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VELOCITY-FREE FAULT ESTIMATION AND COMPENSATION FOR DYNAMIC POSITIONING OF SHIPS WITH DISTURBANCE VIA FUZZY APPROACH

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Abstract. *In this paper, we focus on the output-feedback fault-tolerant controller design for dynamic positioning ships with consideration of actuator faults and external disturbances. Firstly, a Takagi-Sugeno fuzzy model is used to represent the nonlinear dynamic positioning system of ships. Secondly, to cope with potential actuator faults, fuzzy approach and high gain method are employed to generate the fault-tolerant control law. A rigorous closed-loop stability analysis is carried out and a stability condition is posed in a linear matrix inequality framework through which the controller and observer gains are computed directly. Finally, in order to illustrate feasibility and effectiveness of the proposed fuzzy fault-tolerant control approach, some simulations are carried on a dynamic positioning ship with actuator faults and disturbances.*
Keywords: Dynamic positioning, Fault-tolerant control, Takagi-Sugeno fuzzy models, Linear matrix inequality, Fault estimation

1. **Introduction.** In recent years, with the rapid development of ocean engineering, the dynamic positioning (DP) of ships has attracted significant attention [1, 2, 3, 4, 5, 6, 7, 8]. This is because the DP system can automatically keep the ship at the desired target position and heading through its own thrusters and propellers. Compared with the traditional anchor moored positioning, the DP system is more convenient to operate and avoids destroying seabed, and the cost of the DP system does not increase as water depth increases [9]. Based on this, some advanced DP techniques have been proposed to obtain desirable performance and stability, with many results reported in the literature, such as hybrid control [6, 10], model predictive control [1, 11], backstepping control [12, 13] and sliding-mode control [7, 14]. However, most researches on the DP control did not consider the possibility of the occurrence of thruster faults. In practice, the highly disturbed marine environment will lead to aging of the components of ships, which causes inevitable malfunction in actuators [15]. As long as the fault is related to the state of the ship, it will change the structure and even stability of the DP system. Therefore, the fault-tolerant control (FTC) of DP ships is necessary since it can maintain the DP control system stability and safety in the event of thruster faults.

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In [16], a control reconfiguration strategy for dynamic positioning ships is proposed using disturbance decoupling methods, but it is assumed that the failed thruster has already been isolated. Cristofaro and Johansen utilized an unknown input observer (UIO) technique to produce a fault detection and isolation mechanism for an over-actuated marine vessel based on linearized model [17]. An iterative learning observer-based fault detection and fault-tolerant controller are proposed for dynamic positioning of ships via back-stepping technique, but it is assumed that all states of the DP system are measurable [12]. In [18], a thruster robust fault-tolerant control is proposed for ocean surface vessels with parametric uncertainties and unknown environment disturbances, provided that all the states available. Fault detection and isolation mechanism, based on two techniques: the parity space approach and the Luenberger observer, was proposed to guarantee a fault-tolerant robust control for dynamic positioning of ships [7]. Other remarkable results on fault-tolerant control for dynamic positioning of ships can be found in [13, 15, 19]. Unfortunately, it should be noted that all aforementioned fault-tolerant control methods for the DP system depend, more or less, on fault detection and isolation (FDI) block, which causes the time delay problem. Moreover, the measurements of velocity cannot always be obtained in practice, sometimes in order to reduce weight and cost, or even because of sensor failures [9]. Therefore, the full state feedback fault-tolerant control schemes cannot be directly applied to the DP ships without velocity measurements. In addition, most of the existing fault-tolerant control schemes are only available to the linear model of ships. However, the dynamic positioning ships process nonlinear characteristics, and research on fault-tolerant control of the nonlinear dynamic positioning systems has more practical significance.

It is well recognized that Takagi-Sugeno (T-S) fuzzy models are nonlinear systems described by a set of IF-THEN rules which gives a local linear representation of an underlying nonlinear system, and each local-model contributes to the global behavior of the nonlinear system through a weighting function [20, 21]. Thanks to the convex sum property of the weighting functions, it is possible to generalize some tools developed in the linear domain to nonlinear systems. During the last decades, some attention has been attracted to describing nonlinear dynamic positioning system of ships using T-S fuzzy models and some results have been reported in [22, 23, 24, 25], but most of these results are about fuzzy stabilization control without considering faults.

Based on the above discussions, an adaptive fuzzy fault-tolerant controller is proposed to enhance the performance of post-fault DP system. To the best of the authors' knowledge, it is the first time that fuzzy theory is applied to the fault-tolerant control of dynamic positioning. First of all, a high-gain observer is used to generate the auxiliary derivative outputs which are fed back to the fuzzy observer. Secondly, an adaptive fuzzy observer is constructed to estimate unmeasured velocity states, unknown time-varying disturbances and thruster faults. Finally, an adaptive fuzzy fault-tolerant controller is designed to deal with the thruster faults and unknown disturbances.

The main contribution of this paper is the proposed velocity-free fault estimation and compensation for DP ships with disturbance. The novelty of the approach with respect to existing results is summarized as follows.

i) Unlike in [7, 16, 17], where fault detection and isolation mechanism are needed, in this paper, neither real-time fault diagnosis nor fault isolation is required; consequently, there is no delay between the fault occurrence and corresponding actions.

ii) Compared with the FTC in [12, 13, 18, 19], the FTC scheme in this paper has more practical application value due to the following two facts: 1) applying the fuzzy model to describing the ship's nonlinearity; 2) assuming that the velocity states are not measurable.

The rest of this paper is organized as follows. In Section 2, the problem formulation and preliminaries are provided for preparation. The nonlinear adaptive fuzzy output-feedback fault-tolerant control design is presented for the DP system of ships with unmeasured velocity states, unknown time-varying environment disturbances and thruster faults in Section 3. In Section 4, we illustrate the effectiveness of the proposed method via simulation on a DP ship. Conclusions and future works are summarized in Section 5.

Notation: Throughout the article, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum and maximum eigenvalues of A, respectively; $A > 0$ ($A < 0$) denotes that A is positive (negative) definite matrix; $*_i$ represents the i-th component of the vector $*$, and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors; $[A]_s$ is defined by $[A]_s = A + A^T$; $[A]^T$ is defined by $A^T A$; $\dim(x)$ denotes the dimension of x; $\|.\|$ represents Euclidean norm of the vector or induced spectral norm of the matrix; the symbol $\star$ within a matrix represents the symmetric entry; $\sum_i^n h_i$ is defined by $\sum_i^n h_i = h_1 + h_2 + \cdots + h_n$.

2. Preliminaries and Problem Formulation.

2.1. Modeling of ships.

For the horizontal motion of a surface vessel, let $\eta = \begin{bmatrix} x & y & \psi \end{bmatrix}^T$ be the three degree-of-freedom (DOF) position (x, y) and heading ψ of the vessel in an Earth-fixed inertial frame, and let $\nu = \begin{bmatrix} u & v & r \end{bmatrix}^T$ be the corresponding surge and sway velocities (u, v) and yaw rate r in the body-fixed frame. The 3-DOF DP ship's equations are described mathematically by the kinematic and kinetic equations as follows [9]

$$\begin{cases} \dot{\eta} = J(\psi)\nu \\ M\dot{\nu} = \tau - D\nu + J^T(\psi)b + \tau_f \end{cases} \tag{1}$$

where $J(\psi)$ is the state dependent rotation matrix from the body-fixed frame to the Earth-fixed inertial frame given in the form

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where $J(\psi)$ is non-singular for all state and that $J^{-1}(\psi) = J^T(\psi)$; $M \in R^{3*3}$, $D \in R^{3*3}$ denote the inertia matrix and damping matrix, respectively; $\tau = \begin{bmatrix} \tau_1 & \tau_2 & \tau_3 \end{bmatrix}^T$ represents the total forces and moments vector produced by thruster system; $b \in R^{3*1}$, $\tau_f \in R^{3*1}$ are unknown time-varying environmental disturbance due to wind, waves and ocean currents, and unknown thruster faults, respectively. The Earth-fixed inertial frame and body-fixed frame are depicted by Figure 1.

By defining the state $\xi = \begin{bmatrix} x & y & \psi & u & v & r \end{bmatrix}^T$, the state-space model can be written in compact form as

$$\begin{cases} \dot{\xi}(t) = A(\xi)\xi(t) + B\tau(t) + Bf(t) \\ y = C\xi \end{cases} \tag{3}$$

where $f = J^T(\psi)b + \tau_f$ represents the sum of external disturbances and thruster failures;

$$A(\xi) = \begin{bmatrix} 0_{3*3} & J(\xi_3) \\ 0_{3*3} & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3*3} \\ M^{-1} \end{bmatrix}, \quad C = \begin{bmatrix} I_{3*3} & 0_{3*3} \end{bmatrix}.$$

The control objective of this paper is to design an adaptive fuzzy output-feedback fault tolerant control law for dynamic positioning of ships (3) in the absence of velocity measurements and subject to thruster faults, for the purpose that the DP ship can maintain the desired position and heading without degrading the desired performance.

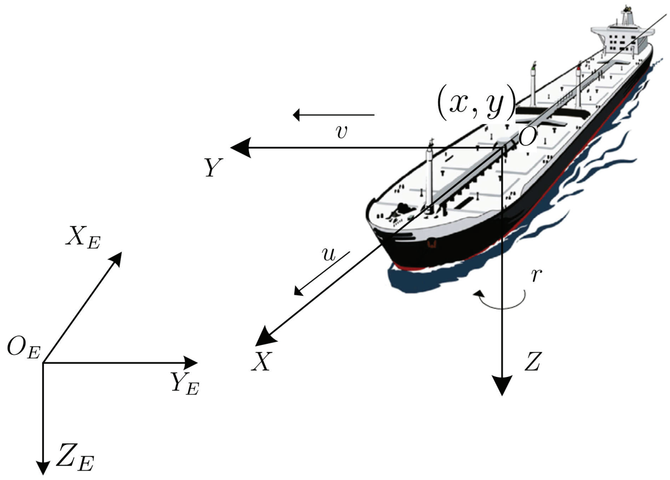

FIGURE 1. The Earth-fixed inertial frame and body-fixed frame

2.2. **Preliminaries.** Throughout this paper, the following assumptions are made.

Assumption 2.1. *The parameters* $M = M^T > 0$*, and* D *is positive matrix (may be not symmetric)* [26]*.*

Remark 2.1. *It should be noted that Assumption 2.1 implies that the ship is sailing at lower speed, which is a common and reasonable assumption in the relevant paper of dynamic positioning control, such as* [15, 17]*. Actually, the nonlinear damping term can be neglected since the linear term dominates at lower speed situation* [9]*.*

Assumption 2.2. *The faults/disturbances signal, states and control input of the system have a norm bounded second time derivative except for a set of measure zero, i.e.,* $\left\|\ddot{i}(t)\right\| \leq i_M < \infty$*,* $\forall i \in (f, \xi, \tau)$*.*

Remark 2.2. *Since the ocean environment is constantly changing and has finite energy, the disturbances acting on the ship can be viewed as the unknown time-varying yet bounded signals with the finite changing rates. Moreover, it should be noted that any energy in nature available to a given system is always limited on account of its physical structure feature. Accordingly, the external disturbance, system state, and actuator input being regarded as bounded in this paper are reasonable. In addition, Assumption 2.2 emphasizing that the changes of these signals are very slow is reasonable in practice* [27, 28]*.*

Remark 2.3. *In general, the environmental disturbances acting on the ship generate two separate movements. The sea waves of the first order generate high-frequency movements, while the slowly changing forces generate low-frequency movements. Only the slowly-varying disturbances should be counteracted by the propulsion system, whereas the oscillatory movements generated by the waves (wave disturbances of the first order) should not enter the control system loop* [9]*. Therefore, we only consider the low-frequency motion model of ships* (1) *in this paper. For the dynamic positioning control method with consideration of high-frequency wave force, please refer to the relevant literature* [10, 29]*.*

Assumption 2.3. *For any state* ξ*, the dynamic positioning system* $(A(\xi), B, C)$ *is minimum phase, that is, all the invariant zeros of the triple* $(A(\xi), B, C)$ *lie in the left half plane.*

Remark 2.4. *This paper does not assume rank*$(CB) = \dim(\tau)$*, so called observer matching condition, which is not satisfied in the DP system of ships* [30]*.*

Before giving the main results, we recalled some lemmas which will be utilized in the subsequent control development and analysis.

Lemma 2.1. *(Schur complement lemma [31]) Suppose* $S = \begin{bmatrix} S_1 & S_2 \\ \star & S_4 \end{bmatrix}$ *is a given symmetric matrix, where* $S_1 \in R^{m*m}$*. Then the following three conditions are equivalent.*

$$\begin{cases} S < 0 \\ S_1 < 0, \quad S_4 - S_2^T S_1^{-1} S_2 < 0 \\ S_4 < 0, \quad S_1 - S_2 S_4^{-1} S_2^T < 0 \end{cases} \tag{4}$$

Lemma 2.2. *(Young's inequality [32]) Given two matrices L and R with approximate dimensions, there exists a positive definite symmetric matrix* $F = F^T > 0$ *such that the following inequality holds*

$$L^T R + R^T L \leq L^T F L + R^T F^{-1} R \tag{5}$$

Lemma 2.3. *Consider a negative definite matrix* $N = N^T < 0$*. Given a symmetric matrix* $X = X^T$ *of appropriate dimension, then there exists* λ *such that*

$$X^T N X \leq -2\lambda X - \lambda^2 N^{-1} \tag{6}$$

Proof: Since $N < 0$, we can always choose a λ such that

$$\left(X + \lambda N^{-1}\right)^T N \left(X + \lambda N^{-1}\right) < 0$$

then

$$XNX + 2\lambda X + \lambda^2 N^{-1} < 0$$

Lemma 2.3 is thus proved.

Lemma 2.4. *(UIO existence conditions [33]) There exist a matrix L, G and a symmetric positive definite matrix* $P = P^T > 0$ *such that*

$$\begin{cases} P(A + LC) + (A + LC)^T P < 0 \\ B^T P = GC \end{cases} \tag{7}$$

if and only if the rank$(CB) =$ *rank*(B)*, and the invariant zeros of* (A, B, C) *lie in the left half plane.*

Lemma 2.5. *(Comparison Lemma) Consider the scalar differential equation*

$$\dot{u} = f(t, u), \quad u(t_0) = u(0) \tag{8}$$

where $f(t, u)$ *is continuous in* t *and local Lipschitz in* u*, for all* $t > 0$ *and all* u*. Let* $[t_0, T)$ *(T could be infinity) be the maximal interval of existence of the solution* $u(t)$*. Let* $v(t)$ *be a continuous function whose upper right-hand derivative* $D^+v(t)$ *satisfies the differential inequality*

$$D^+v(t) \leq f(t, v(t)), \quad v(t_0) \leq u_0 \tag{9}$$

Then, $v(t) \leq u(t)$ *for all* $t \in [t_0, T)$*.*

3. **Main Results.** The schematic diagram of the dynamic positioning fault-tolerant control system is shown in Figure 2. The position and heading of the ship are the output variables $\eta = [x, y, \psi]^T$ of the system. The ship operating in the ocean is subjected to the environmental disturbances and actuator faults, which cause the deviation in the ship's position and heading from their desired values. First, in this paper, the T-S fuzzy model is employed to approximate the nonlinear dynamic of ships. Then, the adaptive fuzzy observer is designed to reconstruct the fault/disturbance signal quickly and accurately by

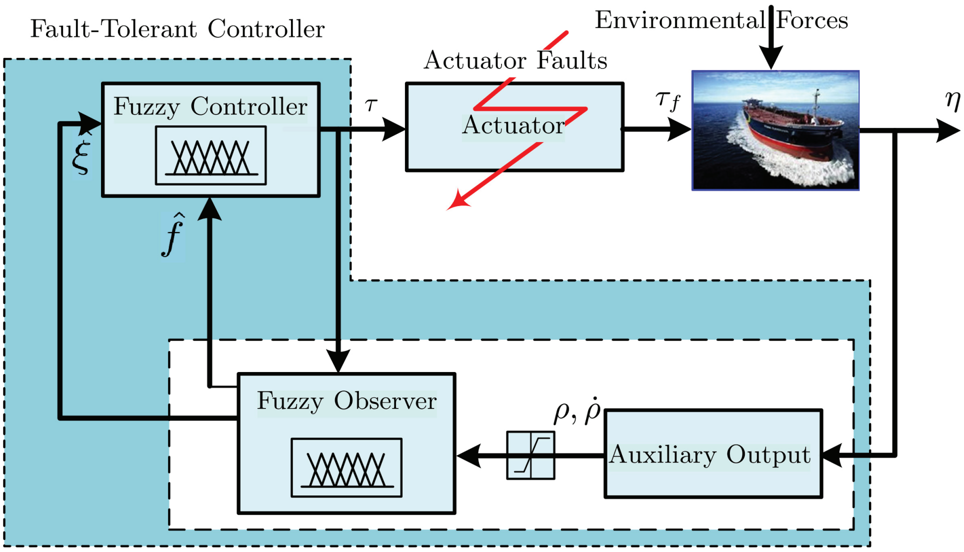

FIGURE 2. Schematic of the dynamic positioning fault-tolerant control system

using auxiliary derivative outputs. Finally, the fuzzy controller can calculate the appropriate forces and moments that the thrusters and propellers must generate in order to eliminate the positioning deviations of the ship.

3.1. **T-S modeling of the nonlinear DP ships.** There are many methods to obtain the T-S model such as: (i) linearization around some operating points and using adequate weighting functions; (ii) identification of unknown parameters of the model using sparse Bayesian approach [34]; (iii) nonlinear sector transformation [35]. In this section, we will use the linearization method to obtain the fuzzy model which represents the nonlinear DP ships (3). Hence, under actuator faults, it follows that

$$\begin{aligned} &\text{Plant Rule i: IF } \theta_1 \text{ is } \mu_{i1}, \ldots, \theta_p \text{ is } \mu_{ip} \\ &\qquad \text{THEN } \dot{\xi} = A_i\xi + B\tau + Bf, \quad \eta = C\xi \end{aligned}$$

where μ_{ij} is the fuzzy set and $\theta_1, \theta_2, \ldots, \theta_p$ are the measurable premise variables; $i = 1, 2, \ldots, n_r$ and n_r is the number of fuzzy rules; A_i is the locally linearized system matrix. Then the total fuzzy system is represented as follows [24, 25]:

$$\begin{cases} \dot{\xi} = \sum_{i}^{n_r} h_i \left(A_i\xi + B\tau + Bf(t) \right) \\ \eta = C\xi \end{cases} \tag{10}$$

where $h_i = \frac{\Pi_j^p \mu_{ij}(\theta_j)}{\sum_i^{n_r} \Pi_j^p \mu_{ij}(\theta_j)}$ is the weighting functions depending on the premise variables $\theta = [\theta_1, \theta_2, \ldots, \theta_p]$, which is, in the sequel of this paper, the heading state ξ_3. It is obvious that the fuzzy weighting function h_i, $i = 1, 2, \ldots, n_r$ satisfies the following convex sum properties.

$$\begin{cases} \sum_{i}^{n_r} h_i = 1 \\ 0 \le h_i \le 1 \end{cases} \tag{11}$$

Theorem 3.1. *(Universal Approximation Theorem* [20]*) For any given real continuous function $F(\xi)$ on a compact $U \subset R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy system $G(\xi)$*

in the form of (10) *such that*

$$\sup_{\xi \in U} \|F(\xi) - G(\xi)\| \leq \varepsilon \tag{12}$$

Remark 3.1. *By Theorem 3.1, with the increase of fuzzy rules, the fuzzy T-S system* (10) *can approximate the nonlinear dynamic positioning ships* (3) *with any precision. However, it also can be seen from Theorem 3.3 below that with the increase of fuzzy rules, the computation load of the system will increase simultaneously, which will cause the time delay in the control system. In* [21]*, Zhang et al. designed the consequent parameters by using sparse Bayesian technology and proposed a method to adjust the number of fuzzy rules according to the consequent parameters.*

Remark 3.2. *The main contribution of the proposed work consists in the approach development that is oriented on the simultaneous design of the fault estimation and compensation systems for marine surface vehicles under the framework of fuzzy systems, which is different from* [22]*. In* [22]*, a single back-stepping controller is designed by using fuzzy system to approximate unknown disturbances. Instead, in this paper, fuzzy rules are used to model the nonlinear controlled plant, marine surface vehicles. Then a group of adaptive observers and controllers are designed to estimate the disturbance and compensate the actuator faults. In summary, the proposed control scheme in this paper is fundamentally different from the result in* [22] *due to the fact that both controllers and plant are essentially designed by fuzzy logic. In addition, compared with* [22]*, the proposed method in this manuscript has another two advantages: a) In comparison with the result in* [22] *where the upper bound of disturbances is assumed to be known, such restrictive assumption is largely relaxed and only the first derivative of disturbance is assumed to be bounded in the proposed work; b) Stability conditions developed in this paper can be posed in a linear matrix inequality framework through which the controller/observer gains can be computed directly.*

3.2. **Auxiliary derivative outputs for the DP system of ships.** The ship's velocity states, in most instances, are not available for the DP control system design. In order to obtain the speed-related information of ships, an auxiliary system is designed to obtain the auxiliary derivative outputs (ADOs) from the position states of the system by using the high gain method. The dynamics of the auxiliary systems are constructed as follows

$$\begin{cases} \dot{\rho} = \bar{A}\rho + \bar{B}\eta \\ \rho_o = \bar{C}\rho \end{cases} \tag{13}$$

where $\rho = \left[\rho_1^T, \rho_2^T, \rho_3^T, \rho_4^T\right]^T \in R^{12}$ are the states of auxiliary system and ρ_o is the auxiliary derivative outputs. By choosing a number $\epsilon > 0$, the definitions of $\bar{A}$, $\bar{B}$, $\bar{C}$ are as follows

$$\bar{A} = \frac{1}{\epsilon}\begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -I & -a_3 I & -a_2 I & -a_1 I \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\epsilon} I \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & \dfrac{J^T}{\epsilon} & 0 & 0 \end{bmatrix}.$$

Theorem 3.2. *Under Assumption 2.2, the auxiliary derivative outputs of high-gain observer constructed by* (13) *can provide the estimations of the position η and velocity vector ν of the ship* (3) *by properly choosing the parameters a_1, a_2 and ϵ such that the matrix $\bar{A}$ is Hurwitz. There exists T such that for all $t > T$, the estimation errors $\rho_o - \xi$ are guaranteed to be bounded.*

Proof: After some manipulations, Equation (13) can be rewritten as

$$\begin{aligned}\rho_1 - \eta &= -a_3\rho_2 - a_2\rho_3 - a_1\rho_4 - \epsilon\dot{\rho}_4 \\ &= -\epsilon\left(a_3\dot{\rho}_1 + a_2\dot{\rho}_2 + a_1\dot{\rho}_3 + \dot{\rho}_4\right) \\ &= -\epsilon\begin{bmatrix}a_3I & a_2I & a_1I & I\end{bmatrix}\rho^{(1)} \end{aligned} \tag{14}$$

Similarly, we can prove that

$$\frac{\rho_2}{\epsilon} - \dot{\eta} = -\epsilon\begin{bmatrix}a_3I & a_2I & a_1I & I\end{bmatrix}\rho^{(2)} \tag{15}$$

Then we analyze the dynamics of

$$\rho^{(i)} = \bar{A}\rho^{(i-1)} + \bar{B}\eta^{(i-1)}, \quad \forall i = 1,2,3,4 \tag{16}$$

By defining a new variable $Y = \rho^{(i-1)}$ and $U = \eta^{(i-1)}$, we can convert the original Equation (16) into

$$\frac{d}{dt}Y = \bar{A}Y + \bar{B}U \tag{17}$$

Then

$$Y(t) = \Phi(t)Y(0) + \int_0^t \Phi(t-\tau)\bar{B}U(\tau)d\tau \tag{18}$$

where $\Phi(t) = e^{\bar{A}t}$ is the fundamental matrix solution. By Assumption 2.2, $U(t) < U_M$ can be obtained. Therefore, according to Lemma 2.5, we can find a T such that for all $t > T$ the $Y(t)$ is bounded by Y_M, where $Y_M = \Phi(t)Y(0) + \frac{U_M\gamma}{\lambda_{\min}(-\bar{A})}$, γ is a positive constant related to the eigenvector of $\bar{A}$. In the light of the definition of $Y(t)$, we have $\left\|\rho^{(i-1)}\right\| \le \rho_M$. Combining with (14) and (15) we can deduce that

$$\begin{aligned}\|\rho_o - \xi\| &= \left\|\begin{bmatrix}I & 0 \\ 0 & J^T\end{bmatrix}\begin{bmatrix}\rho_1 - \eta \\ \dfrac{\rho_2}{\epsilon} - \dot{\eta}\end{bmatrix}\right\| \\ &\le \left\|-\epsilon * \operatorname{diag}\left(\begin{bmatrix}a_3I & a_2I & a_1I & I\end{bmatrix}, \begin{bmatrix}a_3I & a_2I & a_1I & I\end{bmatrix}\right) * \begin{bmatrix}\rho^{(1)} \\ \rho^{(2)}\end{bmatrix}\right\| \\ &\le \epsilon\beta_2 \end{aligned} \tag{19}$$

where $\beta_2 = \beta_1\rho_M$, $\beta_1 = \max\{a_3, a_2, a_1, 1\}$.

Remark 3.3. *(Peaking phenomenon* [36]*) Although the estimation error can be limited to a sufficiently small neighborhood of desired target values, the initial error will lead to the so called peak phenomenon (Detailed reasons for peak phenomenon please refer to* [36]*). To eliminate the peaking phenomenon of the high-gain observer, we introduce saturation mechanism on the auxiliary derivative outputs (see Figure 2).*

3.3. **Fuzzy FTC for the DP system of ships.** In this subsection, an adaptive fuzzy output feedback fault-tolerant controller is designed based on the fuzzy observer for the DP system of ships. The fuzzy observer-based fault-tolerant controller is constructed as

$$\begin{cases}\tau = \left(\sum_i^{n_r} -h_iK_i\hat{\xi}\right) - \hat{f} \\ \dot{\hat{\xi}} = \sum_i^{n_r} h_i\left(A_i\hat{\xi} + B\tau + B\hat{f} + L_i\left(\rho_o - \hat{\xi}\right)\right) \\ \dot{\hat{f}} = P_3^{-1}F\left(\dot{\rho}_o - \dot{\hat{\xi}} + \sigma\left(\rho_o - \hat{\xi}\right)\right)\end{cases} \tag{20}$$

where L_i, F are the observer gain matrix, and K_i is controller gain; $P_3 = P_3^T > 0$ is a positive definite matrix; $\hat{\xi}$ and $\hat{f}$ are the estimate of states and fault signals, respectively; ρ_o is the auxiliary derivative outputs, and $\sigma > 0$.

Theorem 3.3. *The dynamic positioning system described in* (10) *is considered, and Assumptions 2.1-2.3 are supposed. If the observer-based fault-tolerant controller is designed as* (20) *with the parameters* $K_i = X_{2i}X_1^{-1}$, $L_i = X_4^{-1}X_{3i}$, *where* X_1, X_{2i}, X_{3i}, X_4 *are the solutions of the following linear matrix inequalities (LMIs):*

$$\begin{aligned}&\min \gamma\\ &\gamma > 0\\ &\begin{bmatrix} -\gamma I & X_4B - F^T \\ \star & -I \end{bmatrix} < 0\end{aligned}$$

$$\sum_i^{n_r}\sum_j^{n_r}\begin{bmatrix} [A_iX_1 - BX_{2j}]_s & BX_{2j} & B & 0 & 0 \\ \star & -2\lambda X_1 & 0 & \lambda I & 0 \\ \star & \star & -2\lambda I & 0 & \lambda I \\ \star & \star & \star & [X_4A_i - X_{3i}]_s + \epsilon & \dfrac{-1}{\sigma}\left(A_i^TX_4B - X_{3i}^TB\right) \\ \star & \star & \star & \star & \dfrac{-2}{\sigma}B^TX_4B + \dfrac{1}{\sigma} + 2\epsilon \end{bmatrix} < 0$$

then the state-estimation error, fault-estimation error and state vector of the entire closed-loop system are uniformly ultimately bounded.

Proof: Denote $e_\xi = \xi - \hat{\xi}$, $e_f = f - \hat{f}$. Consider the Lyapunov function candidate V for $t \geq T$ as

$$V(X) = X^T\Lambda X$$

where $\Lambda = \operatorname{diag}(P_1, P_2, P_3/\sigma)$, $P_i = P_i^T > 0$, $i = 1, 2, 3$, $X = \begin{bmatrix}\xi^T & e_\xi^T & e_f^T\end{bmatrix}^T$. According to the convex sum properties (11), it can be verified that

$$\begin{cases} \dot{\xi} = \displaystyle\sum_i^{n_r}\sum_j^{n_r} h_ih_j\left((A_i - BK_j)\,\xi + BK_je_\xi + Be_f\right) \\ \dot{e}_\xi = \displaystyle\sum_i^{n_r} h_i\left((A_i - L_i)\,e_\xi + Be_f - L_i\,(\rho_o - \xi)\right) \end{cases} \tag{21}$$

Based on the two equations (21), then by taking the first time derivative of the Lyapunov function V, one can achieve that

$$\begin{aligned}\dot{V} &= \frac{d}{dt}\left(\xi^TP_1\xi + e_\xi^TP_2e_\xi + e_f^T\frac{P_3}{\sigma}e_f\right)\\ &= \sum_i^{n_r}\sum_j^{n_r} h_ih_j\xi^T([P_1(A_i - BK_j)]_s\xi + [P_1BK_j]_s\,e_\xi + [P_1B]_s\,e_f) + e_\xi^T\left([P_2\,(A_i - L_i)]_s\,e_\xi\right.\\ &\quad + [P_2B]_se_f - [P_2L_i]_s(\rho_o - \xi)) + e_f^T\frac{2P_3}{\sigma}\left(\dot{f} - P_3^{-1}F\left(\dot{\rho}_o - \dot{\hat{\xi}} + \sigma\left(\rho_o - \hat{\xi}\right)\right)\right)\\ &= \sum_i^{n_r}\sum_j^{n_r} h_ih_j\left(\xi^T[P_1(A_i - BK_j)]_s\xi + \xi^T[P_1BK_j]_se_\xi + \xi^T[P_1B]_se_f\right.\\ &\quad + e_\xi^T[P_2(A_i - L_i)]_se_\xi + e_\xi^T[P_2B]_se_f - e_\xi^T[P_2L_i]_s(\rho_o - \xi) + e_f^T\frac{2P_3}{\sigma}\dot{f} - e_f^T\frac{2}{\sigma}F\left(\dot{\rho}_o - \dot{\xi}\right)\end{aligned}$$

$$- e_f^T \frac{2}{\sigma} F \dot{e}_\xi - e_f^T \frac{2}{\sigma} F \sigma(\rho_o - \xi + e_\xi) \Big) \tag{22}$$

For simplicity, define the new term $\tilde{\rho} := \rho_o - \xi$. According to Theorem 3.2, we have

$$\left\| \frac{\rho_3}{\epsilon^2} - \eta^{(2)} \right\| = \left\| -\epsilon \begin{bmatrix} a_3 I & a_2 I & a_1 I & I \end{bmatrix} \rho^{(3)} \right\| \leq \epsilon \beta_2 \tag{23}$$

and then the following inequalities are obtained

$$\begin{aligned} \|\dot{\tilde{\rho}}\| &= \left\| \begin{bmatrix} \dot{\rho}_1 - \dot{\eta} \\ \frac{J^T}{\epsilon} \dot{\rho}_2 + \frac{J'^T}{\epsilon} \rho_2 - J'^T \dot{\eta} - J^T \ddot{\eta} \end{bmatrix} \right\| \\ &= \left\| \begin{bmatrix} \dot{\rho}_1 - \dot{\eta} \\ J^T \left(\frac{\rho_3}{\epsilon^2} - \ddot{\eta} \right) + J'^T \left(\frac{\rho_2}{\epsilon} - \dot{\eta} \right) \end{bmatrix} \right\| \\ &\leq \epsilon \bar{\beta}_1 \end{aligned} \tag{24}$$

where $\bar{\beta}_1 = \bar{\beta}_2 \rho_M$, $\bar{\beta}_2$ is a positive constant depending on β_2. Besides, according to Assumption 2.3 and Lemma 2.4, it can be verified that there exists a matrix F such that

$$P_2 B = F^T \tag{25}$$

Then, the derivative of the Lyapunov function (22) can be further rewritten as

$$\begin{aligned} \dot{V} = \sum_i^{n_r} \sum_j^{n_r} h_i h_j \Big(& \xi^T \left[P_1 (A_i - BK_j) \right]_s \xi + \xi^T \left[P_1 B K_j \right]_s e_\xi + \xi^T [P_1 B]_s e_f \\ & + e_\xi^T [P_2 (A_i - L_i)]_s e_\xi - e_\xi^T [P_2 L_i]_s \tilde{\rho} + e_f^T \frac{2P_3}{\sigma} \dot{f} - e_f^T \frac{2}{\sigma} F \dot{\tilde{\rho}} - e_f^T \frac{2}{\sigma} F \dot{e}_\xi - e_f^T 2F \tilde{\rho} \Big) \end{aligned} \tag{26}$$

Then, substitute the differential equation of $\dot{e}_\xi$ (21), the following is obtained:

$$\begin{aligned} \dot{V} = \sum_i^{n_r} \sum_j^{n_r} h_i h_j \Big(& \xi^T [P_1 (A_i - BK_j)]_s \xi + e_\xi^T [P_2 (A_i - L_i)]_s e_\xi + \xi^T [P_1 B K_j]_s e_\xi \\ & + \xi^T [P_1 B]_s e_f - e_f^T \frac{2}{\sigma} F B e_f - e_f^T \frac{2}{\sigma} F (A_i - L_i) e_\xi + e_f^T \frac{2P_3}{\sigma} \dot{f} - e_\xi^T [P_2 L_i]_s \tilde{\rho} \\ & - e_f^T \frac{2}{\sigma} F \dot{\tilde{\rho}} + e_f^T 2F \left(\frac{L_i}{\sigma} - I \right) \tilde{\rho} \Big) \end{aligned} \tag{27}$$

According to Lemma 2.2 we have

$$\begin{aligned} & - e_\xi^T [P_2 L_i]_s \tilde{\rho} - e_f^T \frac{2}{\sigma} F \dot{\tilde{\rho}} + e_f^T 2F \left(\frac{L_i}{\sigma} - I \right) \tilde{\rho} \\ = & - 2 e_\xi^T \frac{1}{\sqrt{\epsilon}} \sqrt{\epsilon} P_2 L_i \tilde{\rho} - 2 e_f^T \frac{1}{\sqrt{\epsilon}} \sqrt{\epsilon} \frac{1}{\sigma} F \dot{\tilde{\rho}} + 2 e_f^T \frac{1}{\sqrt{\epsilon}} \sqrt{\epsilon} F \left(\frac{L_i}{\sigma} - I \right) \tilde{\rho} \\ \leq & \ \epsilon \|e_\xi\|^2 + \epsilon \kappa_{1i} \beta_2^2 + \epsilon \|e_f\|^2 + \frac{\epsilon}{\sigma} \kappa_2 \bar{\beta}_1^2 + \epsilon \|e_f\|^2 + \epsilon \kappa_{3i} \beta_2^2 \end{aligned}$$

where $\kappa_{1i} = \lambda_{\max} \left[L_i^T P_2 \right]^T$, $\kappa_2 = \lambda_{\max} [F]^T$, $\kappa_{3i} = \lambda_{\max} \left[F \left(\frac{Li}{\sigma} - I \right) \right]^T$. Then, the derivative of the Lyapunov function (27) becomes

$$\dot{V} \leq \sum_i^{n_r} \sum_j^{n_r} h_i h_j \Big(\xi^T [P_1 (A_i - BK_j)]_s \xi + e_\xi^T \left([P_2 (A_i - L_i)]_s + \epsilon \right) e_\xi - e_f^T \Big(\frac{2}{\sigma} F B$$

$$- \frac{1}{\sigma} - 2\epsilon\Big) e_f + \xi^T[P_1BK_j]_s e_\xi + \xi^T[P_1B]_s e_f - e_f^T \frac{2}{\sigma} F(A_i - L_i)e_\xi + \kappa_i\Big) \tag{28}$$

where $\bar{\kappa}_i = \epsilon\kappa_{1i}\beta_2^2 + \frac{\epsilon}{\sigma}\kappa_2\bar{\beta}_1^2 + \epsilon\kappa_{3i}\beta_2^2 + \kappa_4$, $\kappa_4 = \frac{\lambda_{\max}(P_3^2)}{\sigma} f_M^2$.

Therefore, (28) can be further transformed into compact form as

$$\begin{aligned} \dot{V} &\le X^T \sum_i^{n_r}\sum_j^{n_r} h_i h_j \begin{bmatrix} [P_1(A_i - BK_j)]_s & P_1BK_j & P_1B \\ \star & [P_2(A_i - L_i)]_s + \epsilon & \dfrac{-1}{\sigma}(A_i - L_i)^T F^T \\ \star & \star & -\left(\dfrac{2}{\sigma}FB - \dfrac{1}{\sigma} - 2\epsilon\right) \end{bmatrix} X \\ &\quad + \sum_i^{n_r} h_i\bar{\kappa}_i \\ &\triangleq X^T \sum_i^{n_r}\sum_j^{n_r} h_i h_j \Xi_{ij} X + \sum_i^{n_r} h_i \bar{\kappa}_i \end{aligned} \tag{29}$$

So as long as we choose $\sum_i^{n_r}\sum_j^{n_r} h_i h_j \Xi_{ij} < 0$, (29) can be transformed into

$$\dot{V} \le -\kappa_0\|X\|^2 + \sum_i^{n_r} \bar{\kappa}_i \tag{30}$$

where $\kappa_0 = \lambda_{\min}(-\sum_i^{n_r}\sum_j^{n_r} \Xi_{ij})$. It follows that $\|X\|^2 > \frac{\sum_i^{n_r}\bar{\kappa}_i}{\kappa_0}$ renders $\dot{V} < 0$. It is obviously observed that V is uniformly ultimately bounded. According to the definition of V, the signal of X is bounded too.

Next, we will analyze how to select parameters to satisfy the condition provided in Theorem 3.3 using LMIs.

Firstly, for the condition (25), we can rewrite it as

$$\text{Trace}\left(\left(P_2B - F^T\right)^T\left(P_2B - F^T\right)\right) = 0 \tag{31}$$

Further, by Lemma 2.1, the design problem of $P_2B = F^T$ is now transformed into an optimization problem with LMIs constraints:

$$\min \gamma > 0 \tag{32}$$

$$\begin{bmatrix} -\gamma I & P_2B - F^T \\ \star & -I \end{bmatrix} < 0 \tag{33}$$

Secondly, for the condition $\sum_i^{n_r}\sum_j^{n_r} h_i h_j \Xi_{ij} < 0$, post-multiplying and pre-multiplying it by diag $\left(P_1^{-1}, P_1^{-1}, I\right)$ yields

$$\sum_i^{n_r}\sum_j^{n_r} h_i h_j \begin{bmatrix} \Omega_{ij} & \Omega_j \\ \star & W^T N_i W \end{bmatrix} < 0 \tag{34}$$

where $\Omega_{ij} = (A_i - BK_j)P_1^{-1} + P_1^{-1}(A_i - BK_j)^T$, $\Omega_j = \left[BK_jP_1^{-1}, B\right]$, $W = \text{diag}\left(P_1^{-1}, I\right)$,

$$N_i = \begin{bmatrix} (2P_2(A_i - L_i) + \epsilon) & \dfrac{1}{\sigma}(A_i - L_i)^T F^T \\ \star & -\left(\dfrac{2}{\sigma}FB - \dfrac{1}{\sigma} - 2\epsilon\right) \end{bmatrix}$$

According to Lemma 2.3, there exists λ such that $W^T N_i W \leq -2\lambda W - \lambda^2 N_i^{-1}$. In addition, according to Lemma 2.1, we can obtain the equivalent condition of (34) as follows

$$\sum_{i}^{n_r}\sum_{j}^{n_r} h_i h_j \begin{bmatrix} \Omega_{ij} & \Omega_j & 0 \\ \star & -2\lambda W & \lambda \\ \star & \star & N_i \end{bmatrix} < 0 \tag{35}$$

Let $P_1^{-1} = X_1$, $K_i P_1^{-1} = X_{2i}$, $P_2 L_i = X_{3i}$, $P_2 = X_4$, and then we can obtain the LMIs in Theorem 3.3.

Remark 3.4. *In the design of the fault-tolerant controller, the above LMIs only deduced one possible solution, which may not satisfy the required convergence rate. To solve this problem, a new LMI, $X_1 - Q < 0$, where $Q > 0$ is a positive definite matrix determined by designer, can be added to ensure the appropriate convergence rate by increasing control gain $K_i = X_{2i} X_1^{-1}$.*

4. **Simulation Research.** In this section, the numerical simulations of the nonlinear dynamic positioning ships are carried out to demonstrate the efficiency of the proposed fuzzy output-feedback fault-tolerant control approach. Readers can access all the data supporting the conclusions of the study in [37]. We consider a DP ship given in [37] as

$$M = 10^7 * \begin{bmatrix} 0.027521 & 0 & 0 \\ 0 & 0.076348 & -0.073803 \\ 0 & -0.073803 & 6.690963 \end{bmatrix}$$

$$D = 10^6 * \begin{bmatrix} 0.000025 & 0 & 0 \\ 0 & 0.009865 & 0.001375 \\ 0 & 0.000711 & 2.813355 \end{bmatrix}$$

It is necessary to determine the working region of ξ_3 for finding the linearized subsystems for the T-S fuzzy model. In this numerical example, the working region is chosen as $\xi_3 \in (-\pi/2, \pi/2)$. As mentioned in Remark 3.1, the number of fuzzy rules is closely related to the control accuracy and computation burden of system. Referring to [24, 25], in this paper, three operating points are chosen, and the corresponding membership functions are summarized in Table 1. Using parameters of M and D given above, the T-S fuzzy model was computed and obtained as

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0.035 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0.035 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0131 & -0.0429 \\ 0 & 0 & 0 & 0 & -0.0002 & -0.0425 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & -0.035 & 0 \\ 0 & 0 & 0 & 0.035 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0131 & -0.0429 \\ 0 & 0 & 0 & 0 & -0.0002 & -0.0425 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0.035 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0.035 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0131 & -0.0429 \\ 0 & 0 & 0 & 0 & -0.0002 & -0.0425 \end{bmatrix}, \quad B = 10^{-5} * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.3634 & 0 & 0 \\ 0 & 0.1324 & 0.0015 \\ 0 & 0.0015 & 0.0015 \end{bmatrix}$$

TABLE 1. Membership function

	μ_1	μ_2	μ_3
$\left(0, \frac{\pi}{2}\right)$	$\frac{2x_3}{\pi}$	$1 - \mu_1$	0
$\left(\frac{-\pi}{2}, 0\right)$	0	$1 - \mu_3$	$-\frac{2x_3}{\pi}$

It is easy to verify Assumption 2.3 and there are no invariant zeros in the dynamic positioning system. Further, by choosing the parameters $\sigma = 0.8$, $\lambda = 10$, $\epsilon = 0.01$, $a_1 = a_3 = 4.5$, $a_2 = 7$, the gain matrices of the fuzzy observer and controller are calculated by MATLAB LMI toolbox as follows

$$L_1 = \begin{bmatrix} 1.8768 & 0.0112 & -0.0001 & 0.1818 & -0.4925 & 0.0015 \\ -0.0336 & 1.9591 & 0.0010 & 0.6258 & 0.2749 & -0.0004 \\ 0.0044 & 0.0021 & 2.0639 & 0.0013 & -0.0352 & 0.7265 \\ 0.1425 & 0.0982 & -0.0011 & 1.1610 & -0.0143 & -0.0002 \\ -0.0657 & 0.2666 & -0.0364 & -0.0244 & 0.4247 & -0.0284 \\ 0.0011 & -0.0005 & 0.4089 & 0.0004 & -0.0108 & 0.2567 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1.8868 & 0.0003 & 0.0001 & 0.7903 & -0.0209 & 0.0001 \\ -0.0007 & 1.9930 & 0.0007 & 0.0218 & 0.7190 & -0.0019 \\ 0.0002 & -0.0021 & 2.0640 & 0.0000 & -0.0321 & 0.7265 \\ 0.2336 & 0.0010 & -0.0000 & 1.1366 & -0.0001 & -0.0000 \\ -0.0061 & 0.3265 & -0.0370 & -0.0010 & 0.3685 & -0.0284 \\ 0.0001 & -0.0015 & 0.4089 & 0.0000 & -0.0101 & 0.2567 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 1.8758 & -0.0103 & 0.0002 & 0.1815 & 0.4841 & -0.0014 \\ 0.0348 & 1.9591 & 0.0010 & -0.6261 & 0.2752 & -0.0004 \\ -0.0043 & 0.0021 & 2.0639 & -0.0014 & -0.0353 & 0.7265 \\ 0.1427 & -0.1027 & 0.0011 & 1.1604 & 0.0130 & 0.0002 \\ 0.0581 & 0.2666 & -0.0364 & 0.0235 & 0.4261 & -0.0284 \\ -0.0010 & -0.0005 & 0.4089 & -0.0004 & -0.0109 & 0.2567 \end{bmatrix}$$

$$K_1 = 10^6 * \begin{bmatrix} 0.1469 & 0.0000 & -0.0000 & 1.5368 & -0.0000 & -0.0000 \\ -0.0000 & 0.3114 & -0.0078 & -0.0000 & 3.3301 & -0.1451 \\ 0.0000 & -0.0097 & 0.4026 & -0.0000 & -0.1119 & 5.1954 \end{bmatrix}$$

$$K_2 = 10^6 * \begin{bmatrix} 0.1470 & 0.0000 & -0.0000 & 1.5106 & -0.0000 & -0.0000 \\ -0.0000 & 0.3143 & -0.0078 & -0.0000 & 3.0391 & -0.1310 \\ 0.0000 & -0.0097 & 0.3959 & -0.0000 & -0.1117 & 5.0178 \end{bmatrix}$$

$$K_3 = 10^6 * \begin{bmatrix} 0.1469 & 0.0000 & -0.0000 & 1.5236 & -0.0000 & -0.0000 \\ -0.0000 & 0.3203 & -0.0078 & -0.0000 & 3.3301 & -0.1451 \\ 0.0000 & -0.0093 & 0.4106 & -0.0000 & -0.1119 & 5.3814 \end{bmatrix}$$

$$F = 10^{13} * \begin{bmatrix} -0.2437 & 0.0023 & 0.0000 & 2.8563 & 0.0006 & 0.0000 \\ 0.0072 & -0.4647 & 0.0456 & 0.0002 & 3.3922 & 0.0754 \\ 0.0001 & -0.0051 & -0.0091 & 0.0000 & 0.0377 & 0.0559 \end{bmatrix}$$

Remark 4.1. *How to select the value of parameters σ, λ and ϵ to make the LMIs in Theorem 3.3 solvable is a key problem. The specific values of parameters can be selected by experience and theoretical proof. It is worth noting that the selection of ϵ should not be too large or too small; otherwise the control accuracy will be reduced or peak phenomenon will occur* [36].

4.1. **Simulation of proposed control method in the constant fault case.** For the simulation purposes, the initial conditions are selected as $\eta(t_0) = \begin{bmatrix}10\text{m} & 10\text{m} & 1\text{rad}\end{bmatrix}^T$, and the expected conditions are taken as $\eta_d = \begin{bmatrix}0\text{m} & 0\text{m} & 0\text{rad}\end{bmatrix}^T$. In order to illustrate the efficiency of the proposed fault-tolerant control scheme, the constant faults and time-varying faults are simulated respectively. First, we consider the constant fault situations for actuators, which is denoted as Case-1 in the following and is created as

$$f(t) = \begin{cases} f_1 = f_2 = f_3 = 0; & \text{if } 0 < t \leq 20 \\ f_1 = f_2 = f_3 = 7 * 10^5; & \text{otherwise} \end{cases} \tag{36}$$

Simulation results of the proposed fuzzy fault-tolerant control strategy for the constant actuator faults are depicted in Figures 3-5. It is shown from Figure 3 that the proposed controller can force the ship to the desired target value $\eta_d = [0, 0, 0]^T$ in around 20s. In Figure 4 the velocity estimation errors $[e_u, e_v, e_r]^T$ are shown to be uniformly ultimately bounded with respect to a ball whose radius is sufficiently small. Figure 5 describes fault

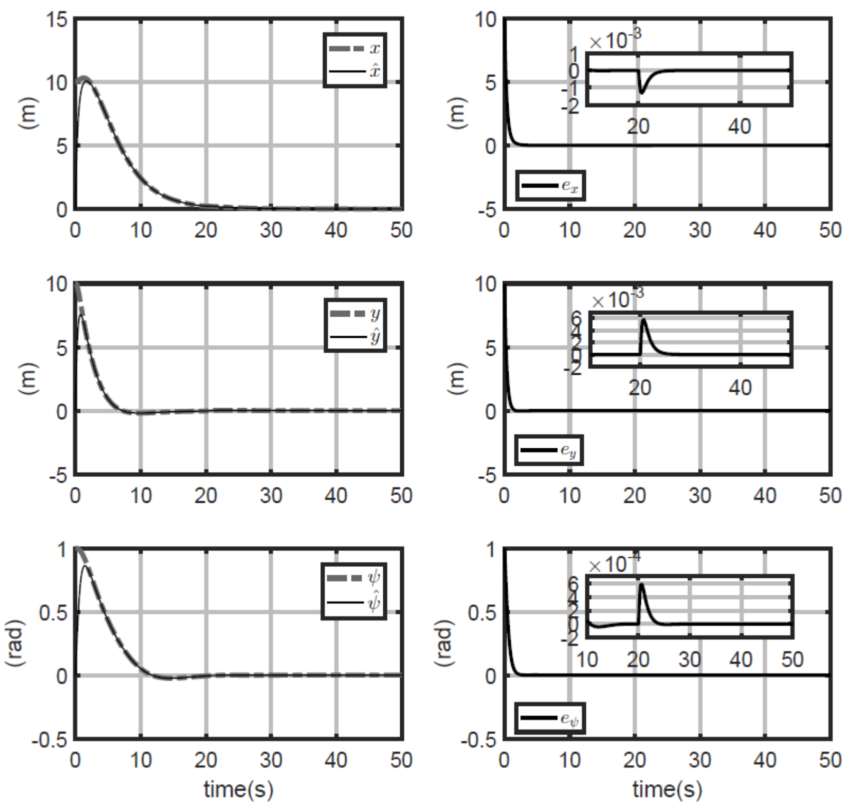

FIGURE 3. Actual and estimated position, and their estimation errors (Case-1)

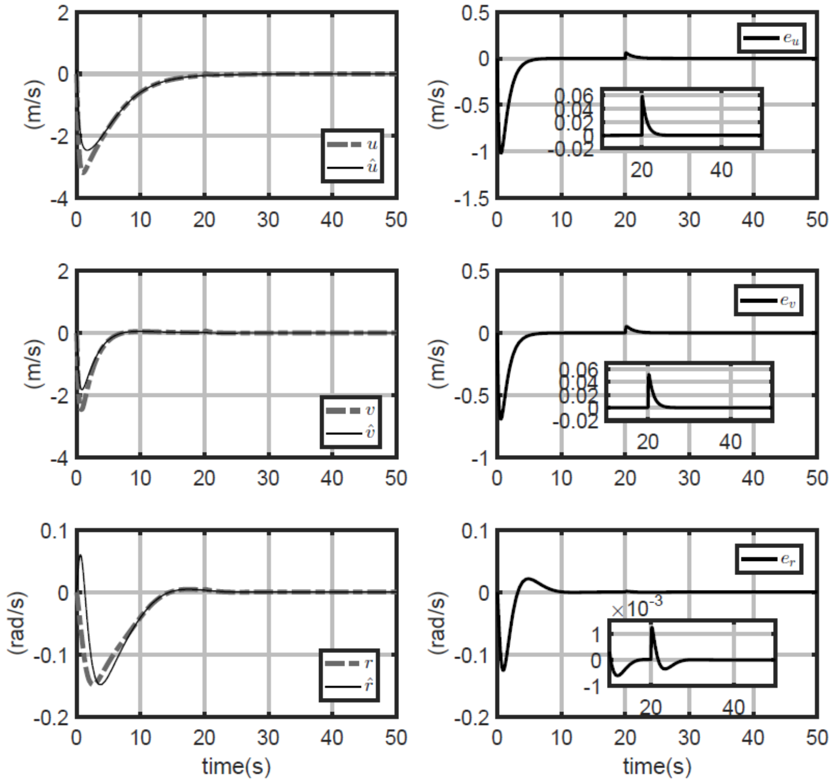

FIGURE 4. Actual and estimated velocity, and their estimation errors (Case-1)

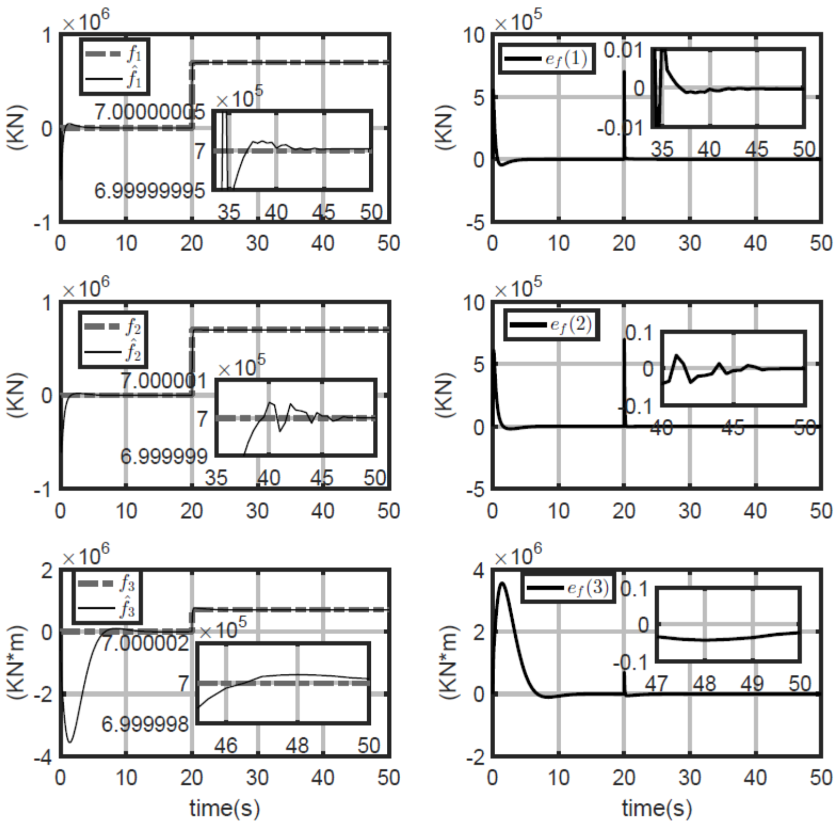

FIGURE 5. Actual and estimated fault, and their estimation errors (Case-1)

estimation of the proposed fuzzy observer. Note that although the oscillation occurs at initial stage, the fault estimation error e_f converges to a sufficiently small neighborhood of zero within a finite time quickly. Therefore, from the simulation results, it is seen that the proposed nonlinear adaptive fuzzy output-feedback fault-tolerant controller has a good robustness against unknown constant faults.

4.2. **Simulation of proposed control method in the time-varying fault case.** Further, it is assumed that a time-varying faults $f(t)$ occur in the actuator, which is denoted as Case-2, i.e.,

$$f(t) = \begin{cases} f_1 = 10^6 \sin(0.1t)\cos(0.1t)^2, \quad f_2 = 0, \quad f_3 = 0; & \text{if } 0 < t \leq 10 \\ f_2 = 10^6 \sin(0.1t)\cos(0.1t)^2, \quad f_1 = 0, \quad f_3 = 0; & \text{if } 10 < t \leq 30 \\ f_3 = 7 * 10^6 \sin(0.1t)\cos(0.1t)^2, \quad f_1 = 0, \quad f_2 = 0; & \text{otherwise} \end{cases} \tag{37}$$

Simulation results of the proposed fuzzy fault-tolerant control strategy for the time-varying actuator faults are depicted in Figures 6-8. It can be observed from Figures 6 and 7 that the fuzzy output-feedback fault-tolerant controller can force the ship to the desired target value $\eta_d = [0, 0, 0]^T$, also, the velocity estimation errors $[e_u, e_v, e_r]^T$ can converge to a sufficiently small neighborhood of zero. The reconstructed time-varying fault $\hat{f}$ is shown in Figure 8, and it clearly reveals that the fuzzy observer can accurately and quickly reconstruct the time-varying faults.

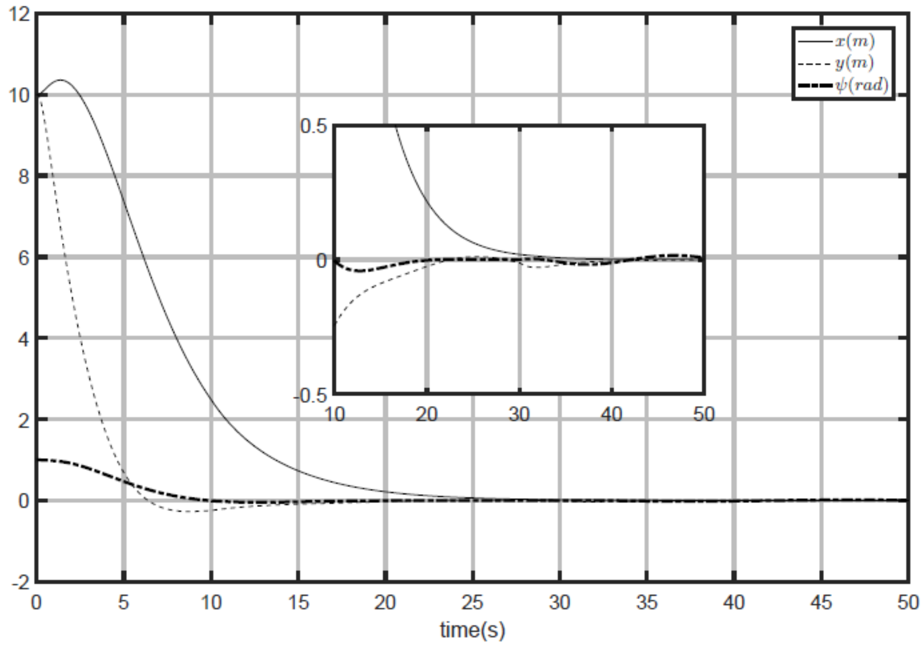

Figure 6. Position and heading of DP ships (Case-2)

It is worth noting that, in practical application, the marine environment is complex and the disturbance forces caused by wind, wave and current cannot be ignored. A frequently used disturbance model for marine control applications is the first-order Markov process [10, 38]

$$\dot{b} = -A_b^{-1} b + B_b \omega_b \tag{38}$$

where $b \in R^3$ is the vector representing the changing disturbance forces and moment, $\omega_b \in R^3$ is the vector of the Gaussian white noise with zero mean value, and A_b and B_b are the diagonal matrix of positive time constants and the diagonal matrix scaling

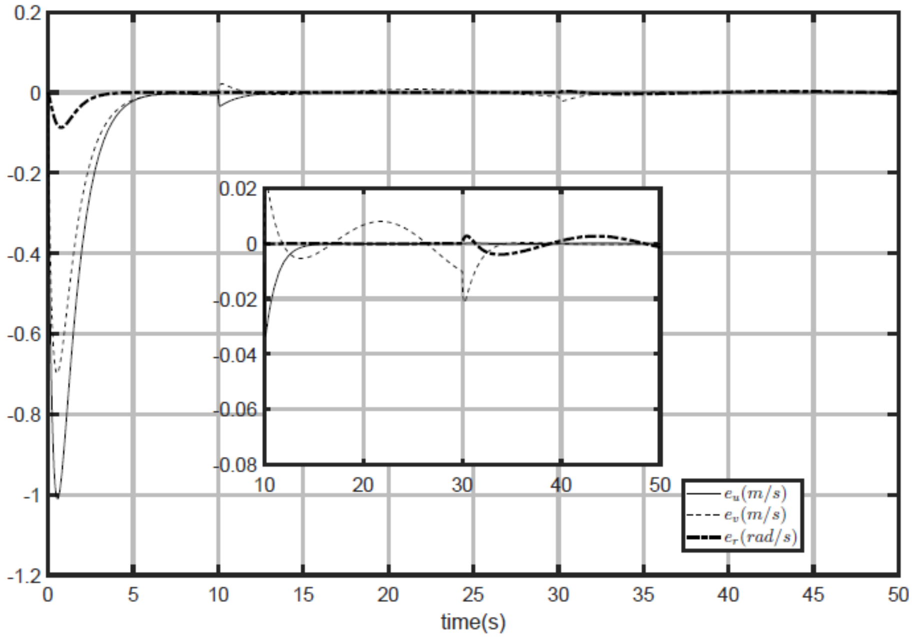

FIGURE 7. Velocity estimation errors (Case-2)

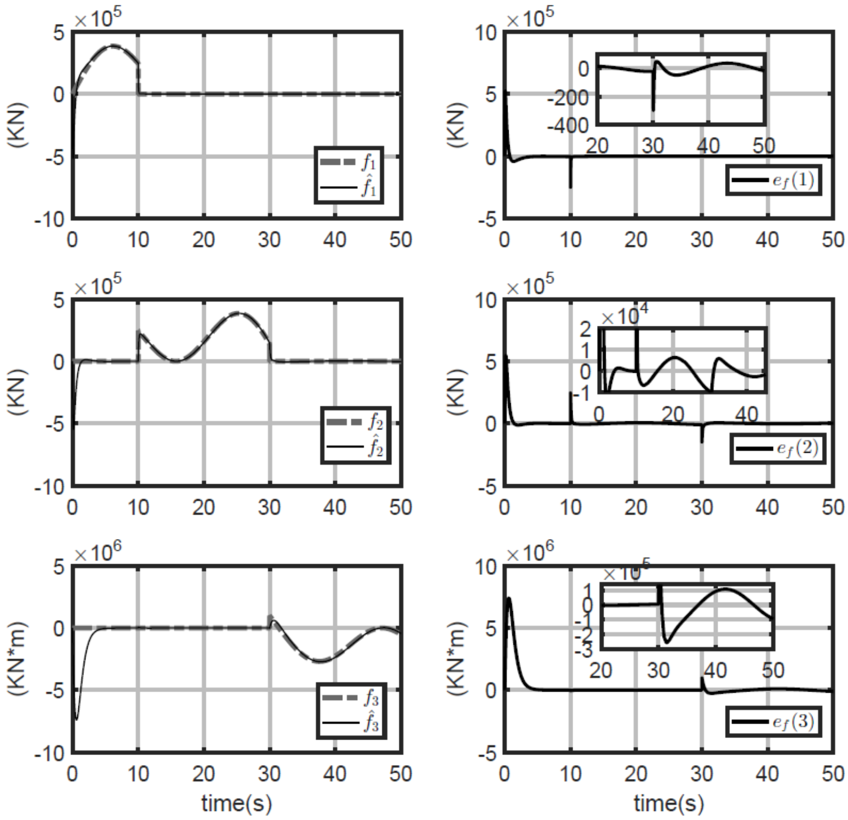

FIGURE 8. Actual and estimated fault, and their estimation errors (Case-2)

the amplitude of Gaussian white noise. For the simulation purposes, the parameters are selected as $A_b = \text{diag}(3000, 3000, 3000)$, and $B_b = \text{diag}(1000, 1000, 1000)$.

Figures 9-11 display the simulation results when the above disturbance (38) is taken into account, and demonstrate that the proposed fuzzy fault-tolerant controller is also

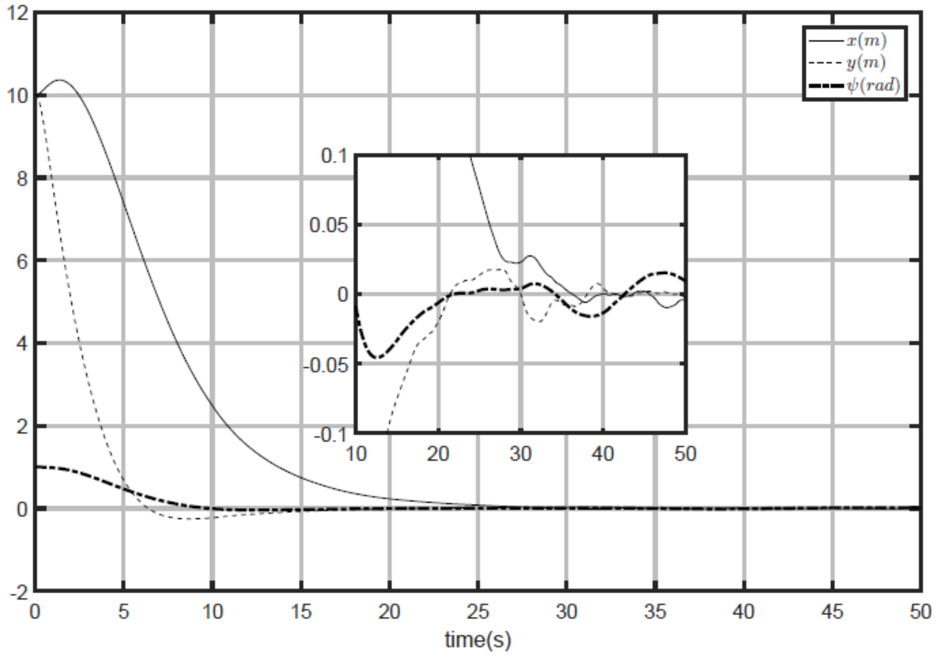

FIGURE 9. Position and heading of DP ships (Case-3)

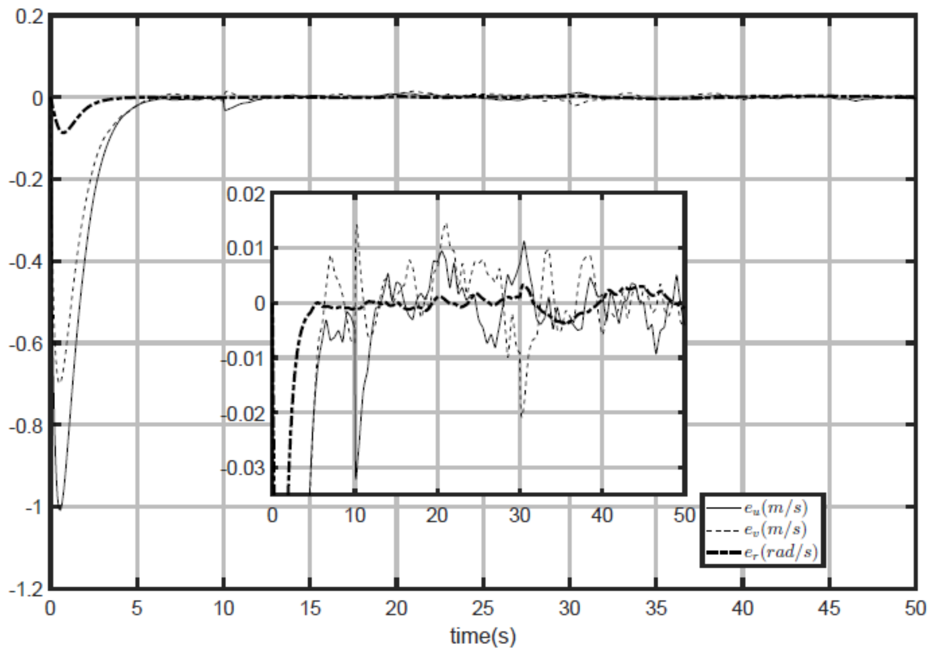

FIGURE 10. Velocity estimation errors (Case-3)

effective for the case of simultaneous presence of actuator time-varying faults (37) and environmental disturbances (38), which is denoted as Case-3, even though the heading of ship is slightly chattering. Moreover, from the simulation results in Figures 8 and 11, it is quite obvious that estimation errors of fault signal in Case-3 are much bigger than those in Case-2, which is caused by the fact that $\bar{\kappa}_i$ in (30) of Case-3 increases as f_M increases. Nevertheless, the ship position (x, y) and heading ψ can also arrive at the desired target value $\eta_d = [0, 0, 0]^T$ in around 25s. Consequently, it is concluded that the proposed fuzzy

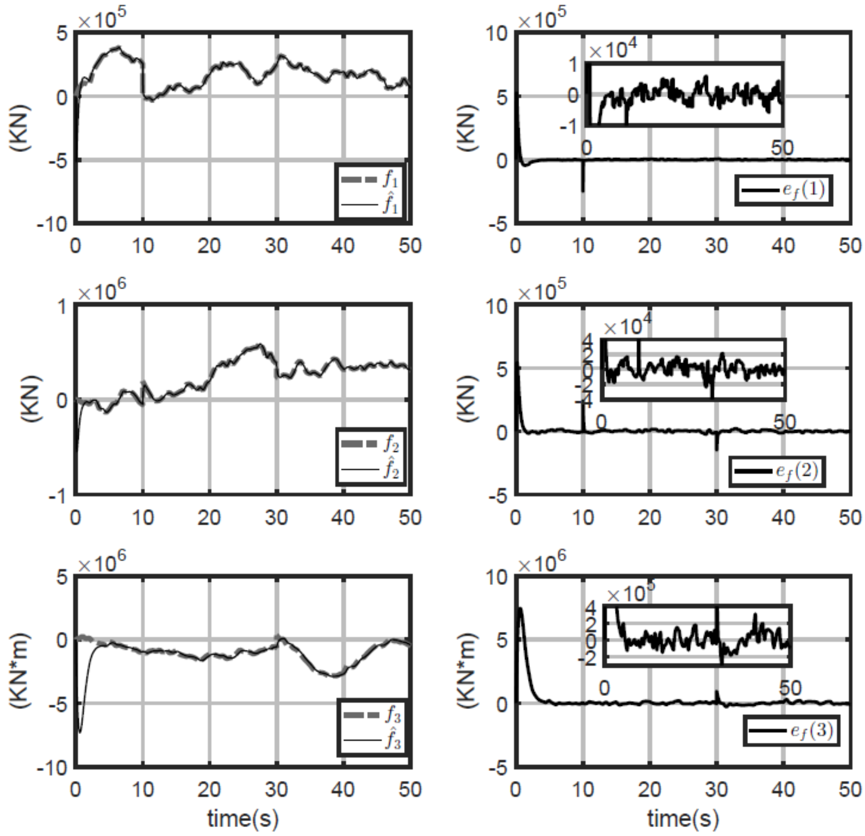

Figure 11. Actual and estimated fault, and their estimation errors (Case-3)

output-feedback fault-tolerant controller possesses not only excellent fault tolerance but also great disturbance rejection ability for dynamic positioning of ships.

5. **Conclusions.** An adaptive fuzzy output-feedback fault-tolerant control scheme for nonlinear dynamic positioning of ships with actuator faults is investigated in this paper. The proposed fault-tolerant control scheme only depends on the measurements of the ship position and heading. Closed-loop stability of the proposed fault-tolerant control system is established by the Lyapunov stability theory. The effectiveness of the fault-tolerant control scheme for DP ships in the presence of disturbance is demonstrated through simulations. In simulations, it has been shown that the fault effect has been negated completely without degrading the system performance significantly which confirms the effectiveness of the proposed scheme. In addition, due to the fact that the saturation limits and loss-of-effectiveness always exist in actuators, the FTC strategy designing for DP ships with multiplicative faults under more actuator constraints is another interesting and practical issue, which will be investigated in our future work.

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