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A FINITE MEMORY STRUCTURE SMOOTHING FOR DISCRETE-TIME SUSPENSION SYSTEM WITH UNKNOWN ROAD DISTURBANCE

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Abstract. *In this paper, the finite memory structure (FMS) smoother is applied for the state estimation of automotive suspension system with random walk road disturbance. The quarter-car automotive suspension system is expressed in the discrete-time state-space model. The road disturbance is treated as the auxiliary state and thus the augmented state-space model is derived. Through computer simulations for a bus suspension system, it is shown that the FMS smoother based estimation can be better than both infinite memory structure (IMS) and FMS filter based estimations. In addition, the FMS smoother based estimation can be comparable to IMS and FMS filter based estimations after the effect of the road disturbance completely disappears.*

Keywords: Automotive suspension system, Finite memory structure filter, Infinite memory structure filter, Road disturbance

1. Introduction.

In recent years, wheeled motor vehicles industry has shown a tendency of replacing electromechanical components by mechatronic systems with intelligent and autonomous properties. An active suspension system, one of the critical components in the presence of motor vehicles, has attracted many researchers in the past few decades [1, 2].

For the state-space model of the automotive suspension system with noises, state estimation filters such as IMS filter and FMS filter should be applied to estimating the proper state of the controller and used the updated ideal states in each step in order that the suspension performance will be improved. The IMS filter, such as well-known Kalman filter [3], has been successfully applied for the automotive suspension system [4, 5]. In contrast to the IMS filter, the FMS filter using only finite measurements on the most recent window has been known inherently to be bounded input/bounded output stable and more robust against temporary uncertainties such as model uncertainty, unknown input, and incomplete measurement information [6, 7, 8, 9, 10]. Therefore, the FMS filter was also applied successfully for the state estimation filtering of automotive suspension systems with random walk road disturbance [11].

Meanwhile, because the FMS filter is a causal filter providing estimates for states at given times based only on the relative past, the estimates exhibit a delay. Hence, the FMS smoother has been developed for estimation problems where there is a fixed delay between a measurement and the availability of its estimate [12, 13, 14, 15]. This fixed delay is associated only with the availability of the estimate – not with an error in the actual

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estimates, as is the case with the FMS filter. Although FMS smoothers in [12, 13, 14, 15] have their own unique features, they have the following common advantages. The smoother generally utilizes more measurement information than the filter to provide state estimates, which can give more accurate estimation performance than the filter. In addition, since the smoother provides state estimates at the delayed time using measurement information up to the current time, measurement information can be reflected in advance in the presence of the state change, which can give more fast convergence than the filter. If these superiorities of the FMS smoother in [12, 13, 14, 15] can be verified in practical applications such as the automotive suspension system, this might be very informative for engineers and researchers in control and estimation areas, which is a main motivation of this paper.

Therefore, this paper proposes an FMS smoother based state estimation of the automotive suspension system with random walk road disturbance. The quarter-car automotive suspension system is expressed in the discrete-time state space model. The road disturbance considered as one of temporary uncertainties is treated as the auxiliary state and thus the augmented state-space model is derived. Finally, simulation results for a bus suspension system show that the FMS smoother based estimation can be superior to both FMS and IMS filter based estimations. Meanwhile, the FMS smoother based estimation can be comparable to the other two estimations after the effect of the road disturbance completely disappears.

This paper has the following structure. In Section 2, an augmented state-space model and its FMS smoothing algorithm are described for the discrete-time quarter-car automotive suspension system. In Section 3, computer simulations for a bus suspension system are performed. Then, concluding remarks are given in Section 4.

2. Augmented State-Space Model and FMS Smoothing Algorithm for Quarter-Car Automotive Suspension System.

A continuous-time state-space model of the quarter-car automotive suspension system can be represented by

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c \omega(t), \\ z(t) &= C_c x(t), \end{aligned} \tag{1}$$

where variables are expressed by

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} X_1(t) \\ \dot{X}_1(t) \\ Y_1(t) \\ \dot{Y}_1(t) \end{bmatrix}, \ \omega(t) = \begin{bmatrix} u(t) \\ d(t) \end{bmatrix},$$
$$z(t) = Y_1(t) = X_1(t) - X_2(t),$$

with the body displacement $X_1(t)$, the suspension displacement $X_2(t)$, the relative displacement between body and suspension $Y_1(t)$, the desired input force by the cylinder $u(t)$, the road disturbance $d(t)$. System matrices A_c, B_c, C_c, are obtained from physical parameters such as the body mass, the suspension mass, the spring constant of suspension system, the spring constant of wheel and tire, the damping constant of suspension system, the damping constant of wheel and tire as shown in [2].

In actual situations, there can be system and measurement noises in the automotive suspension system. Then, using a sampling period T the discretized automotive suspension system (1) can be extended by the ultimate discrete-time state-space model with road disturbance d_i as well as system and measurement noises w_i, v_i as follows:

$$\begin{aligned} x_{i+1} &= \Phi x_i + B_u u_i + B_d d_i + B_w w_i, \\ z_i &= H x_i + v_i, \end{aligned} \tag{2}$$

where Φ, B_u, B_d, and H are obtained by

$$\begin{aligned} \Phi &= e^{A_c T}, \\ \begin{bmatrix} B_u & B_d \end{bmatrix} &= \left(\int_0^T e^{A_c \varepsilon} d\varepsilon \right) B_c = \left(e^{A_c T} - I \right) A_c^{-1} B_c, \\ H &= C_c, \end{aligned}$$

and the initial state $\hat{x}_{i_0}$ is a random variable with a mean $\bar{x}_{i_0}$ and a covariance Σ_{i_0}. The system noise w_i and the measurement noise v_i are zero-mean white Gaussian whose covariances Q and R are assumed to be positive definite matrix. The matrix B_w for the system noise is assumed to be $[B_u\ B_d]$.

Since the road disturbance d_i is the unknown signal, it can be represented by random walk processes as

$$d_{i+1} = d_i + \delta_i,$$

where the road disturbance noise δ_i is a zero-mean white Gaussian random process with covariance Q_δ. It is noted that the random-walk process provides a general and useful tool for the analysis of unknown time-varying parameters and has been widely used in the detection and identification area. The random walk road disturbance can be treated as an auxiliary state and then the state-space model (2) can be rewritten as an augmented state-space model as

$$\begin{aligned} \begin{bmatrix} x_{i+1} \\ d_{i+1} \end{bmatrix} &= A \begin{bmatrix} x_i \\ d_i \end{bmatrix} + B u_i + G \begin{bmatrix} w_i \\ \delta_i \end{bmatrix}, \\ z_i &= C \begin{bmatrix} x_i \\ d_i \end{bmatrix} + v_i, \end{aligned} \tag{3}$$

where

$$A = \begin{bmatrix} \Phi & B_d \\ 0 & I \end{bmatrix}, \quad B = \begin{bmatrix} B_u \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} B_w & 0 \\ 0 & I \end{bmatrix}, \quad C = \begin{bmatrix} H & 0 \end{bmatrix},$$

and the system noise and the road disturbance noise term $\left[w_i^T\ \delta_i^T\right]^T$ is a zero-mean white Gaussian random process with covariance $Q = \text{diag}([Q_w\ Q_\delta])$.

The state estimation filtering has been applied to estimating two vertical velocities, body velocity and damper velocity, and relative displacement between body and suspension of the quarter-car automotive suspension with noises as well as road disturbance. The IMS filtering, such as well-known Kalman filtering [3], has been successfully applied [4, 5]. As an alternative to the IMS filter, the FMS filter has been also applied using only the most recent finite measurements on the window [11].

The FMS smoother to estimate the augmented state $\left[x_{i-l}^T\ d_{i-l}^T\right]^T$ at the delayed time $i-l$ is developed under a weighted least square criterion using only finite measurements as well as inputs on the most recent window $[i-M, i]$. The delayed time $i-l$ means there is a fixed delay between the measurement and the availability of its estimate. The positive integer d is the delay length satisfying $0 \leq l < M$ and equal to the number of discrete time steps between the delayed time $i-l$ at which the state is to be estimated and the current time i of the last measurement used in estimating it. The window initial time $i-M$ will be denoted by i_M hereafter for simplicity. Finite measurements and inputs on the most recent window $[i_M, i]$ are denoted by Z_i and U_i, respectively, and represented by

$$Z_i \triangleq \left[z_{i_M}^T \; z_{i_M+1}^T \cdots z_{i-1}^T\right]^T, \; U_i \triangleq \left[u_{i_M}^T \; u_{i_M+1}^T \cdots u_{i-1}^T\right]^T. \tag{4}$$

Using Z_i and U_i, the augmented discrete-time state-space model (3) can be expressed in the following regression form

$$Z_i - \bar{\Xi} U_i = \bar{\Gamma} \begin{bmatrix} x_{i_M} \\ d_{i_M} \end{bmatrix} + \bar{\Lambda} W_i + V_i, \tag{5}$$

where W_i and V_i have the same form as (4) for w_i, v_i, respectively, and matrices $\bar{\Gamma}$, $\bar{\Xi}$, $\bar{\Lambda}$ are as follows:

$$\bar{\Gamma} \triangleq \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{M-2} \\ CA^{M-1} \end{bmatrix}, \; \bar{\Xi} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{M-3}B & CA^{M-4}B & \cdots & 0 & 0 \\ CA^{M-2}B & CA^{M-3}B & \cdots & CB & 0 \end{bmatrix},$$

$$\bar{\Lambda} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CG & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{M-3}G & CA^{M-4}G & \cdots & 0 & 0 \\ CA^{M-2}G & CA^{M-3}G & \cdots & CG & 0 \end{bmatrix}.$$

From the discrete-time state-space model (2), the augmented state $[x_{i-l}^T \; d_{i-l}^T]^T$ at the delayed time $i - l$ is represented by

$$\begin{bmatrix} x_{i-l} \\ d_{i-l} \end{bmatrix} = A^{M-l} \begin{bmatrix} x_{i_M} \\ d_{i_M} \end{bmatrix} + \tilde{\Xi} U_i + \tilde{\Lambda} W_i, \tag{6}$$

where

$$\tilde{\Xi} \triangleq \left[A^{M-l-1}B \; \cdots \; AB \; B \; \overbrace{0 \; 0 \; \cdots \; 0}^{d} \right],$$

$$\tilde{\Lambda} \triangleq \left[A^{M-l-1}G \; \cdots \; AG \; G \; \overbrace{0 \; 0 \; \cdots \; 0}^{d} \right].$$

Therefore, using (6), the regression form (5) can be expressed in terms with $\left[x_{i-l}^T \; d_{i-l}^T\right]^T$ at the delayed time $i - l$ as follows:

$$Z_i - \Xi U_i = \Gamma \begin{bmatrix} x_{i-l} \\ d_{i-l} \end{bmatrix} + \Lambda W_i + V_i, \tag{7}$$

where

$$\Gamma \triangleq \bar{\Gamma} A^{-(M-l)},$$
$$\Lambda \triangleq \bar{\Lambda} - \bar{\Gamma} A^{-(M-l)} \tilde{\Lambda},$$
$$\Xi \triangleq \bar{\Xi} - \bar{\Gamma} A^{-(M-l)} \tilde{\Xi}.$$

The noise term $\Lambda W_i + V_i$ in (7) is zero-mean white Gaussian with covariance Π given by

$$\Pi \triangleq \Lambda \left[\text{diag}(\overbrace{Q\ Q\ \cdots\ Q}^{M}) \right] \Lambda^T + \left[\text{diag}(\overbrace{R\ R\ \cdots\ R}^{M}) \right], \tag{8}$$

where $\text{diag}(Q\ Q\ \cdots\ Q)$ and $\text{diag}(R\ R\ \cdots\ R)$ denote block-diagonal matrices with M elements of Q and R, respectively.

Now, using (7) and (8), to get the FMS smoother $\left[\hat{x}_{i-l}^T\ \hat{d}_{i-l}^T\right]^T$ given finite measurements Z_i and inputs U_i on the most recent window $[i_M, i]$, the following weighted least square cost function must be minimized:

$$\left\{ \left(Z_i - \Xi U_i\right) - \Gamma \begin{bmatrix} x_{i-l} \\ d_{i-l} \end{bmatrix} \right\}^T \Pi^{-1} \left\{ \left(Z_i - \Xi U_i\right) - \Gamma \begin{bmatrix} x_{i-l} \\ d_{i-l} \end{bmatrix} \right\}. \tag{9}$$

Taking a derivation of (9) with respect to $\left[x_{i-l}^T\ d_{i-l}^T\right]^T$ and setting it to zero, the proposed FMS smoother $\left[\hat{x}_{i-l}^T\ \hat{d}_{i-l}^T\right]^T$ is given by following simple matrix form:

$$\begin{bmatrix} \hat{x}_{i-l} \\ \hat{d}_{i-l} \end{bmatrix} = \left[\left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T \Pi^{-1} \right] \left[Z_i - \Xi U_i \right]. \tag{10}$$

As shown in (10), gain matrices $\left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T \Pi^{-1}$ and $-\left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T \Pi^{-1} \Xi$ for the FMS smoother require computation only on the interval $[0, M]$ once and are time-invariant for all windows. Thus, the on-line computation of the FMS smoother based estimation requires only filter updates. In addition, unlike the IMS smoother such as the fixed-lag Kalman smoother [16] based estimation, the FMS smoother based estimation does not need the combining of two algorithms for before and after estimated time $i - l$, the memory requirement for the save of intermediate values, and the initialization algorithm. Moreover, the FMS smoother is known to have several inherent properties such as unbiasedness, deadbeat, time-invariance, robustness [12, 13, 14, 15].

3. **Computer Simulations.** In this section, a bus suspension system is considered for simulations. Through the discretization with physical parameters in [2], system matrices for discrete-time state-space model (3) for the quarter-car automotive suspension system with road disturbance d_i can be obtained by

$$\Phi = \begin{bmatrix} 0.8628 & 0.0958 & -0.0044 & -0.0013 \\ -2.8740 & 0.8628 & 0.1998 & -0.0178 \\ 0.7829 & 0.0864 & -0.0648 & -0.0021 \\ -9.7230 & 0.4498 & 1.9570 & -0.1183 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0.0000018 \\ 0.0000337 \\ 0.0000036 \\ 0.0001143 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.1372 \\ 2.8740 \\ -0.7829 \\ 9.7230 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.0000018 & 0.1372 \\ 0.0000337 & 2.8740 \\ 0.0000036 & -0.7829 \\ 0.0001143 & 9.7230 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}. \tag{11}$$

System and measurement noise covariances are taken by $Q_w = \text{diag}\left(\left[0.02^2\ 0.02^2\right]\right)$ and $R = 0.04^2$ respectively. The road disturbance noise covariance is taken by $Q_\delta = 0.01^2$. The IMS filter, the FMS filter with $M = 10$, and the FMS smoother with $M = 10$ and $l = 3$ are compared. To make a clearer comparison of estimation performances, simulations of 30 runs are performed using different system and measurement noises, and each single simulation run lasts 400 samples.

The random walk road disturbance d_i in the automotive suspension system is emulated by step-type unknown input

$$d_i = \begin{cases} 0.4 & \text{if } 100 \leq i \leq 200, \\ 0 & \text{otherwise.} \end{cases}$$

That is, although three kinds of filters are computed by the nominal discrete-time state-space model (11), actual measurements for these three filters are obtained from the system (2) with the road disturbance.

The body velocity and the damper velocity are important variables in the automotive suspension control design. A feedback controller is designed in order that the output for the relative displacement between body and suspension should meet design specifications such as an overshoot and a settling time. Thus, the simulation work focuses on estimation filtering and smoothing of two kinds of vertical velocities and relative displacement between body and suspension. Upper figures of Figures 1-3 show RMS estimation errors of two kinds of vertical velocities and relative displacement between body and suspension for 30 simulations. In addition, lower figures show estimation errors for one of 30 simulations. As shown in result figures, the FMS smoother based estimation can be better than IMS and FMS filter based estimations in terms of error magnitude and error convergence. For body and damper velocities, relative displacement between body and suspension, the estimation error of the FMS smoother based estimation is smaller than those of IMS and FMS filter based estimations on the interval where the unknown road disturbance exists. In addition, the convergence of estimation error is faster than IMS and FMS filter based estimations after the effect of the road disturbance disappears. Therefore, the FMS smoother based estimation for the automotive suspension system can be more robust than the other two estimations when applied to the quarter-car automotive suspension with the

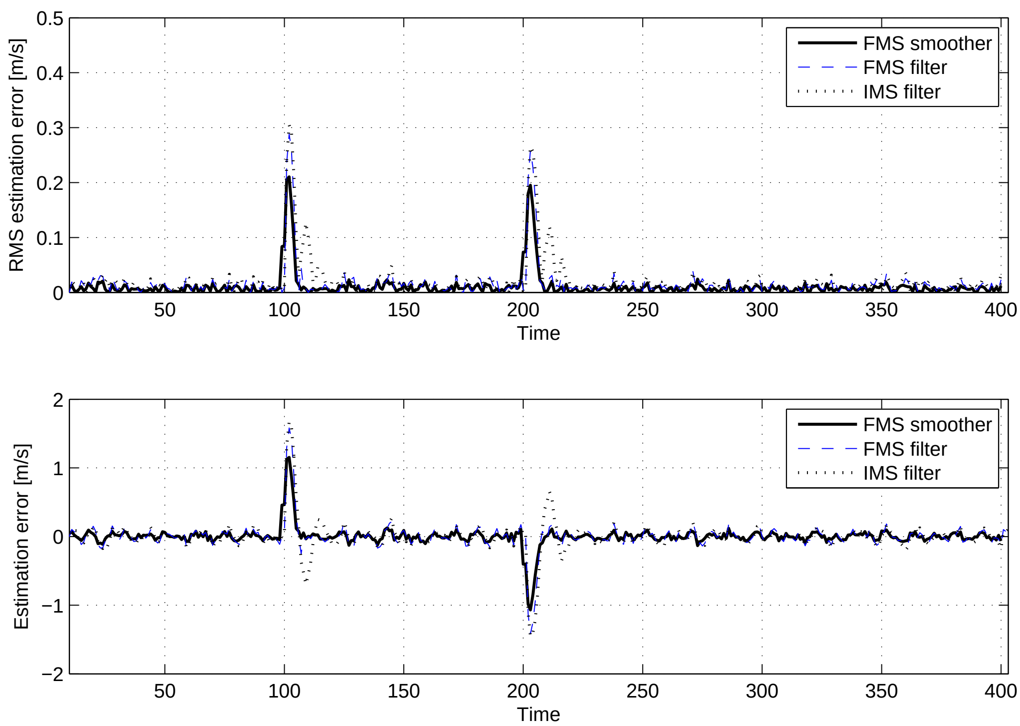

FIGURE 1. Simulation results for body velocity

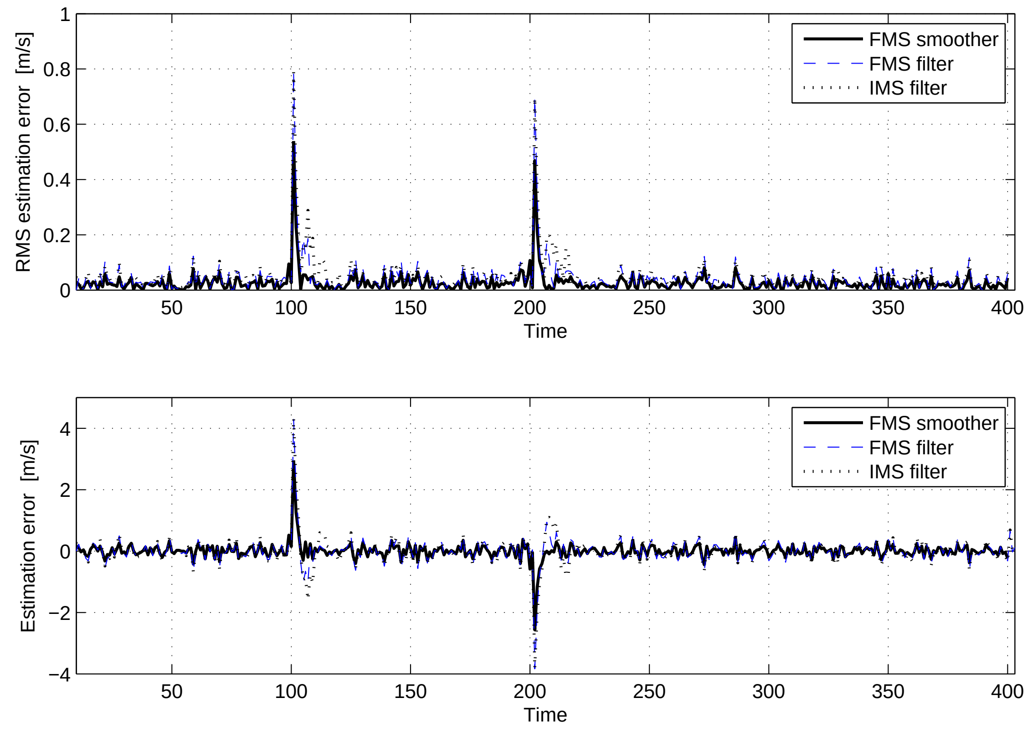

FIGURE 2. Simulation results for damper velocity

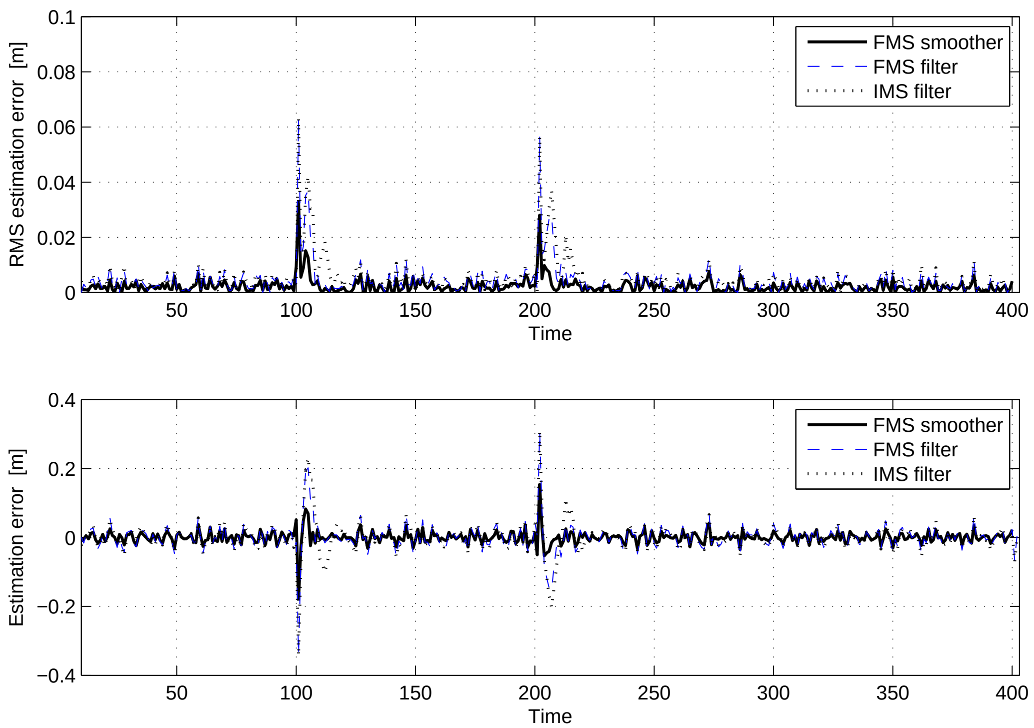

FIGURE 3. Simulation results for relative displacement between body and suspension

unknown road disturbance, although the FMS smoother is designed with no consideration of robustness.

4. **Conclusions.** This paper has applied the FMS smoother for the state estimation of quarter-car automotive suspension system where the road disturbance is considered as one of temporary uncertainties. The augmented discrete-time state-space model for the quarter-car automotive suspension system has been derived to treat the road disturbance as the auxiliary state. Simulation results for a bus suspension system have shown that the FMS smoother based estimation can be better than both FMS and IMS filter based estimations. Meanwhile, when the effect of the road disturbance completely disappears, the performance of the FMS smoother based estimation is similar to the other two estimations. This means that the FMS smoother based estimation can show superior performance to the temporary uncertain automotive suspension system with the road disturbance rather than the nominal system that does not have temporary uncertainty.

As mentioned before, there can be temporary uncertainties such as a model uncertainty, an unknown input, and incomplete measurement information, for the state-space model of the automotive suspension system. Therefore, alternative state estimation smoothers should be researched to diminish the effects of all kinds of temporary uncertainties in the future although this paper has dealt with the unknown input.

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