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MATHEMATICAL MODELING AND RISK MANAGEMENT OF PRODUCTION SYSTEMS WITH JUMP PROCESS VIA STOCHASTIC ANALYSIS

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Abstract. *The jump diffusion process is mathematically modeled in the field of finance such as US dollar to Japanese yen exchange rate fluctuations. In the field of small and medium-sized manufacturing, a significant increase in manufacturing costs has a significant impact on management. We propose a mathematical model with the jump in the rate of return, which is the stochastic diffusion model, and evaluate the system. A delta function is introduced to the jump process that is based on empirical data. We also propose the Black-Scholes model of mathematical finance as a business risk management strategy for the system evaluation. Finally, we show that the risk premium can be used to evaluate the target rate of return with a jump process.*
Keywords: Jump process, Delta function, Stochastic differential equation, Lead time, Black-Scholes (BS) equation

1. **Introduction.** We aim at mathematically clarifying various problems that occur in the manufacturing industry of control devices and contribute to the improvement of profitability and productivity in the future. Regarding a mathematical modeling of production systems, the analysis was made focusing attention on rate of return deviation [1]. As a result, we have reported that rate of return deviation has power-law characteristics [2, 3]. Generally, disnormality of rate of return deviation in business is well known about a stock price fluctuation model, although with conditions. For example, there exists widely-known Lévy process [4, 5].

A mathematical model (option pricing model) for which is one of the representative products of financial derivatives, was also introduced by Black and Scholes in the early 1970s [6, 7]. In the BS model with the assumption that the movement of the underlying price follows the diffusion process, the theoretical price of the option is expressed by a relatively simple formula. It is well known that the emergence of the BS model has been the main driver of the rapid development of the options market since then. However, the BS model has a limitation that it cannot capture the various features of the real market. For this reason, many option pricing models have been developed that can better represent real market characteristics by relaxing or changing the assumptions that the BS model has.

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However, almost all of the reported actual data was entirely limited to stock price data. The jump process occurs frequently in the yen-dollar exchange rate fluctuation. In addition, in the price theory of financial derivatives, Black-Scholes model (hereinafter referred to as "BS model") in which a jump is considered is proposed [8].

As another example, also in applying Real Option, many of the return fluctuation models are of a log-normal stochastic differential equation, and there is also one that handles a jump process [8].

We have mainly worked on mathematical modeling and system evaluation of production process targeting a small-to-medium-sized equipment manufacturing industry. We have reported on this subject over the past several years. Mathematical models can be roughly divided into deterministic and stochastic systems. This paper is about stochastic systems.

In the first thing of previous study, we had worked on a physical model of the production process using one-dimensional diffusion equations with respect to mathematical modeling of deterministic systems [9, 10]. Then, in our previous studies related to this topic, we reported a production propagation model as a deterministic system and subsequently proposed a lead-time analysis method [11].

With respect to the stochastic system, many concerns that occurred in the supply chain are major problems facing production efficiency and business profitability. A stochastic bi-linear partial differential equation with time-delay was derived for outlet processes. The supply chain was modeled by considering with a time delay system [12]. Moreover, the analysis of production processes in stochastic system based on financial engineering, we have proposed that a production throughput rate was able to be estimated by utilizing Kalman filter theory based on the stochastic differential equation [13]. We have also proposed a stochastic differential equation (SDE) for the mathematical model describing production processes from the input of materials to the end. We utilized a risk-neutral principal in stochastic calculus based on the SDE [14, 15].

In this study, while promoting the production system business, some orders will substantially exceed the estimated cost once every fiscal year, causing a significant downward jump in the rate of return. A jump corresponds to a phenomenon where an unexpected cost is incurred for a certain order-made product, significantly impacting the target profit and turning to a deficit. Herein, we present the stochastic model with a significant downward jump in the rate of return in production systems. A delta function is empirically introduced for mathematical models that consider jumps. This is because the introduction of the delta function can be regarded as the limit of the probability density function of normal distribution. Delta functions are useful for representing special normal distributions. We model this problem and perform a stochastic analysis to enable system evaluation. In addition, it is possible to verify the jump process based on actual data, thereby avoiding future risks. We propose the Merton model of finance along with the Black-Scholes model for system evaluation. In the proposed evaluation equation, a large year-end evaluation value C_0 implies that the advantage with respect to the set value K (target rate of return) is also large, and hence the evaluation is high. In addition, if the evaluation risk is reduced and K is decreased, the value of C_0 increases. Therefore, the evaluation value of safe rate of return will be lowered. This method is very useful to evaluate the target rate of return. For our small and medium-sized equipment manufacturers, costs that are higher than expected will cause a large loss in management. We propose the mathematical model that incorporates a jump model for the rate of return in the production process. Furthermore, we evaluate based on the magnitude of the deviation between the system value derived from the rate of return at the fiscal year-end and the set value (K). To our knowledge, the stochastic analysis of the production process that takes into account the jump process has not been previously undertaken.

2. **Production Systems in the Production Equipment Industry.** We refer to the production system in manufacturing equipment industry studied in this paper. This is not a special system but "Make-to-order system with version control". Make-to-order system is a system which allows necessary manufacturing after taking orders from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

However, effective utilization of the production forecast information on the orders may suppress certain amount of "variation", but the complete suppression of variation will be difficult. In other words, "volatility" in monthly cash flow occurs and of course influences a rate of return in these companies. The production management system, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, is called as "product number management system" and is widely used.

All productions are controlled with numbered products and instructions are given for each numbered product.

Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial numbers in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks.

Therefore, careful management of the lead time or production date may not suppress "volatility" in manufacturing (production).

The company in this study is the "supplier" in Figure 1 and "factory" here. Companies are under the assumption that there are N (numbers of) suppliers; however, this study deals with one company because no data is published for the rest of the companies $(N-1)$.

2.1. **Production flow process.** A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow process, which manufactures low

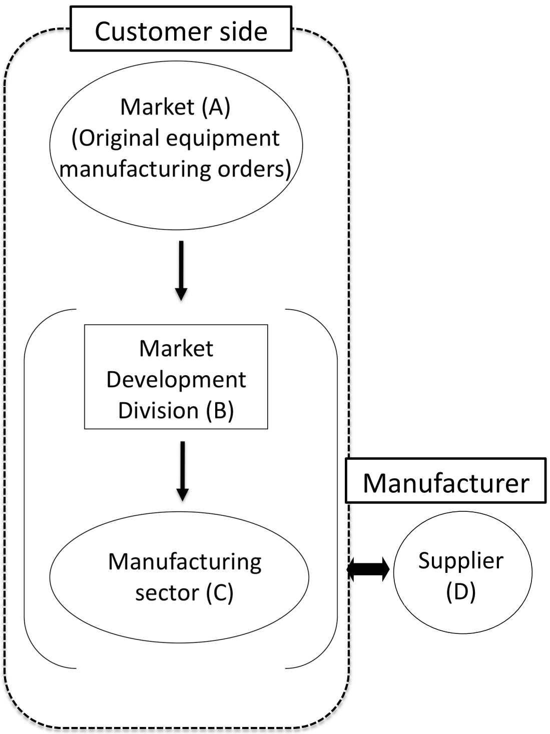

FIGURE 1. Business structure of company of research target

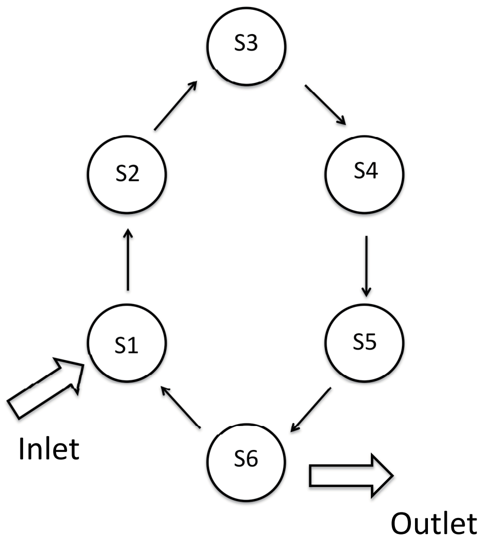

FIGURE 2. Production flow process

volumes of a wide variety of products, is produced through several stages in the production process. In Figure 2, the process consists of six production stages (S1-S6). In each step S1-S6 of the manufacturing process, materials are being produced.

The direction of the arrow represents the direction of the production flow. In this system, production materials are supplied from the inlet and the end product will be shipped from the outlet.

Assumption 2.1. *The manpower work in each process becomes non-linear because it varies depending on the ability of the worker.*

Assumption 2.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system). Production work is completed from S1 to S6 in Figure 2.*

Assumption 2.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption 2.1 reflects the realistic production structure and is somewhat valid. Assumption 2.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 2.2 is reasonable because new production starts from S1.

Based on the control equipment, the product can be manufactured in one cycle. The production throughput required to maintain 6 pieces of equipment/day is as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{1}$$

where the throughput of the previous process is set as 20 (min). In Equation (1), "28" represents the throughput of the previous process plus the idle time for synchronization. "8" is the number of processes and the total number of all processes is "8" plus the previous process. "60" is given by 20 (min) $\times$ 3 (cycles).

One process throughput (20 min) in full synchronization is

$$T_s = 3 \times 120 + 40 = 400 \text{ (min)} \tag{2}$$

Therefore, a throughput reduction of about 10% can be achieved. However, the time between processes involves some asynchronous idle time.

Please refer to Appendix. Test run results and Tables 5-10 are shown in Appendix A.

3. **Distribution System and Diffusion Equation of the Production Process.** From Figure 3, we refer to the network capacity (i.e., a statically acceptable amount of production) in an interprocess network (a production field) as R. An interprocess network indicates a sequential flow from one process to the other after the completion of the current process. Here assuming that the production density function for the i-th equipment is $S_i(x,t)$, $S_i(x,t)$ is expressed by

$$[J(x,t)dt - J(x+dx,t)dt]R = [S_i(x,t+dt) - S_i(x,t)]Rdx \tag{3}$$

where J is the production flow [9, 16].

We define production flow as the displacement of a production density function in the unit-production direction. The production density function is proportional to the cost necessary for production; thus, it can be considered as the production cost per unit

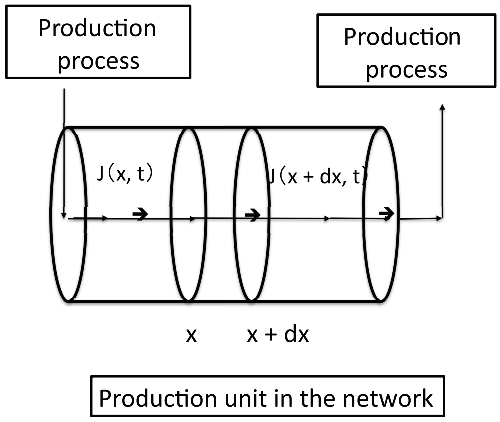

FIGURE 3. Network interprocess division of worker

production. Furthermore, as production leads to returns, the production density function can be considered as returns.

$$\frac{\partial S_i(x,t)}{\partial t} = D\frac{\partial^2 S_i(x,t)}{\partial x^2} \tag{4}$$

where D is the diffusion coefficient, t is the time variable, and x is the spatial variable.

This equation is equivalent to the diffusion equation derived from the minimization condition of free energy in a production field, indicating that the connections between processes can be treated as a diffusive propagation of products (refer to Figure 3) [9, 16, 17].

A model of the production process, which is connected in one dimension, is described as follows. The process of production is indicated by the movement of production units from one process (node) to another. This production flow is equivalent to transmission rate, which is defined as the rate of data flow between connected nodes in communication engineering. Accordingly, we formulate the production model in a manner similar to heat propagation in physics. Thus, the production process is modeled mathematically using a continuous diffusion type of partial differential equation consisting of time and spatial variables [9].

Setting the network capacity (the available static production volume) to R in an interprocess network (production field, equivalent to a stochastic field), we obtain the following:

$$[J(x)dt - J(X+dx)dt]R = [S(t+dt) - S(t)]Rdx \tag{5}$$

where J is the production flow and S is the production density.

In the present model, the production flow indicates the displacement of production processes in the direction related to the production density. In other words, the production cost per production is as follows.

Definition 3.1. *Production cost per unit production*

$$J = -D\frac{\partial S}{\partial x} \tag{6}$$

where D is a diffusion coefficient.

From Equation (5), we obtain

$$-\frac{\partial J}{\partial x} = \frac{\partial S}{\partial t} \tag{7}$$

From Equations (6) and (7), we obtain

$$\frac{\partial S}{\partial t} = D\frac{\partial^2 S}{\partial x^2} \tag{8}$$

where $t \in [0, T]$, $x \in [0, L] \equiv \Omega$, $S(0, x) = S_0(x)$, $B_x S(t, x)|_{x=\partial\Omega}$.

This equation is equivalent to the diffusion equation derived from the minimization condition of free energy in a production field [9, 10]. The connections between processes can be treated as a diffusive propagation of products (refer to Figure 3).

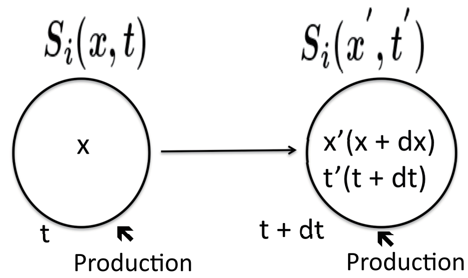

FIGURE 4. Unit of production by changing the excitation force

As shown in Figure 4, X represents the production elements that constitute a unit production and varies $X \rightarrow X'$ at $[t + dt]$. In other words, the unit production varies by exciting the external force and is the basis for revenue generation (an increase of potential energy). Therefore, in the transition $S_i(x, t) \rightarrow S_i(x', t')$, the production cost, which is the cumulated external force, increases. The connections between production processes are referred to as "joints".

In the general idea of production flow, we define the joint propagation model at multiple stages in the production process and the potential energy in the production field.

Thereafter, we can construct a control system, which increases the process throughput, by calculating the gradient function in the autonomous distributed system. The gradient function is described in the next opportunity.

$$\frac{\partial S}{\partial t} + \Delta(v \cdot S) = \frac{1}{2}\Delta\left(D^2 S\right) + \lambda \tag{9}$$

where, λ denotes a forced external force function and v denotes a production propagation speed. Here, λ is omitted here.

We assume that S is defined as follows: S represents a production density with a fluctuation, and v also causes a fluctuation in throughput. As a result, a production is proportional to the slope of production density.

4. **Jump Process Occurring in a Production Business.** Up to Section 3, by going through production flow process $S1$ to $S6$, we will proceed with the construction of a stochastic model that takes into account the jump process in Section 4.

4.1. **Delta function.** Jumps correspond to the delta function shown in Figure 5. We empirically introduce mathematical models that consider jumps. This is because the introduction of the delta function can be regarded as the limit of the probability density function of normal distribution. Delta functions are useful for representing special normal distributions.

Definition 4.1. *Delta function*

$$\delta(t) = \lim_{\sigma\to 0}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{t^2}{2\sigma^2}\right) = a\lim_{n\to\infty}\sqrt{\frac{n}{\pi}}\exp\left(-nt^2\right) \approx a\sqrt{\frac{n}{\pi}}\exp\left(-nt^2\right) \tag{10}$$

where, $n = \frac{1}{\sigma}$*.* a *and* n $(\gg 0)$ *are a delta function strength and a constant parameter* $(= 1000)$*.*

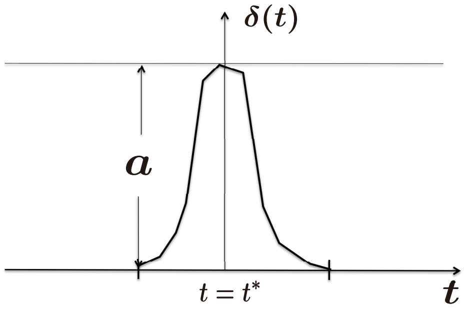

FIGURE 5. Delta function

4.2. **Jump process.** In Figure 6, $w(t)$ and t denote a rate of return and starting time of manufacturing respectively. t is denoted as follows.

$$t \in [T_1, T_2, T_3, \ldots, T_i, \ldots] \tag{11}$$

In a previous paper, we have applied the mathematical finance methods to the event cost by using the Poisson process to model the events occurring and have reported the cost evaluation similar to Figure 6 [18]. Therefore, the term jump is added to the $w(t)$ model to evaluate the production return.

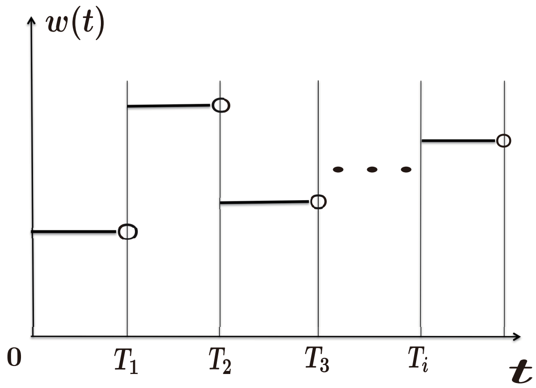

FIGURE 6. Jump model

Definition 4.2. *Stochastic differential equation with jump*

$$\frac{dw(t)}{w(t)} = \hat{\mu}dt + \hat{\sigma}dZ(t) \tag{12}$$

where $w(t)$*,* $\hat{\mu} = \mu - \delta(t)$*,* $\hat{\sigma} = \sigma + \delta(t)$ *and* $Z(t)$ *are the rate of return, average, volatility and Wiener process respectively* [19].

Definition 4.3. *Jump process in production processes. Downward jumps in the rate of return are because of upward jumps in the manufacturing costs.*

The cost, gross profit, and the rate of return are as follows.

- Manufacturing cost ≈ Manufacturing cost + Overhead cost
- Gross profit ≈ Estimated price − Manufacturing cost + Factory cost
- Rate of return ≈ Gross profit/Amount of sales

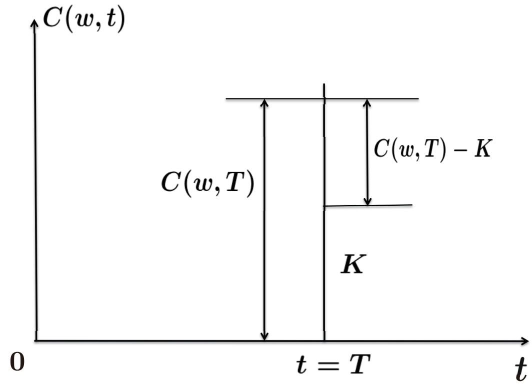

FIGURE 7. System evaluation at the end of the period

In Figure 13, the average lead time is 0.366 and the volatility is 0.236. In Figures 13 and 14, the horizontal axis represents the start time series, while the vertical axis represents the normalized lead time and normalized lead time deviation, respectively. Furthermore, Figures 15 and 16 show that the mathematical model of such a time series has "power distribution" characteristics and a fluctuation spectrum as seen in our previous reports. Therefore, it can be represented using a lognormal stochastic differential equation. It can be regarded as exponential distribution judging from the start time series actual data of batch type production system. Therefore, the delta function is converted from the actual data (Figure 16) to the following equation:

$$\frac{dw(t)}{w(t)} = (\mu_w - \lambda k)dt + \sigma_w dZ + dY(t) \tag{13}$$

where, $Y(t) = \sum_n (J_n - 1)$ and let $n = 1$ for simplicity.

J is a stochastic variable according to the same distribution independent of one another and represents the rate of jump width, λ is a Poisson strength, k is an expected jump width rate and $N(t)$ is Poisson distribution of the strength λ. Moreover,

$$E[J-1]dN(t) = \lambda k dt \tag{14}$$

$$E[dY(t)] = \lambda k dt \tag{15}$$

For example, if the jump intensity is adjusted to 0.8 in Figure 9, $[J-1]$ becomes $0.8-1 = -0.2$. That is, it jumps 20% below. However, fluctuation terms for volatility are not considered in this study. In Equations (10) and (11), $t \in [T_1, T_2, \ldots, T_i]$ represents the lead time series, n represents the number of jumps. $w(t)$ represents the magnitude of the rates of return generated from the time series in Equations (13). In the event risk evaluation presented in our previous paper [18], we evaluated the system based on the number of lead time deviation occurrences for each event which is a certain set value.

In this paper, we evaluate the system by the difference between the system value derived from the return rate at the end of the term and the set value.

Definition 4.4. *Derivative value in finance $C(w,t)$*

$$\frac{dC(w,t)}{C(w,t)} = (\mu_c - \lambda k_c)dt + \sigma_c dZ(t) + (J_c - 1)dN(t) \tag{16}$$

where k_c and J_c are as follows respectively.

$$k_c = E[J_c - 1],\ J_c = \frac{C(Jw,t)}{C(w,t)},\ \sigma_c = \frac{\left(\frac{\partial C(w,t)}{\partial w}\sigma_w w\right)}{C(w,t)} \tag{17}$$

Let $w(t)$ behave according to the following equation:

$$\frac{dw(t)}{w(t)} = (\mu_w - \lambda k)dt + \sigma_w dZ(t) + (J-1)dN(t) \tag{18}$$

where J is assumed that $w(t)$ satisfies the technical conditions for maintaining a solution [5].

Applying $C(w,t)$ to Ito's lemma, $C(w,t)$ must follow Equation (19).

$$\begin{aligned} C(w,t)\mu_c = & \frac{1}{2}\sigma_w^2 w^2 \frac{\partial^2 C(w,t)}{\partial w^2} + (\mu_w - \lambda k)w\frac{\partial C(w,t)}{\partial w} + \frac{\partial C(w,t)}{\partial t} \\ & + \lambda E[C(Jw,t) - C(w,t)] \end{aligned} \tag{19}$$

5. **Derivative Evaluation of the Production System Based on BS Equation.** We evaluate based on the magnitude of the deviation between the system value derived from the rate of return at the fiscal year-end and the set value (K) in Figure 7 in Section 4. We use the Merton model in finance as a system evaluation method.

Assumption 5.1. *There is no risk premium. Because, there is no transaction between the rate of return $w(t)$ and the derivative value $C(w,t)$ derived from it. However, the safe interest rate r exists.*

Therefore, the portfolio $V(\Theta_t)$ composed of the rates of return $w(t)$ and $C(w,t)$ is expressed by the following equation as in the delta hedge of the BS model [19].

$$\Theta_t \sigma_w + (1-\Theta_t)\sigma_c = 0 \tag{20}$$

where Θ_t is the distribution ratio, which corresponds to the investment ratio in finance.

$$\begin{aligned} \frac{dV(\Theta_t)}{V(\Theta_t)} = & \left[\Theta_t(\mu_w - \lambda k) + \{1-\Theta_t\}(\mu_c - \lambda k_c)\right] dt \\ & + \left[\Theta_t(J-1) + \{1-\Theta_t\}(J-1)\right] dN(t) \end{aligned} \tag{21}$$

The following equation is obtained from the above assumption.

$$\Theta_t \mu_w + (1-\Theta_t)\mu_c - r = 0 \tag{22}$$

We obtain from Equation (20) and Equation (22) as follows.

$$\frac{\mu_w - r}{\sigma_w} = \frac{\mu_c - r}{\sigma_c} \tag{23}$$

Substituting Equation (23) into Equation (16),

$$\begin{aligned} & \frac{1}{2}\sigma_w^2 w^2 \frac{\partial^2 C(w,t)}{\partial w^2} + (r - \lambda k)w\frac{\partial C(w,t)}{\partial w} + \frac{\partial C(w,t)}{\partial t} \\ & - rc\lambda E[C(Jw,t) - C(w,t)] = 0 \end{aligned} \tag{24}$$

5.1. **Calculation of option value by risk neutralization method.** We calculate the option value by risk neutralization method [15, 19].

Assumption 5.2. *Judging from the two points of expected return and risk, price formation in the market becomes complicated. Therefore, if all market investors are risk neutral, the market becomes very simple. This is because the risk-neutralists judge the value of a product based on the expected return level, i.e. pricing, so in such a market, if the expected return of one product is higher than the expected return of another product, the product with the lower return cannot be selected. Therefore, we introduce the risk-neutral measure Q^* to construct the lognormal stochastic differential equation as the mathematical model.*

Assumption 5.3. *The log-normal stochastic differential equation model is assumed using the concept of mathematical finance. Here, the jump term is added. $w(t)$ behaves the next stochastic process under the risk neutral measure Q^*.*

$$\frac{dw(t)}{w(t)} = (\mu_w - \lambda^* k^*)dt + \sigma_w dZ^*(t) + (J^* - 1)dN^*(t) \tag{25}$$

where $Z^*(t)$, $N^*(t)$ are Wiener process, Poisson process respectively [5].

In the production system, we set the final option evaluation value by the risk neutral method as follows.

$$C_0 = E^*\left[e^{-rT}C_T\right] \tag{26}$$

where, E^*, r and C_T denote the expectation operator, the risk free rate and the number of jumps that occur by the end of period T.

Next, the final option evaluation value is given by the following equation, and let P_n be the probability that the jump will occur n times by the end of period T

$$C_0 = \sum_{n=0}^{\infty} E^*\left[e^{-rT}C_T^n\right] P_n \tag{27}$$

where, P_n follows the Poisson process as follows.

$$P_n = \frac{e^{-rT}(\lambda^* T)^n}{n!} \tag{28}$$

where, λ^* denotes the jump occurrence probability.

We obtain the evaluation value C_0 at the end of period T by substituting Equation (28) into Equation (27).

$$C_0 = \sum_{n=0}^{\infty} \frac{e^{-rT}(\lambda^* T)^n}{n!} E^*\left[e^{-rT}C_T^n\right] \tag{29}$$

Further, Equation (29) can be modified as follows. For more information please refer to Reference [19].

$$E^*\left[e^{-rT}C_T^n\right] = E^*\left[e^{-rT}\{e^Y - K\}_+\right] \tag{30}$$

where, $Y = \Phi(\mu_Y, \sigma_Y^2)$. $\Phi(\cdot,\cdot)$ is the standard normal distribution. μ_Y and σ_Y^2 are the average and voratility respectively.

$$E^*\left[e^{-rT}\left\{e^Y - K\right\}_+\right] = e^{(r_n - r)T + \ln w_0} \times E^*\left[\left(e^{Y_n} - K_n\right)\mathbf{1}_{\{e^{Y_n} \geq K_n\}}\right] \tag{31}$$

where, w_0 is the rate of return at $t = 0$. r_n is the risk free rate for n jumps. Y_n is the standard normal distribution at the n jumps as follows.

$$r_n = r - \lambda^* k^* + \frac{n\mu_j^*}{T} \tag{32}$$

$$Y_n \sim \Phi\left(-\sigma_n^2 T/2, \sigma_n 2^T\right) \tag{33}$$

$$K_n = e^{-r_n T - \ln w_0} K \tag{34}$$

Further, Equation (31) can be modified as follows [19].

$$\begin{aligned} E^*\left[e^{-rT}\left\{e^Y - K\right\}_+\right] &= e^{(r_n - r)T + \ln w_0} \times E^*\left[e^{Y_n}\mathbf{1}_{\{e^{Y_n} \geq K_n\}}\right] \\ &\quad - e^{(r_n - r)T + \ln w_0} K_n \times E^*\left[\mathbf{1}_{\{e^{Y_n} \geq K_n\}}\right] \end{aligned} \tag{35}$$

Here, the first term and the second term on the right side of Equation (35) can be modified as follows [19].

$$
\begin{aligned}
&(\textit{Second term}) \\
&\quad e^{\{(r_n-r)T+\ln w_0\}} K_n E^*\left[\mathbf{1}_{\{e^{Y_n}\geq K_n\}}\right] \\
&= e^{-rT} K_n P\left\{Y_n \geq \ln K_n\right\} \\
&= e^{\{(r_n-r)T+\ln w_0\}} K_n P\left\{\frac{Y_n+\sigma^2 T/2}{\sigma_n\sqrt{T}} \geq \frac{\ln K_n+\sigma^2 T/2}{\sigma_n\sqrt{T}}\right\} \\
&= e^{\{(r_n-r)T+\ln w_0\}} K_n \Phi\left(\frac{\ln(1/K_n)-\sigma^2/2}{\sigma_n\sqrt{T}}\right) \\
&= e^{-rT} K\Phi\left\{\frac{\ln\left(\frac{w_0}{K}\right)+\left(r_n+\frac{\sigma_n^2}{2}\right)T}{\sigma_n\sqrt{T}} - \sigma_n\sqrt{T}\right\}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
&(\textit{First term}) \\
&\quad e^{(r_n-r)T+\ln w_0} \times E^*\left[e^{Y_n}\mathbf{1}_{\{e^{Y_n}\geq K_n\}}\right] \\
&= e^{(r_n-r)T+\ln w_0} P\left\{e^{X_n} \geq K_n\right\} \\
&= w_0 e^{(r_n-r)T}\Phi\left\{\frac{\ln\left(\frac{w_0}{K}\right)+\left(r_n+\frac{\sigma_n^2}{2}\right)T}{\sigma_n\sqrt{T}}\right\}
\end{aligned} \tag{37}
$$

where, we utilize Equation (33) to obtain Equation (36), and also utilize X_n to obtain Equation (37). Here, the standard normal distribution $X_n \sim \Phi\left(\sigma_n^2 T/2, \sigma_n^2 T\right)$.

Therefore, Equation (38) is derived as follows using Equation (36) and Equation (37) [19].

$$
E^*\left[e^{-rT}\left\{e^Y - K\right\}_+\right] = e^{-rT}\left[e^{r_n T} w_0 \Phi(d_1) - K\Phi(d_2)\right] \tag{38}
$$

where,

$$
d_1 = \frac{\ln\left(\frac{w_0}{K}\right)+\left(r_n+\frac{\sigma_n^2}{2}\right)T}{\sigma_n\sqrt{T}} \tag{39}
$$

$$
d_2 = d_1 - \sigma_n\sqrt{T} \tag{40}
$$

Thus, substituting Equation (38) into Equation (29) results in:

$$
C_0 = e^{-rT}\sum_{n=0}^{\infty}\frac{e^{-\lambda^* T}(\lambda^* T)^n}{n!}\left[e^{r_n T} w_0 \Phi(d_1) - K\Phi(d_2)\right] \tag{41}
$$

6. **Numerical Calculation of the Evaluation Value in Case of Both No Jump and One-Time Jump.** Numerical examples show for the case where there is one-time jump or no jump in the batch system. Regarding with the production flow process, the production efficiency is better the production flow process than the batch process. However, with respect to comparison with the case where there is a jump, we consider sufficient to compare with the batch system.

6.1. **Numerical calculation of the rate of return of the batch system.** Figures 8, 9, 10, and 11 show the rates of return of a batch-type production system based on actual data. The horizontal axis shows the estimated number of man-hour as a variable, but here is the time axis. The vertical axis shows the rate of return. Figure 8 shows the event without jumps. In Figure 8 with no jump, the lowest rate of return is almost 0.2. However, in Figures 9, 10, and 11 where there is a jump, the minimum value of the rate of return has dropped to near zero. From Figures 9, 10, and 11, it can be confirmed that if a jump occurs, the rate of return declines extremely by the downward jump. Figure 12 shows the Wiener process, which are $Z^*(t)$ and $N^*(t)$ in Equation (25).

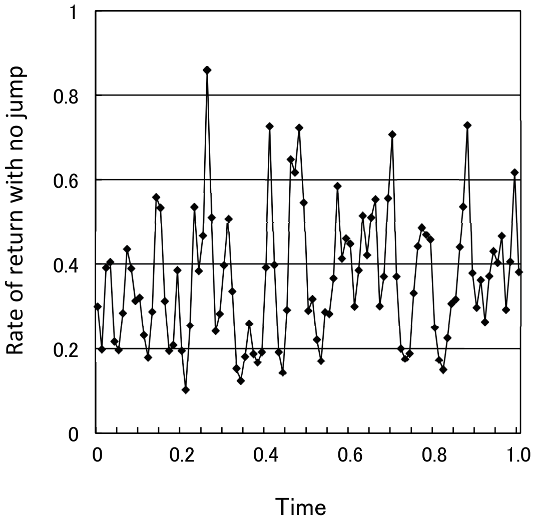

FIGURE 8. Rate of return of batch production system without jump

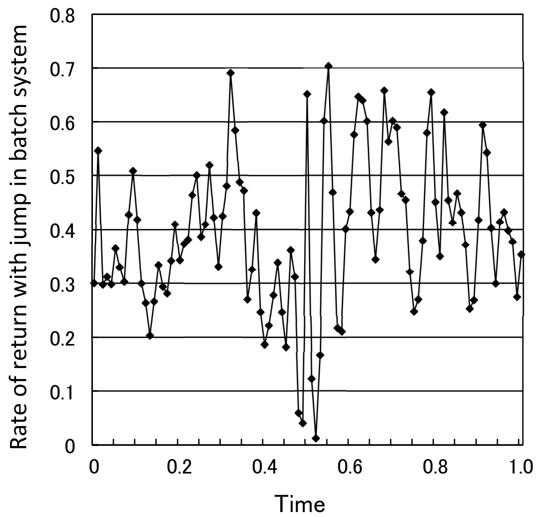

FIGURE 9. Rate of return of batch production system with jump (Different average)

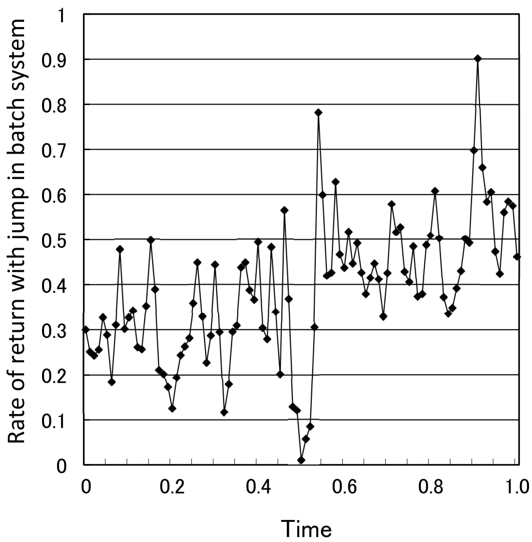

FIGURE 10. Rate of return of batch production system with jump (Different average)

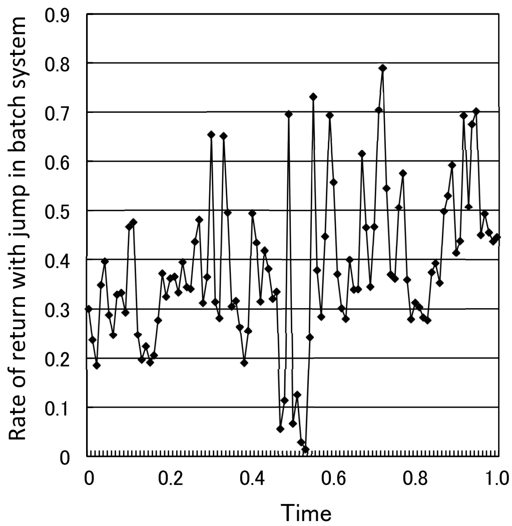

FIGURE 11. Rate of return model of batch production system with jump (Different average)

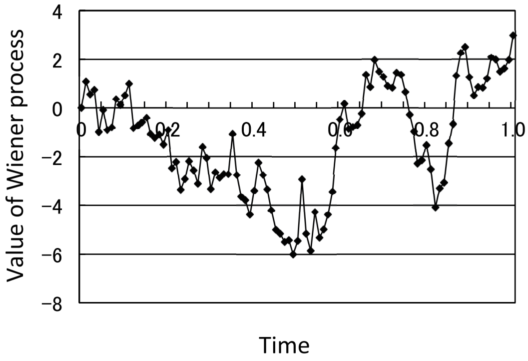

Figure 12. Value of Winner process

Table 1. Rate of return with or without jump in batch system

Figure No.	Initial value	Average	Volatility
8	0.3	0.4	0.3
9	0.3	0.4 − Delta Func. value	0.3 + Delta Func. value
10	0.3	0.4 − Delta Func. value	0.3 + Delta Func. value
11	0.3	0.6 − Delta Func. value	0.3 + Delta Func. value

Table 1 shows the initial value, average value, and volatility parameter value of the rate of return figure. For a figure having a jump, the value is obtained by subtracting the delta function value from the mean value, and for the volatility, the value is obtained by adding the delta function value.

Regarding with parameters a and n, $a = 0.08$ and $n = 1000$ in delta function of Figure 9, $a = 0.15$ and $n = 1000$ in delta function of Figure 10, $a = 0.1$ and $n = 1000$ in delta function of Figure 11. a and n denote the parameters in Equation (10). Figure 13 shows the normalized lead time versus the period batch production process in time series. Here, there are conspicuous lead time delays at the lead time series outset of 10, 34, 37, and 47. The standardized lead time is obtained by dividing lead time by 25. Figure 14 shows the normalized lead time deviation adjacent to time series outset.

- No jump

$$C_0 = e^{-rT}\left[e^{r_0 T} w_0 \Phi(d_1) - K\Phi(d_2)\right] \tag{42}$$

where $r_0 = r$.

Namely,

$$C_0 = w_0\Phi(d_1) - Ke^{-rT}\Phi(d_2) \tag{43}$$

$$d_1 = \frac{1}{\sigma_w\sqrt{T}}\ln\left(\frac{w_0}{K}\right) + \left(r + \frac{\sigma_w^2}{2}\right)T \tag{44}$$

$$d_2 = d_1 - \sigma_w\sqrt{T} \tag{45}$$

- One-time jump

$$C_0 = e^{-rT}\left(e^{-\lambda^* T(\lambda^* T)}\right) \times \left[e^{r_1 T} w_0 \Phi(d_1) - K\Phi(d_2)\right] \tag{46}$$

where

$$\lambda^* = \lambda(1+K)$$

$$K = e^{\mu_1 - \frac{\sigma_1^2}{2}}$$

$$r_1 = r - \lambda^* K + \ln\left(\frac{\mu_1}{T}\right)$$
$$\sigma_1^2 = \sigma_w^2 + \frac{\sigma_1^2}{T}$$

where μ_1 and σ_1^2 are both of the average and the volatility of jump distribution respectively.

$$d_1 = \frac{1}{\sigma_w \sqrt{T}}\left[\ln\left(\frac{w_0}{K}\right) + \left(r + \frac{\sigma_1^2}{2}\right)T\right]$$
$$d_2 = d_1 - \sigma_1 \sqrt{T}$$

Here, (μ_1, σ_1) represents the average and volatility of the distribution with respect to J.

Table 2 shows the numerical values used to calculate the system evaluation C_0 (option price). Table 3 shows the resulting target values K for no jumps, one jump, and more than one jump. Here, the values for one or more jumps are estimated by reference to the value for one jump. Table 4 shows the parameter values used to calculate the system evaluation (option price) with and without jumps. Thus, the evaluation value with respect to no jump or existing of jump is as follows. In the production system, jumps for each period are about once, and it is important that the evaluation value uses the value of K as an evaluation parameter.

TABLE 2. Parameter value at one-time jump

	Calculated value
w_0	0.1
r	0.2
λ^*	1.11 ($\lambda = 1.0$)
r_1	0.184
σ_1	0.3 ($= \sigma_w$)
T	1.0
e^{-rT}	0.818
$e^{-\lambda^* T}$	0.329
$e^{r_1 T}$	1.202

TABLE 3. Evaluation value

	No jump	One-time jump	More than one-time jump
Evaluation value ($K = 0.4$)	0.690	0.245	0.240
Evaluation value ($K = 0.6$)	0.491	0.191	0.187

TABLE 4. Parameter value at jump from zero to multiple times

	Initial value	K	μ	σ	r
Figure 18	0.1	0.6	0.4	0.3	0.2
Figure 19	0.1	0.4	0.4	0.3	0.2
Figure 20	0.1	0.4	0.4	0.3	0.2

As described above, it is useful to be able to evaluate the target rate of return for the parameters of the production system. However, it is not possible to make a relative evaluation between no jump and jump existing.

7. **Conclusion.** A jump corresponds to the phenomenon where an unexpected cost is incurred for a certain order-made product, sufficiently impacting the target profit and turned into a deficit. We certainly recognize the jump to some extent once a year. Such

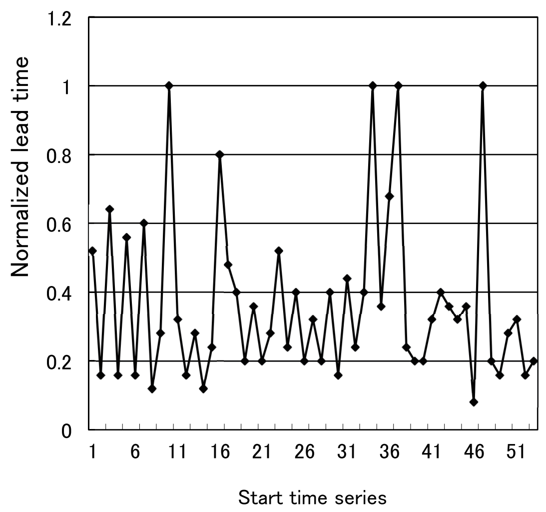

FIGURE 13. Normalized lead time of production system in batch process

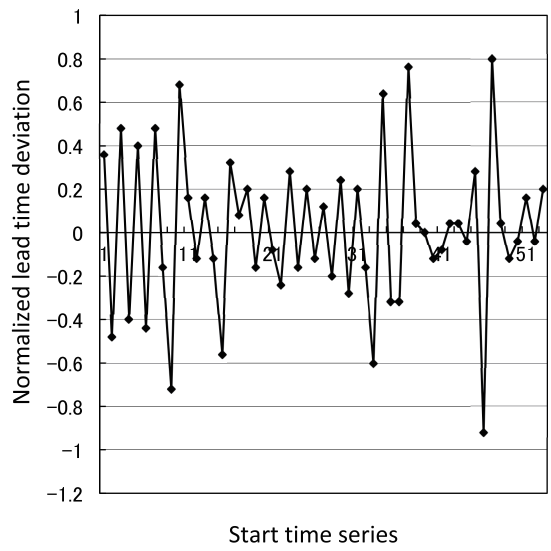

FIGURE 14. Normalized lead time deviation of production system in batch process

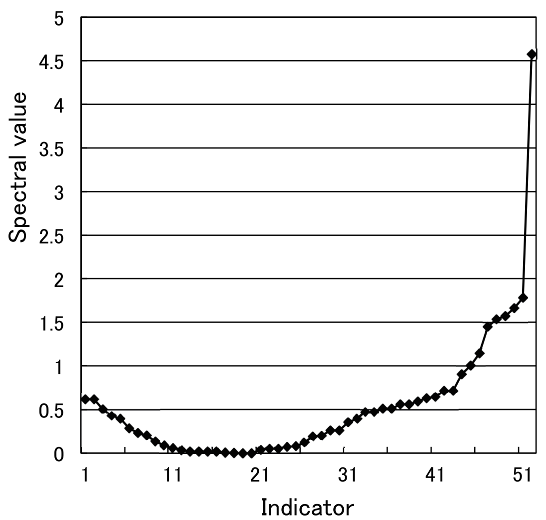

FIGURE 15. Fluctuation spectrum on the batch production normalized lead time

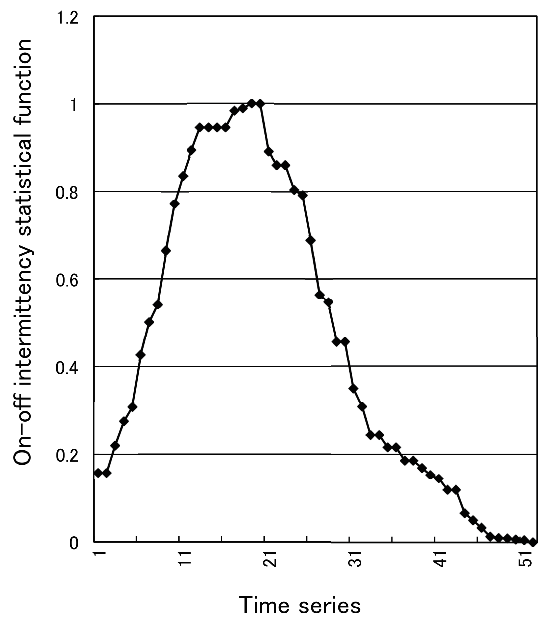

FIGURE 16. On-off intermittency statistical function $\exp(-t \cdot S(\Phi))$ on the batch production normalized lead time

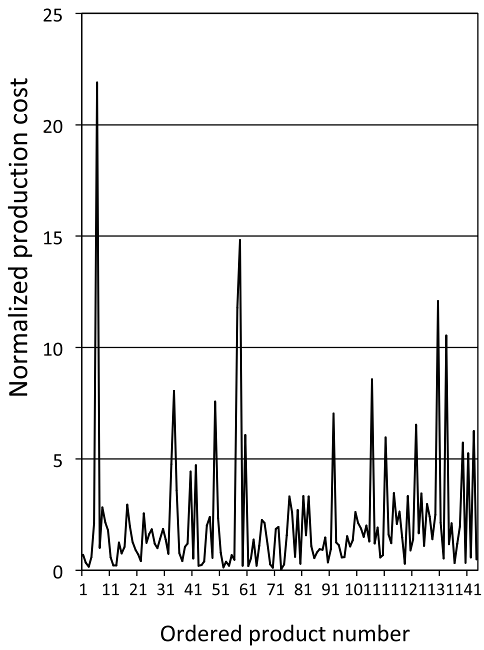

FIGURE 17. Normalized production cost (Average cost = 2.204)

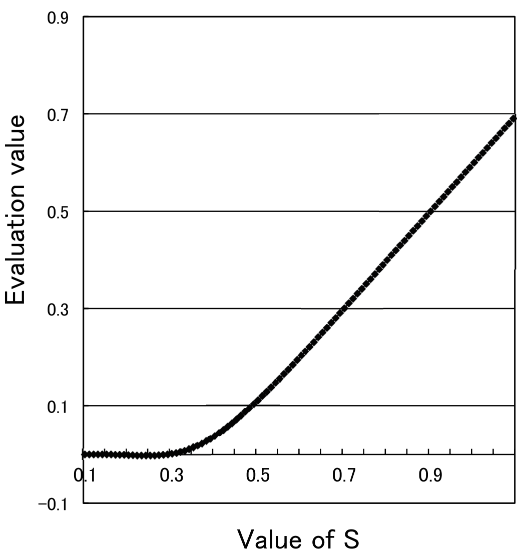

FIGURE 18. Solution calculation of option evaluation equation (No jump)

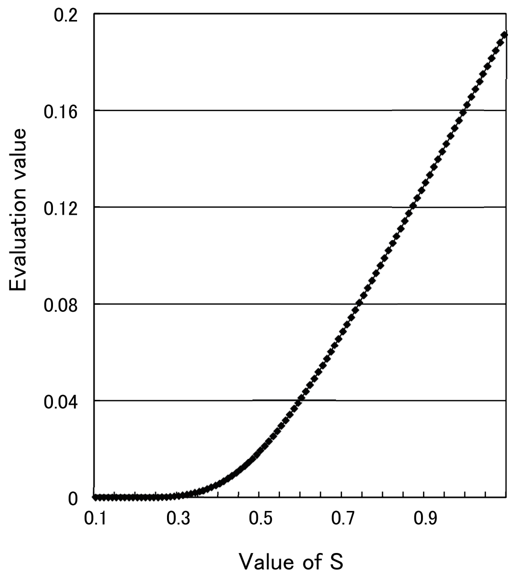

FIGURE 19. Solution calculation of option evaluation equation (One-time jump)

issues can not only occur in production processes but also in a wide variety of other fields. In this paper, we modeled this problem and performed stochastic analysis for system evaluation. In addition, it is possible to verify the jump process based on actual data, thereby avoiding future risks.

In our proposed evaluation equation, a large year-end evaluation value C_0 implies that the advantage with respect to the set value K (target return rate) is also large, and hence the evaluation is high. In addition, if the evaluation risk is reduced and K is decreased, the value of C_0 increases. Hence, the evaluation value of safe rate of return will be lower.

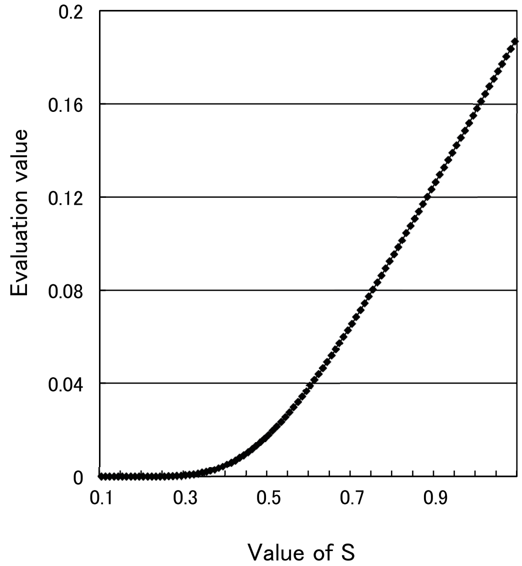

FIGURE 20. Solution calculation of option evaluation equation (More than one-time jump)

Hence, this method is very useful to evaluate the target rate of return. However, we cannot make a relative evaluation between the evaluations with and without a jump.

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Appendix A. Analysis of Actual Data in the Production Flow System.

- (test run1): Each throughput in every process (S1-S6) is asynchronous, and its process throughput is asynchronous. Table 5 represents the manufacturing time (min) in each process. Table 6 represents the variance in each process performed by workers. Table 5 represents the target time, and the theoretical throughput is given by $3 \times 199 + 2 \times 15 = 627$ (min).

  In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. Figure 21 is a graph illustrating the measurement data in Table 5, and it represents the total working time for each worker (K1-K9). The graph in Figure 22 represents the variance data for each working time in Table 5.
- (test run2): Set to synchronously process the throughput.

  The target time in Table 7 is 500 (min), and the theoretical throughput (not including the synchronized idle time) is 400 (min). Table 8 represents the variance data of each working process (S1-S6) for each worker (K1-K9).
- (test run3): The process throughput is performed synchronously with the reclassification of the process. The theoretical throughput (not including the synchronized idle time) is 400 (min) in Table 9. Table 10 represents the variance data of Table 9.

  "WS" in the measurement tables represents the standard working time. This is an empirical value obtained from long-term experiments.

Table 5. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 6. Volatility of Table 5

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

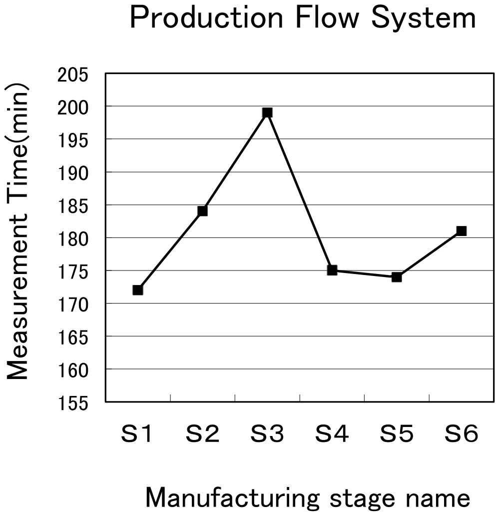

FIGURE 21. Total work time for each stage (S1-S6) in Table 5

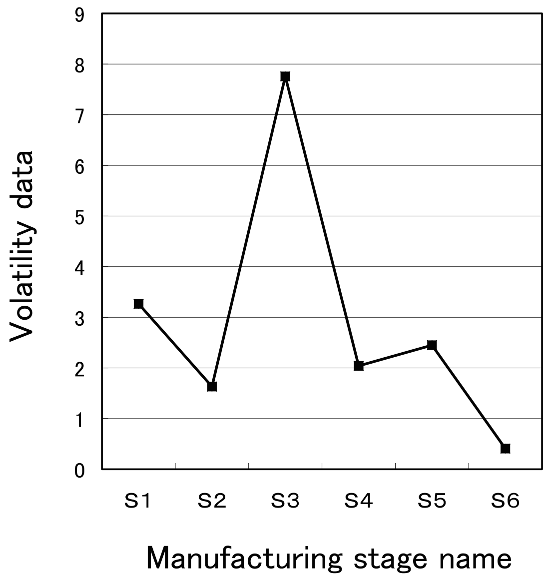

FIGURE 22. Volatility data for each stage (S1-S6) in Table 5

TABLE 7. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

TABLE 8. Volatility of Table 7

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 9. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180

TABLE 10. Volatility of Table 9

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.33	3	3	0	0	0
K5	1.33	1.33	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.33	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0