International Journal of Innovative
Computing, Information and Control
Volume 16, Number 1, February 2020

ICIC International ©2020 ISSN 1349-4198
pp. 29–43

SECOND-ORDER SLIDING MODE FOR POSITION AND ATTITUDE TRACKING CONTROL OF QUADCOPTER UAV: SUPER-TWISTING ALGORITHM

Djihad Matouk[1], Foudil Abdessemed[1], Oussama Gherouat[2]
and Younes Terchi[3]

[1]L.E.A Laboratory
Department of Electronics
Mostefa Ben Boulaid University of Batna 2
53, Route de Constantine, Fésdis, Batna 05078, Algeria
d.matouk@univ-batna2.dz

[2]L.S.I Laboratory
Department of Electronics
Ferhat Abbas University of Setif 1
Cité el maabouda, Sétif 19000, Algeria
oussama.gherouat@univ-setif.dz

[3]Department of Electronics
Ferhat Abbas University of Setif 1
Cité el maabouda, Sétif 19000, Algeria
terchi.younes@gmail.com

Received May 2019; revised September 2019

Abstract. *Currently, tracking control of UAVs type quadcopter is a hot spot for researchers. In order to solve this control problem, the choice of the appropriate controller, according to the desired objectives, is a fundamental concern. Regardless of the harmful chattering phenomena, sliding mode control (SMC) has shown acceptable performance. In this paper, trajectory tracking control of the quadcopter is carried out through second-order sliding mode control (2-SMC). It is one of the alternative solutions that preserves the advantages of the conventional SMC while avoiding the undesirable chattering effect. Specifically, the super-twisting algorithm, which is modified 2-SMC that prevents the need for any sliding variable derivative, is adopted. For the sake of ensuring stability and enhancing the quadcopter tracking trajectory, a global block control based on the super-twisting algorithm is designed. The proposed technique offers high stability since it allows the derivation of the appropriate control law for each position and attitude state. Simulation results illustrate the efficiency of this approach in terms of stability and tracking control. A comparison study with classical SMC and type-2 fuzzy logic controller is given to clarify the effectiveness of the proposed 2-SMC.*

Keywords: Quadcopter UAV, Full trajectory tracking, Nonlinear control, Second-order sliding mode control, Super-twisting algorithm

1. **Introduction.** Over the past two decades, quadcopters have attracted great interest in the research community. This type of mini-rotorcraft is emerging as a widely used platform for many purposes, mainly to satisfy the military needs but also those of the civilians. Research on quadcopters seeks to facilitate human life. Nowadays, they are capable of fulfilling more complex and dangerous missions. The extensive application that quadcopters provide has put forward higher requirements for good performance [1,2]. The particular characteristic that quadcopters own highlights their potential use ranging from

DOI: 10.24507/ijicic.16.01.29

mapping to rescue applications. However, their small size and maneuverability is also a problem when attempting to exclude human interference.

Among a wide range of challenging items, research on the control of quadcopters is of great significance. The aforementioned advantages of quadcopters require accurate control strategy to achieve effective autonomy. In this area of quadcopter control, the existing literature demonstrates that this problem has been treated through three main control strategies [3]. Initial attempts were based on linear control such as PID [4-6], LQ/LQR [7,8] and H_∞ [9]. The authors in [10] and [11] have developed controllers based on the linear dynamics model and neglected inherent nonlinearity of attitude dynamics. However, performance of the linear control strategy is restricted and valid only for some conditions [12,13]. Therefore, model-based nonlinear controllers have been developed to deal with the limitations set by the linear methods. For instance, backstepping [14-17] and sliding mode control (SMC) [18-20] are nonlinear control techniques that have mainly been applied for the stabilization and trajectory tracking of the quadcopter as they both have been proved to fulfill this task efficiently. Nevertheless, researchers have shifted the focus to the SMC since it is efficient for handling systems with large uncertainties, time-varying properties, and nonlinearities. In [21], backstepping control and SMC have been examined in a mission of tracking the desired path. The superiority of SMC under model uncertainties and external disturbances was shown. Another comparison between backstepping and SMC has been conducted in [22] and has shown that performance of SMC is smoother and faster. However, in [23] the application of the SMC to the OS4 model has provided average results partly due to the switching nature of the controller. The third category is learning-based control methods which disengage from the system model. Fuzzy logic-based controller (FLC) [24,25] has good adaptability, strong robustness and fault tolerance. However, it suffers from low accuracy in the steady-state.

Second-order sliding mode control (2-SMC) is one of the important techniques which have the ability to overcome the undesired chattering in classical SMC while preserving its main advantages. Various 2-SMC algorithms have been proposed and applied on the quadcopter. Zheng et al. have tackled the tracking trajectory problem in [26] based on 2-SMC. In their flight control architecture, the user-defined inputs are the desired values of the fully actuated states z^d and ψ^d along with the four coupled states $\left(x^d, \theta^d\right)$ and $\left(y^d, \phi^d\right)$. Reference [27] proposes a 2-SMC where the sliding surface is based on the PID dynamics. For the position states (x and y), a simplified form of their coupled expression is used as detailed in [28]. The obtained results demonstrate that the PID-based 2-SMC achieves superior performance compared with conventional SMC. In [29], a 2-SMC using super-twisting algorithm is combined with the terminal sliding mode in order to design a non-singular sliding mode controller, which is used for the attitude tracking of the quadcopter and the avoidance of chattering. The effectiveness of this methodology has been demonstrated through simulation, and the tracking error has been shown to converge to zero in finite time. Aiming at shortening the transient time, Sumantri et al. proposed in [30] a 2-SMC for a robust tracking controller by employing a nonlinear sliding surface, which allows for the closed-loop dynamics to have a variable damping ratio. For the same aim, and by employing linear sliding manifold, a robust 2-SMC has been proposed for altitude tracking in [31]. The controllers proposed in [27] and [31] handle hover flight only, whereas those proposed in [28] and [29] handle attitude control only. However, the control of both position and attitude of the aircraft is required in most real-life applications of quadcopters like rescue, search, and mapping. Moreover, the full control allows powerful and flexible structure of the controller [28]. Nevertheless, the underactuated property of quadcopters leads to strong coupling between longitudinal/lateral motion and rotary motions and hence the control of the quadcopter is a difficult task [32].

Due to the requirement of the full control of position and attitude in most real-life applications of quadcopters, and the efficiency of the 2-SMC over other controllers, the main objective of the present work is to investigate the 2-SMC for quadcopter 3D trajectory tracking control and stabilization. For this purpose, we develop a set of six controllers based on the 2-SMC that fully control the quadcopter's navigation in space. Because of the underactuated property of the quadcopter, we adopt a control structure that consists of two loops, namely, inner and outer loops. The latter is dedicated to the control of roll, pitch, and yaw movements as well as the altitude, whereas the former is devoted to the control of the longitudinal and lateral motions. Furthermore, in order to deal with the coupling between longitudinal/lateral translation and rotary motions, we use a correction block that provides a connection between the inner and outer loops and restricts the proposed controller to operate within the boundaries of the underactuated property of the quadcopter. That is, the correction block is used to satisfy the coupling between longitudinal/lateral translation and rotary motions.

The remainder of this paper is structured as follows. In Section II, some preliminaries are prepared to give the dynamic model of the quadcopter UAV according to Newton-Euler formalism and to formulate the problem. Section III is devoted to the synthesis of the control laws based on 2-SMC for stabilizing and tracking of a quadcopter. Simulation results are demonstrated and discussed in Section IV and a conclusion in Section V completes the work.

2. **Preliminaries and Problem Statement.** Figure 1 depicts the schematic of a quadcopter. It is a rotary-wing UAV composed of four fixed pitch propellers mounted on four arms in a cross form. Despite the possibility of the plus "+" form, the cross "x" configuration remains preferred [33]. Controlling the rotational speeds of blades permits movement manipulation of the quadcopter. It can be lifted, propelled forward and laterally, and the hover position control is achieved by maintaining a constant value of the total thrust force. The sense of rotation of each rotor is very unique. Two rotors of the same arm spin in a direction while the two others spin in the opposite direction. This is to cancel the yawing moment and to create the desired yaw motion.

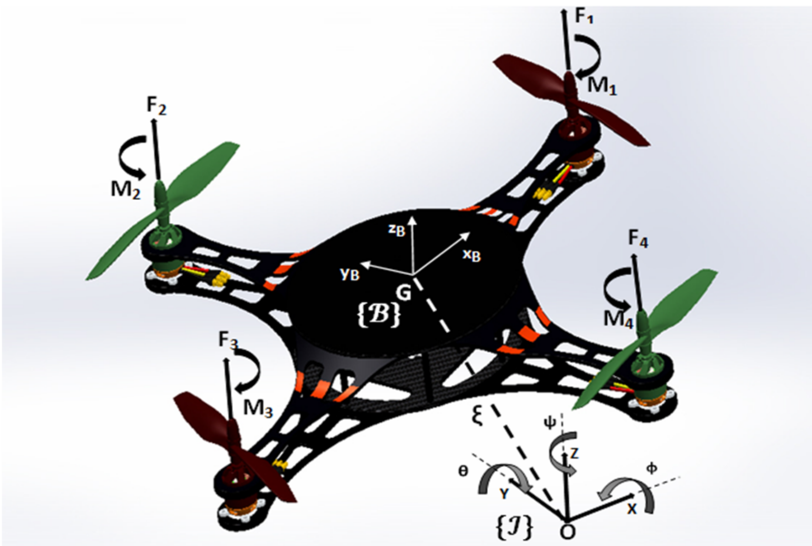

FIGURE 1. Structure of the quadcopter model

To describe the motion of the quadcopter, six degrees of freedom (DoF) are required. These are three translation and three rotation motions. However, as it is well-known, the quadcopter has only four control inputs, i.e., the speeds of rotors ω_i, $i = 1, 2, 3, 4$. Because of this property, the quadcopter is considered a nonlinear underactuated complex system.

In order to model the dynamics of the system, let us consider the inertial frame of reference $\{\mathcal{I} = O; X, Y, Z\}$ and a coordinate system attached to the aircraft $\{\mathcal{B} = G; x_B, y_B, z_B\}$, where G is the vehicle's center of gravity (see Figure 1). The states that define the quadcopter are described by: the position ξ and orientation η vectors of the body-fixed frame referring to the inertial frame, and their time derivatives given by the velocity vector V and the angular rates Ω in the mobile frame.

$$\rho = [\xi, \eta]^T;\ \xi = [x, y, z]^T;\ \eta = [\phi, \theta, \psi]^T$$

$$\vartheta = [V, \Omega]^T;\ V = \left[V_x^B, V_y^B, V_z^B\right]^T;\ \Omega = [\Omega_x, \Omega_y, \Omega_z]^T$$

The relationships between the variables expressed in the fixed and mobile frames are obtained through the transformation matrices R_t and R_r given by (1) and (2), which ensure the transformation of linear and angular variables respectively.

Remark 2.1. *Because of the large dimension of the matrix R_t, the notations c and s are used only in (1) to denote, respectively,* cos *and* sin *functions.*

$$R_t = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{1}$$

$$R_r = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{2}$$

Remark 2.2. *It should be noted that for small angles, R_r can be taken approximately as the identity matrix. That is, $\Omega \approx \dot{\eta}$.*

In regards to the system modeling, we go deeply into the influencing factors on the quadcopter dynamics. The objective is to introduce a quadcopter dynamics model as realistically as possible. We consider five effects; the gravity F_G, the total thrust force T and the reactive torques M_i induced by the propulsion system, the drag force F_D and the gyroscopic effect.

The four actuators produce lift forces and moments where both the lift force F_i and the moment M_i of each actuator are proportional to the square of its angular speed ω_i. Moreover, the moments are around the z_b axis and their directions oppose the sense of rotation, whereas each force is perpendicular to the plane of the appropriate propeller i.

$$F_i = k_F \sum_{i=1}^{4} \omega_i^2 \text{ and } M_i = k_M \omega_i^2$$

with the coefficients k_F and k_M being the lift and drag factors. The thrust force T is the total force generated by the propeller system:

$$T = \sum_{i=1}^{4} F_i$$

We continue on the rest of the forces and moments acting on the quadcopter. The gravitational force $F_G = -mg$ and the drag force $F_D = -k_d\dot{\xi}$ where k_d is the drag coefficient. The main moments that influence the dynamics of the vehicle are:

- The roll actuators action is a rotation around the x-axes due to the torque τ_ϕ resultant from the difference between the lift forces F_2 and F_4, thus $\tau_\phi = l \cdot (F_4 - F_2)$.
- The pitch actuators action is a rotation around the y-axes due to the torque τ_θ resultant from the difference between the lift forces F_1 and F_3, thus $\tau_\theta = l \cdot (F_3 - F_1)$.
- The rotation of the quadcopter around the z-axes is due to the counter-torque unbalance τ_ψ; it is given by:

$$\tau_\psi = \sum_{i=1}^{4} (-1)^i M_i = -M_1 + M_2 - M_3 + M_4.$$

We consider in this work the aerodynamic friction torque $\tau_a = -k_a \Omega^2$, where k_a is the aerodynamic friction coefficient. Also, we consider the gyroscope torque τ_g that gather both the propellers gyroscope torque τ_{gp} and the gyroscopic torque τ_{gq} due to the quadcopter movement. It is given along the three axes by:

$$\tau_g = \tau_{gp} + \tau_{gq}$$
$$\tau_g = \sum_{i=1}^{4} \Omega \wedge I_r \begin{bmatrix} 0 & 0 & (-1)^{i+1}\omega_i \end{bmatrix}^T + \Omega \wedge I\Omega$$

The description of the system dynamics can be obtained by Newton laws as the system of equations given by

$$\begin{cases} V = \dot{\xi} \\ m\dot{V} = R_t T + F_G + F_D \\ \Omega = R_r \dot{\eta} \\ I\Omega = -\Omega \times I\Omega + \sum M \end{cases} \tag{3}$$

which, can be rewritten as

$$\begin{cases} \ddot{x} = \dfrac{1}{m}\left[(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) U_z - k_{dx}\dot{x}\right] \\ \ddot{y} = \dfrac{1}{m}\left[(\cos\phi \sin\theta \sin\psi - \sin\phi \sin\psi) U_z - k_{dy}\dot{y}\right] \\ \ddot{z} = \dfrac{1}{m}\left[\cos\phi \cos\theta U_z - mg - k_{dz}\dot{z}\right] \\ \ddot{\phi} = \dfrac{1}{I_x}\left[(I_y - I_z)\dot{\theta}\dot{\psi} - I_r\overline{\omega}\dot{\theta} + U_\phi - k_{a\phi}\dot{\phi}^2\right] \\ \ddot{\theta} = \dfrac{1}{I_y}\left[(I_z - I_x)\dot{\phi}\dot{\psi} + I_r\overline{\omega}\dot{\phi} + U_\theta - k_{a\theta}\dot{\theta}^2\right] \\ \ddot{\psi} = \dfrac{1}{I_z}\left[(I_x - I_y)\dot{\phi}\dot{\theta} + U_\psi - k_{a\psi}\dot{\psi}^2\right] \end{cases} \tag{4}$$

where m is the total mass, $I = diag\,(I_x, I_y, I_z)$ is the moment of inertia for the quadrotor, I_r is the inertia of the rotors, $\sum M$ is the total torque, $\overline{\omega} = -\omega_1 + \omega_2 - \omega_3 + \omega_4$ is the signed sum of the angular velocities of the propellers, and $[U_z, U_\phi, U_\theta, U_\psi]^T = [T, \tau_\phi, \tau_\theta, \tau_\psi]^T$ is the control input vector whose elements are given in terms of the angular speeds by

$$\begin{cases} U_z = k_F\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ U_\phi = l \cdot k_F\left(\omega_4^2 - \omega_2^2\right) \\ U_\theta = l \cdot k_F\left(\omega_3^2 - \omega_1^2\right) \\ U_\psi = k_M\left(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2\right) \end{cases} \tag{5}$$

with l being the distance from G to the center of a rotor.

The control objective is to ensure asymptotic convergence of the state variables to the desired time-varying route in the space. This requires the design of a fly controller capable of generating the appropriate control inputs for the system given by (4). Each control variable has a different effect on the quadcopter dynamics; U_z decides the altitude while U_ϕ, U_θ, and U_ψ adjust the rotational motions (rolling, pitching and yawing) along the three axes.

3. **Controller Design.** The basic idea of the classical SMC is first to attract the tracking errors of the system's state variables into a suitably selected region $s(X,t) = 0$, then design a control law U_{sw} that always maintains the system in that region. In summary, an SMC consists of two parts like in (6). The equivalent control U_{eq} is determined by the model of the system. It is designed with the equivalent control method, whose principle is based on the determination of the system behavior when it is on the sliding surface s.

$$U = U_{eq} + U_{sw} \tag{6}$$

In the case of systems with relative degree one ($RD = 1$), even though the control problem can be solved by the classical first-order SMC, high order sliding mode controllers (HO-SMC) are preferred in order to avoid the chattering effect. One of the major problems for the implementation of the HO-SMC algorithms is that the number of necessary information increases with the order of the sliding regime. For this reason, 2-SMC is a good solution. Super-twisting is among the most used 2-SMC algorithms by researchers. This is owing to its chattering reduction capability while maintaining the robustness of conventional SMC. Moreover, the super-twisting algorithm does not need any derivative of the sliding surface, and thus it is simple to implement.

Figure 2 shows the control strategy proposed for the underactuated system given in (4). The control of the position and the attitude is achieved through two cascade loops and a correction block. The inner loop is designed in order to ensure the asymptotic convergence of the attitude and altitude motions to their desired values ϕ^d, θ^d, ψ^d and z^d. For this purpose, four controllers are established based on 2-SMC. On the other hand, the outer loop is devoted to the control of the longitudinal and lateral motions for which two controllers are developed based on 2-SMC as well. The outer loop has as inputs the desired positions x^d and y^d chosen directly by the user as well as the altitude controller U_z. It provides the corresponding desired controllers U_x and U_y. These latter serve as

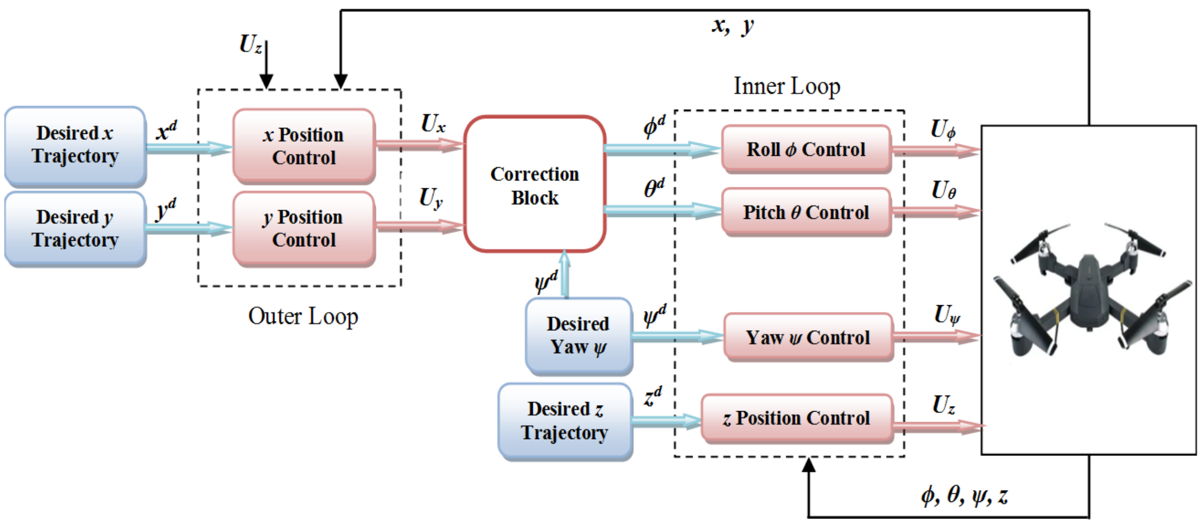

FIGURE 2. Overview of the adopted control structure

inputs for the correction block that adjusts the desired roll and pitch rotations ϕ^d and θ^d according to the desired yaw ψ^d. The obtained values of the desired roll and pitch angles are used as inputs for the inner loop where the desired yaw and desired altitude remain as assigned.

Let us define U_x and U_y as virtual inputs.

$$\begin{cases} U_x = (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ U_y = (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \end{cases} \tag{7}$$

Then, the correction block aims at finding ϕ^d and θ^d corresponding to U_x and U_y by using (7) as

$$\begin{cases} \phi^d = \arcsin\left(U_x\sin\left(\psi^d\right) - U_y\cos\left(\psi^d\right)\right) \\ \theta^d = \arcsin\left(\dfrac{U_x\cos\left(\psi^d\right) + U_y\sin\left(\psi^d\right)}{\cos(\phi^d)}\right) \end{cases} \tag{8}$$

Considering the dynamics of the quadcopter system in the state space representation

$$\dot{X} = f(X,t) + g(X,t)U(t)$$

where $X \in \mathbb{R}^n$ is the system state and $U \in \mathbb{R}^p$ is the control input. The quadcopter states are usually represented by the vector $X = [x_1, x_2, \ldots, x_i, \ldots, x_{12}]^T$ given by

$$X = \left[x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}\right]^T \tag{9}$$

Then the dynamical system, in state-space form, is the equivalent of

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(X,t) + g_1(X,t)U_x \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(X,t) + g_2(X,t)U_y \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3(X,t) + g_3(X,t)U_z \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = f_4(X,t) + g_4(X,t)U_\phi \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = f_5(X,t) + g_5(X,t)U_\theta \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = f_6(X,t) + g_6(X,t)U_\psi \end{cases} \tag{10}$$

where the functions $f_i(X,t)$ and $g_i(X,t)$, $i \in \{1,2,3,4,5,6\}$ can easily be obtained from (4). In 2-SMC, the objective of the control problem is to enforce the system to evolve on the sliding manifold $s(X,t) = 0$ and to obtain in finite time $s(X,t) = ds(X,t)/dt = 0$. For the trajectory tracking problem of the quadcopter, let $s_t = [s_x(X,t), s_y(X,t), s_z(X,t)]^T$ and $s_r = [s_\phi(X,t), s_\theta(X,t), s_\psi(X,t)]^T$ be the vectors containing the sliding surfaces defined for the tracking control problem of the translational and rotational subsystems in (10). Then, it is suitable to take the vector $s = [s_t(X,t), s_r(X,t)]^T$ as the output variable of the quadcopter dynamics system. The first-order time derivative of $s(X,t)$:

$$\begin{aligned} \frac{d}{dt}s(X,t) &= \frac{\partial}{\partial t}s(X,t) + \frac{\partial}{\partial X}s(X,t)\frac{dX}{dt} \\ &= \frac{\partial}{\partial t}s(X,t) + \frac{\partial}{\partial X}s(X,t)\left[f(X,t) + g(X,t)U(t)\right] \end{aligned} \tag{11}$$

and the second-order time derivative of $s(X,t)$:

$$\begin{aligned}\frac{d^2}{dt^2}s(X,U,t) &= \frac{\partial}{\partial t}\dot{s}(X,U,t) + \frac{\partial}{\partial X}\dot{s}(X,U,t)\frac{dX}{dt} + \frac{\partial}{\partial U}\dot{s}(X,U,t)\frac{dU}{dt} \\ &= \frac{\partial}{\partial t}\dot{s}(X,U,t) + \frac{\partial}{\partial X}\dot{s}(X,U,t)\left[f(X,t)+g(X,t)U(t)\right] \\ &\quad + \frac{\partial}{\partial U}\dot{s}(X,U,t)\dot{U}\end{aligned} \tag{12}$$

Therefore, by defining $\Delta(X,U,t)$ and $\nabla(X,U,t)$ as

$$\begin{cases}\Delta(X,U,t) = \dfrac{\partial}{\partial t}\dot{s}(X,U,t) + \dfrac{\partial}{\partial X}\dot{s}(X,U,t)\left[f(X,t)+g(X,t)U(t)\right] \\ \nabla(X,U,t) = \dfrac{\partial}{\partial U}\dot{s}(X,U,t)\end{cases} \tag{13}$$

the 2-SMC problem is the equivalent to the stabilization problem of the following system

$$\begin{cases} y_1 = s(X,t) \\ \dot{y}_1 = \dot{s}(X,t) \\ \dot{y}_2 = \Delta(X,U,t) + \nabla(X,U,t)W(t)\end{cases} \tag{14}$$

where $W(t)$ is the auxiliary control input and it is the time derivative of the actual system control $U(t)$. The position and attitude dynamics of the quadcopter is characterized by $RD = 1$ with respect to the sliding surface. This is explained by the fact that the control input U arises in the equation of the first derivative of the sliding surface.

Remark 3.1. *Unlike the conventional SMC that acts on the first-time derivative of the sliding variable, the 2-SMC acts directly on its second-time derivative.*

The continuous super-twisting control law $U_{st}(t)$ comprises two parts. The first part denoted by $W(t)$ is defined by its discontinuous first-time derivative $dW(t)/dt$, whereas the second part is a continuous function of the sliding variable [34,35].

The ST-SMC is given by (15), and it is a simplified form derived from the fact that the system is linearly dependent on the control [36].

$$\begin{cases} U_{st}(t) = W(t) - c_1 \left|s\right|^{1/2}\operatorname{sign}(s) \\ \dot{W}(t) = -c_2\operatorname{sign}(s)\end{cases} \tag{15}$$

The sufficient conditions of finite-time convergence are:

$$\begin{cases} \left|\Delta(X,U,t)\right| < \wp \\ 0 \le k_{\min} \le \dfrac{\partial \dot{s}}{\partial U} \le k_{\max}\end{cases} \tag{16}$$

Then, the parameters of the super-twisting algorithm can be bounded as

$$\begin{cases} c_1^2 \ge \dfrac{4\wp}{k_{\min}^2}\cdot\dfrac{k_{\max}\left(c_2+\wp\right)}{k_{\min}\left(c_2-\wp\right)} \\ c_2 > \dfrac{\wp}{k_{\min}}\end{cases} \tag{17}$$

For systems defined as in (10) and (14), the general form considered for the selected sliding variable is a linear combination of the tracking error of the state variable and its derivative [26,31]. The zero-order sliding variables considered for the control of translational and rotational dynamics are, respectively, given by

$$s_t = \begin{bmatrix} s_x(X,t) \\ s_y(X,t) \\ s_z(X,t)\end{bmatrix} = \begin{bmatrix}\dot{e}_x \\ \dot{e}_y \\ \dot{e}_z\end{bmatrix} + \begin{bmatrix}\lambda_x & 0 & 0 \\ 0 & \lambda_y & 0 \\ 0 & 0 & \lambda_z\end{bmatrix}\begin{bmatrix} e_x \\ e_y \\ e_z\end{bmatrix} \tag{18}$$

and

$$s_r = \begin{bmatrix} s_\phi(X,t) \\ s_\theta(X,t) \\ s_\psi(X,t) \end{bmatrix} = \begin{bmatrix} \dot{e}_\phi \\ \dot{e}_\theta \\ \dot{e}_\psi \end{bmatrix} + \begin{bmatrix} \lambda_\phi & 0 & 0 \\ 0 & \lambda_\theta & 0 \\ 0 & 0 & \lambda_\psi \end{bmatrix} \begin{bmatrix} e_\phi \\ e_\theta \\ e_\psi \end{bmatrix} \tag{19}$$

with $\lambda = [\lambda_x, \lambda_y, \lambda_z, \lambda_\phi, \lambda_\theta, \lambda_\psi]$ being positive coefficients that represent the switching surface coefficients.

The control laws of the state variables can now be derived based on the ST-SMC by taking the first-time derivatives of the corresponding sliding surfaces given in (18) and (19). Since all derivations of the control laws follow the same steps, and without loss of generality, we demonstrate here the derivation of the control law of the yaw angle ψ. As mentioned earlier, e_ψ is the error between the real and the desired ψ position, that is

$$e_\psi = \psi^d - \psi \tag{20}$$

and from (19), the corresponding sliding surface for the angle ψ is given by

$$s_\psi = \dot{e}_\psi + \lambda_\psi e_\psi \tag{21}$$

for which, the first-time derivative of the sliding surface is

$$\dot{s}_\psi = \ddot{\psi}^d - (f_6(X,t) + g_6(X,t)U_\psi) + \lambda_\psi \dot{e}_\psi \tag{22}$$

Hence, by setting $\dot{s}_\psi = 0$ in (22), the equivalent control law for the state variable ψ is obtained as follows

$$U_{eq}^\psi = \frac{\ddot{\psi}^d - f_6(X,t) + \lambda_\psi \dot{e}_\psi}{g_6(X,t)} \tag{23}$$

As can be seen from (6), the control law of the state ψ is the sum of its equivalent control law and switching function U_{sw}^ψ. In our case, the latter is represented by the super-twisting switching function U_{st}^ψ which can be obtained by replacing s_ψ from (21) in (15) as

$$\begin{cases} U_{st}^\psi(t) = -c_1^\psi \left|s_\psi\right|^{1/2} \operatorname{sign}(s_\psi) + W_\psi(t) \\ \dot{W}_\psi(t) = -c_2^\psi \operatorname{sign}(s_\psi) \end{cases} \tag{24}$$

Finally, it is worth noting that the sufficient conditions given by (16) can be reformulated by using (22) as

$$\begin{cases} \left| \dfrac{\partial}{\partial t} \dot{s}_\psi(X,U,t) + \dfrac{\partial}{\partial X} \dot{s}_\psi(X,U,t) \times [f_6(X,t) + g_6(X,t)U_\psi(t)] \right| < \wp_\psi \\ 0 \le k_{\min}^x \le g_6(X,t) \le k_{\max}^\psi \end{cases} \tag{25}$$

4. **Simulation Results and Discussion.** In order to validate the designed control laws and evaluate the proposed controllers based on ST-SMC, MATLAB/Simulink is used. We hold the same numerical values of the quadcopter parameters employed in [26]. These are summarized in Table 1. In the present work, the values of the desired trajectories are fixed for the position and yaw angle. That is, the desired position (x^d, y^d, z^d) and desired yaw angle ψ^d come directly from the user. The corresponding values of the desired angles, ϕ^d and θ^d are calculated through the correction block. Finally, the control laws based on the proposed algorithm are applied for the six state variables (position and attitude). The control parameters are adjusted referring to the result obtained via classical SMC and the simulation time is set to 100 s.

In this simulation, the proposed controller is examined for a fly following different types of trajectories, namely, ramp, horizontal flight and sinusoidal trajectory. For the purpose of comparison and validation of the proposed controller, both the type-2 FLC and SMC are carried out for the same flight scenario.

TABLE 1. Quadcopter parameters

Parameter	*Value*
m	1.1 kg
I_x, I_y	1.22 kg.m^2
I_z	2.2 kg.m^2
I_r	0.2 kg.m^2
l	0.21 m
k_F	5 N.s^2/rad^2
k_M	2 N.m.s^2/rad^2
k_d	0.1 kg/s
k_a	0.12 kg.m^2/rad
g	9.81 m/s^2

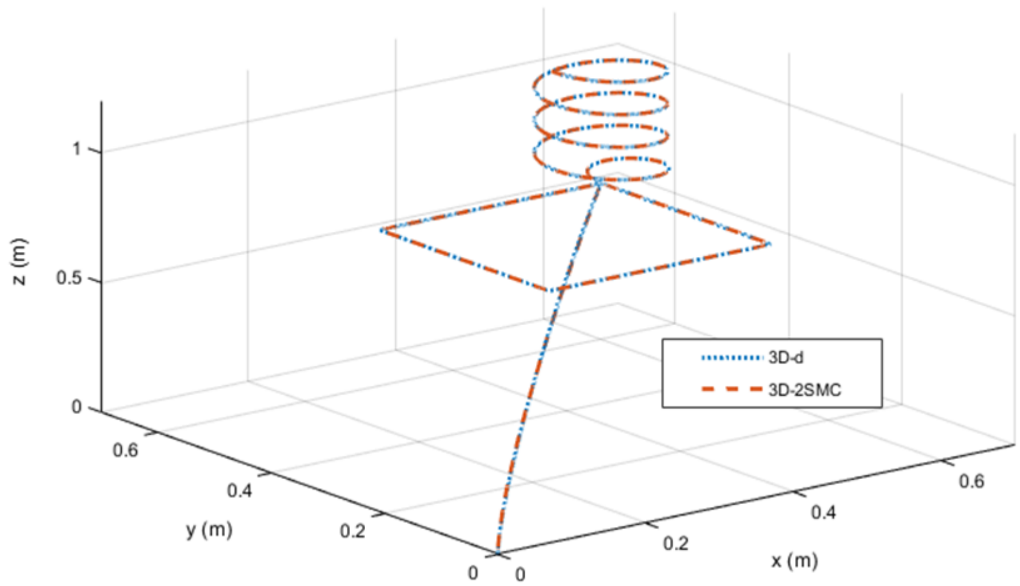

FIGURE 3. Quadcopter path tracking in 3D: desired and real trajectories, in the case of 2-SMC

Figure 3 represents the path tracking in 3D of the quadcopter aircraft based on the proposed ST-SMC. As seen, the quadcopter aircraft has perfectly tracked the desired path without deviations. Figures 4 and 5 show the resulting tracking outputs for the positions x, y and z and the attitude: roll, pitch and yaw. It is easy to observe that the quadcopter position, as well as the orientation, converges to their desired values. In Figures 6 and 7, the resulting errors between the desired setpoint value and the quadcopter real trajectories are presented, with the miniatures on the figures highlighting the important differences between the performance of the controllers. It can be seen from these figures that (i) for all the motion the errors tend to zero for the proposed ST-SMC as well as for the SMC, (ii) in contrast to the SMC and ST-SMC, the type-2 FLC suffers from low accuracy in the steady-state, and (iii) in the transient state, the ST-SMC is better than SMC in terms of overshoot and settling time. Furthermore, it is well-known that SMC has the drawback of chattering. For instance, this drawback can clearly be seen from Figure 8, which depicts the variations of the control input for the yaw motion. The same figure demonstrates the effectiveness of the ST-SMC in the removal of chattering. With the ST-SMC, we have shorter settling time, reduced overshot and satisfactory chattering elimination.

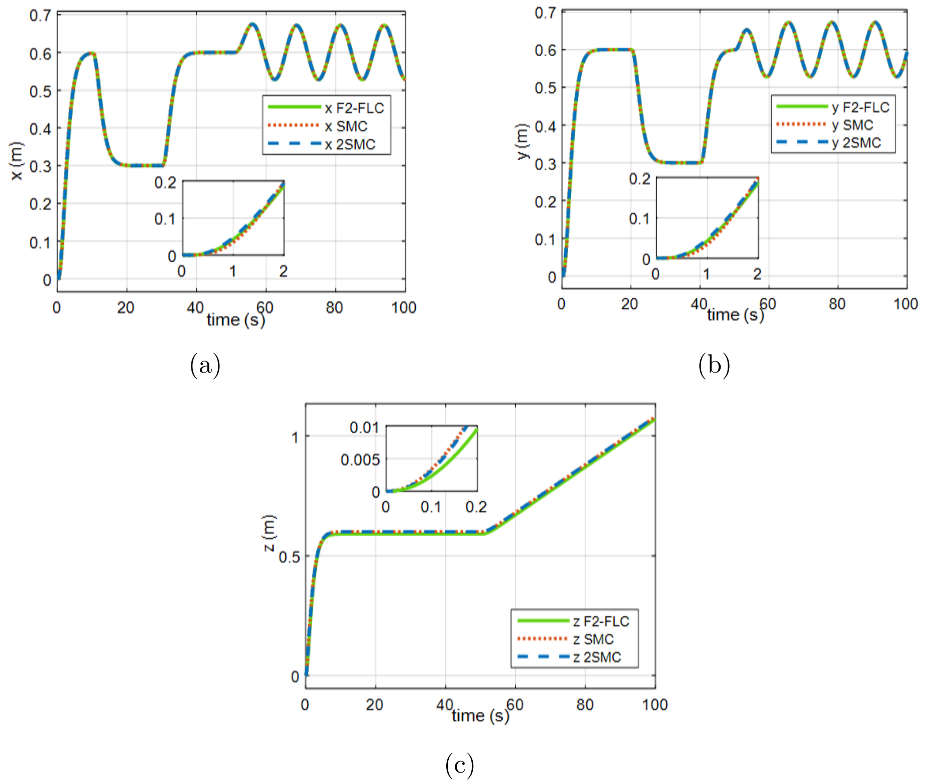

FIGURE 4. Position control results, comparison between classical SMC, type-2 FLC, and 2-SMC

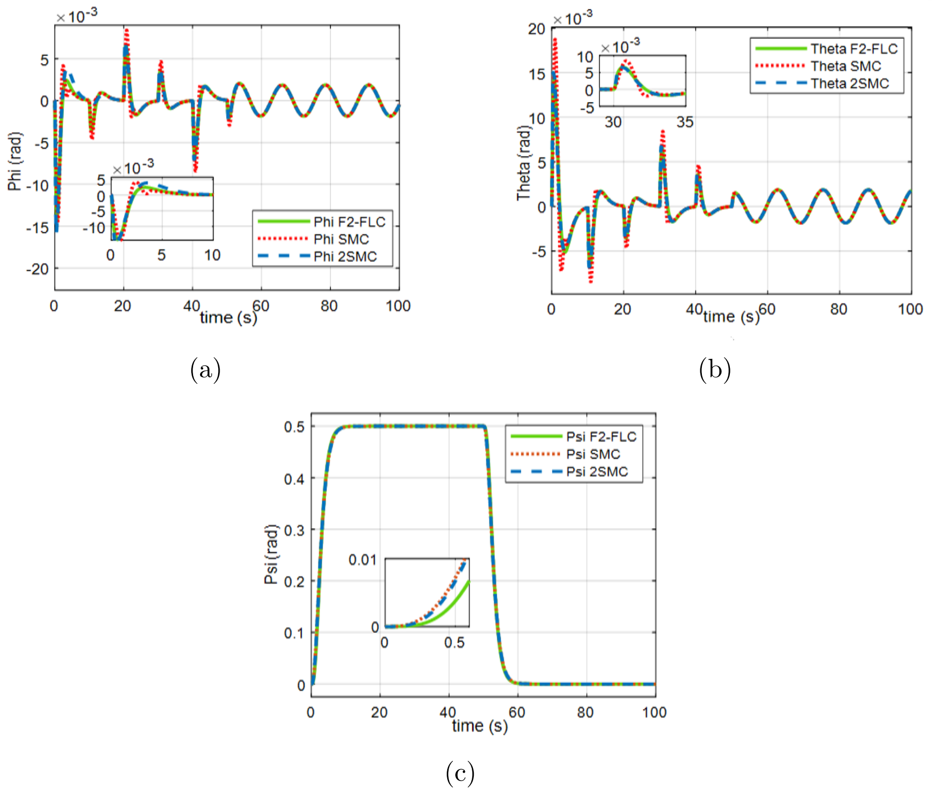

FIGURE 5. Attitude control results, comparison between classical SMC, type-2 FLC, and 2-SMC

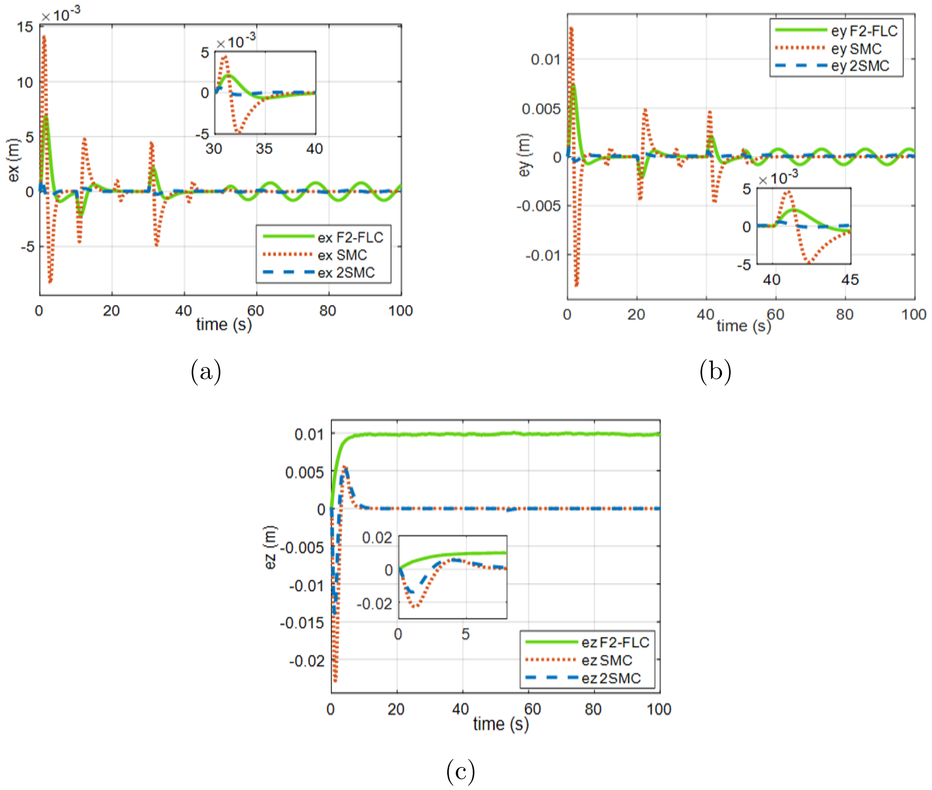

FIGURE 6. Position errors, comparison between classical SMC, type-2 FLC, and 2-SMC

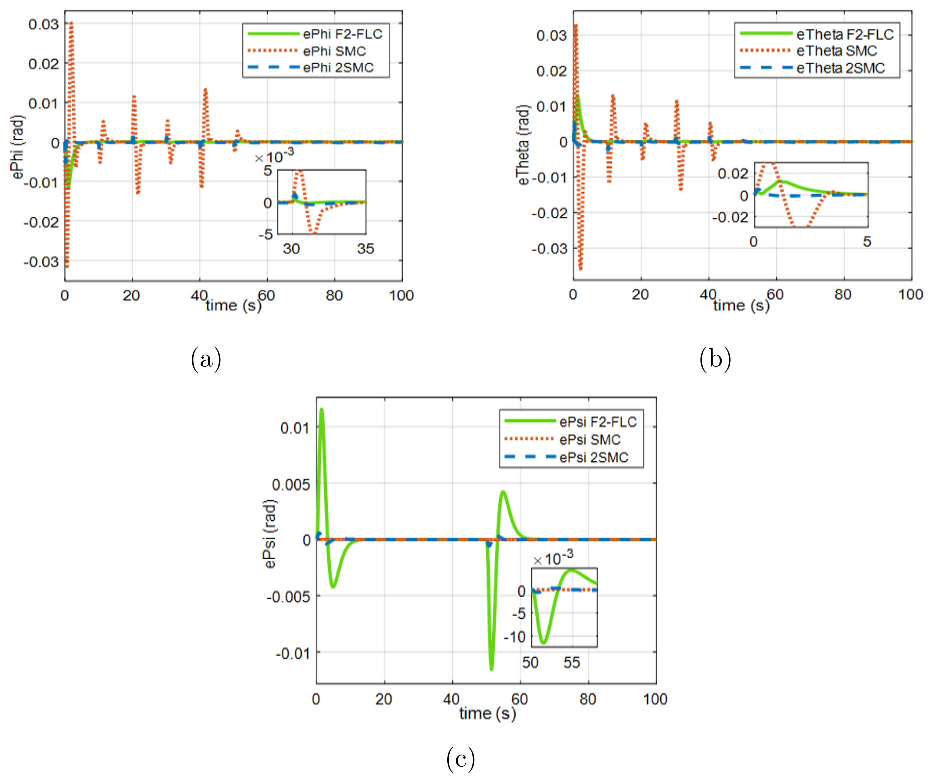

FIGURE 7. Attitude errors, comparison between classical SMC, type-2 FLC, and 2-SMC

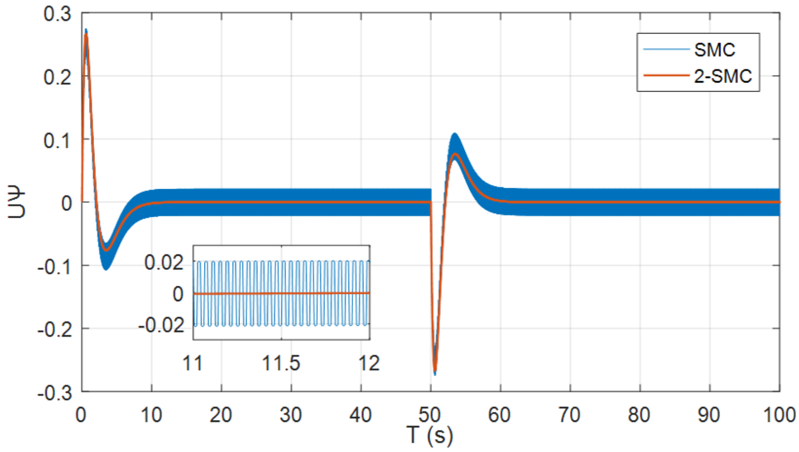

FIGURE 8. Chattering results of the SMC and 2-SMC (the case of yaw control input)

As seen from the obtained results, the proposed 2-SMC controller is a free-chattering controller. Therefore, it would make the quadcopter flight and positioning stable without any vibrations. The stability of the proposed controller, together with its good path tracking performance, would allow the quadcopter to perform rescue and retrieve operations in very tight places without the risk of collision with the surrounding obstacles. In addition, being a free-chattering controller, the proposed controller protects the motors from the damage that can be caused by the chattering and overshoot effects. Furthermore, due to the fact that the proposed controller is SMC-based, it is robust against disturbances, unpredicted inaccuracies, and parameter variations that can occur in practical situations. As a consequence of the relevant advantages given above, the proposed controller is both suitable and desirable from a practical viewpoint.

5. **Conclusions.** In this paper, a control block based on a super-twisting second-order sliding mode controller has been investigated for the steering of a quadcopter UAV to achieve the desired position and attitude. Six controllers have been developed for the corresponding six system states. The global system is divided into two subsystems, namely, the position control subsystem, and the attitude and altitude control subsystem. This strategy has been adopted in order to obtain efficient control even though the under-actuation property is still present. The 2-SMC technique is chosen due to its useful advantages, i.e., it guarantees robustness against parameter fluctuations, model uncertainties, and random external disturbances. Moreover, 2-SMC acts directly on the second-derivative of the sliding manifold, and thus chattering is avoided. MATLAB/Simulink simulations have been conducted to assess the proposed controller. The simulation results have shown the efficiency of the proposed controller in path tracking even under sudden changes. In addition, these results have demonstrated the superiority of the proposed controller in comparison with the SMC and type-2 FLC.

As for future work, our aim is to introduce external disturbances with variable parameters, generalize the dynamic model to include the situation of a real system in the wind field, and test the system experimentally.

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