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FAULT DIAGNOSIS AND FAULT-TOLERANT CONTROL OF NETWORKED MANIPULATOR SYSTEM WITH ACTUATOR FAULT

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Abstract. *In this paper, a fault diagnosis and fault-tolerant control scheme is proposed for the actuator fault in the networked manipulator system. The advantage of this scheme is that the observer has a simple design structure, and the designed controller can eliminate not only the impact of the fault, but also the time-delay. In the fault diagnosis, the fault estimation law based on the adaptive observer is given by the residual, to diagnose the fault estimation accurately and quickly, and the residual is established by the time-delay state due to the time-delay in the network system. By using the fault information, a PD controller plus the compensation fault-tolerant controller is designed to compensate for the impact of the actuator fault and time delay. And the sliding mode controller is introduced to be combined with the PD controller to further improve the control performance, which ensures that the effects of time-delays are eliminated while the effects of fault are tolerated. Finally, the effectiveness of the above scheme is verified by the simulation results.*
Keywords: Fault diagnosis, Fault-tolerant control, Manipulator, Networked control system

1. **Introduction.** The application of the manipulator has brought many conveniences to the fields of industry, medical treatment, and aerospace. The safety and reliability of the manipulator have also become the focus of the research. In practice, due to the external interference or abnormal conditions, some key components such as actuator and sensor of the manipulator deviate from normal working conditions and even fault. The development of fault diagnosis and fault-tolerant control theory provides a theoretical basis and solution for the fault in [1-5].

Fault diagnosis of the actuator fault of manipulator has been the focus of researchers. Among the existing research results, it is attractive to design the fault diagnosis observer based on the system model. For example, in [6,7], the high-order sliding mode observer is designed for the actuator fault based on the mathematical model of the manipulator. In [8], the fault is reconstructed by the observer with precise and fast properties. In [9], an algorithm for robust fault diagnosis in the uncertain manipulator system by using a neural sliding mode observer is investigated.

Fault-tolerant control can be divided into active fault-tolerant control and passive fault-tolerant control [10-13]. Robust control is one of the usual passive fault-tolerant control schemes [11], and in the active fault-tolerant control, the fault information obtained in fault diagnosis is used to reconstruct the controller and compensate for the effect of the fault on the system performance. In [6], a super-twisting second-order sliding mode

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controller is designed to accommodate not only fault but also uncertainties. A controller is introduced in [12] to compensate for the fault, which combines the properties of the PD controller and the sliding mode controller. A novel control methodology for tracking control of manipulators is developed in [14] based on a novel adaptive back-stepping non-singular fast terminal sliding mode control, which performs an outstanding performance for fault-tolerant control.

Recently, research on networked control systems and their stability has gradually increased [15-17]. Networked control of the robotic manipulator system based on a second-order sliding mode observer is presented in [18], and the proposed approach is robust to the network delays. In [19], the nonlinear observer is designed to provide the estimation of the unmeasurable state and the modeling uncertainty to construct the fault estimation algorithm, and a fault-tolerant control method is proposed to make the faulty system stable by considering the delay time through the network. To compensate for the effect of the sensor fault and time-delay in a class of networked control systems, a feed-forward and feedback sensor fault-tolerant controller can be found in [20] based on the fault information from the fault diagnosis observer.

With the rapid development of the Internet technology and its widespread application, the combination of the network and manipulators is becoming a new research highlight in the field of robotics. In the networked manipulator control system, faults in the manipulator and time-delays in the network communication cause huge impact on the control effectiveness. It is attractive to consider the problem of how to diagnose and tolerate the fault that occurs in the networked manipulator system. In the current literature, fault diagnosis and fault-tolerant control for the networked control system and the manipulator system are discussed respectively. However, fault diagnosis and fault-tolerant control of the networked manipulator system has not yet attracted corresponding attentions. Because of the time-delay caused by the network, the state of the networked manipulator system is delayed by a delayed moment. It will be a challenge to perform the fault diagnosis and fault-tolerant control by using the delay state when the fault occurs in the networked manipulator systems. The scheme of the fault diagnosis and fault-tolerant for the actuator fault in the networked manipulator system under the existence of the time-delay is considered in this paper. The main contributions of this paper can be summarized as follows: Different from the time-delay control (TDC) in [21-23], the time-delay state is used to construct the residual to design the fault estimation law; the fault-tolerant controller is designed by the fault information obtained from fault diagnosis, and the controller is constructed with the property that the impact of the time-delay can be accommodated.

The structure of the following contents is shown as follows: the model of the actuator fault in the networked manipulator system is introduced in Chapter 2; the fault diagnosis observer is designed in Chapter 3 to diagnose the fault occurrence time and amplitude; the fault-tolerant controller and its improved design are described in Chapter 4; simulation results are given in Chapter 5 to verify the feasibility of the above scheme; concluding remarks and future research are provided in Chapter 6.

2. Model Establishment.

2.1. Manipulator fault system model. The dynamic model of the manipulator actuator fault system can be described as follows

$$M(q_t)\ddot{q}_t + C(q_t, \dot{q}_t)\dot{q}_t + g(q_t) = \tau + Nf + Dd(t) \tag{1}$$

where q_t, $\dot{q}_t$, $\ddot{q}_t$ represent the angle, angular velocity and angular acceleration at the time t respectively, $\tau \in R^n$ is the control input, $M(q_t)$ and $C(q_t, \dot{q}_t)\,\dot{q}_t$ are the inertia matrix and the torques of the centripetal and Coriolis, and $g(q_t)$ denotes the gravitational torques. $f \in R^p$ is the actuator fault. The norm of the fault satisfies the conditions: $\|f\| \le f_{\max}$ and $\left\|\dot{f}\right\| \le \alpha$, and its distribution matrix is denoted as $N \in R^{n\times p}$, d is the disturbance and $\|d\| \le \bar{d}$, $D \in R^{n\times p}$ denotes the disturbance distribution matrix. These properties are stated as follows [24].

Property 2.1. *The positive-definite and symmetric inertia matrix, satisfies the following inequalities:*

$$m_1\|\kappa\|^2 \le \kappa^T M(q_t)\kappa \le m_2\|\kappa\|^2, \quad \forall \kappa \in R^n$$

where $m_1, m_2 \in R$ are known positive bounding constants.

Property 2.2. *The time derivative of the inertia matrix and the centripetal-Coriolis matrix satisfy the following skew symmetric relationship:*

$$\kappa^T\left[\dot{M}(q_t) - 2C(q_t,\dot{q}_t)\right]\kappa = 0, \quad \forall \kappa \in R^n$$

In order to use the estimation information of the fault, the manipulator dynamic model will be transformed into the state space model.

Define the vector $x_1(t) = q_t$, $x_2(t) = \dot{q}_t$, $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in R^{2n}$, $u = \tau$, then the dynamic model of the manipulator actuator fault system (1) can be transformed into the state space model [12]

$$\begin{cases} \dot{x}(t) = Ax(t) + B(x(t),u) + N(x(t))f(t) + D(x(t))d(t) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where $A = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$, $B(x(t),u(t)) = \begin{bmatrix} 0_n \\ M(x_1(t))^{-1}[u(t) - C(x_1(t),x_2(t))x_2(t) - g(x_1(t))] \end{bmatrix}$ is the nonlinear part, $D(x(t)) = \begin{bmatrix} 0_n \\ M(x_1(t))^{-1}D \end{bmatrix}$ represents the disturbance part, $N(x(t)) = \begin{bmatrix} 0_n \\ M(x_1(t))^{-1}N \end{bmatrix}$ denotes the distribution matrix of the fault, and $C = \begin{bmatrix} I_n & 0_n \end{bmatrix}$ is the output matrix.

2.2. **Networked manipulator fault system model.** Because of the time delay occurring in the networked system, the state at time t can only receive the information of the state at time $t - t_d$. So, the dynamic model of the manipulator actuator fault system can be described as follows

$$M(q_{t-t_d})\ddot{q}_{t-t_d} + C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} + g(q_{t-t_d}) = \tau + Ff + Dd \tag{3}$$

where $t_d \in R$ denotes the time-delay. q_{t-t_d} represents the state with time-delay at the time t. $f \in R^p$ is the actuator fault. The norm of the fault satisfies the conditions: $\|f\| \le f_{\max}$ and $\left\|\dot{f}\right\| \le \alpha$, and its distribution matrix is denoted as $N \in R^{n\times p}$, d is the disturbance and $\|d\| \le \bar{d}$, $D \in R^{n\times p}$ denotes the disturbance distribution matrix.

Define $x^1_{t-t_d} = q_{t-t_d}$, $x^2_{t-t_d} = \dot{q}_{t-t_d}$, $x_{t-t_d} = \begin{bmatrix} x^1_{t-t_d} \\ x^2_{t-t_d} \end{bmatrix} \in R^{2n}$, and then (3) can be rewritten as follows

$$\begin{cases} \dot{x}_{t-t_d} = Ax_{t-t_d} + B(x_{t-t_d},u) + N(x_{t-t_d})f + D(x_{t-t_d})d \\ y_{t-t_d} = Cx_{t-t_d} \end{cases} \tag{4}$$

where $A = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$, $B(x_{t-t_d}, u) = \begin{bmatrix} 0_n \\ M(x_{t-t_d})^{-1}\left[u - C\left(x^1_{t-t_d}, x^2_{t-t_d}\right)x^2_{t-t_d} - g\left(x^1_{t-t_d}\right)\right] \end{bmatrix}$ is the nonlinear part, $N(x_{t-t_d}) = \begin{bmatrix} 0_n \\ M(x^1_{t-t_d})^{-1}F \end{bmatrix}$ represents the disturbance part, $D(x_{t-t_d}) = \begin{bmatrix} 0_n \\ M\left(x^1_{t-t_d}\right)^{-1} D \end{bmatrix}$ denotes the distribution matrix of the fault, and $C = \begin{bmatrix} I_n & 0_n \end{bmatrix}$ is the output matrix.

3. **Fault Diagnosis Scheme.** To estimate the fault, the adaptive observer can be designed as (5) for the faulty system (4)

$$\begin{cases} \dot{\hat{x}}_{t-t_d} = A\hat{x}_{t-t_d} + B\left(\hat{x}_{t-t_d}, u\right) + N\left(\hat{x}_{t-t_d}\right)\hat{f} + L\left(y_{t-t_d} - \hat{y}_{t-t_d}\right) \\ \hat{y}_{t-t_d} = C\hat{x}_{t-t_d} \end{cases} \tag{5}$$

where $L \in R^{2n\times n}$ is the observer gain, $\hat{x}_{t-t_d}$ is the estimation of the state x_{t-t_d}, $\hat{y}_{t-t_d}$ is the output of the observer, and the $\hat{f}$ is the estimation of the fault.

From (4) and (5), the fault estimation error can be defined as $\Delta f = \hat{f} - f$, and the state error can be written as $e_{t-t_d} = x_{t-t_d} - \hat{x}_{t-t_d}$, and the residual of the system can be given as $\varepsilon_{t-t_d} = C(x_{t-t_d} - \hat{x}_{t-t_d}) = Ce_{t-t_d}$. Then the state error dynamics equation is obtained as

$$\begin{aligned} \dot{e}_{t-t_d} &= \dot{x}_{t-t_d} - \dot{\hat{x}}_{t-t_d} \\ &= Ax_{t-t_d} + B(x_{t-t_d}, u) + N(x_{t-t_d})f + D(x_{t-t_d})d - A\hat{x}_{t-t_d} - B\left(\hat{x}_{t-t_d}, u\right) \\ &\quad - N\left(\hat{x}_{t-t_d}\right)\hat{f} - L\left(y_{t-t_d} - \hat{y}_{t-t_d}\right) \\ &= (A - LC)e_{t-t_d} + B(x_{t-t_d}, u) - B\left(\hat{x}_{t-t_d}, u\right) + \left[N(x_{t-t_d}) - N\left(\hat{x}_{t-t_d}\right)\right]f \\ &\quad - N\left(\hat{x}_{t-t_d}\right)\Delta f + D(x_{t-t_d})d \end{aligned} \tag{6}$$

Before introducing the main process of the fault diagnosis scheme proposed in this paper, some significant lemma and assumptions can be given as follows.

Lemma 3.1. [12, 25]. *There exists a scale $\mu > 0$ making Inequality (7) be satisfied*

$$2a^Tb \le \frac{1}{\mu}a^Ta + \mu b^Tb, \quad a, b \in R^n \tag{7}$$

Assumption 3.1. *It is assumed that for the manipulator actuator fault augmented system, the Lipschitz condition is satisfied. There exist the Lipschitz constants γ_1, γ_2 making the inequality be satisfied*

$$\left\|B(x_{t-t_d}, u) - B\left(\hat{x}_{t-t_d}, u\right)\right\| \le \gamma_1 \left\|x_{t-t_d} - \hat{x}_{t-t_d}\right\|,$$

$$\left\|N(x_{t-t_d}) - N\left(\hat{x}_{t-t_d}\right)\right\| \le \gamma_2 \left\|x_{t-t_d} - \hat{x}_{t-t_d}\right\|.$$

Assumption 3.2. *For the fault distribution matrix $N\left(\hat{x}_{t-k}\right)$, there exists a constant matrix signed N chosen specially making the following conditions hold*

$$N_i \ge N\left(\hat{x}_{t-t_d}\right)_i, \quad |N_i| \ge \left|N\left(\hat{x}_{t-t_d}\right)_i\right|, \quad \|N_i\| \ge \left\|N\left(\hat{x}_{t-t_d}\right)_i\right\|.$$

Assumption 3.3. *For the disturbance, the inequality $\|D(x_{t_d}, d)\| \le \bar{D}\|d\|$ holds and $\bar{D} > 0$.*

Theorem 3.1. *For the manipulator augment system, the above assumptions hold. Given the scalars M_x, M_s, M_d and ρ, there exist a positive definite symmetric matrix $P \in R^{2n\times 2n}$, and matrices $L \in R^{2n\times n}$ and $R_a \in R^{p\times n}$ making the following conditions be satisfied*

$$\int e_{t-t_d}^T M_x e_{t-t_d} ds - \rho \int \Delta f^T M_a \Delta f ds - \rho \int d^T M_d d ds \le 0 \tag{8}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & 0 \\ * & * & \Pi_{33} \end{bmatrix} < 0 \tag{9}$$

where $\Pi_{11} = A^T P + PA - C^T Y^T - YC + 2\gamma P + M_x$, $\Pi_{12} = PN + C^T R_a^T$, $\Pi_{13} = P\bar{D}$, $\Pi_{22} = \frac{2}{\mu} - \rho M_a$, $\Pi_{33} = -\rho M_d$, $Y = PL$, $\gamma = \gamma_1 + \gamma_2 f_{\max}$, and $*$ denotes the symmetric elements in the symmetric matrix. The fault estimation law can be given as

$$\dot{\hat{f}} = \Gamma_a R_a \varepsilon_{t-t_d} \tag{10}$$

where $\Gamma_a > 0$ is a pre-specified matrix denoting the tuning rate, and R_a is the gain matrix to be designed.

Proof: Consider the Lyapunov function $V = e_{t-t_d}^T P e_{t-t_d} + \Delta f^T \Gamma_a^{-1} \Delta f$ and differentiate V with respect to time

$$\begin{aligned}
\dot{V} &= e_{t-t_d}^T P e_{t-t_d} + \Delta f^T \Gamma_a^{-1} \Delta f \\
&= \dot{e}_{t-t_d}^T P e_{t-t_d} + e_{t-t_d}^T P \dot{e}_{t-t_d} + \dot{\Delta f}^T \Gamma_a^{-1} \Delta f + \Delta f^T \Gamma_s^{-1} \dot{\Delta f} \\
&= [(A - LC)e_{t-t_d} + B(x_{t-t_d}, u) - B(\hat{x}_{t-t_d}, u) + [N(x_{t-t_d}) - N(\hat{x}_{t-t_d})] f \\
&\quad - N(\hat{x}_{t-t_d}) \Delta f + D(x_{t-t_d}) d]^T P e_{t-t_d} + e_{t-t_d}^T P [(A - LC)e_{t-t_d} \\
&\quad + B(x_{t-t_d}, u) - B(\hat{x}_{t-t_d}, u) + [N(x_{t-t_d}) - N(\hat{x}_{t-t_d})] f \\
&\quad - N(\hat{x}_{t-t_d}) \Delta f + D(x_{t-t_d}) d] + 2\Delta f^T \Gamma_a^{-1} \dot{\Delta f}
\end{aligned} \tag{11}$$

According to Assumptions 3.1, 3.2 and 3.3, (11) can be obtained as

$$\begin{aligned}
\dot{V} &\le e_{t-t_d}^T \left[(A - LC)^T P + P(A - LC) + 2\gamma P\right] e_{t-t_d} + 2e_{t-t_d}^T PN\Delta f \\
&\quad + 2e_{t-t_d}^T P\bar{D}d + 2\Delta f^T \Gamma_a^{-1} \dot{\hat{f}} - 2\Delta f^T \Gamma_a^{-1} \dot{f}
\end{aligned} \tag{12}$$

According to Lemma 3.1, it can be shown that

$$\Delta f^T (-\Gamma_a^{-1}) \dot{f} \le \frac{1}{\mu} \Delta f^T \Delta f + \mu \dot{f}^T \left(-\Gamma_a^{-1}\right) \left(-\Gamma_a^{-1}\right) \dot{f} \le \frac{1}{\mu} \Delta f^T \Delta f + \delta_k \tag{13}$$

where $\delta_k = \mu\alpha^2 \lambda_{\max} (\Gamma_a^{-1} \Gamma_a^{-1})$. Then (12) can be further written as follows

$$\begin{aligned}
\dot{V} &\le e_{t-t_d}^T Q e_{t-t_d} + \frac{2}{\mu} \Delta f^T \Delta f + 2e_{t-t_d}^T PN\Delta f + 2\Delta f^T R_a C e_{t-t_d} + 2e_{t-t_d}^T P\bar{D}d + \delta_a \\
&= e_{t-t_d}^T Q e_{t-t_d} + 2e_{t-t_d}^T \left(PN + C^T R_a^T\right) \Delta f + \frac{2}{\mu} \Delta f^T \Delta f + 2e_{t-t_d}^T P\bar{D}d + \delta_k
\end{aligned} \tag{14}$$

where $Q = (A - LC)^T P + P(A - LC) + 2\gamma P$. Define

$$H = \dot{V} + e_{t-t_d}^T M_x e_{t-t_d} - \rho \Delta f^T M_a \Delta f - \rho d^T M_d d \tag{15}$$

Then it can be obtained that

$$\begin{aligned}
H &\le e_{t-t_d}^T Q e_{t-t_d} + 2e_{t-t_d}^T \left(PN + C^T R_a^T\right) \Delta f + \frac{2}{\mu} \Delta f^T \Delta f + \delta_k \\
&\quad + 2e_{t-t_d}^T P\bar{D}d + e_{t-t_d}^T M_x e_{t-t_d} - \rho \Delta f^T M_a \Delta f - \rho d^T M_d d \\
&\le e_{t-t_d}^T (Q + M_x) e_{t-t_d} + 2e_{t-t_d}^T \left(PN + C^T R_a^T\right) \Delta f
\end{aligned}$$

$$+\Delta f^T\left(\frac{2}{\mu}-\rho M_a\right)\Delta f+2e_{t-t_d}^T P\bar{D}d-d^T(\rho M_d)d+\delta_k \tag{16}$$

Define a new state $\Xi=\left[\begin{array}{ccc} e_{t-t_d}^T & \Delta f^T & d^T \end{array}\right]^T$, and it can be obtained from (9) that

$$H\le \Xi^T\Pi\Xi+\delta \tag{17}$$

It can be further formulated that

$$H\le -\pi\left\|\Xi\right\|^2+\delta \tag{18}$$

where $\pi=\lambda_{\min}(-\Pi)$, $\delta=\delta_k$. When $\pi\|\Xi\|^2>\delta$ holds,

$$\dot{V}+e_{t-t_d}^T M_x e_{t-t_d}-\rho\Delta f^T M_a\Delta f-\rho d^T M_d d\le 0 \tag{19}$$

What can be indicated from (18) and (19) is that the new state Ξ is uniformly bounded and converges to a small set $\Psi=\left\{\Xi\middle|\|\Xi\|^2\le\delta/\pi\right\}$ under the Lyapunov stability theory, which represents the state error and fault estimation error converge and are bounded.

Based on the fault estimation obtained in Chapter 3, an active fault-tolerant control scheme can be designed on the basis of the fault estimation information. The active fault tolerant controller is constructed for the fault system by using the fault information obtained, which can compensate the impact of the fault on the system.

4. **Fault-Tolerant Controller Design.** The design of the active fault-tolerant controller for the manipulator fault system, which can still maintain stability of the system and achieve the expected trajectory, is based on the manipulator control algorithm, such as the PD controller, and the sliding mode controller. The existing manipulator control algorithm provides convenience for design of the fault-tolerant controller.

Define q_d as the expected trajectory, because of the networked time-delay, the controller can only obtain the status information q_{t-t_d} of time $t-t_d$ at time t. In order to keep the state trajectory still follow the desired trajectory after fault occurs, the active fault-tolerant controller is designed by using the fault information. Define the tracking error and its derivative as:

$$\theta=q_d-q_{t-t_d} \tag{20}$$

$$\dot{\theta}=\dot{q}_d-\dot{q}_{t-t_d} \tag{21}$$

4.1. **Design of PD plus compensation controller.** PD controller is a common control algorithm in manipulator control, which has the simple structure and good effect. Based on the PD controller, the term of fault compensation and time-delay compensation is added to achieve the fault-tolerant control effect of the networked manipulator fault system.

The PD plus compensation controller can be designed as follows:

$$\begin{cases} u_{PD}=u_{eq}-N\hat{f}+K_k\tanh\left(\dot{\theta}\right) \\ u_{eq}=M(q_{t-t_d})\dot{q}_d+C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_d+K_P\theta+K_D\dot{\theta}-D\bar{d}+g(q_d) \end{cases} \tag{22}$$

where $K_P\in R^{n\times n}>0$ and $K_D\in R^{n\times n}>0$ are the proportional gain matrix and the differential gain matrix, respectively. $K_k\in R^{n\times n}>0$, $\tanh(\bullet)$ is designed as the term of time delay compensation. Then the closed-loop system can be written as

$$M(q_{t-t_d})\ddot{\theta}+C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{\theta}+K_P\theta+K_D\dot{\theta}+K_k\tanh\left(\dot{\theta}\right)=0 \tag{23}$$

Proof: Consider the Lyapunov function $V_{PD}=\frac{1}{2}\dot{\theta}^T M(q_{t-t_d})\dot{\theta}+\frac{1}{2}\theta^T\varepsilon\theta$ and $\forall\varepsilon>0$. Differentiate V_{PD} with respect to time as

$$\begin{aligned}\dot{V}_{PD} &= \frac{1}{2}\ddot{\theta}^T M(q_{t-t_d})\dot{\theta} + \frac{1}{2}\dot{\theta}^T M(q_{t-t_d})\ddot{\theta} + \frac{1}{2}\dot{\theta}^T \dot{M}(q_{t-t_d})\dot{\theta} + \frac{1}{2}\dot{\theta}^T \varepsilon\theta + \frac{1}{2}\theta^T \varepsilon\dot{\theta} \\ &= \dot{\theta}^T M(q_{t-t_d})\ddot{\theta} + \frac{1}{2}\dot{\theta}^T \dot{M}(q_{t-t_d})\dot{\theta} + \dot{\theta}^T \varepsilon\theta \end{aligned} \tag{24}$$

According to the characteristics of the manipulator system, (24) can be written as

$$\begin{aligned}\dot{V}_{PD} &= \dot{\theta}^T M(q_{t-t_d})\ddot{\theta} + \frac{1}{2}\dot{\theta}^T \dot{M}(q_{t-t_d})\dot{\theta} + \dot{\theta}^T \varepsilon\theta \\ &= \dot{\theta}^T M(q_{t-t_d})\ddot{\theta} + \dot{\theta}^T C\left(q_{t-t_d}, \dot{q}_{t-t_d}\right)\dot{\theta} + \dot{\theta}^T \varepsilon\theta \\ &= \dot{\theta}^T \left(M(q_{t-t_d})\ddot{\theta} + C\left(q_{t-t_d}, \dot{q}_{t-t_d}\right)\dot{\theta}\right) + \dot{\theta}^T \varepsilon\theta \end{aligned} \tag{25}$$

Substituting (23) to (25), it can be obtained as follows:

$$\begin{aligned}\dot{V}_{PD} &= \dot{\theta}^T \left(M(q_{t-t_d})\ddot{\theta} + C\left(q_{t-t_d}, \dot{\theta}_{t-t_d}\right)\dot{\theta}\right) + \dot{\theta}^T \varepsilon\theta \\ &= \dot{\theta}^T \left(-K_P\theta - K_D\dot{\theta} - K_k \tanh\left(\dot{\theta}\right)\right) + \dot{\theta}^T \varepsilon\theta \\ &= -K_P\dot{\theta}^T\theta - K_D\dot{\theta}^T\dot{\theta} - K_k\dot{\theta}^T \tanh\left(\dot{\theta}\right) + \dot{\theta}^T \varepsilon\theta \\ &= \dot{\theta}^T \left(-K_P\theta - K_D\dot{\theta} - K_k \tanh\left(\dot{\theta}\right) + \varepsilon\theta\right) \end{aligned} \tag{26}$$

When K_P, K_D, K_k and ε are chosen to satisfy $K_P + K_D \left\|\frac{\dot{\theta}}{\theta}\right\| + K_k \left\|\frac{\tanh(\dot{\theta})}{\theta}\right\| \geq \varepsilon$, the inequality $\dot{V}_{PD} \leq 0$ holds, which indicates that the system is convergent.

4.2. **Design of the PD plus SMC controller.** SMC (sliding mode controller) is widely used in the control of nonlinear systems, and can also achieve good control effects in the manipulator system. In order to further improve the PD plus compensation fault-tolerant controller in 4.1, a networked controller combining PD and SMC is designed here.

Define $z_1 = \theta$, $z_2 = \dot{z}_1 + cz_1$, and then choose a PD controller u_{PS} as follows

$$u_{PS} = K_P z_1 + K_D \dot{z}_1 \tag{27}$$

where $K_P \in R^n$ and $K_D \in R^n$ are the proportional coefficient and the differential coefficient matrix, respectively.

Construct the sliding surface as follows

$$s = u_{PS} + z_2 = K_P z_1 + K_D \dot{z}_1 + z_2 \tag{28}$$

Differentiate the sliding surface s with respect to time as

$$\begin{aligned}\dot{s} &= K_P\dot{z}_1 + K_D\ddot{z}_1 + \dot{z}_2 = (K_P + c)\dot{z}_1 + (K_D + I)\ddot{z}_1 \\ &= (K_P + c)\dot{z}_1 + (K_D + I)\ddot{q}_d - (K_D + I)\ddot{q}_{t-t_d} \\ &= (K_P + c)\dot{z}_1 + (K_D + I)\ddot{q}_d - (K_D + I)M(q_{t-t_d})^{-1}[u + Nf + Dd - (C\left(q_{t-t_d}, \dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} \\ &\quad + g(q_{t-t_d}))] \\ &= (K_P + c)\dot{z}_1 + (K_D + I)\ddot{q}_d - (K_D + I)M(q_{t-t_d})^{-1}u - (K_D + I)M(q_{t-t_d})^{-1}\left[Nf + Dd\right. \\ &\quad \left.- \left(C\left(q_{t-t_d}, \dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} + g(q_{t-t_d})\right)\right] \end{aligned} \tag{29}$$

Then the fault-tolerant controller can be designed as follows

$$u_{ftc}(t) = u_{eq}(t) + u_f(t) \tag{30}$$

The equivalent control law u_{eq} is constructed as (31):

$$u_{eq}(t) = M(q_{t-t_d})\left[(K_D + I)^{-1}(K_P + c)\dot{z}_1 + (K_D + I)^{-1}K_s s(t) + \ddot{q}_d\right] - D\bar{d}$$

$$+C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_{t-t_d}+g\left(\dot{q}_{t-t_d}\right) \tag{31}$$

where $I \in R^{n\times n}$ is the unit matrix, and $K_s \in R^{n\times n} > 0$ is the pre-specified auxiliary parameter.

The fault compensation controller can be designed as (32) to compensate for the effect of the fault

$$u_f(t) = -N\hat{f} \tag{32}$$

Proof: Consider the candidate Lyapunov function as $V_c = \frac{1}{2}s^Ts + \frac{1}{2}z_1^Tz_1$. Differentiate V_c with respect to time

$$\dot{V}_c = z_1^T\dot{z}_1 + s^T\dot{s} = z_1^T(z_2 - cz_1) + s^T\dot{s} = z_1^Tz_2 - z_1^Tcz_1 + s^T\dot{s} \tag{33}$$

It can be further rewritten as

$$\begin{aligned}\dot{V}_c = z_1^Tz_2 - z_1^Tcz_1 + s^T\big\{&(K_P+c)\dot{z}_1 + (K_D+I)\ddot{q}_d - (K_D+I)M(q_{t-t_d})^{-1}u \\ &-(K_D+I)M(q_{t-t_d})^{-1}\left[Nf + Dd - \left(C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} + g(q_{t-t_d})\right)\right]\big\}\end{aligned} \tag{34}$$

Substituting (30) into (34), it can be obtained as

$$\begin{aligned}\dot{V}_c &= z_1^Tz_2 - z_1^Tcz_1 + s^T\big\{(K_P+c)\dot{z}_1 + (K_D+I)\ddot{q}_d - (K_D+I)M(q_{t-t_d})^{-1}u_{ftc} \\ &\quad -(K_D+I)M(q_{t-t_d})^{-1}\left[Nf + Dd - \left(C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} + g(q_{t-t_d})\right)\right]\big\} \\ &= z_1^Tz_2 - z_1^Tcz_1 + s^T\big\{(K_P+c)\dot{z}_1 + (K_D+I)\ddot{q}_d - (K_D+I)M(q_{t-t_d})^{-1}(u_{eq}+u_f) \\ &\quad -(K_D+I)M(q_{t-t_d})^{-1}\left[Nf + Dd - \left(C\left(q_{t-t_d},\dot{q}_{t-t_d}\right)\dot{q}_{t-t_d} + g(q_{t-t_d})\right)\right]\big\}\end{aligned} \tag{35}$$

Substituting (31) and (32) into (35), it can be obtained as follows

$$\dot{V}_c = z_1^Tz_2 - z_1^Tcz_1 - s^TK_ss \tag{36}$$

Define $Z = \left[\begin{array}{cc} z_1^T & z_2^T \end{array}\right]^T$, and (36) can be rewritten as

$$\dot{V}_c = -Z^T\Phi Z = -\varphi\left\|Z^TZ\right\| \tag{37}$$

where $\Phi = \left[\begin{array}{cc}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{array}\right]$, $\varphi = \lambda_{\min}(\Phi)$, and $\Phi_{11} = c + K_P^TK_sK_P - K_P^TK_sK_Dc - c^TK_DK_sK_P + c^TK_DK_sK_Dc$, $\Phi_{12} = K_P^TK_sK_D + K_P^TK_s - c^TK_DK_sK_D - c^TK_DK_s - \frac{1}{2}I$, $\Phi_{21} = \Phi_{12}^T$, $\Phi_{22} = K_s + K_sK_D + K_D^TK_s + K_D^TK_sK_D$.

$\dot{V}_c \leq 0$ can be satisfied when c, K_s and controller parameters K_P and K_D are selected appropriately making $\varphi > 0$ holds, which indicates that the tracking error converges under the fault-tolerant controller.

5. **Computer Simulation Results.**

In order to verify the effectiveness of the proposed fault diagnosis and fault-tolerant control scheme, a manipulator model is introduced to perform the simulation. According to the structure of Equation (1), the coefficient matrix of the manipulator model can be given as follows:

$$M(q) = \begin{pmatrix}(m_1+m_2)l_1^2 + m_2l_2^2 + 2m_2l_1l_2\cos(q_2) & m_2\left(l_2^2 + l_1l_2\cos(q_2)\right)\\ m_2\left(l_2^2 + l_1l_2\cos(q_2)\right) & m_2l_2^2\end{pmatrix}$$

$$C(q,\dot{q})\dot{q} = \begin{pmatrix}-2m_2l_1l_2\sin(q_2)\dot{q}_1\dot{q}_2 - m_2l_1l_2\sin(q_2)\dot{q}_2^2\\ m_2l_1l_2\sin(q_2)\dot{q}_2^2\end{pmatrix}$$

$$g(q) = \begin{pmatrix}(m_1+m_2)l_1g\cos(q_1) + m_2l_2g\cos(q_1+q_2)\\ m_2l_2g\cos(q_1+q_2)\end{pmatrix}$$

where $l_1 = 1$ m, $l_2 = 0.8$ m, $m_1 = m_2 = 1$ kg. The scale of the time delay is set as $t_d = 0.5$ s.

5.1. **The simulation for the fault diagnosis scheme.** To compute the LMI, the parameters of the adaptive observer can be selected as $M_x = 0.1$, $M_d = 0.4$, $M_f = 0.4$, $\rho = 3.0$. Choosing $\mu = 5$, the observer gain and the adaptive gain matrix is solved respectively as follows.

$$L = \begin{bmatrix} 67.0853 & 6.5372 \\ 6.5151 & 64.6919 \\ 734.9859 & 7.8199 \\ 7.5746 & 732.1320 \end{bmatrix}, \quad R_a = \begin{bmatrix} 13.3204 & 11.1004 \end{bmatrix}.$$

In order to fully prove the effectiveness of the proposed fault diagnosis scheme, three types of fault cases are considered here, including the constant value fault, the slow time-varying fault and the fast time-varying fault. The forms are shown as follows

Case 1: Constant fault $f = \begin{cases} 13.5 & t \in [30, 60] \\ 0 & \text{else} \end{cases}$;

Case 2: Slow time-varying fault $f = \begin{cases} 13.5\sin(0.5t) & t \in [25, 75] \\ 0 & \text{else} \end{cases}$;

Case 3: Fast time-varying fault $f = \begin{cases} 0.7(t-20) & t \in [20, 50] \\ 24 - 0.7(t-20) & t \in [50, 80] \\ 0 & \text{else} \end{cases}$.

In Figure 1, a constant value is designed to describe the constant fault, and the fault estimation can track the fault with a high accuracy. For the time-varying fault, a sinusoidal function is used in case 2 to describe the slow-change fault. A linear function with a large slope is used in case 3 to describe the fast-change faults. In each case, the fault observer can estimate the fault quickly and accurately.

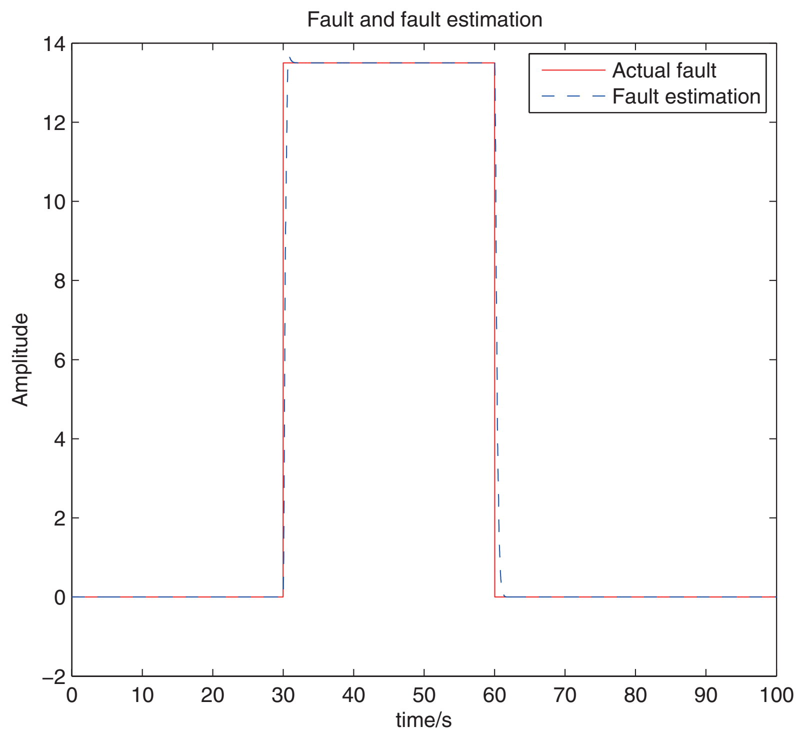

FIGURE 1. Constant fault and its estimation

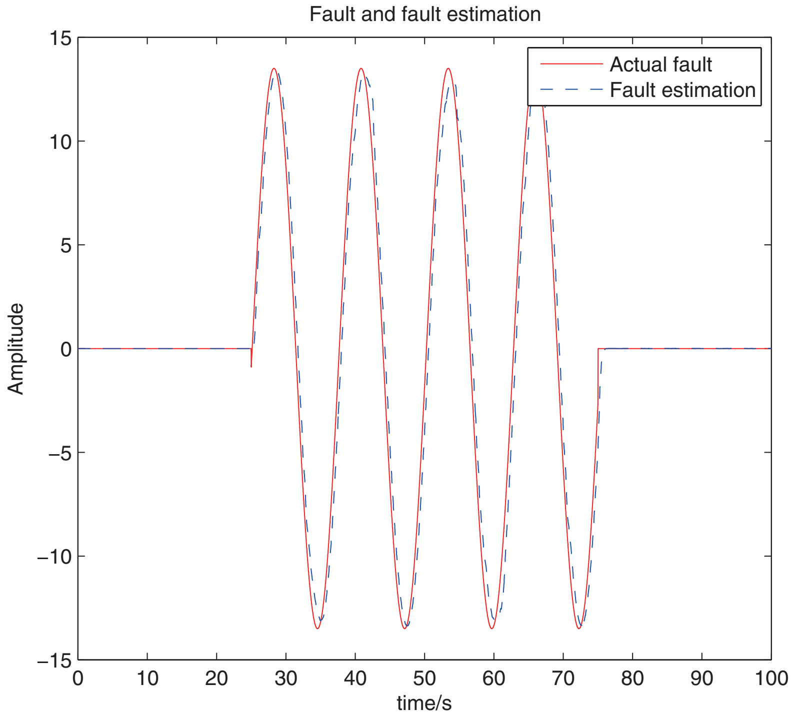

FIGURE 2. Slow time-varying fault and its estimation

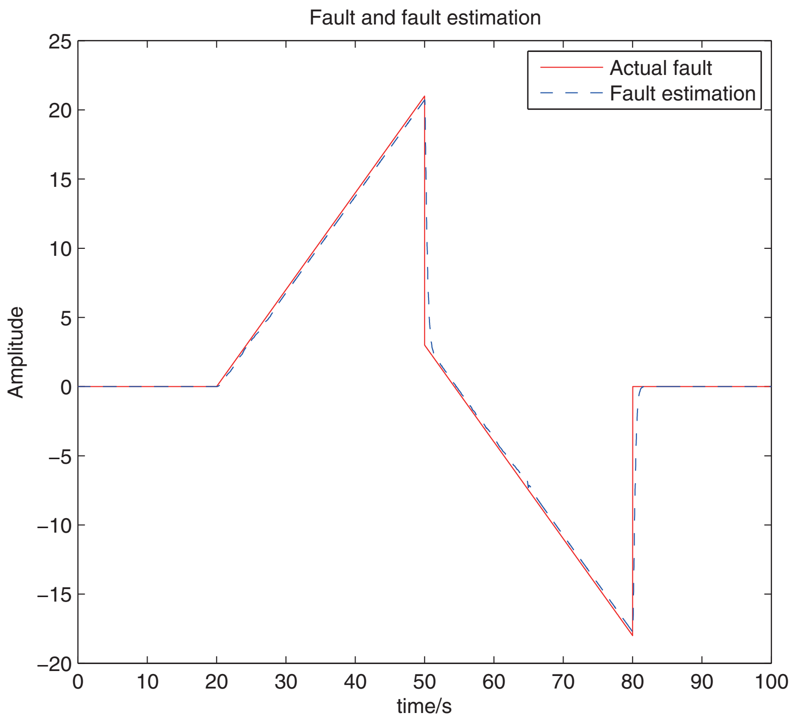

FIGURE 3. Fast time-varying fault and its estimation

5.2. **The simulation for the fault-tolerant scheme.** Taking fault case 3 as the example to perform the simulation to verify the validity of the fault-tolerant controller, and the comparison results of the two fault-tolerant control schemes are given. The desired trajectory is set as $q_d(t) = \begin{bmatrix} 5.4\sin(0.3t) \\ 3.6\sin(0.2t) \end{bmatrix}$.

In Chapter 4.1, the proportional gain matrix, the differential gain matrix, and the gain matrix of the time-delay compensation, of the PD plus compensation controller is given as follows, respectively:

$$K_P = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \quad K_D = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}, \quad K_k = \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}.$$

In Chapter 4.2, the proportional gain matrix, the differential gain matrix, and the auxiliary parameters, of the PDSMC controller are shown as follows, respectively:

$$K_P = \begin{bmatrix} 20 & 0 \\ 0 & 30 \end{bmatrix}, \quad K_D = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, \quad c = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad K_s = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}.$$

The control results can be found as follows.

In Figure 4 and Figure 5, the fault-tolerant control effect of the fault system under the action of PD plus compensation fault-tolerant controller is described. It can be seen that the controller compensates the effect of fault and time-delay on the system performance, and the control effect is good; Figure 6 and Figure 7 show the fault-tolerant results under the PDSMC fault-tolerant controller, which can be obviously compared. The control effect of PDSMC fault-tolerant controller is better than that of the PD plus compensation fault-tolerant controller. This can also be demonstrated by the mean and norms of errors in Table 1.

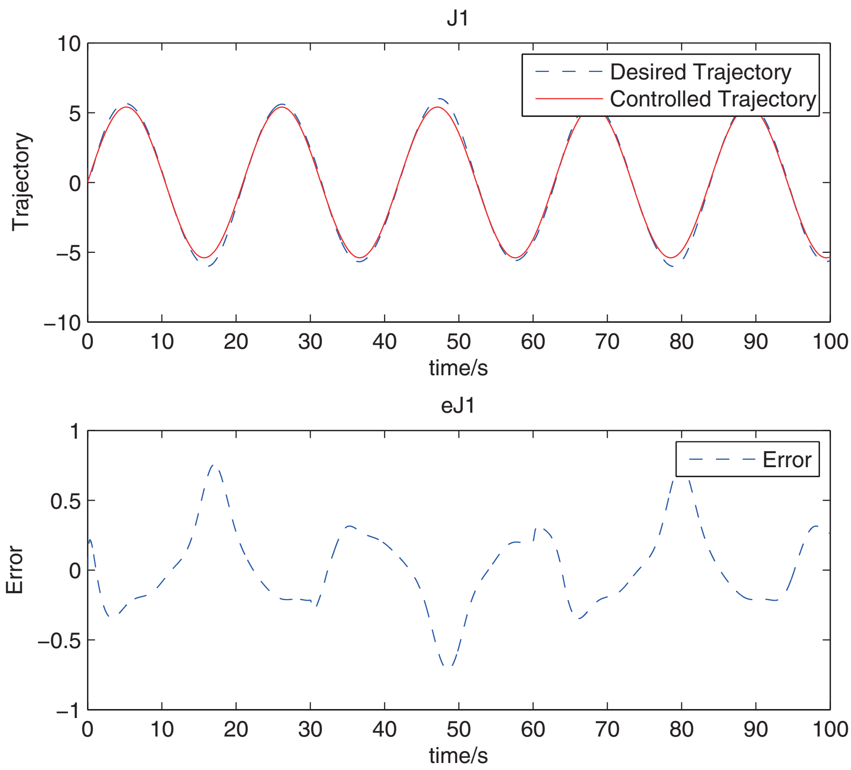

Figure 4. Desired trajectory, trajectory with PD, and tracking error of joint 1

6. **Conclusions.** In this paper, a scheme including fault diagnosis and fault-tolerant control is proposed for the actuator fault in the networked manipulator system. In the fault diagnosis scheme, considering the influence of time-delay, the fault estimation law based on the adaptive observer for the actuator fault is designed by the residual of the time-delay state. Using the actuator fault information, a PD plus the compensation fault-tolerant controller is designed to compensate for the effect of the fault and network

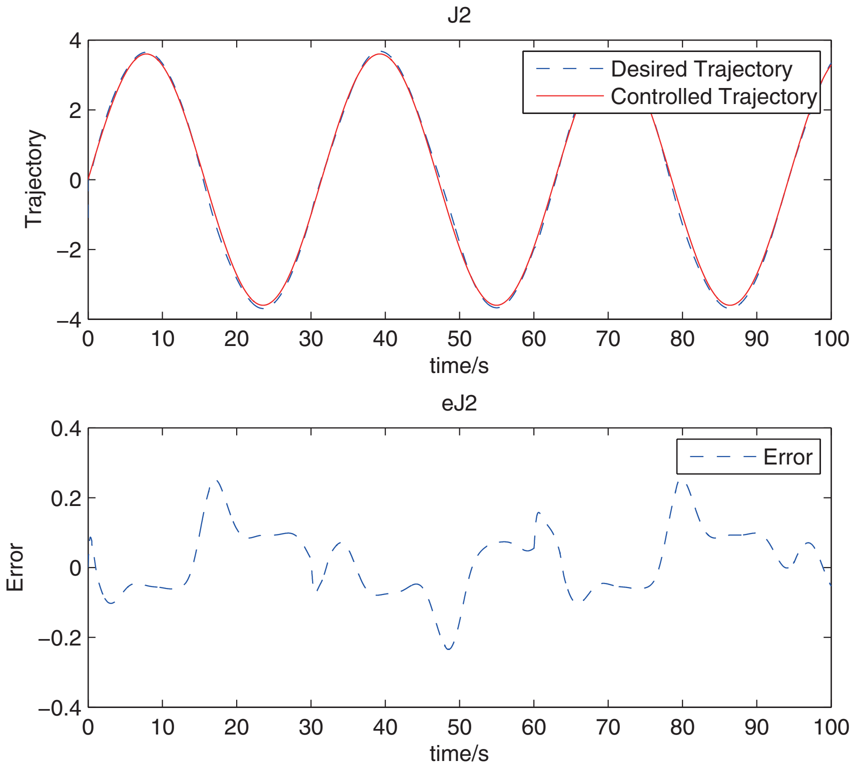

FIGURE 5. Desired trajectory, trajectory with PD, and tracking error of joint 2

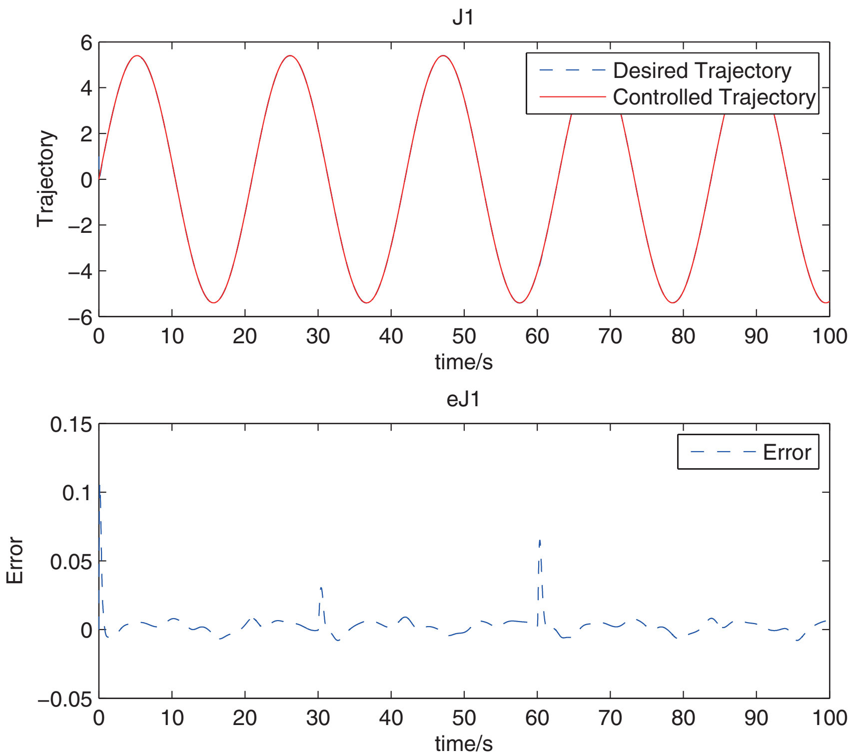

FIGURE 6. Desired trajectory, trajectory with PDSMC, and tracking error of joint 1

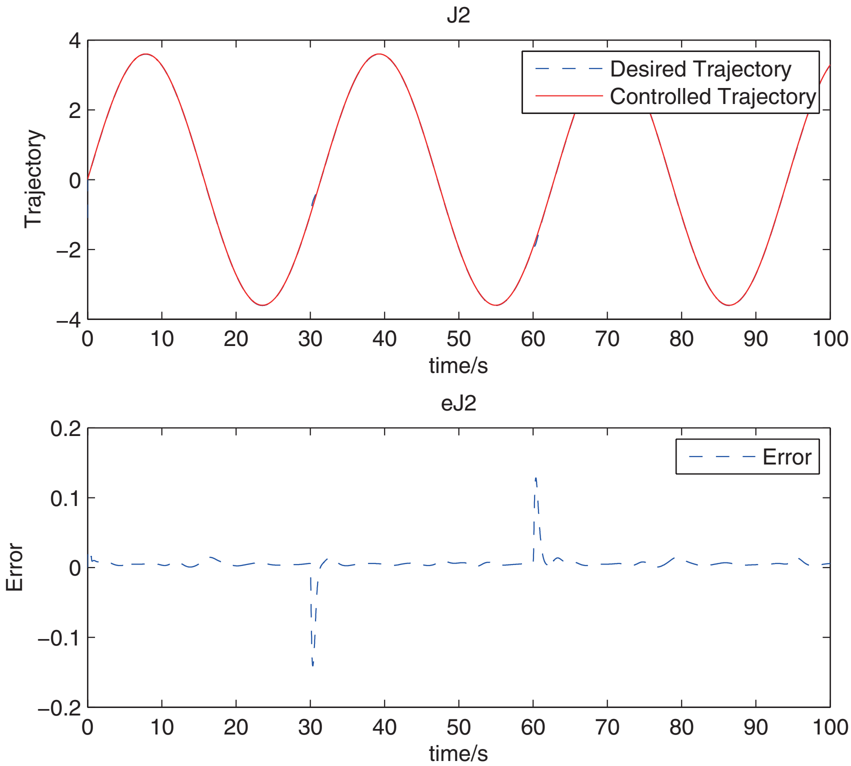

FIGURE 7. Desired trajectory, trajectory with PDSMC, and tracking error of joint 2

TABLE 1. Mean and norm of the track error after fault tolerance

	PD plus compensation	PDSMC
Mean	0.0351	0.0080
Norm	248.306	59.827

time-delay. Further, the sliding mode controller is combined with the PD controller for the control improvement, resulting in better control results. The effectiveness and feasibility of the proposed design are proved through the simulation. The next research work will pay attention to the networked multiple manipulator cooperative system, and the problem of fault diagnosis and fault-tolerant control will be studied from the perspective of practical application.

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