OPTIMAL LOCATION AND PRICING OF DUOPOLY ONLINE RETAIL PRODUCTS: A SPATIAL SEARCH MODEL

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ABSTRACT. The “top-of-the-web effect” in Internet advertising has attracted widespread attention, but the current theories are not satisfactory in explaining this phenomenon. The main reason is that same as offline markets the online market also has significant spatial attributes. In the past, most researches mainly focused on ranking search with time attribute as the core rather than page distance search. They ignore the impact of spatial search costs, can only analyze product page rankings, but cannot study product web site location. In this paper, by defining the buyer’s expected return function of spatial search and search selection revenue function, we construct a model based on spatial search cost, and solve the problem of web site location and pricing of competitive online products. The research has found that approaching the top of the web page and choosing a location are not the optimal decision for all products. Platforms and online retail companies have endogenous motivations for quality false reports, and market competitiveness has nothing to do with search costs.

Keywords: Online retail products, Spatial search costs, Web site location, Pricing

1. Introduction. Online advertising is the most important source of value for the Internet economy. In the fierce market competition, due to the particularity of consumer web search behavior, the key factor that determines the value of online advertising is the location of the product on the e-commerce platform web page, i.e., the location of the web page [1]. Despite the growing investment in online advertising, there has been a great divide in market opinion on online advertising [2]. Among them, what annoy the most people is selling the product display position to those who conduct false advertising and these inducements of indulgence. Baidu’s Zexi Wei incident eventually intensified these contradictions. One of the immediate consequences of this event is to promote the Development and Implementation by the Chinese Government of the Interim Measures for the Administration of Internet Advertising [3] (SAIC, 2016) in 2016 and the implementation of a special crackdown on Internet advertising in early 2018 [4] (SAIC, 2018). Web platforms are beginning to get involved in the management of advertising content, but the way they are involved has been the subject of a lot of criticism [5] (E-Race Moments, 2016). This event reflects that the location and pricing of online advertising have a great impact on the competitive behavior of the product market, so the web site location and pricing of online advertising have become a hot issue in the academic community.

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The main factor affecting the location of product on web pages is the search behavior of online consumers. Hotchkiss et al. found that the viewer’s concern for the content of the web page showed a clear top-down and left-to-right attenuation [6]. The empirical study of online behavior by Benartzi and Lehrer showed that the above behavior has a significant regularity: the buyer’s distribution in the online market is very uneven, and the transfer cost of the buyer’s change of focus is very small [7]. Bell argued that the root cause of this regularity is that people generally face “search for resistance” or “search cost” when conducting online search activities such as web browsing or clicking [8]. Research on the cost of search has been a hot topic since Stigler’s classic paper was published in 1961, especially in the age of the Internet economy [9,10]. Representative research by Rizwan et al. suggests that “false advertising” has the incentive to buy a top position on a web page, referred to simply as “top” [11]. Chen and Li studied the impact of product quality on search advertising bidding, and believed that e-commerce platforms have incentives to reduce the evaluation of product quality [12]. Jiang and Wang used empirical methods to study the relative position of products on the web page on the effect of search engine marketing, and thought that the top is the best choice [13]. If there is product competition, a large relative distance can help reduce competition. However, the research by Rhodes et al. focused primarily on time-based searches (i.e., searches for rankings) without taking account of spatial attributes of the web market (i.e., the actual distance of the product on the web page) [14]. These researches ignore the impact of space search costs, so they can analyze the web page ranking and price discrete phenomena of online products, but can not help solve the problem of product-specific web site location. It is also relatively rare to study the rationality of spatial search costs in web site location and product pricing, which is out of touch with a great deal of empirical research and practical conclusions [7]. Shy proposed a simple method to solve the problem of time search by defining the expected price drop function and loss function, but this method is only suitable for the search problem of homogeneity products [15].

In view of these problems, according to Shy’s method, this paper constructs the product page location and pricing model based on the cost of spatial search by defining the buyer’s spatial search expected revenue function and the search selection revenue function [15]. The main difference with shy’s model is that the buyer’s search cost is defined by our research. In shy’s study, the search cost of all buyers is the same, but in our study, it is different from each other [16]. It tries to balance the relative ranking and absolute distance selection of competitive online products on the market size and profitability of sellers and platforms. This method maintains the simplicity of the Shy method and it is suitable for heterogenetic products, taking account of time and space search issues [15].

The following contents of this paper are organized as follows: The second part shows assumptions and modeling, the third part is game equilibrium, the fourth part is equilibrium analysis, and the fifth part is case analysis and the last part presents conclusions.

2. Assumptions and Modeling.

2.1. Assumptions. In a network economy, the most typical decision-making entities are buyers, sellers, and platforms. These entities are rational and only take their own profit maximization as their decision-making goal. When a seller chooses to sell its products on an e-commerce platform, the location and pricing of the product in the web market will affect the buyer’s willingness to pay, which in turn affects the seller’s market size and profit. In view of this interactive behavior, the basic assumptions of this article are as follows: there is a third partial platform of EC, and the web page market operated by the platform is a linear market with complete information \( x \in [0, \infty) \). The starting position,
that is, the top position of the product display is \( l_T > 0 \). There are \( n \) oligopoly sellers on it, all of which sell only one homogeneous product \( i \in \{1, 2, \ldots, n\} \). Product \( i \) has the same retention value for all buyers, which is equal to its exogenous quality \( s_i = s \), market position is \( x_i \), price is \( p_i \), and sales volume is \( q_i \). Obviously, \( p_i \leq s_i = s \). The location rent \( q w \), paid by the seller to the platform according to the product sales volume; here, \( w \) is the location rent. The total sales volume of the platform is \( Q = \sum_{i=1}^{n} q_i \). Both platform operation and product sales have a constant marginal cost, which is set to 0. The cost per sale (CPS) model used here has the same nature as the cost per click (CPC) model. CPC divided by the percent conversion is equal to CPS. There are a total of 2 heterogeneous buyers. The unit cost \( t \) that different buyers need to pay when searching for products is different. For the convenience of analysis, suppose \( t \) is uniformly distributed on \([0, 1]\). Any buyer \( t \in [0, 1] \) has at most unit demand for the product. All buyers start to search for products from market 0. The search cost for buyer \( t \) to find market \( x_i \) is \( c(t, x_i) = c(t(x_i)) = \alpha t x_i \). The parameter \( \alpha \) measures the search cost in the buyer’s preference importance. The utility of the buyer \( t_j(x_i) \) is \( u_i = u_j(t_j(x_i)) = s_i - p_i - \alpha t_j x_i \); here, \( t_j \) represents the indifference search cost of buyer about whether to buy the product \( j \) or not. The seller’s profit is \( \pi_i \). The profit of the platform is \( \Pi \).

**Definition 2.1.** Search expected return function: \( v(p_i) \) is the buyer’s expected total return obtained by searching for \( x_{i+1} \) after the buyer knows the product price \( p_i \) at \( x_i \).

Let \( u_{i+j} = s_{i+j} - p_{i+j} - \alpha t_{i+j} x_i \), \( j \in \{0, 1, 2, \ldots, n - i\} \). Formally, the probability of obtaining each return \( P(u_{i+j}|i \in \{1, 2, \ldots, n\}) = 1/n \), so

\[
v(p_i) = \sum_{j=i}^{n} \frac{u_{i+j}}{n} \tag{1}
\]

Formula (1) means that while mastering a seller’s bid, the expected return of a product search is performed.

**Definition 2.2.** Search and selection revenue function: \( U(t_{i+1}, p_{i+1}) \) represents the revenue selection that buyers of type \( t_{i+1} \) face when they choose whether to continue searching at \( x_{i+1} \).

By Definition 2.2

\[
U(t_{i+1}, p_{i+1}) = \begin{cases} 
u_i & \text{If you choose to buy at } x_i \\ v(p_i) & \text{If you continue to search for } x_{i+1} \end{cases} \tag{2}
\]

Obviously, if a buyer does not continue to search, he knows that buying at \( x_i \) will gain \( u_i \). If he continues to search, he knows that buying at \( x_{i+1} \) will get the expected return of \( v(p_i) \). A rational buyer will choose to maximize his own profit, so when \( u_i > v(p_i) \), the buyer stops searching. When \( u_i < v(p_i) \), the buyer continues to search. When \( u_i = v(p_i) \), there is no difference between stopping the search and continuing the search.

2.2. **Modeling.** To simplify the analysis, suppose that there are only duopoly sellers in the market, \( i \in \{1, 2\} \).

2.2.1. **Buyer behavior.**

1) When \( x = 0 \), \( u_0 = 0 \), \( v(0) = \sum_{j=1}^{2} \frac{u_j}{2} = s - \frac{p_1 + p_2}{2} - \alpha t_1 x_1 \), by Formula (2), then

\[
U(t_1, x_1) = \begin{cases} 0 & \text{If you stop searching at } x = 0 \\ s - \frac{p_1 + p_2}{2} - \alpha t_1 x_1 & \text{If you continue to search } x_1 \text{ place} \end{cases}
\]
According to Definition 2.2, $s - \frac{p_1 + p_2}{2} - \alpha t_1 x_1 < 0$, that is, when $t_1 > \frac{2s - p_1 - p_2}{2\alpha x_1}$, the buyer stops searching. When $t_1 < \frac{2s - p_1 - p_2}{2\alpha x_1}$, the buyer continues to search. When $t_1 = \frac{2s - p_1 - p_2}{2\alpha x_1}$, the buyer has no difference between stopping the search and continuing the search. $t_1$ is the buyer who is willing to buy product 2 for the largest search cost.

2) When $x = x_1$, $u_1 = u(t_2(x_1)) = s - p_1 - \alpha t_2 x_1$, $v(p_1) = \sum_{j=2}^{2} \frac{u_j}{2} = \frac{s - p_2 - \alpha t_2 x_2}{2}$, from Formula (2), then

$$U(t_2, x_2) = \begin{cases} s - p_1 - \alpha t_2 x_1 & \text{If you stop searching at } x = x_1 \\ \frac{s - p_2 - \alpha t_2 x_2}{2} & \text{If you continue to search } x_2 \end{cases}$$

According to Definition 2.2, $s - p_1 - \alpha t_2 x_1 > \frac{s - p_2 - \alpha t_2 x_2}{2}$, that is, when $t_2 > \frac{2p_1 - s - p_2}{\alpha(x_2 - 2x_1)}$, the buyer stops searching. When $t_2 < \frac{2p_1 - s - p_2}{\alpha(x_2 - 2x_1)}$, the buyer continues to search. When $t_2 = \frac{2p_1 - s - p_2}{\alpha(x_2 - 2x_1)}$, the buyer has no difference between stopping the search and continuing the search. $t_2$ is the buyer who is willing to buy product 2 for the largest search cost.

From the above, for any buyer $t \in [0, 1]$, when $t \in [0, t_1)$, she will buy product 2. When $t \in [t_1, t_2]$, she will buy product 1. When $t \in [t_2, 1]$, she will not browse the web, let alone buy any products. For the convenience of calculation, set $x_1 = l \geq l_T$, $x_2 = ml$, $m \geq 0$. Obviously, $l$ measures the absolute distance between the products. There is a demand function:

$$q_1 = \bar{t}_1 - \bar{t}_2 = \frac{2s - p_1 - p_2}{2\alpha l} - \frac{2p_1 - s - p_2}{(m - 2)\alpha l}$$

$$q_2 = \bar{t}_1 - \bar{t}_2 = \frac{2p_1 - s - p_2}{(m - 2)\alpha l}$$

(3)

2.2.2. **Seller behavior.** Because the seller needs to pay the location rent to the platform, the profit obtained by selling the advertising location to the buyer is the product of the difference between the location price and the rent and the sales quantity. The seller’s profit function is

$$\pi_i = (p_i - w)q_i$$

(4)

2.2.3. **Platform behavior.** The profit function of the platform is the total sum of the product of the rent of each location and the number of the locations

$$\Pi = \sum_{i=1}^{2} wq_i$$

(5)

3. **Game Equilibrium.** The above-mentioned market game is a sequential process, which is divided into three steps. First, the seller selects the location of the product based on the quality of the product. Secondly, the platform sets prices for the location of the web page based on the seller’s location. Again, the seller sets the price for the product. In order to obtain all the perfect subgame Nash equilibriums, a backward induction method can be adopted.

3.1. **Equilibrium price.** Putting Formula (3) into Formula (4), there is a profit function:

$$\pi_1 = (p_1 - w) \left[ \frac{2s - p_1 - p_2}{2\alpha l} - \frac{2p_1 - s - p_2}{(m - 2)\alpha l} \right]$$

$$\pi_2 = (p_2 - w) \left[ \frac{2p_1 - s - p_2}{(m - 2)\alpha l} \right]$$

(6)
It is easy to verify the negative definite Hessian matrix, so there is an optimal equilibrium price. Solve, \( \frac{\partial \bar{p}_1}{\partial p_1} = 0 \). Easy to know \( \bar{p}_1 = \frac{(5m-8)s+(m+8)w}{6m} \), \( \bar{p}_2 = \frac{(m-4)s+2(m+2)w}{3m} \). Bring it into (3), \( \bar{q}_1 = \frac{(5m-8)(s-w)}{6mal} \), \( \bar{q}_2 = \frac{(m-4)(s-w)}{3m(m-2)al} \), \( \bar{Q} = \bar{q}_1 + \bar{q}_2 = \frac{(5m+16)(s-w)}{6mal} \).

3.2. Equilibrium rent. Put the total product sales volume \( \bar{Q} \) obtained in Section 3.1 into Formula (5), and there is a platform rental income function:

\[
\Pi = \frac{(5m+16)(s-w)w}{6mal} \tag{7}
\]

Obviously, \( \frac{d\Pi}{dl} < 0 \). Therefore, when \( \frac{d\bar{Q}}{dl} = 0 \), there is an optimal rent: \( w^* = \frac{s}{2} \).

Substituting \( w^* \) into \( \bar{p}_1, \bar{p}_2, \bar{q}_1, \bar{q}_2, \bar{Q} \), and then substituting these results into formulas (6) and (7), the following conclusions are obtained.

**Lemma 3.1.** In the linear web market, if the buyer’s search cost is evenly distributed between \([0, 1]\), the search intensity is \( \alpha \). When the web site of the duopoly product with the same quality as \( s \) is selected as \( l, ml, m > 4 \), the market will have the following Nash equilibrium:

\[
\hat{p}_1 = \frac{(11m-8)s}{12m}, \quad \hat{p}_2 = \frac{2(m-1)s}{3m}, \quad \hat{q}_1 = \frac{(5m-8)s}{24mal}, \quad \hat{q}_2 = \frac{(m-4)s}{6m(m-2)al}, \quad \hat{Q} = \frac{(5m+16)s}{24mal^2},
\]

\[
\hat{\bar{\lambda}}_1 = \frac{(5m-8)^2s^2}{9m^2al^2}, \quad \hat{\bar{\lambda}}_2 = \frac{(m-4)^2s^2}{36m^2(m-2)al}, \quad \hat{\bar{\Pi}} = \frac{(5m+16)s}{24mal^2}.
\]

3.3. Equilibrium location. The equilibrium result obtained from Section 3.2 has \( \frac{d\bar{\lambda}_1}{dl} = -\frac{(m-1)(5m-8)s^2}{9m^2al^2} < 0 \). Therefore, the optimal strategy for seller 1 is to minimize the product’s web page position, that is, the most preferred address of product 1 is the top position, that is, \( x_1^* = l^* = l_T \).

Similarly, simple calculation shows that when \( m > 4 \), \( \frac{d^2\bar{\lambda}_2}{dl^2} < 0 \), then when \( \frac{d\bar{\lambda}_2}{dl} = 0 \), \( m^* = 6 + 2\sqrt{5} \), \( \bar{\lambda}_2 \) has the largest value at the time. The most preferred address for product 2 is \( x_2^* = (6 + 2\sqrt{5})l_T \).

Substituting \( l^*, m^* \) into the results obtained in Section 3.2, we have the following conclusions.

**Lemma 3.2.** In the linear web market, if the buyer’s search cost is evenly distributed between \([0, 1]\), the search intensity is \( \alpha \). When duopoly products with the same quality of \( s \) choose their respective web page positions, the market will have the following Nash equilibrium:

\[
x_1^* = l_T, \quad x_2^* = (6 + 2\sqrt{5})l_T, \quad p_1^* = \frac{(29+11\sqrt{5})s}{12(3+\sqrt{5})}, \quad p_2^* = \frac{4(3+\sqrt{5})s}{6(3+\sqrt{5})}, \quad w^* = \frac{s}{2},
\]

\[
q_1^* = \frac{(11+\sqrt{5})s}{12(3+\sqrt{5})al_T}, \quad q_2^* = \frac{(1+\sqrt{5})s}{12(3+\sqrt{5})(2+\sqrt{5})al_T}, \quad Q^* = \frac{(23+5\sqrt{5})s}{12(3+\sqrt{5})al_T}, \quad \bar{\lambda}_1^* = \frac{(11+\sqrt{5})^2s^2}{9(3+\sqrt{5})^2al_T}, \quad \bar{\lambda}_2^* = \frac{(1+\sqrt{5})^2s^2}{36(3+\sqrt{5})^2al_T}, \quad \bar{\Pi}^* = \frac{(23+5\sqrt{5})^2s^2}{24(3+\sqrt{5})^2al_T}.
\]

4. Equilibrium Analysis.

4.1. Sensitivity analysis. From Lemma 3.1, through simple calculation, \( \hat{p}_1, \hat{p}_2, w^* \) have nothing to do with \( l, \alpha, \hat{q}_1, \hat{q}_2, Q, \hat{\bar{\lambda}}_1, \hat{\bar{\lambda}}_2; \bar{\Pi} \) was negatively correlated with \( l \) and \( \alpha \). All equilibrium results are positively correlated with \( s; w^* \) has nothing to do with \( m, \hat{p}_1, \hat{p}_2, \hat{q}_1; \hat{\bar{\lambda}}_1 \) is positively correlated with \( m, Q; \bar{\Pi} \) is negatively correlated with \( m \). Only \( \hat{q}_2 \) and \( \hat{\bar{\lambda}}_2 \) are more complicated. When \( m = 4 + 2\sqrt{2} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} = 0 \). When \( m < 4 + 2\sqrt{2} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} > 0 \). When \( m > 4 + 2\sqrt{2} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} < 0 \). When \( m = 6 + 2\sqrt{5} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} = 0 \). When \( m < 6 + 2\sqrt{5} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} > 0 \). When \( m > 6 + 2\sqrt{5} \), \( \frac{\partial \bar{\lambda}_2}{\partial m} < 0 \). Obviously, the product price has nothing to do with the search cost and whether the product is at the top, but positively related to the relative distance between products. It shows that increasing the relative distance helps to reduce the competitiveness of the market, while reducing the search cost does not contribute
significantly to improving the competitiveness of the market. High quality products will enhance the income of all market entities, especially for the platform; it is more concerned about how to attract high-quality products to the platform. The closer the absolute position of the product is to the top of the web page, the greater the sales volume and profit of the platform and all products, which indicates that there is a strong incentive for the product or platform to set the top. The relative distance between products is an important decision-making factor. Compared with product 1, product 2 also pays attention to maintaining a certain distance from product 1. Therefore, she will approach the top, but will not pursue the top. The optimal location decision of product 2 is the result of the trade-off among being close to the top, increasing production and increasing price. Although this trade-off decision of product 2 is beneficial to both products, it is not optimal for the platform. The platform has sufficient motivation to encourage all products to top or close to the top. Therefore, the following conclusions are drawn.

Proposition 4.1. For homogeneous duopoly online retail products, 1) product pricing is independent of search cost and absolute location, but positively related to relative distance and product quality. Platform pricing is only positively related to product quality. 2) Sales volume is negatively correlated with search cost and absolute position, and positively correlated with quality. The sales volume of product 1 is positively correlated with the relative distance, and the sales volume of product 2 is the largest when the relative distance is moderate. The sales volume of the platform is negatively correlated with search cost, absolute location and relative distance, and positively correlated with quality. 3) Product profit is negatively correlated with search cost and absolute position. The profit of product 1 is positively correlated with the relative distance, and the profit of product 2 is the largest when the relative distance is moderate. The profit of the platform is negatively correlated with search cost, absolute location and relative distance.

4.2. Comparative analysis. From Lemma 3.2, through simple calculation, we know that $p_1^* > p_2^*$, $q_1^* > q_2^*$, $\pi_1^* > \pi_2^*$. The results show that the price, sales volume and profit of the products will be higher if the price is set at the top. Therefore, the following conclusions are drawn.

Proposition 4.2. For homogeneous duopoly online retail products, in the process of website location, they will strive to compete for the top position. It is only a helpless choice for the losers to make a suboptimal decision.

5. Case Analysis. The conclusion of this paper is verified by the actual situation of Perrier water in the web market of Jingdong Mall (see Figure 1).

Jingdong Mall adopts the mode of price competitive ranking, and the location and pricing of product in web pages are the result of market competition. For the two homogeneous Perrier water products in Figure 1, their page location (row) is $x_1 = 1$, $x_2 = 13$ (Filter out the other products, so the location of product 1 is considered as 1), the product price (RMB) is $p_1 = 129$, $p_2 = 119$. Therefore, $m^t = \frac{2}{x_1} = 13$, $P^t = \frac{p_1}{p_2} = 1.08$. Because this paper adopts the game analysis method based on spatial search, the web page location and pricing of product are the result of mechanism design. For these two homogeneous Paris aquatic products, the theoretical calculation results are as follows $x_1 = 1$, $x_2 = 6 + 2\sqrt{5}$, the prices are $p_1^* = \frac{2(3+\sqrt{5})s}{12(3+\sqrt{5})}$, $p_2^* = \frac{2(3+\sqrt{5})s}{3(3+\sqrt{5})}$. Therefore, $m^d = \frac{2}{x_1} = 10.5$, $P^d = \frac{p_1}{p_2} = 1.29$. Here, $m^t$, $P^t$ are actual values, and $m^d$, $P^d$ are theoretical values. Error comparison between them is as follows: location error is $\left| \frac{m^d - m^t}{m^t} \right| = 19.2\%$, pricing error is $\left| \frac{P^d - P^t}{P^t} \right| = 19.4\%$. Although there are errors, they are within the acceptable range.
It shows that the location and pricing method based on spatial search cost is close to the reality of online retail market. It also shows that the conclusions of this paper can provide effective decision support for the operation of online enterprises and e-commerce platforms.

6. Conclusions. The two propositions obtained in this paper have certain reference value for the market competition of online retail enterprises, the decision-making of e-commerce platform, and the effective governance of e-commerce market. 1) It is not the optimal decision for all products to approach the top location of web pages. Although the top page has a huge attraction for every product, the price is too high which only can be acceptable for only a few products. For other product locations, the rational choice is to...
keep a moderate relative distance with the top product. This conclusion is different from the common sense that it should be as close as possible to the top of the web page, and also different from the conclusion of some studies that we should keep away from the top products. However, the platform always has sufficient motivation to encourage retailers to pursue the top of products. 2) Both the platform and the online retail enterprises have the endogenous motivation of quality false report. In the free competition environment, if the cost of false quality report is low, retailers and platforms have enough incentive to pursue quality false report. There is no theoretical basis for the popular view that the platform is responsible for managing the retailer’s false quality reports. 3) Market competitiveness has nothing to do with search cost. Reducing search cost does not help to improve the competitiveness of online market, which means that improving search engine technology and reducing search cost have no obvious contribution to improving market competition. This is different from some previous studies. Market competition is more likely to be a market problem than a technical one.

The innovations of this paper are as follows: in view of the fact that there is spatial attribute in online market, the market behavior model of relevant subjects in online retail market is established by setting the expected return function of spatial search and search selection revenue function. It is found that it is not the optimal decision for all products to reach the top of the web page. This paper presents a solution to the optimal location of the non-top products.

REFERENCES