \sin\theta}{\cos\theta} q + \frac{\cos\phi \sin\theta}{\cos\theta} r
\end{aligned}
$$

$$\begin{aligned}
\dot{\theta} &= \cos\phi q - \sin\phi r \\
\dot{\psi} &= \frac{\sin\phi}{\cos\theta} q + \frac{\cos\phi}{\cos\theta} r \\
\dot{p} &= -qr(I_y - I_z)/I_x + L_a a + L_b b + \Delta p + d_p \\
\dot{q} &= -pr(I_z - I_x)/I_y + M_a a + M_b b + \Delta q + d_q \\
\dot{r} &= -pq(I_x - I_y)/I_z + N_r r + N_{col} u_{col} + N_{ped} u_{ped} + \Delta r + d_r
\end{aligned} \tag{1}$$

where $z \in R$ indicates the altitude, and $\phi \in R$, $\theta \in R$, $\psi \in R$ indicate the attitude angles. $v \in R$ represents the vertical velocity, and $p \in R$, $q \in R$, $r \in R$ represent the attitude angle velocity. $u_{col} \in R$, $u_{ped} \in R$ stand for collective pitch and pedal control input, respectively. $a \in R$, $b \in R$ are the longitudinal and lateral flapping angles of main rotor. $m \in R$ is the helicopter mass. $g \in R$ is the acceleration of gravity. $I_x \in R$, $I_y \in R$, $I_z \in R$ mean moments of inertia. Meanwhile, $L_a \in R$, $L_b \in R$, $M_a \in R$, $M_b \in R$, $N_r \in R$, $N_{col} \in R$, $N_{ped} \in R$, $Z_{col} \in R$ indicate the aerodynamic parameters. $\Delta v \in R$, $\Delta p \in R$, $\Delta q \in R$, $\Delta r \in R$ represent system uncertainties. $d_v \in R$, $d_p \in R$, $d_q \in R$, $d_r \in R$ denote the external disturbance. In addition, $u^f = [u_{col}, u_{ped}, a, b]^T$ stands for the control input vector with actuator gain fault. The fault can be expressed as

$$u_i^f = \delta_i u_i, \quad i = 1, 2, 3, 4 \tag{2}$$

where δ_i indicates the remaining control rate coefficient, which can be written as [13]

$$\delta_i = \frac{1}{1 + \beta_i e^{-\zeta_i}}, \quad i = 1, 2, 3, 4 \tag{3}$$

where β_i stands for the unknown constant, and ζ_i indicates the unknown bounded function of state dependent nonlinear actuator fault. Define $\Phi(u) = u(t) - \delta u(t)$, which stands for the unknown nonlinear function. Thus, considering the actuator gain fault, the control input can be rewritten as

$$u^f = u(t) - \Phi(u) \tag{4}$$

In this manuscript, the control objective is that a prescribed performance based adaptive fault-tolerant tracking control will be designed for altitude and attitude system of a UH with system uncertainties, disturbance and actuator gain faults.

Assumption 2.1. [13] *The ideal system reference tracking signal y_d is known and bounded; meanwhile, its time derivative is also known and bounded.*

Assumption 2.2. [7] *The roll angle and pitch angle satisfy inequality constraint conditions $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.*

Lemma 2.1. [13] *Using a linear parameterized radial basis function neural networks (RB FNNs) to approximate the unknown function F, the approximation result can be written as*

$$\hat{F}(\xi) = \hat{W}^T S + \varepsilon \tag{5}$$

where $\xi \in R^n$ indicates the input variable of NN, S is the radial basis function, and $\hat{W}$ is the weight value of NN. Based on the approximation theory of NN, the optimal weight value of NN is given by

$$W^* = \arg\min_{\hat{W}\in\Omega} \left[\sup_{z\in S_z} \left| \hat{F}(\xi) - F(\xi) \right| \right] \tag{6}$$

where $\Omega = \left\{ \hat{W} : \left\| \hat{W} \right\| \leq \bar{W} \right\}$, $\bar{W} > 0$ is the upper boundary of NN weight value. And, the NN optimal approximation result indicates

$$F(\xi) = W^{*T} S + \varepsilon^* \tag{7}$$

where ε^* *is the NN optimal estimate error, and there has* $\|\varepsilon^*\| < \bar{\varepsilon}$, $\bar{\varepsilon} > 0$ *is a positive constant which is the maximum value of approximation error.*

Lemma 2.2. [15] *For all given bounded initial conditions, assume that there has a positive definite and continuous Lyapunov function* $V(x)$ *defined in* C^1 *and two class* K *functions* π_1, π_2*:* $R^n \to R$*, which satisfy the conditions* $\pi_1(\|x\|) \le V(x) \le \pi_2(\|x\|)$*. For the positive constants* c_1 *and* c_2*, if the condition* $\dot{V}(x) \le -c_1 V(x) + c_2$ *is guaranteed, then the solution* $x(t)$ *is uniformly bounded.*

3. **Fault Tolerant Tracking Controller Design.** To facilitate controller design, we give the following definitions:

$$x_1 = [z, \phi, \theta, \psi]^T \in R^4 \quad x_2 = [v, p, q, r]^T \in R^4 \tag{8}$$

Then, considering (4) and (8), Equation (1) can be written as

$$\begin{aligned} \dot{x}_1 &= F_1(x_1) + G_1(x_1)x_2 \\ \dot{x}_2 &= F_2(\bar{x}_2) + G_2(\bar{x}_2)u + \Phi(u) + \Delta F(\bar{x}_2) + d \end{aligned} \tag{9}$$

where $G_1(x_1) \in R^{4\times4}$ and $G_2(\bar{x}_2) \in R^{4\times4}$ are the nonlinear control matrices, and $F_1(x_1) \in R^4$ is a function vector which is a zero vector here. $\Delta F(\bar{x}_2) \in R^4$ is the system uncertainty and $d \in R^4$ is an unknown disturbance vector.

$$G_1(x_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{\sin\phi\sin\theta}{\cos\theta} & \dfrac{\cos\phi\sin\theta}{\cos\theta} \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix},$$

$$G_2(\bar{x}_2) = \begin{bmatrix} -\dfrac{1}{m} g\cos\theta\cos\phi Z_{col} & 0 & 0 & 0 \\ 0 & 0 & L_a & L_b \\ 0 & 0 & M_a & M_b \\ N_{col} & N_{ped} & 0 & 0 \end{bmatrix},$$

$$F_2(\bar{x}_2) = \begin{bmatrix} g \\ -qr(I_y - I_z)/I_x \\ -pr(I_z - I_x)/I_y \\ -pq(I_x - I_y)/I_z + N_r r \end{bmatrix}, \quad \Delta F(\bar{x}_2) = \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}, \quad d = \begin{bmatrix} d_v \\ d_p \\ d_q \\ d_r \end{bmatrix}.$$

In this section, we will propose a prescribed performance fault tolerant tracking control based on NN disturbance observer. Firstly, for tackling output performance constraint problem, the prescribed performance function technique is employed.

Define the tracking error of altitude and attitude angles as

$$Z_1 = x_1 - y_d \tag{10}$$

where y_d indicates the reference output signals. The time derivative of tracking error Z_1 can be derived as

$$\dot{Z}_1 = F_1(x_1) + G_1(x_1)x_2 - \dot{y}_d \tag{11}$$

Meanwhile, define the performance function as [18]

$$\bar{Z}_{1i} = \ln\frac{\underline{\chi}_i + Z_{1i}/\rho_i}{\bar{\chi}_i - Z_{1i}/\rho_i}, \quad i = 1, 2, 3, 4 \tag{12}$$

where $\bar{\chi}_i$ and $\underline{\chi}_i$ are the corresponding bounds of tracking errors when the convergence time is arrived. ρ_i indicates the boundaries of prescribed performance, which satisfy the following form [18]:

$$\dot{\bar{Z}}_{1i} = \frac{\bar{\chi}_i - Z_{1i}/\rho_i}{\underline{\chi}_i + Z_{1i}/\rho_i}\left(\frac{\dot{Z}_{1i}/\rho_i}{\bar{\chi}_i - Z_{1i}/\rho_i} + \frac{\left(\underline{\chi}_i + Z_{1i}/\rho_i\right)\dot{Z}_{1i}/\rho_i}{\left(\bar{\chi}_i - Z_{1i}/\rho_i\right)^2}\right) - \frac{\bar{\chi}_i - Z_{1i}/\rho_i}{\underline{\chi}_i + Z_{1i}/\rho_i}\left(\frac{Z_{1i}\dot{\rho}_i/\rho_i^2}{\bar{\chi}_i - Z_{1i}/\rho_i} + \frac{\left(\underline{\chi}_i + Z_{1i}/\rho_i\right)Z_{1i}\dot{\rho}_i/\rho_i^2}{\left(\bar{\chi}_i - Z_{1i}/\rho_i\right)^2}\right) \tag{13}$$

In order to facilitate the presentation of the following content, the definitions of matrices are given as follows.

$$M_i\left(Z_{1i}\right) = \frac{\bar{\chi}_i + \underline{\chi}_i}{\left(\underline{\chi}_i + Z_{1i}/\rho_i\right)\left(\bar{\chi}_i - Z_{1i}/\rho_i\right)\rho_i} \tag{14}$$

$$N_i\left(Z_{1i}\right) = \frac{\bar{\chi}_i + \underline{\chi}_i}{\left(\underline{\chi}_i + Z_{1i}/\rho_i\right)\left(\bar{\chi}_i - Z_{1i}/\rho_i\right)\rho_i^2} Z_{1i}\dot{\rho}_i \tag{15}$$

In addition, we have $M = diag(M_1, M_2, M_3, M_4)$ and $N = [N_1, N_2, N_3, N_4]^T$. Invoking (14) and (15), the time derivative of transformation error can be rewritten as

$$\dot{\bar{Z}}_1 = M\left(Z_1\right)\left(F_1\left(x_1\right) + G_1\left(x_1\right)x_2 - \dot{y}_d\right) - N\left(Z_1\right) \tag{16}$$

Assume that x_2^* indicates the virtual control of velocity loop, and it can be designed in the form of

$$x_2^* = M_1^{-1}\left(Z_1\right)\left(G_1^{-1}\left(x_1\right)\left(-F_1\left(x_1\right) - K_1\bar{Z}_1 + \dot{y}_d\right) + N_1\left(Z_1\right)\right) \tag{17}$$

where $K_1 = K_1^T > 0$ is the design constant.

Substituting (17) into (16) yields

$$\dot{\bar{Z}}_1 = -K_1\bar{Z}_1 + M\left(Z_1\right)G_1\left(x_1\right)\left(x_2 - x_2^*\right) \tag{18}$$

Choose the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\bar{Z}_1^T\bar{Z}_1 \tag{19}$$

Invoking (18), the derivative of V_1 with respect to time yields

$$\dot{V}_1 = -\bar{Z}_1^T K_1\bar{Z}_1 + \bar{Z}_1^T M\left(Z_1\right)G_1\left(x_1\right)\left(x_2 - x_2^*\right) \tag{20}$$

According to the definition of virtual control signal x_2^*, the velocity tracking errors of altitude and attitude angles can be defined in the form of

$$Z_2 = x_2 - x_2^* \tag{21}$$

Substituting (9) into (21), we obtain

$$\dot{Z}_2 = F_2\left(\bar{x}_2\right) + G_2\left(\bar{x}_2\right)u + \Phi\left(u\right) + \Delta F\left(\bar{x}_2\right) + d - \dot{x}_2^* \tag{22}$$

The RBFNNs are employed to approximate the unknown nonlinear function which is given by

$$\Delta F\left(\bar{x}_2\right) + \Phi\left(u\right) = W^{*T}S\left(\bar{x}_2, u\right) + \varepsilon^* \tag{23}$$

where W^* indicates the optimal NN weight, $S\left(\bar{x}_2, u\right)$ represents the basis function, and ε^* means the optimal approximate error. Furthermore, define the compound disturbance

$D = d + \varepsilon^*$, and $\hat{D}$ indicates the disturbance estimate value. Invoking (22) and (23), we obtain

$$\dot{Z}_2 = F_2(\bar{x}_2) + G_2(\bar{x}_2)u + W^{*T}S(\bar{x}_2, u) + D - \dot{x}_2^* \tag{24}$$

The NN disturbance observer can be designed as

$$\dot{\eta} = -\Gamma\eta + \psi\left(\bar{x}_2, u^f, \hat{W}\right) + \hat{D}$$
$$\psi\left(\bar{x}_2, u^f, \hat{W}\right) = \Gamma x_2 + F_2(\bar{x}_2) + G_2(\bar{x}_2)u + L^{-1}\hat{W}^T S(\bar{x}_2, u) \tag{25}$$

where η denotes the state of NN disturbance observer, and L is the NN disturbance observer gain. $\hat{W}$ indicates the NN approximate weight matrix, and its adaptive update law can be designed as

$$\dot{\hat{W}} = \Lambda\left(S(\bar{x}_2, u)L^{-1}(\tilde{\eta} + Z_2) + \sigma_0\hat{W}\right) \tag{26}$$

where $\sigma_0 > 0$ and $\Lambda^T = \Lambda > 0$ are the design parameters. In addition, we define the nominal estimate error of NN disturbance observer as

$$\tilde{\eta} = x_2 - \eta \tag{27}$$

Invoking (25), we obtain

$$\dot{\tilde{\eta}} = -\Gamma\tilde{\eta} - L^{-1}\tilde{W}^T S(\bar{x}_2, u) - \tilde{D} \tag{28}$$

According to the definition of NN, it is obvious that the approximation error is bounded. As can be seen from (25), the structure of the NN disturbance observer contains disturbance estimation values. Next, we will discuss the design of nonlinear disturbance observer, which is designed to estimate composite disturbances D. The nonlinear disturbance observer is proposed as

$$\hat{D} = L(x_2 - \vartheta) \quad \dot{\vartheta} = F_2(\bar{x}_2) + G_2(\bar{x}_2)u + L^{-1}\hat{W}^T S(\bar{x}_2, u) + \hat{D} \tag{29}$$

where $\hat{D}$ represents the estimate output of nonlinear disturbance observer. Define the nonlinear disturbance observer error as

$$\tilde{D} = D - \hat{D} \tag{30}$$

According to the disturbance observer error (30) and dynamic of UH system, we obtain the time derivative of nonlinear disturbance observer error as

$$\dot{\tilde{D}} = \dot{D} - L\left(\dot{x}_2 - \dot{\vartheta}\right) = \dot{D} - \tilde{W}^T S(\bar{x}_2, u) - L\tilde{D} \tag{31}$$

Based on the NN observer (25) and adaptive disturbance observer (29), the prescribed performance based fault-tolerant tracking control law is designed as

$$u = G_2^{-1}(\bar{x}_2)\left(-K_2 Z_2 - F_2(\bar{x}_2) - L^{-1}\hat{W}^T S(\bar{x}_2, u) - \hat{D} + \dot{x}_2^* - G_1^T(x_1)M^T(Z_1)Z_1\right) \tag{32}$$

where $K_2 = K_2^T > 0$ is the design constant.

Choose the Lyapunov function candidate as

$$V_2 = \frac{1}{2}Z_2^T Z_2 + \frac{1}{2}\tilde{\eta}^T\tilde{\eta} + \frac{1}{2}tr\left(\tilde{W}\Lambda^{-1}\tilde{W}\right) + \frac{1}{2}\tilde{D}^T\tilde{D} \tag{33}$$

Invoking (18), the derivative of V_2 with respect to time yields

$$\begin{aligned}\dot{V}_2 = &-Z_2^T K_2 Z_2 - Z_2^T G_1^T(x_1)M^T(Z_1)\bar{Z}_1 + L^{-1}Z_2^T\tilde{W}^T S(\bar{x}_2, u)\\ &+ Z_2^T\tilde{D} - \tilde{\eta}^T\Gamma\tilde{\eta} - L^{-1}\tilde{\eta}^T\tilde{W}S(\bar{x}_2, u) - \tilde{\eta}^T\tilde{D}\\ &- L^{-1}(\tilde{\eta} + Z_2)^T\tilde{W}^T S(\bar{x}_2, u) - \sigma_0 tr\left(\tilde{W}^T\hat{W}\right)\end{aligned}$$

$$+\tilde{D}^T\dot{D} - \tilde{D}^T\tilde{W}^T S(\bar{x}_2, u) - L\tilde{D}^T\tilde{D} \tag{34}$$

Then, we have

$$\begin{aligned}\dot{V}_2 = & -Z_2^T K_2 Z_2 - \tilde{\eta}^T\Gamma\tilde{\eta} - L\tilde{D}^T\tilde{D} - Z_2^T G_1^T(x_1) M^T(Z_1)\bar{Z}_1 \\ & + Z_2^T\tilde{D} - \tilde{\eta}^T\tilde{D} - \sigma_0 tr\left(\tilde{W}^T\hat{W}\right) + \tilde{D}^T\dot{D} - \tilde{D}^T\tilde{W}^T S(\bar{x}_2, u)\end{aligned} \tag{35}$$

From the property of Young inequality, we obtain

$$-\tilde{D}^T\tilde{W}^T S(\bar{x}_2, u) \le \gamma\tau^2\left\|\tilde{D}\right\|^2 + \frac{1}{\gamma}\left\|\tilde{W}\right\|^2 \tag{36}$$

$$-tr\left(\tilde{W}^T\hat{W}\right) \le -\frac{1}{2}\left\|\tilde{W}\right\|^2 + \frac{1}{2}\|W^*\|^2 \tag{37}$$

$$\tilde{D}^T\dot{D} \le \frac{\sigma_d}{2}\left\|\tilde{D}\right\|^2 + \frac{1}{2\sigma_d}\bar{D}^2 \tag{38}$$

$$Z_2^T\tilde{D} \le \frac{1}{2}\|Z_2\|^2 + \frac{1}{2}\left\|\tilde{D}\right\|^2 \tag{39}$$

$$-\tilde{\eta}^T\tilde{D} \le \frac{1}{2}\|\tilde{\eta}\|^2 + \frac{1}{2}\left\|\tilde{D}\right\|^2 \tag{40}$$

where τ is a positive constant which satisfies $\left\|\tilde{W}\right\|^2 \le \tau$. γ and σ_d are the positive design parameters. Thus, the time derivative of V_2 can be rewritten as

$$\begin{aligned}\dot{V}_2 = & -Z_2^T K_2 Z_2 - \tilde{\eta}^T\left(\Gamma - \frac{1}{2}I_m\right)\tilde{\eta} - \left(L - \frac{1}{2} - \frac{\sigma_d}{2} - \gamma\tau^2\right)\tilde{D}^T\tilde{D} \\ & - \left(\sigma_0 - \frac{1}{2} - \frac{1}{\gamma}\right)\left\|\tilde{W}\right\|^2 + \frac{1}{2\sigma_d}\bar{D}^2 - Z_2^T G_1^T(x_1) M^T(Z_1)\bar{Z}_1\end{aligned} \tag{41}$$

According to the Lyapunov function candidate (19) and (33), we propose one total Lyapunov function as

$$V = V_1 + V_2 \tag{42}$$

Invoking (20) and (35), we have

$$\begin{aligned}\dot{V} = & -\bar{Z}_1^T K_1\bar{Z}_1 - Z_2^T K_2 Z_2 - \left(L - \frac{1}{2} - \frac{\sigma_d}{2} - \gamma\tau^2\right)\tilde{D}^T\tilde{D} \\ & - \tilde{\eta}^T\left(\Gamma - \frac{1}{2}I_m\right)\tilde{\eta} - \left(\sigma_0 - \frac{1}{2} - \frac{1}{\gamma}\right)\left\|\tilde{W}\right\|^2 + \frac{1}{2\sigma_d}\bar{D}^2\end{aligned} \tag{43}$$

According to Lemma 2.2, it can be concluded that all signals of closed-loop system are uniformly bounded, and the constrained output tracking errors are never violated.

In summary, the above design procedure of fault-tolerant tracking control scheme using prescribed performance method for the UH system (1) with fault, disturbance and uncertainties can be concluded using the following theorem.

Theorem 3.1. *Consider the altitude and attitude tracking system (1), the fault-tolerant tracking control law using prescribed performance technique is decided by (17) and (32), the NN disturbance observer is designed as (25), the NN weight update law is chosen as (26), the nonlinear disturbance observer is given by (29), then we can conclude that all closed-loop system signals* $\bar{Z}_1$, Z_2, $\tilde{\eta}$, $\tilde{W}$, $\tilde{D}$ *are bounded and the tracking errors of nonlinear UH altitude and attitude system converge to a specified bounded region. In addition, the output tracking errors satisfy the requirement of prescribed performance boundary.*

4. **Simulation Results.** In this section, the simulation results about UH system are utilized for illustrating the effectiveness of the designed fault tolerant control approach in this paper. Consider the altitude and attitude system of the UH with the following parameters.

TABLE 1. Parameters of UH

Symbol	Value	Unit
m	8.2	kg
g	9.8	kg·m^2
I_x	0.305	kg·m^2
I_y	0.684	kg·m^2
I_z	0.787	kg·m^2
L_a	55.86	1/s^2
L_b	708.02	1/s^2
M_a	345.19	1/s^2
M_b	−23.03	1/s^2
N_{col}	256.42	deg/s^2
N_{ped}	2095.16	deg/s^2
Z_{col}	−5.71	deg/s^2
N_r	−11.445	1/s

The controller design parameters can be chosen as follows: $K_1 = diag\{3, 3, 3\}$, $K_2 = \{5, 5, 5\}$, the weight update law design parameters are chosen as $\Gamma = 0.1$, $L = 1$, $\sigma_0 = 10.8$. The width vector of Gaussian function is set as $[-3, -2, -1, 0, 1, 2, 3]^T$, and the center vector of Gaussian function is set as $[1, 1, 1, 1, 1, 2, 3]^T$. The reference trajectory is given by $y_d = [0.5\sin(t)\text{rad}, 0.5\sin(t)\text{rad}, 0.5\sin(t)\text{rad}, 5\text{m}]^T$. In addition, the system uncertainty is set as $\Delta v = 0.2v$, $\Delta p = \Delta q = \Delta r = \cos(\phi)$, and the disturbance is set as $d(t) = 4.5$. The state dependent nonlinear actuator fault function $\zeta_i = (\phi \times \psi)^2$.

Under the proposed adaptive fault tolerant control scheme, the simulation results of UH are presented in Figure 1-Figure 8. Figure 1-Figure 3 indicate the UH attitude tracking results. In Figure 1-Figure 3, the solid lines denote the real attitude response, while the dashed lines denote the desired attitude tracking signals. They show that the

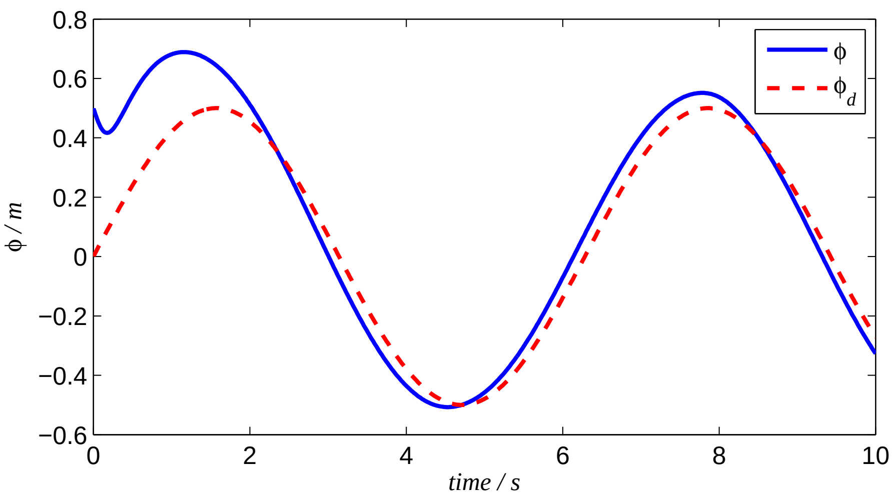

FIGURE 1. The response of roll angle

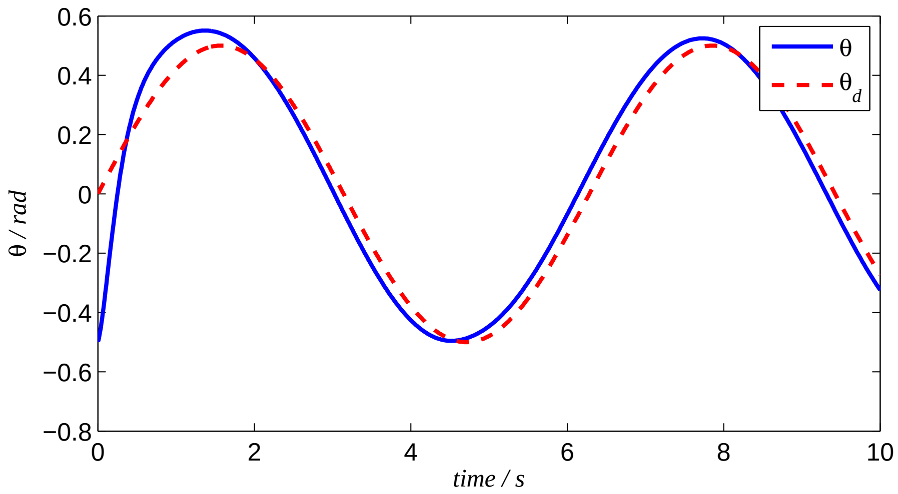

FIGURE 2. The response of pitch angle

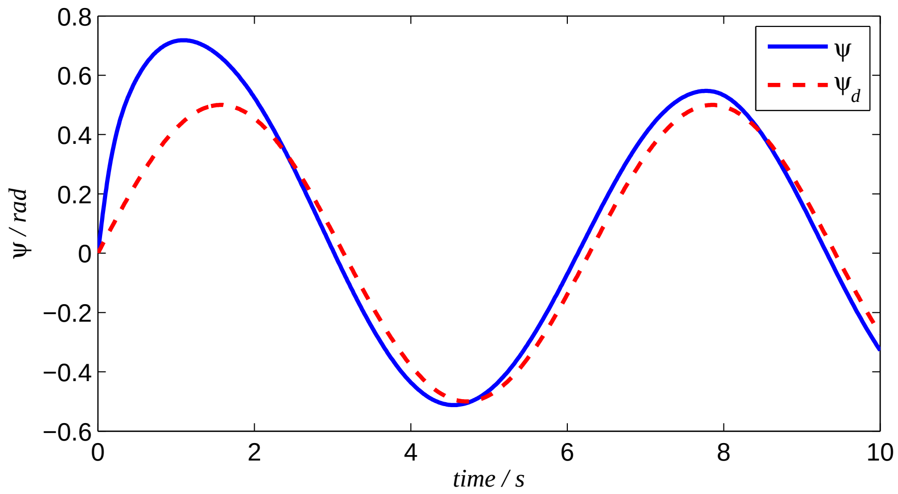

FIGURE 3. The response of yaw angle

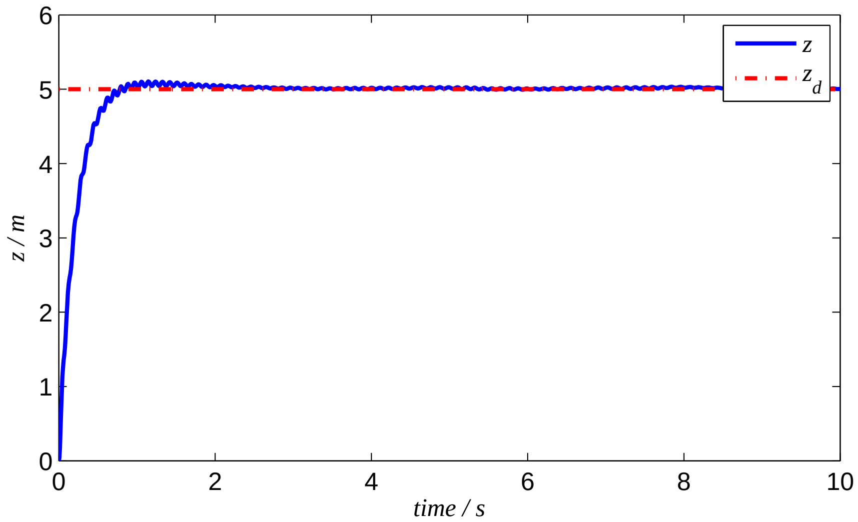

FIGURE 4. The response of altitude

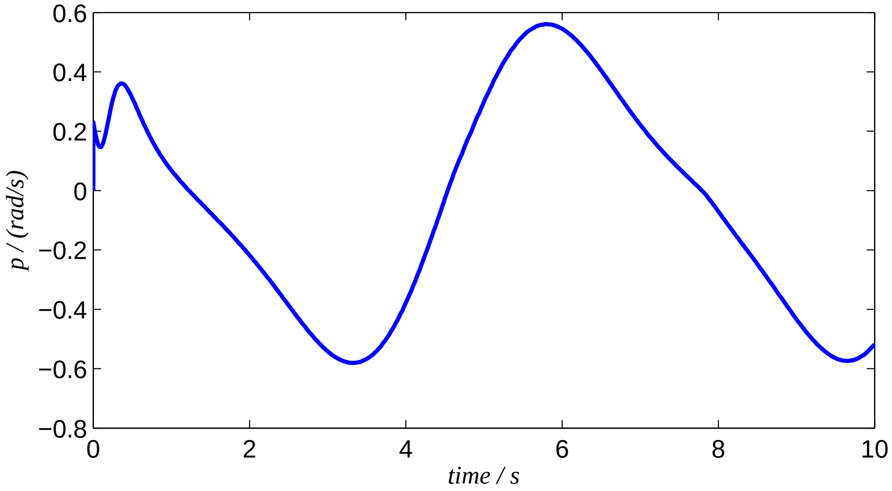

FIGURE 5. The response of roll angle velocity

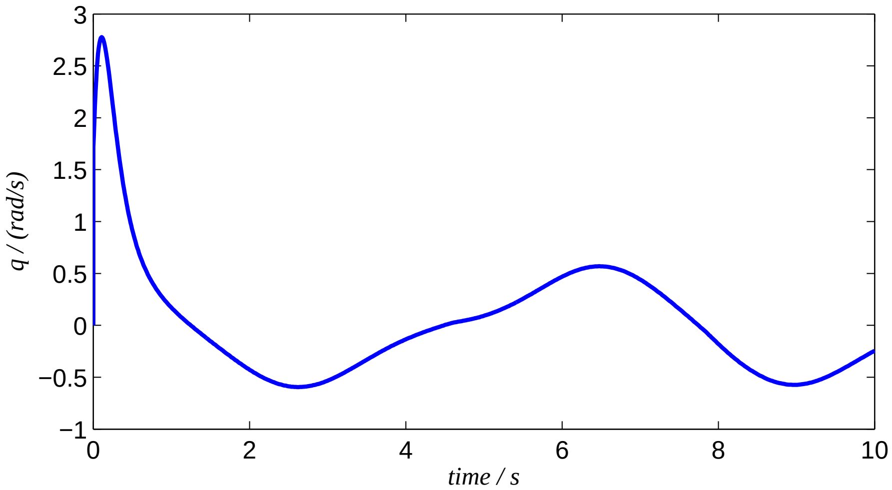

FIGURE 6. The response of pitch angle velocity

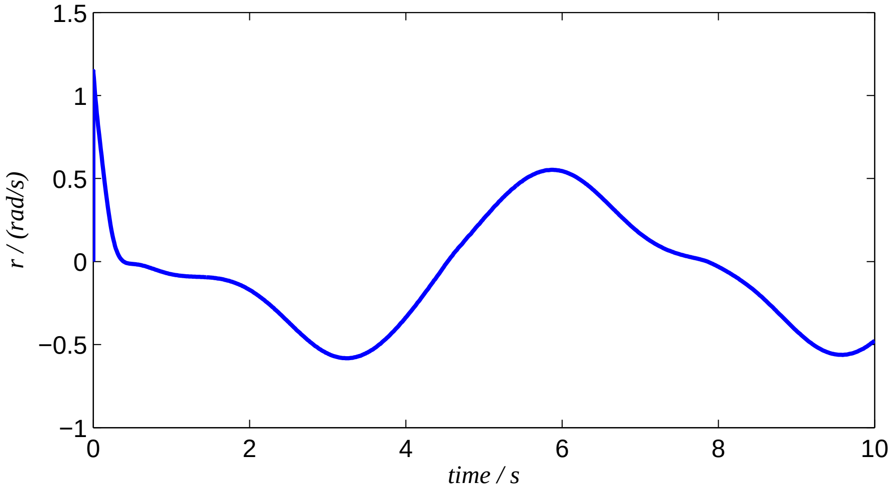

FIGURE 7. The response of yaw angle velocity

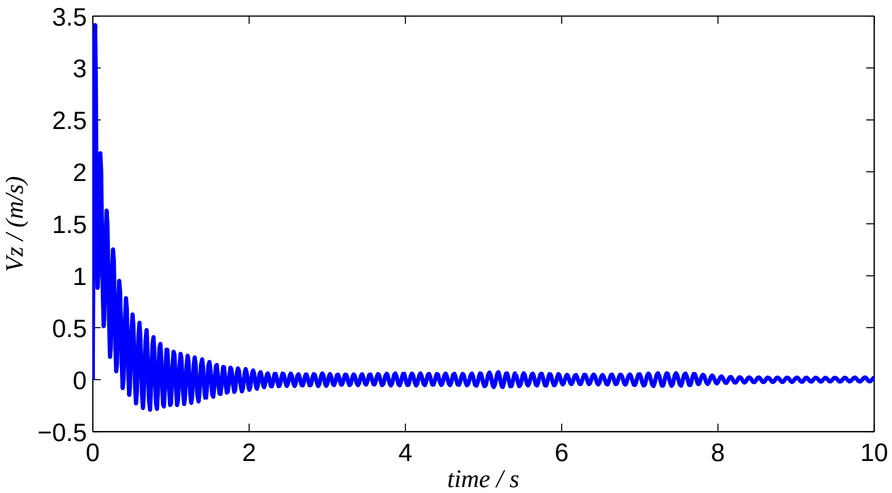

FIGURE 8. The response of altitude velocity

satisfactory tracking control performance is realized for the UH with actuator gain faults. Figure 4 shows the altitude tracking result and the bounded tracking error is guaranteed. Figure 5-Figure 8 present the attitude angle velocities and vertical velocities of the UH, respectively. From the presented simulation results, we can see that all tracking error signals of closed-loop system for the UH system with disturbance and fault-tolerant are asymptotically stable. Thus, the adaptive robust fault-tolerant tracking control method using prescribed performance function method is valid.

5. **Conclusions.** In this paper, an adaptive robust fault-tolerant tracking control method using prescribed performance function method has been proposed for nonlinear altitude and attitude system of the UH with actuator gain faults. The boundness of tracking errors of closed-loop system is proved by Lyapunov stability theory. The simulation results have been given to illustrate that the proposed controller design method has robust ability to the uncertainties for UH systems. In the future work, the adaptive robust fault-tolerant tracking control scheme can be extended to the flight control for the six degrees of freedom dynamics of the UAH with actuator gain faults, system uncertainties and unknown disturbances.

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