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A NOVEL METHOD BASED ON FIXED POINT ITERATION AND IMPROVED TOPSIS METHOD FOR MULTI-ATTRIBUTE GROUP DECISION MAKING

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Abstract. *For the existing problem of determining the expert weights by calculating the deviation between the individual decision matrix and the average decision matrix in multi-attribute group decision-making, the definition of coordinated expert weights is introduced, and an algorithm of coordinated expert weights based on fixed point iteration is proposed. Firstly, by the random given initial weights of decision-makers, the Euclidean distance measure is used to calculate the proximity between each expert decision matrix and the overall decision matrix. And then, the weights of decision-makers are adjusted by the proximity. Through adaptive adjustment repeatedly, the coordinated weights are received. The coordinated weights are determined by the given decision matrices and are not affected by the selection of the initial value of expert weights, which can objectively reflect the scoring level of each expert. According to the coordinated expert weights, an improved TOPSIS method based on information entropy measure is used to rank the alternatives. Numerical experiments are used to demonstrate the effectiveness of the coordinated expert weights algorithm based on fixed point iteration and the improved TOPSIS method.*
Keywords: Group decision-making, Coordinated weights of experts, Fixed point iteration, Improved TOPSIS method

1. **Introduction.** Multi-attribute decision making, one of the well-known branches of decision-making, aims at finding the most suitable solutions from a set of alternatives under conflicting attribute. Due to the increasing complexity of the soci-economic environments, a single decision maker (DM) or expert may be impossible to consider all relevant aspects of a problem. Hence, many real world decision problems tend to be made by groups of decision-makers (DM) rather than individuals. In this case, the multi-attribute decision making problems require to be further extended to multi-attribute group decision making (MAGDM) problems [1, 2]. In group decision making, the rationality of expert weight has a direct influence on the accuracy of decision result. Therefore, in the research of multi-attribute group decision problem, the research of expert weights determination method plays an important role.

The existing literature on expert weights determination methods is mainly divided into subjective weighting method and objective weighting method. Subjective weighting methods mainly include the Delphi method [3], and the analytic hierarchy process method [4, 5]. While objective weighting methods mainly include the grey relational coefficient [6], the entropy value method [7], the maximizing deviation method [8], the Euclidean

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distance method based on average decision matrix [9] and the projection method based on average decision matrix [10, 11, 12]. The other relevant methods can be referred to [13, 14, 15, 16].

Due to the subjective determination method based on prior experiences with strong subjectivity, in most cases, the objective determination methods are preferred. It is a common objective weighting method to determine the weight of experts according to their scoring level. Hence, how to evaluate the accuracy of the decision matrix given by experts is a crucial problem. In [9, 10, 11, 12], the deviation between the individual decision matrix and the ideal decision matrix is used to evaluate the accuracy of the decision matrix given by the expert and then determine the weight of the expert. The closer decision matrix given by the expert to the ideal decision matrix, the bigger weight of the expert, which sounds undoubtedly reasonable.

However, the way of determining the ideal decision matrix in [9, 10, 11, 12] is worth discussing. All of them regard the average decision matrix as the ideal decision matrix, which is obtained based on an underlying assumption that the weights of experts are the same. Paradoxically, the weights of experts are usually different by calculating the deviation between the individual decision matrix and the average decision matrix. When the weights of experts are obtained by the above method, the ideal decision matrix should be renewed. And then the deviation between the individual decision matrix and the ideal decision matrix would be updated. Hence, the weights of experts would be renewed again and it would lead to another ideal decision matrix. To some extent, the definition of the ideal decision matrix above is not reasonable. What is a reasonable definition of the ideal decision matrix? And how to determine the weights of experts by the ideal decision matrix?

To settle the above problem, for the given expert decision matrices, we give the definitions of ideal decision matrix X^* and coordinated expert weights $\lambda^* = (\lambda_1^*, \dots, \lambda_k^*, \dots, \lambda_t^*)$ as follows.

Definition 1.1. *Given t decision matrices $X_1, \dots, X_k, \dots, X_t$, then the coordinated expert weights $\lambda^* = (\lambda_1^*, \dots, \lambda_k^*, \dots, \lambda_t^*)$ and the ideal decision matrix X^* should satisfy the following conditions:*

$$
\begin{aligned}
X^* &= \lambda_1^* X_1 + \cdots + \lambda_k^* X_k + \cdots + \lambda_t^* X_t \\
\lambda_k^* &= \frac{m(X_k, X^*)}{m(X_1, X^*) + \cdots + m(X_k, X^*) + \cdots + m(X_t, X^*)}, \quad k = 1, 2, \dots, t,
\end{aligned} \tag{1}
$$

where $m(X_k, X^*)$ denotes the degree of the decision matrix X_k approaching to the ideal decision X^*. Under the measure m, the coordinated expert weights $\lambda^* = (\lambda_1^*, \dots, \lambda_k^*, \dots, \lambda_t^*)$ satisfying the above conditions should be unique.

In fact, the coordinated expert weights $\lambda^* = (\lambda_1^*, \dots, \lambda_k^*, \dots, \lambda_t^*)$ can be abstracted as the solution to the nonlinear equations $\lambda = G(\lambda)$, where the multivariate function $G(\lambda)$ represents the updated value of each expert's weight under the measure m. Therefore, we can try to use the fixed point iterative algorithm to find the coordinated expert weights.

As a classical multi-attribute decision making method, TOPSIS method [17, 18, 19, 20, 21, 22, 23, 24, 25] considers two benchmarks, one is the distance measure between each alternative and positive ideal solution, and the other one is the distance measure between each alternative and negative ideal solution, and then sorts each alternative according to its relative proximity. In the TOPSIS method, the two distance measures are treated equally. Sometimes it would lead to such case: $A_1 \succ A_2$ by the first distance measure, $A_2 \succ A_1$ by the second distance measure, while $A_1 = A_2$ by the TOPSIS method. At this time, we should consider the ability of the two distance measures to distinguish the alternatives to give a more scientific evaluation outcome. In fact, the abilities of the two distance measures to distinguish the alternatives are usually different. In order to reflect the abilities of the two distance measures to distinguish the alternatives, an improved TOPSIS method based on information entropy is proposed to determine the weights of the two distance measures, based on which a new evaluation criterion is given to rank the alternatives.

As for the shortages mentioned above, this work intends to introduce the definitions of ideal decision matrix and coordinated expert weights and then propose an algorithm based on fixed point iteration to determine the coordinated weights of experts. Furthermore, this paper intends to put forward an improved TOPSIS method to promote the ability of ranking the alternatives.

The rest of this paper is organized as follows. Section 2 reviews some basic notions, operations and aggregation operators related to interval-valued intuitionistic fuzzy number. Section 3 gives the description of the novel method based on fixed point iteration and improved TOPSIS method. Section 4 presents a numerical example and comparison analysis to illustrate the validity of the proposed method. The final section discusses the conclusion and further research of this paper.

2. **Preliminaries.** Since interval-valued intuitionistic fuzzy set is more flexible and applicable in decision making field, this paper mainly presents a novel method for group decision making with interval-valued intuitionistic fuzzy information.

In this section, we briefly review basic concepts related to interval-valued intuitionistic fuzzy set, interval-valued intuitionistic fuzzy matrix, and the Euclidean distance measure of them, respectively.

Atanassov and Gargov [26] introduced the notion of interval-valued intuitionistic fuzzy set as follows.

Definition 2.1. [26] *Let $X = \{x_1, x_2, \ldots, x_m\}$ be a universe of discourse, and then an interval-valued intuitionistic fuzzy set $\widetilde{A}$ on X is defined as:*

$$\widetilde{A} = \{< x_i, \mu_{\widetilde{A}}(x_i), \nu_{\widetilde{A}}(x_i) > | x_i \in X\}, \tag{2}$$

where $\mu_{\widetilde{A}}(x_i) = \left[\mu^l_{\widetilde{A}}(x_i), \mu^u_{\widetilde{A}}(x_i)\right] \subseteq [0,1]$ and $\nu_{\widetilde{A}}(x_i) = \left[\nu^l_{\widetilde{A}}(x_i), \nu^u_{\widetilde{A}}(x_i)\right] \subseteq [0,1]$ are intervals, $\mu^l_{\widetilde{A}}(x_i) = \inf\mu_{\widetilde{A}}(x_i)$, $\mu^u_{\widetilde{A}}(x_i) = \sup\mu_{\widetilde{A}}(x_i)$, $\nu^l_{\widetilde{A}}(x_i) = \inf\nu_{\widetilde{A}}(x_i)$, $\nu^u_{\widetilde{A}}(x_i) = \sup\nu_{\widetilde{A}}(x_i)$, and $\mu^u_{\widetilde{A}}(x_i) + \nu^u_{\widetilde{A}}(x_i) \le 1$, for all $x_i \in X$, and $\pi_{\widetilde{A}}(x_i) = \left[\pi^l_{\widetilde{A}}(x_i), \pi^u_{\widetilde{A}}(x_i)\right]$, where $\pi^l_{\widetilde{A}}(x_i) = 1 - \mu^u_{\widetilde{A}}(x_i) - \nu^u_{\widetilde{A}}(x_i)$, $\pi^u_{\widetilde{A}}(x_i) = 1 - \mu^l_{\widetilde{A}}(x_i) - \nu^l_{\widetilde{A}}(x_i)$, for all $x_i \in X$.

Especially, if $\mu_{\widetilde{A}}(x_i) = \mu^l_{\widetilde{A}}(x_i) = \mu^u_{\widetilde{A}}(x_i)$ and $\nu_{\widetilde{A}}(x_i) = \nu^l_{\widetilde{A}}(x_i) = \nu^u_{\widetilde{A}}(x_i)$, then an interval-valued intuitionistic fuzzy set $\widetilde{A}$ is reduced to an intuitionistic fuzzy set.

Xu and Chen [27] called the pair $\widetilde{\alpha} = (\mu_{\widetilde{\alpha}}, \nu_{\widetilde{\alpha}})$ an interval-valued intuitionistic fuzzy number (IVIFN), and denoted an IVIFN by $\left(\left[\mu^l_{\widetilde{\alpha}}, \mu^u_{\widetilde{\alpha}}\right], \left[\nu^l_{\widetilde{\alpha}}, \nu^u_{\widetilde{\alpha}}\right]\right)$, where $\left[\mu^l_{\widetilde{\alpha}}, \mu^u_{\widetilde{\alpha}}\right], \left[\nu^l_{\widetilde{\alpha}}, \nu^u_{\widetilde{\alpha}}\right]$, $\left[\pi^l_{\widetilde{\alpha}}, \pi^u_{\widetilde{\alpha}}\right] \subseteq [0,1]$, $\mu^u_{\widetilde{\alpha}} + \nu^u_{\widetilde{\alpha}} \le 1$, $\pi^l_{\widetilde{\alpha}} = 1 - \mu^u_{\widetilde{\alpha}} - \nu^u_{\widetilde{\alpha}}$, $\pi^u_{\widetilde{\alpha}} = 1 - \mu^l_{\widetilde{\alpha}} - \nu^l_{\widetilde{\alpha}}$, and $\left[\mu^l_{\widetilde{\alpha}}, \mu^u_{\widetilde{\alpha}}\right]$ and $\left[\nu^l_{\widetilde{\alpha}}, \nu^u_{\widetilde{\alpha}}\right]$ represent the supported interval and the opposed interval about an evaluation object, respectively.

Definition 2.2. [27] *Given two IVIFNs $\widetilde{\alpha} = \left(\left[\mu^l_{\widetilde{\alpha}}, \mu^u_{\widetilde{\alpha}}\right], \left[\nu^l_{\widetilde{\alpha}}, \nu^u_{\widetilde{\alpha}}\right]\right)$ and $\widetilde{\beta} = \left(\left[\mu^l_{\widetilde{\beta}}, \mu^u_{\widetilde{\beta}}\right], \left[\nu^l_{\widetilde{\beta}}, \nu^u_{\widetilde{\beta}}\right]\right)$, then*

1) $\widetilde{\alpha} + \widetilde{\beta} = \left(\left[\mu^l_{\widetilde{\alpha}} + \mu^l_{\widetilde{\beta}} - \mu^l_{\widetilde{\alpha}}\mu^l_{\widetilde{\beta}}, \mu^u_{\widetilde{\alpha}} + \mu^u_{\widetilde{\beta}} - \mu^u_{\widetilde{\alpha}}\mu^u_{\widetilde{\beta}}\right], \left[\nu^l_{\widetilde{\alpha}}\nu^l_{\widetilde{\beta}}, \nu^u_{\widetilde{\alpha}}\nu^u_{\widetilde{\beta}}\right]\right)$,

2) $\lambda\widetilde{\alpha} = \left(\left[1 - \left(1 - \mu^l_{\widetilde{\alpha}}\right)^\lambda, 1 - (1 - \mu^u_{\widetilde{\alpha}})^\lambda\right], \left[\left(\nu^l_{\widetilde{\alpha}}\right)^\lambda, (\nu^u_{\widetilde{\alpha}})^\lambda\right]\right)$, $\lambda > 0$.

Then, Yue [9] gave the definition of the Euclidean distance measure between IVIFNs as follows.

Definition 2.3. [9] *Given two IVIFNs* $\widetilde{\alpha} = \left(\left[\mu^l_{\widetilde{\alpha}}, \mu^u_{\widetilde{\alpha}}\right], \left[\nu^l_{\widetilde{\alpha}}, \nu^u_{\widetilde{\alpha}}\right]\right)$ *and* $\widetilde{\beta} = \left(\left[\mu^l_{\widetilde{\beta}}, \mu^u_{\widetilde{\beta}}\right], \left[\nu^l_{\widetilde{\beta}}, \nu^u_{\widetilde{\beta}}\right]\right)$, *then*

$$D\left(\widetilde{\alpha}, \widetilde{\beta}\right) = \sqrt{\left(\mu^l_{\widetilde{\alpha}} - \mu^l_{\widetilde{\beta}}\right)^2 + \left(\mu^u_{\widetilde{\alpha}} - \mu^u_{\widetilde{\beta}}\right)^2 + \left(\nu^l_{\widetilde{\alpha}} - \nu^l_{\widetilde{\beta}}\right)^2 + \left(\nu^u_{\widetilde{\alpha}} - \nu^u_{\widetilde{\beta}}\right)^2 + \left(\pi^l_{\widetilde{\alpha}} - \pi^l_{\widetilde{\beta}}\right)^2 + \left(\pi^u_{\widetilde{\alpha}} - \pi^u_{\widetilde{\beta}}\right)^2} \tag{3}$$

is called the Euclidean distance between $\widetilde{\alpha}$ *and* $\widetilde{\beta}$, *where* $\pi^l_{\widetilde{\alpha}} = 1-\mu^u_{\widetilde{\alpha}}-\nu^u_{\widetilde{\alpha}}$, $\pi^u_{\widetilde{\alpha}} = 1-\mu^l_{\widetilde{\alpha}}-\nu^l_{\widetilde{\alpha}}$, $\pi^l_{\widetilde{\beta}} = 1 - \mu^u_{\widetilde{\beta}} - \nu^u_{\widetilde{\beta}}$, $\pi^u_{\widetilde{\beta}} = 1 - \mu^l_{\widetilde{\beta}} - \nu^l_{\widetilde{\beta}}$. *In general, the smaller the value* $D\left(\widetilde{\alpha}, \widetilde{\beta}\right)$ *is, the closer the* $\widetilde{\alpha}$ *is to* $\widetilde{\beta}$.

Later, Yue and Jia [28] introduced the interval-valued intuitionistic fuzzy matrix to express the information of attributes in MAGDM problem where all elements in the matrix are IVIFNs.

Definition 2.4. [28] *Given two interval-valued intuitionistic fuzzy matrices* $X_1 = (x^1_{ij})_{m\times n}$ *and* $X_2 = (x^2_{ij})_{m\times n}$, *then the Euclidean distance between* X_1 *and* X_2 *is defined as:*

$$D(X_1, X_2) = ||X_1 - X_2||_2 \tag{4}$$

where $||X_1-X_2||_2 = \left(\sum_{i=1}^m \left(\sum_{j=1}^n \left(\mu^{1l}_{ij} - \mu^{2l}_{ij}\right)^2 + \left(\mu^{1u}_{ij} - \mu^{2u}_{ij}\right)^2 + \left(\nu^{1l}_{ij} - \nu^{2l}_{ij}\right)^2 + \left(\nu^{1u}_{ij} - \nu^{2u}_{ij}\right)^2 + \left(\pi^{1l}_{ij} - \pi^{2l}_{ij}\right)^2 + \left(\pi^{1u}_{ij} - \pi^{2u}_{ij}\right)^2\right)\right)^{1/2}$, $\pi^{1l}_{ij} = 1 - \mu^{1u}_{ij} - \nu^{1u}_{ij}$, $\pi^{1u}_{ij} = 1 - \mu^{1l}_{ij} - \nu^{1l}_{ij}$, $\pi^{2l}_{ij} = 1 - \mu^{2u}_{ij} - \nu^{2u}_{ij}$, $\pi^{2u}_{ij} = 1 - \mu^{2l}_{ij} - \nu^{2l}_{ij}$. *In general, the smaller the value* $D(X_1, X_2)$ *is, the closer the* X_1 *is to* X_2.

3. **Proposed Method.** In this section, we proposed a novel method based on fixed point iteration and improved TOPSIS method for multi-attribute group decision making. In our model, we first use the fixed point iteration algorithm to derive the coordinated weights of DMs. And then we use the improved TOPSIS method to rank the alternatives.

In the process of using the fixed point iteration algorithm to derive the weights of DMs, the initial expert weights can be arbitrarily given. The Euclidean distance measure is used to calculate the proximity between each expert decision matrix and the overall decision matrix. The coordinated weights are determined by the given decision matrices and are not affected by the selection of the initial value of expert weights, which can objectively reflect the scoring level of each expert.

In the process of applying the improved TOPSIS method to rank the alternatives, the information entropy measure is used to determine the weight of the two distance measures, and then give a ranking according to the new evaluation criterion.

For an MAGDM problem, let i: index for alternatives, $i \in M = \{1, 2, \ldots, m\}$, j: index for attributes, $j \in N = \{1, 2, \ldots, n\}$, k: index for DMs, $k \in T = \{1, 2, \ldots, t\}$, a set of m feasible alternatives written as $A = \{A_1, A_2, \ldots, A_m\}$ $(m \geq 2)$, a set of attributes written as $U = \{u_1, u_2, \ldots, u_n\}$, a set of DMs written as $D = \{d_1, d_2, \ldots, d_t\}$.

3.1. **Determining the weights of decision makers based on fixed point iteration.** Firstly, each DM presents individual decision matrix (IDM) as follows:

$$X_k = \left(x_{ij}^k\right)_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{pmatrix} \end{array}, \quad k \in T, \tag{5}$$

where $x_{ij}^k = \left(\left[\mu_{ij}^{kl}, \mu_{ij}^{ku}\right], \left[\nu_{ij}^{kl}, \nu_{ij}^{ku}\right]\right)$, $i \in M$, $j \in N$.

Secondly, construct the overall decision matrix. For random given initial expert weights $\lambda = (\lambda_1, \ldots, \lambda_k, \ldots, \lambda_t)$, the overall decision matrix can be constructed as the weighted average of all DMs X_k, $(k \in T)$ in Equation (6):

$$X^* = \left(x_{ij}^*\right)_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11}^* & x_{12}^* & \cdots & x_{1n}^* \\ x_{21}^* & x_{22}^* & \cdots & x_{2n}^* \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1}^* & x_{m2}^* & \cdots & x_{mn}^* \end{pmatrix} \end{array}, \quad k \in T, \tag{6}$$

where $x_{ij}^* = \sum_{k=1}^t \lambda_k x_{ij}^k$, $i \in M$, $j \in N$, which can be calculated by Definition 2.2.

Thirdly, calculate the weighted Euclidean distance $WD(X^*, X_k)$. Suppose the weight vector of attributes is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, according to Equation (4), the weighted Euclidean distance $WD(X^*, X_k)$ can be calculated as follows:

$$WD(X^*, X_k) = \sqrt{\sum_{i=1}^m \sum_{j=1}^n \omega_j \left(x_{ij}^* - x_{ij}^k\right)^2}, \tag{7}$$

where $\left(x_{ij}^* - x_{ij}^k\right)^2 = \left[D\left(x_{ij}^*, x_{ij}^k\right)\right]^2$, which can be calculated by Equation (3).

Obviously, the smaller distance between IDM X_k and the overall decision matrix X^*, the better evaluation capacity of the decision maker. In other words, the smaller the distance between IDM X_k and the overall decision matrix X^*, the greater the weight of the decision maker. According to the calculation outcome of the distance measure, we get the updated weight of the kth decision maker as follows:

$$\overline{\lambda}_k = \frac{\frac{1}{WD(X^*, X_k)}}{\sum_{k=1}^t \frac{1}{WD(X^*, X_k)}}, \quad k = 1, 2, \ldots, t. \tag{8}$$

Fourthly, calculate the deviation between previous expert weights and current expert weights. The deviation between previous expert weights and current expert weights is defined as follows:

$$L\left(\overline{\lambda}, \lambda\right) = \sqrt{\sum_{k=1}^t \left(\overline{\lambda}_k - \lambda_k\right)^2}. \tag{9}$$

Fifthly, adjust the expert weights. Let us say the threshold is Th. If $L\left(\overline{\lambda}, \lambda\right) \leq Th$, it is considered that the expert weights tend to be stable and consistent. Thus the adjustment process is over and let $\lambda^* = \overline{\lambda}$. Otherwise, let $\lambda = \overline{\lambda}$, go back to the second step. Hence, the expert weights adjustment can be expressed briefly as follows:

$$\begin{cases} \lambda^* = \overline{\lambda}, & \text{if } L\left(\overline{\lambda}, \lambda\right) \leq Th; \\ \lambda = \overline{\lambda}, & \text{otherwise.} \end{cases} \tag{10}$$

Each iteration is equivalent to the experts making a negotiation based on the given decision matrices and the previous expert weights. As negotiation times are large enough, the coordinated expert weights can be received. The initial value only affects the number of iterations. As the number of iterations increases, $L\left(\overline{\lambda}, \lambda\right)$ is monotonically decreasing, and $L\left(\overline{\lambda}, \lambda\right)$ converges to zero as the iteration times are large enough. That is to say, the above iterative sequence is a compressed mapping sequence and a closed sequence. According to the compression mapping theorem [29], the iterative sequence converges to the only fixed point, which is the vector of coordinated expert weights.

3.2. **Ranking the preference of alternatives by improved TOPSIS method.** Firstly, we convert the individual decision matrix X_k, $(k \in T)$ into the alternative decision matrix (ADM) H_i, $(i \in M)$, which can be expressed as follows:

$$H_i = \left(h_{kj}^i\right)_{t\times n} = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_t \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} h_{11}^i & h_{12}^i & \cdots & h_{1n}^i \\ h_{21}^i & h_{22}^i & \cdots & h_{2n}^i \\ \vdots & \vdots & \cdots & \vdots \\ h_{t1}^i & h_{t2}^i & \cdots & h_{tn}^i \end{pmatrix} \end{array}, \tag{11}$$

where

$$h_{kj}^i = x_{ij}^k,\ k = 1, 2, \ldots, t;\ j = 1, 2, \ldots, n. \tag{12}$$

Secondly, determine the positive and negative ideal alternative decision matrices according to the alternative decision matrix H_i, $(i \in M)$.

$$H^+ = \left(h_{kj}^+\right)_{t\times n} = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_t \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} h_{11}^+ & h_{12}^+ & \cdots & h_{1n}^+ \\ h_{21}^+ & h_{22}^+ & \cdots & h_{2n}^+ \\ \vdots & \vdots & \cdots & \vdots \\ h_{t1}^+ & h_{t2}^+ & \cdots & h_{tn}^+ \end{pmatrix} \end{array}, \tag{13}$$

where

$$h_{kj}^+ = \max_{i\in M}\left\{h_{kj}^i\right\} = \left(\left[\max_{i\in M}\{\mu_{kj}^{il}\}, \max_{i\in M}\left\{\mu_{kj}^{iu}\right\}\right], \left[\min_{i\in M}\left\{\nu_{kj}^{il}\right\}, \min_{i\in M}\left\{\nu_{kj}^{iu}\right\}\right]\right) \tag{14}$$

$$H^- = \left(h_{kj}^-\right)_{t\times n} = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_t \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} h_{11}^- & h_{12}^- & \cdots & h_{1n}^- \\ h_{21}^- & h_{22}^- & \cdots & h_{2n}^- \\ \vdots & \vdots & \cdots & \vdots \\ h_{t1}^- & h_{t2}^- & \cdots & h_{tn}^- \end{pmatrix} \end{array}, \tag{15}$$

where

$$h_{kj}^- = \min_{i\in M}\left\{h_{kj}^i\right\} = \left(\left[\min_{i\in M}\left\{\mu_{kj}^{il}\right\}, \min_{i\in M}\left\{\mu_{kj}^{iu}\right\}\right], \left[\max_{i\in M}\left\{\nu_{kj}^{il}\right\}, \max_{i\in M}\left\{\nu_{kj}^{iu}\right\}\right]\right). \tag{16}$$

Thirdly, according to Equation (4), calculate the weighted Euclidean distance $WD(H_i, H^+)$ and the weighted Euclidean distance $WD(H_i, H^-)$, respectively.

$$WD\left(H_i, H^+\right) = \sqrt{\sum_{k=1}^{t} \lambda_k^* \sum_{j=1}^{n} \omega_j \left(h_{kj}^i - h_{kj}^+\right)^2}, \quad i = 1, 2, \ldots, m, \tag{17}$$

where $\left(h_{kj}^i - h_{kj}^+\right)^2 = \left[D\left(h_{kj}^i, h_{kj}^+\right)\right]^2$, which can be calculated by Equation (3).

$$WD\left(H_i, H^-\right) = \sqrt{\sum_{k=1}^{t} \lambda_k^* \sum_{j=1}^{n} \omega_j \left(h_{kj}^i - h_{kj}^-\right)^2}, \quad i = 1, 2, \ldots, m, \tag{18}$$

where $\left(h_{kj}^{i}-h_{kj}^{-}\right)^{2}=\left[D\left(h_{kj}^{i},h_{kj}^{-}\right)\right]^{2}$, which can be calculated by Equation (3).

Fourthly, calculate the information entropy of the two distance measures, respectively.

$$E_1=-\frac{1}{\ln m}\sum_{i=1}^{m}\overline{N}\,WD\left(H_i,H^+\right)\ln\overline{N}\,WD\left(H_i,H^+\right), \tag{19}$$

where $\overline{N}\,WD\left(H_i,H^+\right)=\frac{WD\left(H_i,H^+\right)}{\sum_{i=1}^{m}WD\left(H_i,H^+\right)}$.

$$E_2=-\frac{1}{\ln m}\sum_{i=1}^{m}\overline{N}\,WD\left(H_i,H^-\right)\ln\overline{N}\,WD\left(H_i,H^-\right), \tag{20}$$

where $\overline{N}\,WD\left(H_i,H^-\right)=\frac{WD\left(H_i,H^-\right)}{\sum_{i=1}^{m}WD\left(H_i,H^-\right)}$.

Fifthly, determine the weight of the two distance measures.

$$w_l=\frac{1-E_l}{\sum_{l=1}^{2}(1-E_l)},\quad l=1,2. \tag{21}$$

Inspired by the entropy-based method to determine the weights of attributes [7], we use the entropy-based method to determine the weights of the single methods, which can reflect the fluctuation of the evaluation outcomes given by the single methods. The greater the fluctuation of the evaluation outcome given by a single method, the greater the weight of the single method. The basic principle of the entropy-based method is that the smaller the entropy value of the assessment outcome of alternatives under a single decision method, the bigger the weight should be assigned to the single decision method.

Sixthly, after determining the weights of the single method, we can calculate the combined score value. Obviously, the smaller the Euclidean distance $WD\left(H_i,H^+\right)$, the better the alternative, while the bigger the Euclidean distance $WD\left(H_i,H^-\right)$, the better the alternative. Hence, we should normalize the assessment outcomes, and give the combined score value S_i as follows:

$$S_i=w_1NWD\left(H_i,H^+\right)+w_2NWD\left(H_i,H^-\right),\quad i=1,2,\ldots,m, \tag{22}$$

where

$$NWD\left(H_i,H^+\right)=\frac{\min\limits_{i\in M}WD\left(H_i,H^+\right)}{WD\left(H_i,H^+\right)}, \tag{23}$$

$$NWD\left(H_i,H^-\right)=\frac{WD\left(H_i,H^-\right)}{\max\limits_{i\in M}WD\left(H_i,H^-\right)}. \tag{24}$$

Finally, rank the preference order of alternatives by the following evaluation criteria.

Evaluation criteria: (1) Rank the preference order of alternatives by the combined score value S_i, the bigger, the better. (2) If there exists $A_i\equiv A_j$ by the combined score value S_i, the bigger weight of the distance measure is preferred to rank the alternatives. (3) If $A_i\equiv A_j$ by all of the measures, then $A_i\equiv A_j$.

3.3. **Presented algorithm.** In sum, an algorithm for MAGDM problems, when decision information is expressed in interval-valued intuitionistic fuzzy, using a novel method based on fixed point iteration and improved TOPSIS method, is described as follows.

Step 1. Provide individual decision information.

Each DM d_k provides IDM $X_k=\left(x_{ij}^{k}\right)_{m\times n}$ on alternatives with respect to attributes with interval-valued intuitionistic fuzzy, which is given in Equation (5).

Step 2. Determine the coordinated weights of DMs by repeatedly using Equations (6)-(10), which is a fixed point iteration process.

Step 3. Convert the individual decision matrix X_k, $(k \in T)$ into the alternative decision matrix H_i, $(i \in M)$ by Equations (11) and (12).

Step 4. Determine the ideal decisions of all alternative decision matrices.

For all alternative decision matrix H_i, $(i \in M)$, the positive ideal alternative matrix H^+ and the negative ideal alternative matrix H^- are determined by Equations (13)-(14) and Equations (15)-(16), respectively.

Step 5. Calculate the weighted Euclidean distance $WD\left(H_i, H^+\right)$ by Equation (17) and the weighted Euclidean distance $WD\left(H_i, H^-\right)$ by Equation (18), respectively.

Step 6. Determine the weight of the two distance measures by Equations (19)-(21).

Step 7. Calculate the combined score value S_i by Equations (22)-(24).

Step 8. Rank the preference order of alternatives by the proposed evaluation criteria.

4. **Numerical Illustration and Discussions.** In this section, we give an illustrative example with interval-valued intuitionistic fuzzy information for its flexibility and applicability in decision making field. In fact, the proposed method can be applied in other attribute information forms, such as real numbers, intervals, linguistic variables and intuitionistic fuzzy sets and it can be applied to many MAGDM fields, such as the supplier selection, the strategic alliance partner selection, the robot selection, green supplier development program selection, and software reliability assessment. Furthermore, we give comparison analysis to demonstrate the effectiveness of the proposed method.

4.1. **An illustrative numerical example.** A company is planning to recruit an appropriate supplier. After preliminary examination, four candidates are shortlisted for further evaluation. A committee of four human resource experts has been formed to conduct the interviews and to evaluate all four candidates as part of selection process. Three assessment criteria are introduced for further evaluation process: on time delivery (u_1), flexibility (u_2) and quality (u_3) and the weight vector of attributes is that $\omega = (0.3, 0.4, 0.3)$. The four decision matrixes X_1, X_2, X_3, X_4 evaluated by four human resource experts are shown in Table 1, which is the Step 1 in our model.

By Step 2, for random given initial expert weights, determine the coordinated weights of DMs by using the fixed point iteration algorithm. Without loss of generality, suppose the

Table 1. Four individual decision matrices

IDM	Supplier	u_1	u_2	u_3
X_1	A_1	([0.70, 0.86],[0.11, 0.13])	([0.76, 0.82],[0.05, 0.09])	([0.81, 0.83],[0.01, 0.02])
	A_2	([0.71, 0.75],[0.06, 0.11])	([0.81, 0.82],[0.03, 0.09])	([0.72, 0.75],[0.09, 0.15])
	A_3	([0.77, 0.81],[0.06, 0.08])	([0.71, 0.73],[0.13, 0.19])	([0.62, 0.72],[0.19, 0.21])
	A_4	([0.65, 0.75],[0.15, 0.22])	([0.82, 0.84],[0.06, 0.09])	([0.71, 0.72],[0.11, 0.12])
X_2	A_1	([0.71, 0.75],[0.16, 0.22])	([0.78, 0.83],[0.12, 0.15])	([0.79, 0.83],[0.05, 0.12])
	A_2	([0.72, 0.75],[0.06, 0.11])	([0.81, 0.83],[0.06, 0.09])	([0.69, 0.75],[0.13, 0.18])
	A_3	([0.77, 0.81],[0.05, 0.09])	([0.73, 0.75],[0.15, 0.19])	([0.67, 0.72],[0.17, 0.21])
	A_4	([0.69, 0.75],[0.17, 0.22])	([0.75, 0.77],[0.07, 0.09])	([0.71, 0.75],[0.13, 0.15])
X_3	A_1	([0.67, 0.73],[0.16, 0.23])	([0.75, 0.83],[0.12, 0.14])	([0.81, 0.84],[0.07, 0.12])
	A_2	([0.72, 0.75],[0.11, 0.16])	([0.70, 0.72],[0.06, 0.11])	([0.71, 0.75],[0.14, 0.18])
	A_3	([0.63, 0.75],[0.07, 0.09])	([0.69, 0.75],[0.17, 0.19])	([0.71, 0.73],[0.17, 0.21])
	A_4	([0.61, 0.65],[0.17, 0.23])	([0.79, 0.82],[0.07, 0.11])	([0.71, 0.76],[0.13, 0.17])
X_4	A_1	([0.66, 0.73],[0.12, 0.15])	([0.81, 0.85],[0.05, 0.09])	([0.79, 0.81],[0.05, 0.09])
	A_2	([0.73, 0.76],[0.11, 0.13])	([0.81, 0.82],[0.02, 0.05])	([0.73, 0.75],[0.05, 0.11])
	A_3	([0.67, 0.71],[0.05, 0.07])	([0.65, 0.69],[0.12, 0.15])	([0.75, 0.77],[0.11, 0.13])
	A_4	([0.81, 0.83],[0.03, 0.05])	([0.75, 0.77],[0.09, 0.11])	([0.72, 0.76],[0.13, 0.15])

initial weight vector of experts is that $\lambda = (0.25, 0.25, 0.25, 0.25)$, then the iteration process is shown in Table 2, where $Th = 0.0001$. In this case, after 10 iterations, $L\left(\overline{\lambda}, \lambda\right) < Th$, it is considered that the expert weights tend to be stable and consistent. Hence, let $\lambda^* = \overline{\lambda} = (0.3070, 0.2958, 0.2045, 0.1927)$, which is irrelevant to the initial assignment of expert weights. For further demonstrations, suppose the initial weight vectors of experts are $\lambda = (0.10, 0.10, 0.15, 0.65)$, $\lambda = (0.30, 0.10, 0.20, 0.40)$, and $\lambda = (0.40, 0.15, 0.15, 0.30)$, respectively. The iteration processes are shown in Tables 3-5, where $Th = 0.0001$.

Table 2. The iteration process of the initial expert weights $\lambda = (0.25, 0.25, 0.25, 0.25)$

Number of iterations	0	1	2	3	$\cdots$	9	10
the weight of d_1	0.2500	0.2777	0.2923	0.2997	$\cdots$	0.3070	0.3070
the weight of d_2	0.2500	0.2820	0.2916	0.2946	$\cdots$	0.2958	0.2958
the weight of d_3	0.2500	0.2202	0.2111	0.2075	$\cdots$	0.2045	0.2045
the weight of d_4	0.2500	0.2201	0.2051	0.1982	$\cdots$	0.1927	0.1927
$L\left(\overline{\lambda}, \lambda\right)$		0.0598	0.0248	0.0112	$\cdots$	1.1869e-04	5.5419e-05

Table 3. The iteration process of the initial expert weights $\lambda = (0.10, 0.10, 0.15, 0.65)$

Number of iterations	0	1	2	3	$\cdots$	12	13
the weight of d_1	0.1000	0.1718	0.2239	0.2594	$\cdots$	0.3070	0.3070
the weight of d_2	0.1000	0.1762	0.2272	0.2612	$\cdots$	0.2958	0.2958
the weight of d_3	0.1500	0.1623	0.1943	0.2091	$\cdots$	0.2045	0.2045
the weight of d_4	0.6500	0.4897	0.3546	0.2703	$\cdots$	0.1927	0.1927
$L\left(\overline{\lambda}, \lambda\right)$		0.1918	0.1569	0.0987	$\cdots$	1.2629e-04	5.9206e-05

Table 4. The iteration process of the initial expert weights $\lambda = (0.30, 0.10, 0.20, 0.40)$

Number of iterations	0	1	2	3	$\cdots$	10	11
the weight of d_1	0.3000	0.2657	0.2795	0.2915	$\cdots$	0.3070	0.3070
the weight of d_2	0.1000	0.2356	0.2716	0.2863	$\cdots$	0.2958	0.2958
the weight of d_3	0.2000	0.2046	0.2103	0.2096	$\cdots$	0.2045	0.2045
the weight of d_4	0.4000	0.2942	0.2386	0.2126	$\cdots$	0.1927	0.1927
$L\left(\overline{\lambda}, \lambda\right)$		0.1755	0.0679	0.0322	$\cdots$	1.5833e-04	7.4402e-05

Table 5. The iteration process of the initial expert weights $\lambda = (0.40, 0.15, 0.15, 0.30)$

Number of iterations	0	1	2	3	$\cdots$	8	9
the weight of d_1	0.4000	0.3213	0.3075	0.3054	$\cdots$	0.3070	0.3070
the weight of d_2	0.1500	0.2501	0.2793	0.2898	$\cdots$	0.2958	0.2958
the weight of d_3	0.1500	0.1944	0.2040	0.2054	$\cdots$	0.2045	0.2045
the weight of d_4	0.3000	0.2342	0.2093	0.1994	$\cdots$	0.1927	0.1927
$L\left(\overline{\lambda}, \lambda\right)$		0.1501	0.0419	0.0147	$\cdots$	1.9164e-04	8.9959e-05

As can be seen from the above four tables, even though the expert weights start with different initial values, they eventually stabilize to the same weight, and as the number of iterations increases, $L\left(\overline{\lambda}, \lambda\right)$ is monotonically decreasing.

In order to rank alternatives, we should first convert the individual decision matrix $X_k, (k \in T)$ into the alternative decision matrix H_i, $(i \in M)$ by Step 3, which are given in Table 6.

TABLE 6. Four alternative decision matrices

ADM	Decision maker	u_1	u_2	u_3
H_1	d_1	([0.70, 0.86],[0.11, 0.13])	([0.76, 0.82],[0.05, 0.09])	([0.81, 0.83],[0.01, 0.02])
	d_2	([0.71, 0.75],[0.16, 0.22])	([0.78, 0.83],[0.12, 0.15])	([0.79, 0.83],[0.05, 0.12])
	d_3	([0.67, 0.73],[0.16, 0.23])	([0.75, 0.83],[0.12, 0.14])	([0.81, 0.84],[0.07, 0.12])
	d_4	([0.66, 0.73],[0.12, 0.15])	([0.81, 0.85],[0.05, 0.09])	([0.79, 0.81],[0.05, 0.09])
H_2	d_1	([0.71, 0.75],[0.06, 0.11])	([0.81, 0.82],[0.03, 0.09])	([0.72, 0.75],[0.09, 0.15])
	d_2	([0.72, 0.75],[0.06, 0.11])	([0.81, 0.83],[0.06, 0.09])	([0.69, 0.75],[0.13, 0.18])
	d_3	([0.72, 0.75],[0.11, 0.16])	([0.70, 0.72],[0.06, 0.11])	([0.71, 0.75],[0.14, 0.18])
	d_4	([0.73, 0.76],[0.11, 0.13])	([0.81, 0.82],[0.02, 0.05])	([0.73, 0.75],[0.05, 0.11])
H_3	d_1	([0.77, 0.81],[0.06, 0.08])	([0.71, 0.73],[0.13, 0.19])	([0.62, 0.72],[0.19, 0.21])
	d_2	([0.77, 0.81],[0.05, 0.09])	([0.73, 0.75],[0.15, 0.19])	([0.67, 0.72],[0.17, 0.21])
	d_3	([0.63, 0.75],[0.07, 0.09])	([0.69, 0.75],[0.17, 0.19])	([0.71, 0.73],[0.17, 0.21])
	d_4	([0.67, 0.71],[0.05, 0.07])	([0.65, 0.69],[0.12, 0.15])	([0.75, 0.77],[0.11, 0.13])
H_4	d_1	([0.65, 0.75],[0.15, 0.22])	([0.82, 0.84],[0.06, 0.09])	([0.71, 0.72],[0.11, 0.12])
	d_2	([0.69, 0.75],[0.17, 0.22])	([0.75, 0.77],[0.07, 0.09])	([0.71, 0.75],[0.13, 0.15])
	d_3	([0.61, 0.65],[0.17, 0.23])	([0.79, 0.82],[0.07, 0.11])	([0.71, 0.76],[0.13, 0.17])
	d_4	([0.81, 0.83],[0.03, 0.05])	([0.75, 0.77],[0.09, 0.11])	([0.72, 0.76],[0.13, 0.15])

By Step 4, two ideal alternative decision matrixes, including the positive ideal alternative matrix H^+ and the negative ideal alternative matrix H^-, are calculated and shown in Table 7.

TABLE 7. Ideal alternative decision matrices

IADM	Decision maker	u_1	u_2	u_3
H^+	d_1	([0.77, 0.86],[0.06, 0.08])	([0.82, 0.84],[0.03, 0.09])	([0.81, 0.83],[0.01, 0.02])
	d_2	([0.77, 0.81],[0.05, 0.09])	([0.81, 0.83],[0.06, 0.09])	([0.79, 0.83],[0.05, 0.12])
	d_3	([0.72, 0.75],[0.07, 0.09])	([0.79, 0.83],[0.06, 0.11])	([0.81, 0.84],[0.07, 0.12])
	d_4	([0.81, 0.83],[0.03, 0.05])	([0.81, 0.85],[0.02, 0.05])	([0.79, 0.81],[0.05, 0.09])
H^-	d_1	([0.65, 0.75],[0.15, 0.22])	([0.71, 0.73],[0.13, 0.19])	([0.62, 0.72],[0.19, 0.21])
	d_2	([0.69, 0.75],[0.17, 0.22])	([0.73, 0.75],[0.15, 0.19])	([0.67, 0.72],[0.17, 0.21])
	d_3	([0.61, 0.65],[0.17, 0.23])	([0.69, 0.72],[0.17, 0.19])	([0.71, 0.73],[0.17, 0.21])
	d_4	([0.66, 0.71],[0.12, 0.15])	([0.65, 0.69],[0.12, 0.15])	([0.72, 0.75],[0.13, 0.15])

By Step 5, the weighted Euclidean distances $WD\left(H_i, H^+\right)$ and $WD\left(H_i, H^-\right)$ are calculated and given in Table 8.

By Step 6, the weight vector of the two distance measures by information entropy is determined as follows: $w = (w_1, w_2) = (0.5627, 0.4373)$. Here, w_1 is the bigger, which indicates that the better capacity of the distance measure $WD\left(H_i, H^+\right)$ to distinguish the alternatives for the reason that the fluctuation of the alternatives measured by the distance method $WD\left(H_i, H^+\right)$ is the larger. It is worth noting that the distance measure $WD\left(H_i, H^+\right)$ might not the better measure to distinguish the alternatives in these two distance measures for another numerical example. It means that the weights of the two

TABLE 8. The evaluation outcomes of the two distance measures

Supplier	$WD(H_i, H^+)$	$WD(H_i, H^-)$
A_1	0.1196	0.1894
A_2	0.1330	0.1739
A_3	0.2042	0.1165
A_4	0.1563	0.1434

distance measures depend on actual information given in an MAGDM problem, which makes the evaluation outcome more scientific and reliable.

By Step 7, the assessment outcomes of $WD(H_i, H^+)$ and $WD(H_i, H^-)$ would be normalized, and get the combined score values S_i $(i \in M)$ as listed in Table 9. From Table 9, we can see that if the biggest combined score S_i of some alternative is equal to 1, which means that no matter what the distance measure is used, the alternative is the best. Otherwise, using different distance measures would lead to different best alternatives, and the combined score is a comprehensive evaluation result by the two distance measures.

TABLE 9. The normalized evaluation outcomes and the combined score values

Supplier	$NWD(H_i, H^+)$	$NWD(H_i, H^-)$	S_i
A_1	1.0000	1.0000	1.0000
A_2	0.8991	0.9184	0.9075
A_3	0.5859	0.6153	0.5988
A_4	0.7654	0.7574	0.7619

By Step 8, the preference order of potential suppliers by the proposed evaluation criteria is as follows: $A_1 \succ A_2 \succ A_4 \succ A_3$.

4.2. **Dynamic comparisons.** It is worth noting that only one set of data is used in the above experimental analysis. To give a more convinced result, this subsection shows dynamic comparisons with other methods.

For the evaluation matrix X_1 in Table 1, we set $x_{11}^1 = ([0.70, 0.86], [0.11, 0.13])$ in the above experiment. To show dynamic comparisons, $x_{11}^1 = ([\alpha, 0.86], [0.11, 0.13])$ would be set in this experiment, where the $\alpha \in [0.4, 0.8]$ is a parameter. Other values are the same as in Table 1. When the α increases from 0.4 to 0.8, the curves of weights of DMs based on different methods are shown in Figure 1 and Figure 2, respectively.

From Figure 1 and Figure 2, we can see that the curves of expert weights based on average decision matrix are different from the curves of coordinated weights based on fixed point iteration algorithm. Especially, when $0.65 \le \alpha \le 0.7$, the weight of d_2 is larger than the weight of d_1 by the Euclidean method based on average decision matrix, while the coordinated weight of d_2 is less than the coordinated weight of d_1 by the fixed point iteration algorithm. In fact, the Euclidean method based on average decision matrix by Yue [9] adopts the way of one-time calculation in determining the weights of experts, while the fixed point iteration algorithm adopts the way of repeated adjustment to receive the coordinated weights of experts. Furthermore, the coordinated weights are determined by the given decision matrices and are not affected by the selection of the initial value of expert weights, which can objectively reflect the scoring level of each expert.

According to the coordinated weights of experts, the curves of rankings of A_1, A_2, A_3, A_4 based on traditional TOPSIS method and improved TOPSIS method are shown in Figure 3 and Figure 4, respectively. From Figure 3, we can see that there exists an $\alpha \in (0.55, 0.60)$ resulting in $A_2 = A_1$ by the traditional TOPSIS method, which means

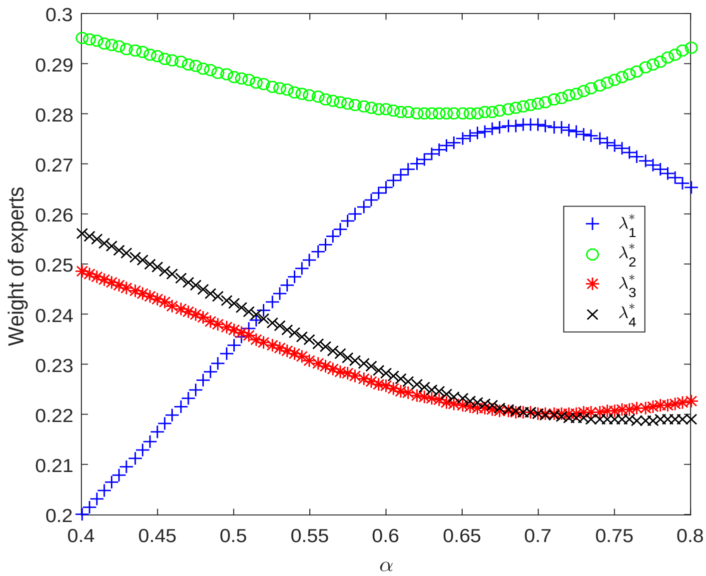

FIGURE 1. Expert weights based on average decision matrix by Yue [9]

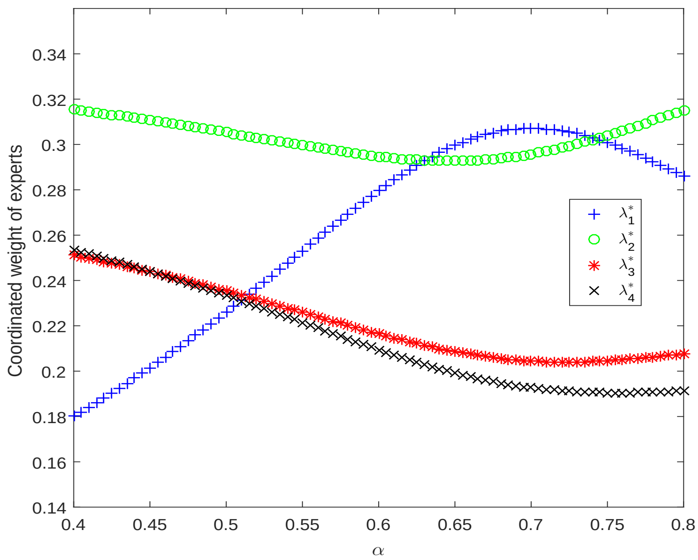

FIGURE 2. Coordinated expert weights based on fixed point iteration algorithm

that the traditional TOPSIS method cannot give a best alternative in this case. However, from Figure 4, we can see that the improved TOPSIS method can give a best alternative all the time by the proposed evaluation criteria. Furthermore, the improved TOPSIS method can reveal whether the optimal alternative based on positive ideal decision matrix is consistent with that based on negative ideal decision matrix. From Figure 4, we can see that when $\alpha \in (0.4, 0.45)$, the biggest combined score S_i is equal to 1, which means that the optimal alternative based on positive ideal decision matrix is consistent with that based on negative ideal decision matrix. Similarly, when $\alpha \in (0.65, 0.80)$, the optimal alternative based on positive ideal decision matrix is also consistent with that based on negative ideal decision matrix.

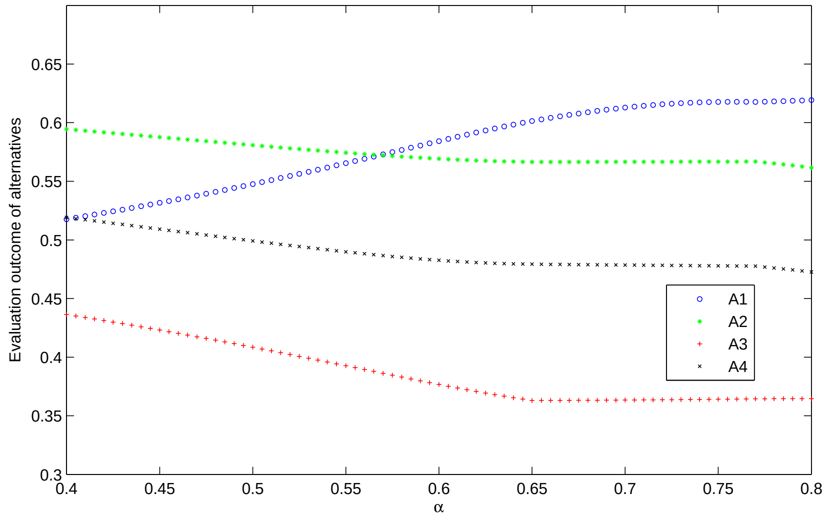

FIGURE 3. Rankings of four alternatives of traditional TOPSIS method

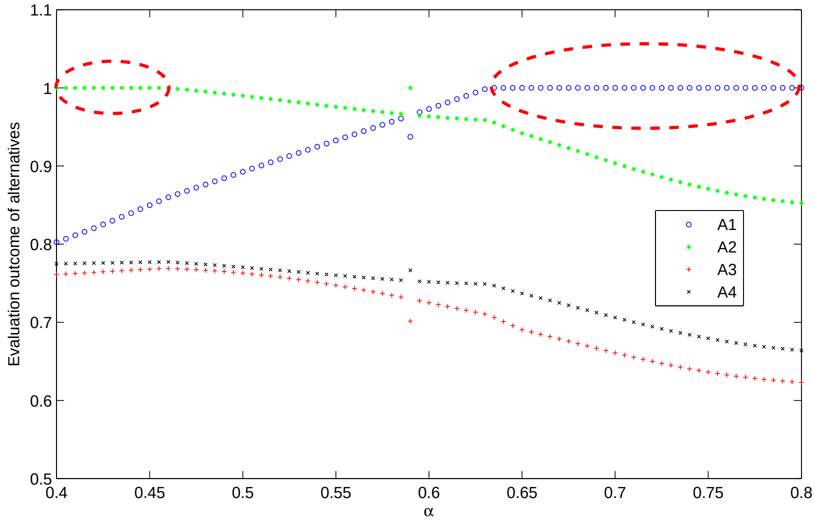

FIGURE 4. Rankings of four alternatives of improved TOPSIS method

In sum, from the above analysis of dynamic comparisons, the improved TOPSIS method is superior to the traditional TOPSIS method to some extent.

5. **Conclusion.** In order to settle the problem of determining the expert weights by calculating the deviation between the individual decision matrix and the average decision matrix in multi-attribute group decision-making, this paper introduces the definitions of ideal decision matrix and coordinated weights of experts. Furthermore, a fixed point iteration algorithm is proposed to derive the coordinated weights of experts. The coordinated weights are determined by the given decision matrices and are not affected by the selection of the initial value of expert weights, which can objectively reflect the scoring level of each expert. At last, the improved TOPSIS method is proposed to promote the ability of ranking the alternatives. Experimental results and comparisons show the validity of the proposed method.

Our research can be further extended along the following lines: 1) to consider another measure instead of the Euclidean distance measure to evaluate the degree of the decision matrix X_k approaching to the ideal decision X^*; 2) to consider other attribute types, such as real numbers, intervals, linguistic variables and intuitionistic fuzzy sets.

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