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ANALYTICAL MECHANICS APPROACH TO CONSERVATION IN PRODUCTION FIELD

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Abstract. *Although it is well known that both of time propulsion symmetry and space propulsion symmetry conservation of energy appear in the field of physics and theoretical applications have made progress by Noether's theorem, there are not many examples where such theoretical analysis has been performed for manufacturing industries. Regarding Testrun data (Testrun1 through Testrun3-1/3-2) collected by the production flow system, it was found that there exist both of time propulsion symmetry and space propulsion symmetry conservation in Testrun data. From the evaluation of Testrun1, the effect of dispersion can be seen sparsely. The production of Testrun2 and Testrun3-1/3-2 is progressing within the range of work variations. In other words, it retains time propulsion symmetry. In addition, Testrun3-1/3-2 can obtain the same product even if it is carried out from different stages. In other words, it retains the spatial propulsion symmetry of* $\{S_1\}$, $\{S_2\}$, $\{S_3\}$ *in Testrun3-1/3-2. From this, we confirmed that the deterministic linear diffusion equation is useful as a mathematical model in the synchronous production method.*
Keywords: Conservation of energy, Noether's theorem, Synchronization of the processes, Analytical mechanics, Diffusion equation

1. **Introduction.** A motive that the present writers and the like started to promote such kind of research during many years of experience of manufacturing operations of control equipment for general industrial machines is as follows. With respect to Japan after Lehman Shock, Japan's economy has been in a slump, and production bases of manufacturing industries keep moving overseas. Business environments of equipment manufacturing companies in Japanese are extremely severe. In Japan, the situation here is that thorough cost reduction is required. Therefore, we thought that, by finding relation between a company size and a production size of a company, and management parameters mathematically, cost reduction becomes possible. Further, as a result of analysis based on Testrun1 through Testrun3-1/3-2 collected over 10 years or more from the abovementioned motive, we have noticed that such data has some kind of law in these data.

We have reported on mathematical modeling (deterministic system, stochastic system), optimization, etc. of production processes in small and medium scale [1, 2, 3, 4, 5]. Previously, we have reported that by creating a state in which the production density of each process corresponds to physical propagation, the manufacturing process is most appropriately described using a diffusion equation [6]. In other words, if the potential of the

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production field (stochastic field) is minimized, the equation is defined by the production density function $\{S_i(t,x), i = 1-n\}$ (i is omitted hereafter) and the constraint is described using an advective diffusion equation to determine the transportation speed ρ [6, 7].

Next, conventionally, it is well known that there exists a fluctuation in physics, and there is research related to current fluctuation in an electric circuit. This is an important theme in mesoscopic physics, and theoretical studies have been conducted since the early 1990s. In general, noise (fluctuation) refers to the amount of variation in the measured values around the average value obtained in this way, that is, the "dispersion" of the measured values [9]. In our previous research, we reported that a delay in the production process is equivalent to a "fluctuation" in physical phenomenon. For example, there is the deviation from the thermal equilibrium state to fluctuations in physics. The propagation of fluctuation (volatility) in each stage delays the entire process. We have mathematically analyzed this phenomenon and assessed whether volatility is encountered during manufacturing [5, 10]. The many concern that occurs in the supply chain is major problem facing production efficiency and business profitability. A stochastic partial bilinear differential equation with time delay was derived for outlet processes. The supply chain was modeled by considering as time delay [5].

On the other hand, there is research related to an entropy which is one of the quantities representing the state of matter in thermodynamics. It also represents the randomness of the particles that make up a substance. If the temperature changes slowly while being held constant, the difference in entropy before and after the change in the heat received divided by the temperature is obtained. When slowly changing while being adiabatic, the entropy has a constant value. The entropy can be determined by the above heat measurement [11]. There is our previous research related to an entropy which is the analysis of Testrun data (Testrun1 through Testrun3-1/3-2). The results of this analysis report that Testrun1 is an asynchronous process and Testrun2 and Testrun3-1/3-2 are synchronous processes [12]. The feature of this research is that it uses Vasicek financial model for the first time as a tool in the rate of return evaluation of a production flow system. Our previous studies showed that synchronization generally increased the rate of return [13]. In an actual production setting, the rate of return exhibits an average regression behavior. The average regression obeys a normal logarithm-type stochastic partial differential equation and can be evaluated at the termination time.

Before discussing symmetry, we will focus on closed systems of production flow system. There is research related to an actual rate of return data using an electrical circuit theory known as multimode vibration theory. We demonstrate that the factor causing reductions in production is a rate of return variations of work [14]. Further, we introduce a potential field that corresponds to an electromagnetic field for analyzing the production process and apply multimode vibration theory to the potential field [14]. There is research related to a multi-vibration mode, which is the theory formed in the field of mechanics in physics. In the multi-vibration problem, eigenvalue (natural frequency), damping, and eigenmode are important. Here, we assume a multi-vibration system that connects single system, with infinite degree of freedom. The most important item for us among these three is the eigenvalue (natural frequency). This eigenvalue (natural frequency) propagates to the connected circuit. This is applied to the production process. In other words, in the production process, the maximum throughput propagates to the stage where the rate of return (throughput) is linked, and it seems that the entire production process appears to be produced with the same throughput. Such a system is called a synchronization system. We clarify the nonlinearity of the production process that occurs due to the standard deviation (STD) of the workers. Within the regions of nonlinearity of the stochastic field of the production process, we clarify the stable conditions that maintain periodic solutions, for each density and nonlinear parameter. Moreover, we establish that the nonlinear model is represented by a van der Pol differential equation [15]. On the basis of actual data, we present that a planned production process, or increasing a rate of return due to variations in the production process, operates in a manner similar to that of phase transitions in physics.

In this paper, we present that the stable regions of nonlinearity of the production process correspond to regions of phase transition [16]. On the basis of actual rate of return data, using an electrical circuit theory known as multimode vibration theory, we demonstrate that the factor causing reductions in production is rate of return variations of work. We introduce a potential field that corresponds to an electromagnetic field for analyzing the production process and apply multimode vibration theory to the potential field. The present analysis has been conducted for increasing the rate of return.

First of all, in our present research, analysis is made focusing attention on linear diffusion partial differential equation. As a result, it is reported the relationship between a production density and a throughput (After this, throughput and rate of return are used in the same meaning). From the result of our previous study, we have reported that the production flow process of production system can be expressed by linear diffusion partial differential equation [6]. The previous research applying fluid mechanics that the trial production of a new concept vertical take-off and landing rotorcraft of flexible kite wing attached multicopter is very interesting [17].

Regarding research related to a classical many-particle system with an external control represented by a time-dependent extensive parameter in a Lagrangian, there is a report in which thermodynamic entropy of the system is uniquely characterized as the Noether invariant associated with a symmetry [11]. In this research, suppose a transformation is applied to the trajectory of a particle that follows a law of motion. If the transformed orbit follows the same law of motion as before, "the law of motion has symmetry with respect to the transformation". For example, rolling a ball on a flat floor without friction draws a trajectory called constant velocity linear motion. This time, if you roll it from a place different from the previous one, it will make a uniform linear motion again. This represents the symmetry of floor uniformity and is called "spatial translational symmetry". Momentum is conserved by Noether's theorem corresponding to this symmetry. In order to prove the temporal and spatial symmetry in the production process, the action integral is introduced according to the proof method in analytical mechanics. Namely, according to Noether's theorem related physics approach, such a production process is a conservation field, and the variation of the action integral is zero. From this, we confirm that the deterministic linear diffusion equation is useful as a mathematical model in the synchronous production method. From the evaluation of Testrun1, the effect of dispersion can be seen sparsely. The production of Testrun2 and Testrun3-1/3-2 is progressing within the range of work variations. In other words, it retains time propulsion symmetry. In addition, Testrun3-1/3-2 can obtain the same product according to space propulsion symmetry, even if it is carried out from different stages. By performing the data analysis, both of time propulsion symmetry and space propulsion symmetry related to conservation of energy become clear to exist. As a consequence of having performed such data analysis, it has become possible to establish a flexible production plan according to the law of spatial propulsion symmetry, even if the production is carried out from different stages.

The report of this paper can be summarized as follows.

1) The manufacturing process is most appropriately described using a diffusion equation.

2) We demonstrate that the factor causing reductions in production is a rate of return variations of work. Further, we introduce a potential field that corresponds to an electromagnetic field for analyzing the production process and apply multimode vibration theory to the potential field.
3) From the evaluation of Testrun1, the effect of dispersion can be seen sparsely. The production of Testrun2 and Testrun3-1/3-2 is progressing within the range of work variations. In other words, it retains time propulsion symmetry. In addition, Testrun3-1/3-2 can obtain the same product according to space propulsion symmetry, even if it is carried out from different stages.

2. Production Business of a Small-to-Midsize Firm.

2.1. Production systems in the production equipment industry. We refer to the production system in manufacturing equipment industry studied in this paper. This is not a special system, but "Make-to-order system with version control". Make-to-order system is a system which allows necessary manufacturing after taking orders from clients, resulting in "volatility" according to its delivery date and lead time. In addition, "volatility" occurs in lead time depending on the contents of make-to-order products (production equipment).

However, effective utilization of the production forecast information on the orders may suppress certain amount of "variation", but the complete suppression of variation will be difficult. In other words, "volatility" in monthly cash flow occurs and of course influences a rate of return in these companies. Production management systems, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, are called as "product number management system" and are widely used.

All productions are controlled with numbered products and instructions are given for each numbered product.

Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial numbers in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks.

Therefore, careful management of the lead time or production date may not suppress "volatility" in manufacturing (production).

The company in this study is the "supplier" in Figure 1 and "factory" here. Companies are under the assumption that there are N (numbers of) suppliers; however, this study deals with one company because no data is published for the rest of the companies ($N-1$).

2.2. Production flow system. A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow processes, which manufacture low volumes of a wide variety of products, are produced through several stages in the production process. In Figure 2, the processes consist of six stages. In each step S1-S6 of the manufacturing process, materials are being produced.

S1 to S6 perform work stages 1 to 6 on the production line in Figure 2. These represent S1-S6 in Tables 4, 6, 8 and 10 in Appendix A. K1-K9 in the tables represent 9 workers. Figure 2 represents a manufacturing process called a production flow system, which is a manufacturing method employed in the production of control equipment. The production flow system, which in this case has six stages, is commercialized by the production of material in steps S1-S6 of the manufacturing process.

The direction of the arrow represents the direction of the production flow. In this system, production materials are supplied from the inlet and the end product will be shipped from the outlet.

Assumption 2.1. *The production structure is nonlinear.*

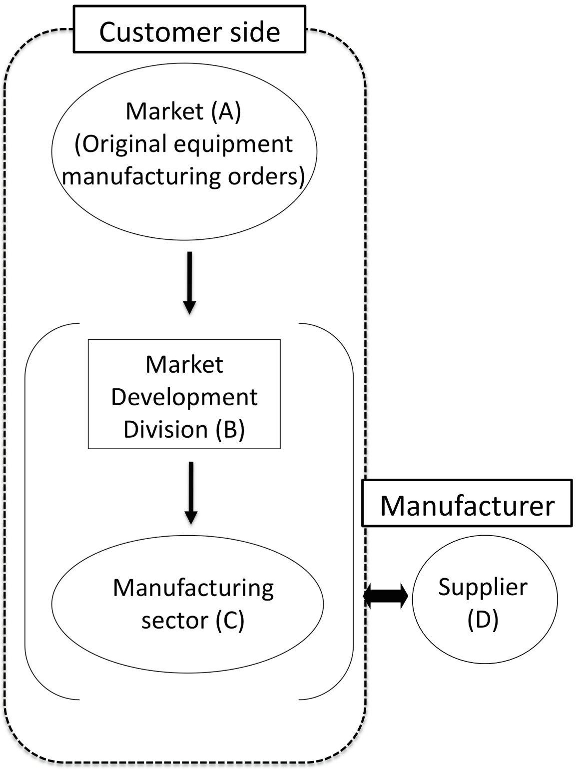


Figure 1. Business structure of company of research target

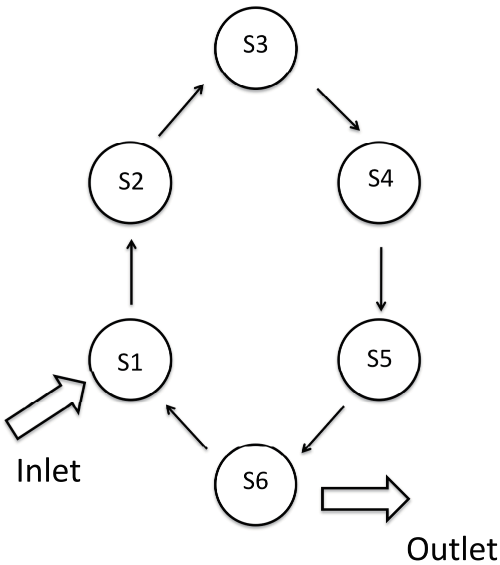


Figure 2. Production flow process

Assumption 2.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system).*

Assumption 2.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the rate of return generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the rate of return, its production structure is nonlinear. Therefore, Assumption 2.1 reflects the realistic production structure and is somewhat valid. Assumption 2.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 2.2 is reasonable because new production starts from S1. For a more detailed analysis, please refer to our Appendix A.

3. **Distribution System and Diffusion Equation of the Production Process.** From Figure 3, a model of the production process, which is connected in one dimension, is described as follows. The process of production is indicated by the movement of production units from one process (node) to another. This production flow is equivalent to transmission rate, which is defined as the rate of data flow between connected nodes in communication engineering. Accordingly, we formulate the production model in a manner similar to heat propagation in physics. Thus, the production process is modeled mathematically using a continuous diffusion type of partial differential equation consisting of time and spatial variables [6].

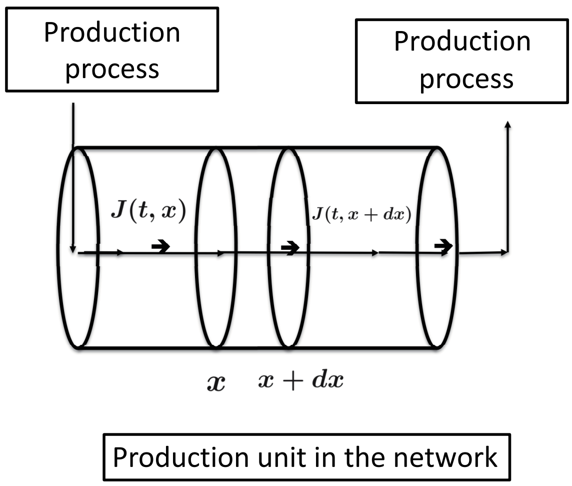


Figure 3. Network inter-process division of worker

Setting the network capacity (the available static production volume) to R in an inter-process network (production field, equivalent to a stochastic field), we obtain the following:

$$[J(t,x)dt - J(t,x+dx)dt]R = [S(t+dt,x) - S(t,x)]Rdx \tag{1}$$

where J is the production flow and S is the production density. $t \in [0,T]$, $x \in [0,L] \equiv \Omega$, $S(0,x) = S_0(x)$, $B_x S(t,x)|_{x=\partial\Omega}$. $B_x S(t,x)|_{x=\partial\Omega}$ indicates the boundary value.

In the present model, the production flow indicates the displacement of production processes in the direction related to the production density. In other words, the production density per production is as follows.

Definition 3.1. *Production density per unit production*

$$J = -D\frac{\partial S}{\partial x} \tag{2}$$

where D is a diffusion coefficient.

From Equation (1), we obtain

$$-\frac{\partial J}{\partial x} = \frac{\partial S}{\partial t} \tag{3}$$

From Equations (2) and (3), we obtain

$$\frac{\partial S(t,x)}{\partial t} = D\frac{\partial^2 S(t,x)}{\partial x^2} \tag{4}$$

where $t \in [0,T]$, $x \in [0,L] \equiv \Omega$, $S(0,x) = S_0(x)$, $B_x S(t,x)|_{x=\partial\Omega}$.

This equation is equivalent to the diffusion equation derived from the minimization condition of free energy in a production field [6, 7]. The connections between processes can be treated as a diffusive propagation of products (refer to Figure 3) [6, 8]. As shown in Figure 4, x represents the production elements that constitute a unit production and varies $x \to x'$ at $[t+dt]$. In other words, the unit production varies by exciting the external force and is the basis for revenue generation (an increase of potential energy). Therefore, in the transition $S(t,x) \to S\left(t', x'\right)$, the production density, which is the cumulated external force, increases. The connections between production processes are referred to as "joints".

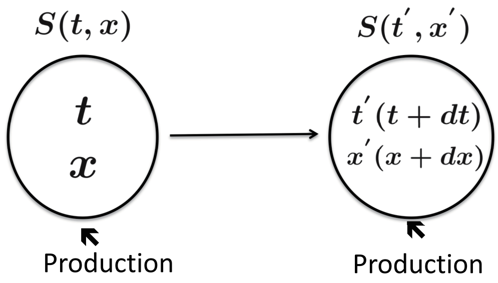


FIGURE 4. Unit of production by changing the excitation force

In the general idea of production flow, we define the joint propagation model at multiple stages in the production process and the potential energy in the production field.

Thereafter, we can construct a control system, which increases the rate of return, by calculating the gradient function in the autonomous distributed system. The gradient function is described in the next opportunity.

$$\frac{\partial S}{\partial t} + \Delta(v \cdot S) = \frac{1}{2}\Delta\left(D^2 S\right) + \lambda \tag{5}$$

where λ denotes a forced external force function and v denotes a production propagation speed. Here, λ is omitted here. Δ represents the Laplacian $\partial^2/\partial x^2$.

3.1. **Potential energy and rate of return of production process.** The description that deviation of free energy produces a rate of return will be made in this Section 3.1.

Assumption 3.1. *Rate of return is created by liquidity of production density function $S(t,x)$. From this, there exists a potential that depends on a production density function.*

Here, the size of potential $F(S(t,x))$ is attributed to inclination of a production density function related to a production unit, that is, liquidity. Therefore, the following equation is

Definition 3.2.

$$\frac{\partial F(S(t,x))}{\partial x} = -\kappa \times grad\ S(t,x) \tag{6}$$

where κ is a constant and "*grad*" is the gradient of production density $S(t,x)$.

$$F(S) = \int_0^L dx\left[f(S) + \frac{D}{2}(\nabla f(S))^2\right] \tag{7}$$

where L represents the production unit, $f(S)$ is the potential function of the variable S, and $(\nabla f(S))^2$ is the "fluctuation" of S [6, 19].

The structure of potential in production density function $S(t,x)$ will be examined [22, 23]. Potential in the present research is defined as "ability to create a return".

By such definition, meaning of Equation (6) has been made clear. In other words, it is considered that inclination related to a production unit of potential of production field $\{S(t,x)\}$ reduces in proportion to inclination related to a production unit of production density function $S(t,x)$, resulting in creating a return (it is considered as a difference between potentials). When considering like this, we define potential energy (free energy) in a production field as follows.

Definition 3.3. *Potential energy in production field*

[*Potential of production field per production density*]
= [*Potential for production unit*] + [*Fluctuation of potential for production unit*]

Such Definition 3.3 is almost equivalent to definition of the potential or free energy of a field in physics.

A transition to the dynamic state, which can be modeled by the Hamilton-Jacobi equation, requires excitation energy, which increases the free energy of the system [19]. To retain profitability in business, a continual input of products to the static field must be present. At the same time, order information must be supplied in the same manner. Figure 5 gives an overview of this production field concept [14]. The number of production units at each stage of a production unit i shifts over time. To function effectively, a production process requires a minimum number of personnel. This situation constitutes a shortest path problem. Production units can be considered to be physically located in mechanical fixtures. The production dynamics enable a company to profit from its business. We consider that revenues are generated by the displacement of the potential in the production field. In other words, the entropy increase contributed by the production unit is another source of revenue. This is the principle of maximum entropy [20].

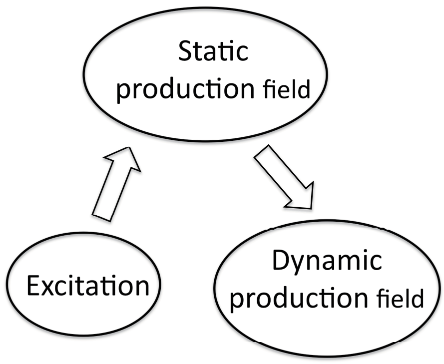


FIGURE 5. Overview of the production field concept

Figure 6 illustrates the transition from a lower-energy production process (energy state C) to the (higher-energy) next process (energy state C'). In Figure 6, the number of production units at each stage of a production unit shifts over time. To function effectively, a production process requires a minimum number of personnel. This situation constitutes a shortest path problem. The displacement of the potential in the production field generates a revenue. From the principle of maximum entropy, the entropy increase contributed by the production unit is another source of revenue [14]. We now derive the model equation that constrains the dynamic behavior of the production density. If the

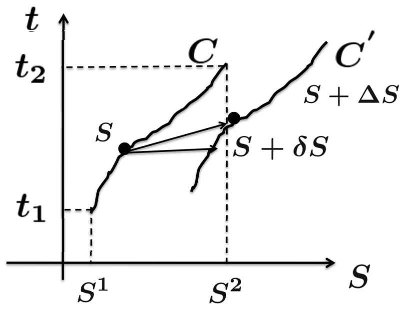


FIGURE 6. Transition from a lower-energy production process to the next process

production field sets $S(t, x)$, introducing sustainable order information and exciting the system with a sustainable target allow the process to progress from a static to a dynamic production field. The free energy of the process is increased by this transition [19]. Please refer to our previous research for more detailed analysis [10, 14].

3.2. **Stability evaluation of the production flow process using multimode vibration.** In this Section 3.2, we introduce a potential field that corresponds to an electromagnetic field for analyzing the production process and apply multimode vibration theory to the potential field. In Figure 7, the mathematical model of the production process is rewritten using the circuit equation; i.e., we obtain the following:

$$L_{12}\frac{di_{12}}{dt} = v_{11} - v_{22} \tag{8}$$

$$L_{23}\frac{di_{23}}{dt} = v_{22} - v_{33} \tag{9}$$

$$C_2\frac{dv_{22}}{dt} + \frac{1}{L_2}\int v_{22}dt + i_{22} = i_{12} - i_{23} \tag{10}$$

$$C_3\frac{dv_{33}}{dt} + \frac{1}{L_3}\int v_{33}dt + i_{33} = i_{23} - i_{34} \tag{11}$$

In the analysis of such simple coupling, Takase discusses the multimode vibration analysis in terms of average potential energy [21]. Kuramitsu and Nishikawa have certified that the structure is derived by the van der Pol equation [22, 23].

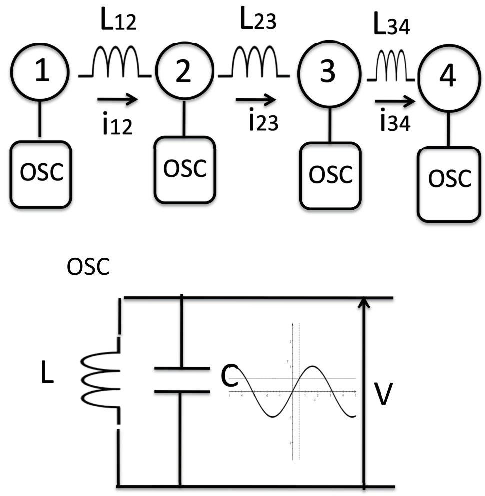


FIGURE 7. Circuit diagram

Moreover, the data gathered from the production flow process indicates that all stages correlate with each other. Thus, in general, according to the method proposed by Kuramitsu and Takase, Figure 8 indicates the production flow process using the circuit diagram of Figure 7 [22, 23]. Here, "OSC" in Figure 8 indicates the working-time delay at each stage in the production process.

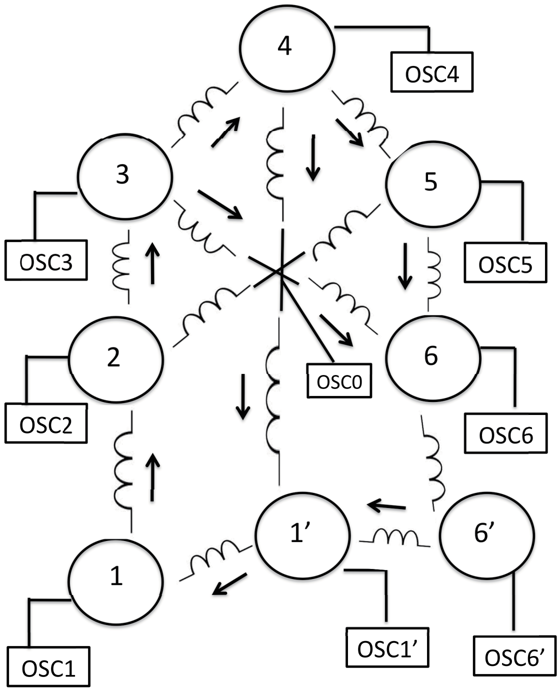


Figure 8. Production flow process modeled like an electrical circuit

If virtual stages "$1'$" and "$6'$" are added, we can apply an analytical method for the target system that describes the lattice-shaped oscillator group studied by Kuramitsu and Takase for biological phenomena [21, 22].

$$L_Y \frac{di_{Y_{i,j}}}{dt} = v_{i,j} - v_{i+1,j} \tag{12}$$

$$C\frac{dv_{i,j}}{dt} + \frac{1}{L_X}\int v_{i,j}dt + i_{i,j} = i_{X_{i,j-1}} - i_{X_{i,j}} + i_{Y_{i-1,j}} - i_{Y_{i,j}} \tag{13}$$

where L_X and L_Y are an inductance and C is a capacitor. $i_{i,j}$ is the current flowing through the coil L_k, and $v_{i,j}$ is the terminal voltage of capacity C.

Therefore, this solution is

$$S_{i,j}(t) = \sum_{l=1}^{3}\sum_{k=1}^{3} r_{lk}(t)_3P_{il} \cdot {}_3P_{jk} \sin\left(w_{il}t + \theta_{jk}(t)\right) \tag{14}$$

Any kind of combination mode [3×3] is a problem regardless of whether it is stable or not in multimode vibration. In the final product equipment, the issue is whether different stages synchronize or not because the synchronization process is the optimal (most appropriate) method.

In the case of coupling [e.g., $\left(1', \mathrm{OSC1}'\right)$, $\left(6', \mathrm{OSC6}'\right)$], combining the previous stage mode and coupling mode or combining the final process mode and coupling mode produces slightly different throughput depending on the stage position; i.e., we obtain the following

[22]:

$$\left({}_3P_{pC}\cdot{}_3P_{q1'}\right)^2+\left({}_3P_{pC}\cdot{}_3P_{q6'}\right)^2=1 \tag{15}$$

where $p=1,\ldots,6$, and $q=1,\ldots,6$.

One stage complements the other; thus, a stable multivibration mode can be established by minimizing the average potential energy.

Table 10 and Table 11 indicate the production times and volatilities at each stage of the production flow process, respectively; these tables indicate "synchronization with preprocess". If Equations (12), (13), and (14) are established, the synchronous vibration mode is possible.

The synchronous vibration of the production process can be obtained depending on the conditions of ${}_3P_{il}\cdot{}_3P_{jk}$. However, our purpose is not to analyze this, but to analyze Testrun data based on this idea. Therefore, it is necessary to confirm that the above theory can be applied. Section 4 will be referred to the purpose of production and the mechanism of production.

4. **The Relation between the Symmetry of the Production Field and the Synchronous Production Method.** This chapter 4 applies Noether's theorem to mentioning that a production flow system retains both of time propulsion symmetry and space propulsion symmetry conservation of energy.

According to Hamilton's principle in analytic mechanics, the actual movement of a material point draws the trajectory in which an amount called an action integral whose value is determined according to its trajectory becomes an extreme value. That is, the action integral (evaluation function) J in production process is defined as follows [18].

Definition 4.1. *Definition of Symmetry*

The Hamilton's principle is applied to the production process. That is, even if the production process is carried out at different time zones under the same initial conditions, the products obtained are the same (time propulsion symmetry). As another applicable item, the products obtained by producing with different production field are the same (spatial propulsion symmetry). Both of time and spatial propulsion symmetry are shown in Figure 9.

4.1. **Analysis for both of time and spatial propulsion symmetry existing by analytical mechanics.** According to Noether's theorem, the production flow system is equivalent to the Euler-Lagrange equation if the potential energy can be introduced [18]. In other words, energy is conserved in the mass system when the force does not depend on time. It means that the phenomenon occurs when the experiment is started under the same initial condition from any time (for example, one month later). From this, the law of conservation of energy expresses the property of the system (time propulsion symmetry) that "the motion does not change even if the origin of time is arbitrarily changed". The potential function at the production field is defined as follows.

Definition 4.2. *Lagrange equation $\mathcal{L}(S,h(t),t)$*

$$\mathcal{L}(S,h(t),t)\equiv\mathcal{F}(S)+\mathcal{G}(h(t)) \tag{16}$$

where $\mathcal{F}(S)$ denotes physical energy (potential with respect to $h(t)$), and $\mathcal{G}(h(t))$ denotes potential energy (production cost of improvement).

The rate of return ($h(t)$) is proportional to the production density. As an example, we consider the production density generated from technical proficiency. Technical proficiency includes improvement power. With regard to applications for Equation (16), please refer to our previous research [14].

Now, when the production factor x is transformed into $x^{'}$ $(= x + \delta x)$, its functional $\phi(x)$ is transformed into $\phi(x^{'})$ $(= \phi(x) + \delta\phi(x))$. That is, Lagrangian $\mathcal{L}(S, h(t), t)$ defines the action integral $J(S)$ as follows. The action integral $\mathcal{L}(S, h(t), t)$ represents the total sum of potentials in the production field.

Here, the action integral $J(S)$ is defined as follows.

Definition 4.3. *Action integral* $J(S)$

$$J(S) = \int_{t_i}^{t_f} dt\mathcal{L}(S, h(t), t) \tag{17}$$

For this production flow system, the time and production density are changed virtually as follows.

$$t^{'} = t + \Delta t \tag{18}$$

$$S^{'}\left(t^{'}\right) = S(t) + \Delta S(t) \tag{19}$$

The action integral $J^{'}\left(S^{'}\left(t^{'}\right)\right)$ which represents the potential, along the production processes after the virtual displacement is as follows.

$$J^{'}\left(S^{'}\left(t^{'}\right)\right) = \int_{t_i^{'}}^{t_f^{'}} dt\mathcal{L}\left(S^{'}\left(t^{'}\right), h^{'}\left(t^{'}\right), t^{'}\right) \tag{20}$$

We evaluate how much Equation (20) changes. Therefore, it is necessary to compare the rate of return $h^{'}\left(t^{'}\right)$ on the process after the virtual change with the original $h(t)$ in Equation (17). Therefore, we proceed the logic as follows [18].

$$\frac{d\delta S(t)}{dt} = \frac{d}{dt}\left(S^{'}(t) - S(t)\right) = \frac{dS^{'}(t)}{dt} - \frac{S(t)}{dt} = \delta\frac{dS(t)}{dt} \tag{21}$$

where $\delta S(t) \equiv S^{'}(t) - S(t)$.

Now, the following equation is obtained by executing the approximate calculation of $S^{'}(t^{'})$.

$$\begin{aligned}
S^{'}\left(t^{'}\right) &= S\left(t^{'}\right) + \delta S\left(t^{'}\right) = S(t + \Delta t) + \delta S(t + \Delta t) \\
&= S(t) + \Delta\frac{dS(t)}{dt} + O\left[(\Delta t)^2\right] + \delta S(t) + \Delta t\frac{d\delta S(t)}{dt} + O\left[(\Delta t)^2 \delta S(t)\right] \\
&= S(t) + \Delta t\frac{dS(t)}{dt} + \delta S(t) + O\left[(\Delta t)^2\right] + O\left[(\Delta t)^2 \delta S(t)\right] \\
&= S(t) + \Delta t\frac{dS(t)}{dt} + \delta S(t) + O[(2)]
\end{aligned} \tag{22}$$

where $(\Delta t)^2$ and $(\Delta t)\delta S(t)$ are represented by $O[(2)]$ together with the secondary small amounts.

Therefore, the following equation is obtained.

$$\Delta S(t) = \delta S(t) + \Delta t\frac{dS(t)}{dt} + O[(2)] \tag{23}$$

Further, the rate of return $dS^{'}\left(t^{'}\right)$ of production density $S(t)$ is obtained as follows.

$$\begin{aligned}
\frac{dS^{'}\left(t^{'}\right)}{dt^{'}} &= \left.\frac{dS^{'}(\tau)}{d\tau}\right|_{\tau=t^{'}} = \left.\frac{dS^{'}(\tau)}{d\tau}\right|_{\tau=t+\Delta t} = \left.\frac{dS^{'}(\tau)}{d\tau}\right|_{\tau=t} + \Delta t\left.\frac{d}{d\tau}\frac{dS^{'}(\tau)}{d\tau}\right|_{\tau=t} + O[(2)] \\
&= \frac{dS(t)}{dt} + \frac{d\delta S(t)}{dt} + \Delta t\frac{d^2 S(t)}{dt^2} + O[(2)]
\end{aligned} \tag{24}$$

The following equation is obtained from Equation (24).

$$\begin{aligned}\Delta\left(\frac{dS(t)}{dt}\right) &\equiv \frac{dS'\left(t'\right)}{dt'} - \frac{dS(t)}{dt} = \frac{d\delta S(t)}{dt} + \Delta\frac{d^2S(t)}{dt^2} + O[(2)] \\ &= \frac{d\left(\Delta S(t) - \dot{S}(t)\Delta t\right)}{dt} + \Delta t\frac{d^2S(t)}{dt^2} + O[(2)] \\ &= \frac{d\Delta S(t)}{dt} - \dot{S}(t)\frac{d\Delta t}{dt} + O[(2)] \end{aligned} \tag{25}$$

With the above preparation, the action integral $J'\left(S'\left(t'\right)\right)$ after the virtual displacement is rewritten by the following two methods.

$$\begin{aligned} J'\left(S'\left(t'\right)\right) &= \int_{t_i}^{t_f} dt\frac{dt'}{dt}\mathcal{L}\left(S'\left(t'\right), h'\left(t'\right), t'\right) \\ &= \int_{t_i}^{t_f} dt\left(1 + \frac{d\Delta t}{dt}\right)\mathcal{L}\left(S'\left(t'\right), h'\left(t'\right), t'\right) \\ &= \int_{t_i}^{t_f} dt\mathcal{L}\left(S'\left(t'\right), h'\left(t'\right), t'\right) + \int_{t_i}^{t_f} dt\frac{d\Delta t}{dt}\mathcal{L}(S,h,t) + O[(2)] \\ &= J(S) + \int_{t_i}^{t_f} dt\left[\Delta\mathcal{L}(S,h,t) + \frac{d\Delta t}{dt}\mathcal{L}(S,h,t)\right] + O[(2)] \end{aligned} \tag{26}$$

where $\Delta\mathcal{L}(S(t),h(t),t)$ denotes as follows.

$$\Delta\mathcal{L}(S(t),h(t),t) \equiv \mathcal{L}\left(S'\left(t'\right), h'\left(t'\right), t'\right) - \mathcal{L}(S(t),h(t),t) \tag{27}$$

With regard to the other method, $J'\left(S'\left(t'\right)\right)$ can be obtained by using another variable t instead of the integral variable t'.

$$\begin{aligned} J'\left(S'\left(t'\right)\right) &= \int_{t_i'}^{t_f'} dt\mathcal{L}\left(S'(t), h'(t), t\right) \\ &= J(S) - \int_{t_i}^{t_f} dt\mathcal{L}(S(t),h(t),t) + \int_{t_f}^{t_f'} dt\mathcal{L}\left(S'(t), h'(t), t\right) \\ &\quad + \int_{t_i}^{t_f} dt\mathcal{L}\left(S'(t), h'(t), t\right) + \int_{t_i'}^{t_i} dt\mathcal{L}\left(S'(t), h'(t), t\right) \\ &= J(S) + \int_{t_i}^{t_f} dt\left[\mathcal{L}\left(S'(t), h'(t), t\right) - \mathcal{L}(S(t),h(t),t)\right] \\ &\quad + \Delta t_f\mathcal{L}(S(t),h(t),t)\Big|_{t=t_f} - \Delta t_i\mathcal{L}(S(t),h(t),t)\Big|_{t=t_i} + O[(2)] \\ &= J(S) + \int_{t_i}^{t_f} dt\left[\delta S(t)\frac{\partial\mathcal{L}(S(t),h(t),t)}{\partial S} + \frac{d\delta S}{dt}\frac{\partial\mathcal{L}(S(t),h(t),t)}{\partial\dot{S}}\right] \\ &\quad [\Delta t\mathcal{L}(S(t),h(t),t)]_{t_i}^{t_f} + O[(2)] \\ &= J(S) + \int_{t_i}^{t_f} dt\delta S(t)\frac{\delta J(S)}{\delta S(t)} + \left[\Delta t\mathcal{L}(S(t),h(t),t) + \delta S\frac{\partial\mathcal{L}(S(t),h(t),t)}{\partial\dot{S}}\right]_{t_i}^{t_f} \\ &\quad + O[(2)] \end{aligned} \tag{28}$$

where $\delta J(S)/\delta S(t)$ is derived as follows.

$$\frac{\delta J(S)}{\delta S(t)} \equiv \frac{\partial \mathcal{L}(S(t),h(t),t)}{\partial S(t)} - \frac{\partial \mathcal{L}(S(t),h(t),t)}{\partial \dot{S}(t)} \tag{29}$$

Therefore, $\delta J(S)/\delta S(t) = 0$ gives the production equation.

From Equations (26) and (28), $\Delta J(S)$ holds for an arbitrary virtual displacement given by the set of two equations (18) and (19).

$$\begin{aligned} \Delta J(S) &\equiv J'\left(S'\right) - J(S) \\ &= \int_{t_i}^{t_f} dt \left[\Delta\mathcal{L}(S(t),h(t),t) + \frac{d\Delta t}{dt} + \mathcal{L}(S(t),h(t),t)\right] + O[(2)] \\ &= \int_{t_i}^{t_f} dt\delta S(t)\frac{\delta J(S)}{\delta S(t)} + \left[\Delta t\mathcal{L}(S(t),h(t),t) + \delta S\frac{\partial \mathcal{L}(S(t),h(t),t)}{\partial \dot{S}}\right]_{t_i}^{t_f} + O[(2)] \end{aligned} \tag{30}$$

From the latter term of Equation (30), we obtain the following equation where the production equation holds.

$$\Delta J(S) = -[N(t_f) - N(t_i)] + O[(2)] \tag{31}$$

$$\begin{aligned} N(t) &\equiv -\Delta t\mathcal{L}(S(t),h(t),t) - \delta S\frac{\partial \mathcal{L}(S(t),h(t),t)}{\partial \dot{S}} \\ &= \Delta t\left(\dot{S}\frac{\partial \mathcal{L}}{\partial \dot{S}} - \mathcal{L}\right) - \Delta S\frac{\partial \mathcal{L}}{\partial \dot{S}} \end{aligned} \tag{32}$$

$N(t)$ in Equation (32) is called Noether current. From Equation (31), $\Delta N(t) = constant$ is derived for the virtual displacement where $\Delta J(S) = 0$. That is, there is a storage amount $N(t)$.

4.2. Temporal and spatial symmetry for a production flow process.

Definition 4.4. *Synchronous process*

1) *The action integral J on the potential of the product remains invariant. That is, $\delta J = 0$. Alternatively, Lagrangian has symmetry.*
2) *When the condition of item 1) is satisfied, it corresponds to (a) of Figure 9 for generalized coordinates (S, h) in the product.*

- Energy conservation (time propulsion symmetry). The action integral J on the potential of the product remains invariant. That is, $\delta J = 0$. Alternatively, Lagrangian has symmetry.

  $$\Delta t = \epsilon \ (constant), \ \Delta S(t) = 0 \tag{33}$$

  Equation (32) indicates Hamiltonian which is derived as follows, that is, the time propulsion symmetry refers to the energy conservation law.

  $$H \equiv \dot{S}\frac{\partial \mathcal{L}}{\partial \dot{S}} - \mathcal{L} \tag{34}$$

- Conservation of momentum (spatial propulsion symmetry)

  $$\Delta t = 0, \ \Delta S(t) = \epsilon \ (constant) \tag{35}$$

  $$p \equiv \frac{\partial \mathcal{L}}{\partial \dot{S}} \tag{36}$$

  That is, the momentum is saved from Equation (36).

Figure 9 shows both of the time propulsion symmetry and the spatial propulsion symmetry. In Figure 9, when the production density is displaced like $S' = S + \delta S$, or when the time is displaced like $t' = t + \delta t$ (displacement of rate of return), the displacement of the nominal orbit in the production field means that "conserved amount" in the independent production field where it is the displacement trajectories of production by displacements of t or S. In other words, the fact that the displacement trajectory of the production field due to t' and S' is independent of the nominal trajectory $J(S)$ of production means the conserved amount for small displacement due to the intrinsic potential of the production field. There is a solution orbit with a maximum or a minimum as shown in Figure 10 in the intrinsic potential of the production field.

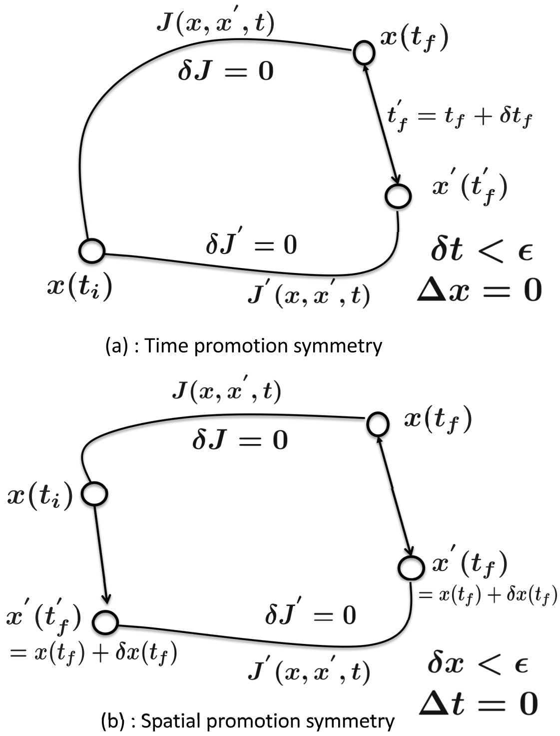
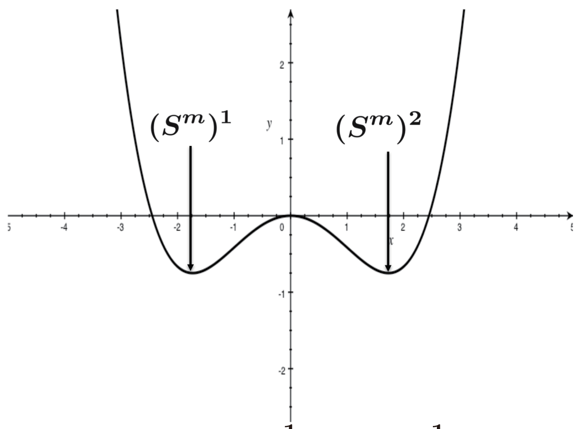


FIGURE 9. Both of time and spatial propulsion symmetry



$$f(S^m) = -\frac{1}{2}(S^m)^2 + \frac{1}{12}(S^m)^4$$

FIGURE 10. Non-production and production are the coexistence.

Production may be delayed due to small displacement of production factors, such as worker's ability, lack of manufactured parts or delay in logistics. In such a situation, the same phenomenon occurs, even in different time zones. On the contrary, there is also the opposite case. In other words, there is a conservation law in production that Noether's theorem has mentioned.

- Purpose of production
  → (1) The simultaneous progress of multiple processes synchronizes and shortens the production lead time.
  (2) A plurality of products are completed by the production flow system.
- Mechanism of production
  → (1) The completion of each product is performed through a plurality of stages.
  (2) The rate of return (lead time) is set for each stage. Multiple workers are used for manufacturing.
  (3) According to the production rule, the work is completed within the set lead time and the process moves to the next process, but the process does not move to the next process until the process lead time (work time) ends.
  (4) The rules regarding workers do not overtake the work of the back end process under any circumstances.

The production flow system was executed under the above purpose, the mechanism of production and rules. In the following, "data" means the measured data by Testrun.

When the rate of return at each production stage matches, it indicates that the synchronization phenomenon is occurring. Therefore, we will carry out an experiment on the actual production line (Testrun) to confirm this synchronization phenomenon.

In Table 8 and Table 10, Testrun3-1/Testrun3-2 indicate a best value for the rate of return in the three types of theoretical working time. Testrun2 is ideal production method. However, because it is difficult for talented worker, Testrun3-1/Testrun3-2 are a realistic method.

Due to the symmetry under the conservation in the production field, the process progress (momentum) in each production process looks exactly the same, indicating that it is indistinguishable. In other words, the synchronization of the processes is progressing in the production system. "Synchronization with preprocess" in Testrun3-2 shown by Figure 11 is a manufacturing method used to increase a rate of return. Because synchronization reduces volatility from the start of production until it finishes, it is the best method available. The conservation in the production field means maintaining symmetry. The symmetry is that, under the same initial conditions, the same phenomenon occurs even if the process moves at different times.

5. **Numerical Analysis of Testrun Data.** We were able to construct the production flow system with $\delta J < \epsilon$ in the following Testrun data 2 and 3-1/3-2 results. This means that the displacement between the "planned potential (action integral)" and the "potential due to the displacement of the throughput executed in the actual process" is extremely small. In other words, it means that there was a conserved amount of potential which means that the production proceeds due to the energy supplied from the outside and the stored entropy increase.

5.1. **Analysis of Testrun1/2/3-1/3-2.** In Testrun1 (asynchronous method), the lead time was set to "WS (working standard)", as shown in Table 4 of Appendix A. When the lead time is small, the WS imposes a strong connection between the stages (S1 to S6). The lead time correlates with the total drift (equal throughput = 0.73). In Testrun2 (synchronous method), where the lead time is large, WS in Table 6 imposes a weak

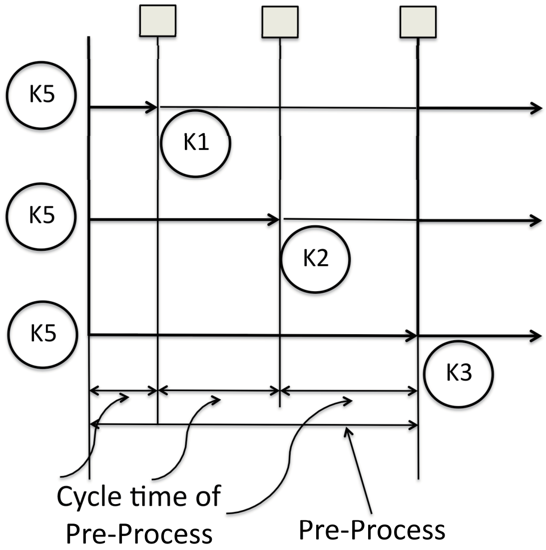


FIGURE 11. Previous process in production equipment

coupling between the stages (S1 to S6). This lead time also correlates with the total drift (equal throughput = 0.92). The total drift values depict the total working time of nine workers (K1-K9) at each stage (S1-S6) (see Tables 4, 6, 8 and 10 in Appendix A).

"(S1)-(S6)" of Tables 4 and 6 in Figures 15 and 16 represent the situation of time-propelled symmetry and spatial propulsion symmetry. Here, "(S3)" of Table 4 in Figure 15 is only affected by the volatility and we have also added "*" to Table 2.

5.2. **Identification of asynchronous and synchronous processes.** Here we examine the relative advantages of the three asynchronous and synchronous patterns Testrun1, Testrun2, and Testrun3-2 by using the multimode vibration theory to determine a matrix. The matrix is derived as follows: The matrix $\boldsymbol{A}$, which is a condition for coexistence with multimode vibration, is given as follows:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix} \tag{37}$$

Equation (37) is a symmetric matrix of $[N \times N]$, (38). However, we use a skew-symmetric matrix of $[6 \times 9]$ in this paper.

$$\boldsymbol{A}' = \begin{bmatrix} a_{11} & \cdots & a_{16} \\ \vdots & \ddots & \vdots \\ a_{91} & \cdots & a_{96} \end{bmatrix} \tag{38}$$

The matrix $\boldsymbol{A}'$, which is a condition for coexistence with multimode vibration, is given as follows:

$$\boldsymbol{A}' = \begin{bmatrix} a_{ll} & a_{l'} \\ a_{l'l} & a_{l'l'} \end{bmatrix} \tag{39}$$

If $\boldsymbol{A}' > 0$, it is well known that the two vibration modes l, l' coexist [22].

Next, for the matrix of (39), we count the number of instances of (1) $\boldsymbol{A}' < 0$, (2) $\boldsymbol{A}' = 0$, and (3) $\boldsymbol{A}' > 0$ for the matrix of (39) using the measurement data retrieved from Table 4, Table 6, and Table 10.

Figure 12 uses the manpower data of Table 4 to produce a diagram for the evaluation of the positive, negative, and zero matrices using any four data values. For example, we pick up the first two values from the first row and the first two values from the second row of Figure 12.

$$\boldsymbol{A}' = \begin{bmatrix} 20 & 20 \\ 22 & 21 \end{bmatrix} \tag{40}$$

By calculating the above matrix, $\boldsymbol{A}' = 20 \times 21 - 20 \times 22 = -20 < 0$. The value is changed in the positive or negative direction from the standard in the asynchronous process. The greater the change is, the greater the process becomes asynchronous. This calculation is shown as Table 1. In our Testruns, Testrun1 is an asynchronous process, Testrun2 is synchronous process, Testrun3-2 is the synchronous-with-preprocess, i.e., the measurement data in Table 4 indicates the data for an asynchronous case and it takes the time to work. With regard to Testrun1 through 3-2, the ranking of throughput is Testrun1 $< 2 <$ 3-2. We should explore the appropriate throughput based on the synchronization-with-preprocess case.

20	20	25	20	20	20
22	21	22	21	19	20
20	26	25	22	22	26
17	15	19	18	16	18
15	20	18	16	15	15
15	15	15	15	15	15
20	20	30	20	21	20
29	33	30	29	32	33
14	14	15	14	14	14

FIGURE 12. Matrix of production process data provided in Table 4

18	19	18	20	20	20
18	18	18	20	20	20
21	21	21	20	20	20
13	11	11	20	20	20
16	16	17	20	20	20
18	18	18	20	20	20
14	14	13	20	20	20
22	22	20	20	20	20
25	25	25	20	20	20

FIGURE 13. Matrix of production process data provided in Table 6

18	19	18	18	18	18
18	18	18	18	18	18
21	21	21	21	21	21
13	11	11	13	13	13
*16	*16	*17	*17	*16	*16
18	18	18	18	18	18
14	14	13	14	14	13
22	22	22	22	22	22
20	20	20	20	20	20

*: Previous process

FIGURE 14. Matrix of production process data provided in Table 10

TABLE 1. Number of (a) $\boldsymbol{A}' > 0$, (b) $\boldsymbol{A}' = 0$, (c) $\boldsymbol{A}' < 0$

	(1) Testrun1	(2) Testrun2	(3) Testrun3-2
(a) $\boldsymbol{A}' > 0$	5	2	2
(b) $\boldsymbol{A}' = 0$	$\boxed{1}$	$\boxed{7}$	$\boxed{8}$
(c) $\boldsymbol{A}' < 0$	6	3	2

5.3. **Evaluation from Testrun1 through Testrun3-2.** Evaluating this result, Testrun1 shows no sync phenomenon. The synchronization phenomenon appears in Testrun2. Testrun3-1/3-2 show that the methods in Tables 8 and 10 are synchronized in the latter half (S4, S5, S6). Further Table 10 in the method of "The shift throughput method in which the set lead time is changed in the middle (workers K4, K5, K6)" is adopted. As a result, the dispersion is distributed in each process and the deviation of the total production time is about the same between stages.

The initial condition is different in the data of Tables 8 and 10; however, the stages $\{S_1\}$, $\{S_2\}$, $\{S_3\}$ represent similar work times. In other words, the stages $\{S_1\}$, $\{S_2\}$, $\{S_3\}$ can be regarded as spatial propulsion symmetry. Namely, the mathematical model for the production density of all Testruns can be expressed as the following equation. Generally, it is represented by the Riemann model (dual flat structure) that we reported [24].

$$\frac{\partial S(t,x)}{\partial t} + v\frac{\partial S(t,x)}{\partial x} = D\frac{\partial^2 S(t,x)}{\partial^2 x^2} \tag{41}$$

where Equation (41) represents a propagation equation with respect to t, x, and v is the parameter of the propagation path between each stage in the process. D is the coefficient.

6. **Conclusion.** We confirmed the linear diffusion equation as a mathematical model of the production process under study. From the evaluation of Testrun1, there are many variations in work. The production of Testrun2 and Testrun3-1/3-2 is progressing within the range of work variations. In other words, it retains time-driven symmetry. In addition, Testrun3-1/3-2 can obtain the same product even if it is carried out from different stages.

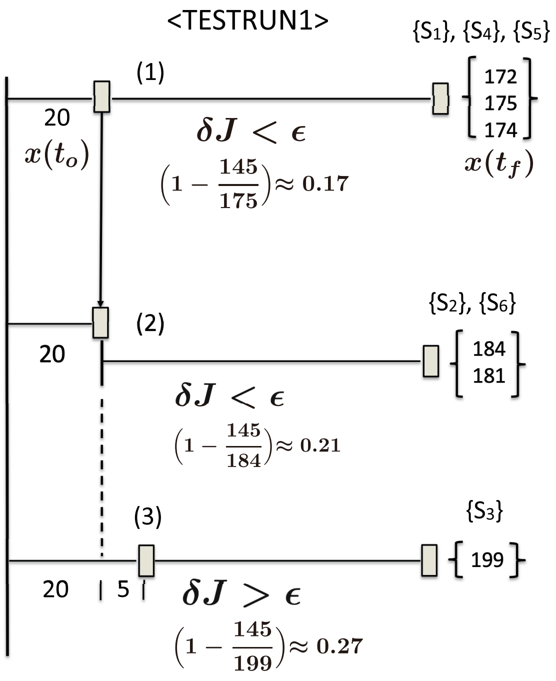


FIGURE 15. Mechanics Testrun1

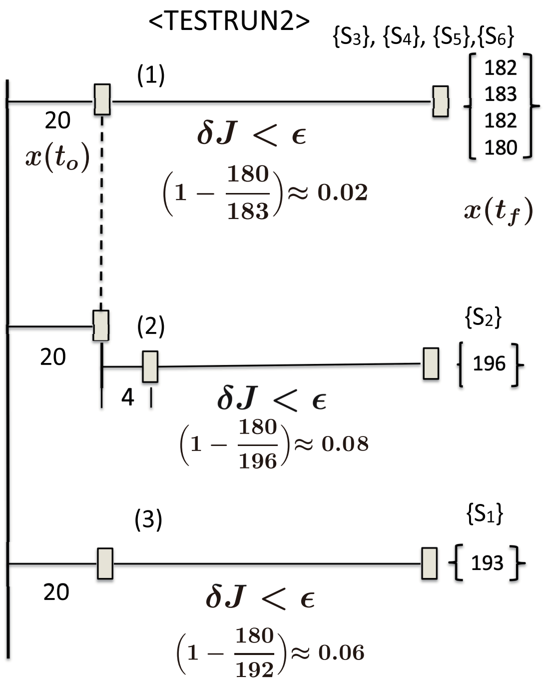


FIGURE 16. Mechanics Testrun2

In other words, it retains the spatial propulsion symmetry of $\{S_1\}$, $\{S_2\}$, $\{S_3\}$ in Testrun3-1/3-2.

Namely, according to Noeta's theorem, such a production process is a conservation field, and the variation of the action integral is zero. From this, we confirmed that the deterministic linear diffusion equation is useful as a mathematical model in the synchronous production method.

Table 2. Symmetry verification result of each process from Testrun data

Testrun	Time-propelled symmetry	Spatial propulsion symmetry
Testrun1	$\{S_1\}$, $\{S_4\}$, $\{S_5\}$	
Testrun1	$\{S_2\}$, $\{S_3\}^*$, $\{S_6\}$	
Testrun2	$\{S_3\}$, $\{S_4\}$, $\{S_5\}$, $\{S_6\}$	
Testrun2	$\{S_1\}$, $\{S_2\}$	
Testrun3-1	$\{S_1\}$, $\{S_2\}$, $\{S_3\}$	$\{S_1\}$, $\{S_2\}$, $\{S_3\}$
Testrun3-1	$\{S_4\}$, $\{S_5\}$, $\{S_6\}$	
Testrun3-2	$\{S_1\}$, $\{S_2\}$, $\{S_3\}$	$\{S_1\}$, $\{S_2\}$, $\{S_3\}$
Testrun3-2	$\{S_4\}$, $\{S_5\}$, $\{S_6\}$	

$\{S_3\}^*$ in Testrun1, the variance value is large due to the lack of worker capacity.

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Appendix A. **Analysis of Actual Data in the Production Flow System.** Based on the control equipment, the product can be manufactured in one cycle. The rate of return required to maintain 6 pieces of equipment/day is as follows.

- (Testrun1): Because the throughput of each process (S1-S6) is asynchronous, the overall process throughput is asynchronous. In Table 4, we list the manufacturing time (min) of each process. In Table 5, we list the volatility in each process performed by the workers. Finally, Table 3 lists the target times. The theoretical throughput is obtained as $3 \times 199 + 2 \times 15 = 627$ (min). In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. In Figure 17, we plot the measurement data listed in Table 4, which represents the total working time of each worker (K1-K9). In Figure 18, we plot the data contained in Table 4, which represents the volatility of the working times.
- (Testrun2): Set to synchronously process the throughput. The target time listed in Table 6 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 7 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Testrun3-1): Introducing a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 8, the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 9 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Testrun3-2): Same as Testrun3-1.

  On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput ($T_s^{'}$) can be obtained using the "Synchronization with preprocess" method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20 \times 6\,(\text{First cycle}) + 17 \times 6\,(\text{Second cycle}) \\ &\quad + 20 \times 6\,(\text{Third cycle}) + 20\,(\text{Previous process}) + 8\,(\text{Idle-time}) \\ &\sim 370 \text{ (min)} \end{aligned} \tag{42}$$

  The full synchronous throughput in one stage 20 (min) is

$$T_s^{'} = 3 \times 120 + 40 = 400 \text{ (min)} \tag{43}$$

  Using the "Synchronization with preprocess" method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed "Synchronization with preprocess" method is realistic and can be applied in flow production systems. Below, we represent for a description of the "Synchronization with preprocess".

In Table 10, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time. Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{44}$$

where the throughput of the preprocess is set to 20 (min). In Equation (44), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) × 3 (cycles).

Table 3. Correspondence between the table labels and the Testrun number

	Table number	Production process	Working time	Volatility
Testrun1	Table 4	Asynchronous process	627 (min)	0.29
Testrun2	Table 6	Synchronous process	500 (min)	0.06
Testrun3-1	Table 8	Synchronous process	470 (min)	0.03
Testrun3-2	Table 10	"Synchronization with preprocess" method	470 (min)	0.03

In Table 8 and Table 10, Testrun3-1/Testrun3-2 indicate a best value for the throughput in the three types of theoretical working time. Testrun2 is ideal production method. However, because it is difficult for talented worker, Testrun3-1/Testrun3-2 are a realistic method.

The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.

Testrun1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73,
Testrun2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92,
Testrun3-1 and Testrun3-2: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95.
Volatility data represent the average value of each Testrun.

Table 4. Testrun1

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 5. Volatility of Table 4

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

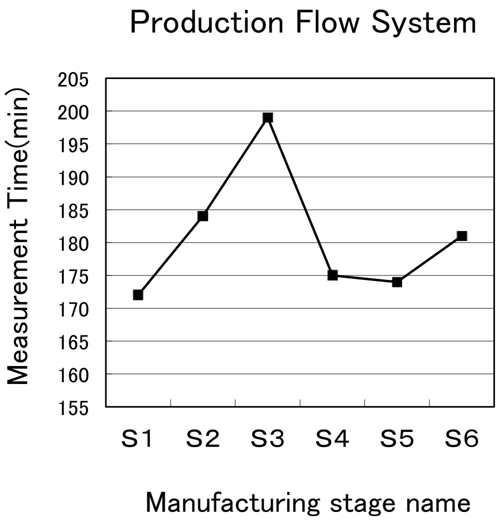

FIGURE 17. Total work time for each stage (S1-S6) in Table 4

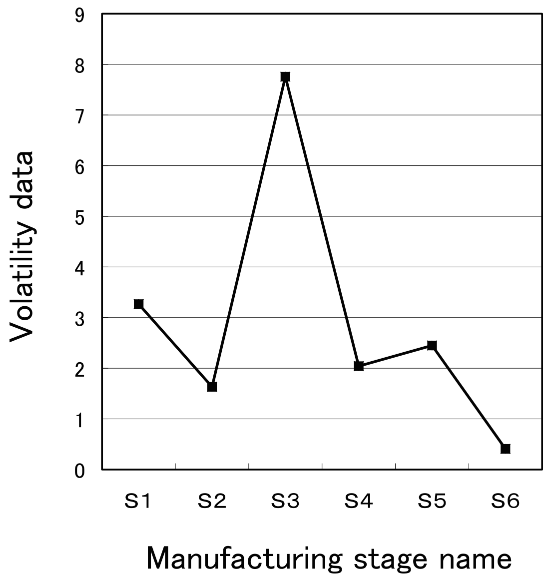

FIGURE 18. Volatility data for each stage (S1-S6) in Table 4

TABLE 6. Testrun2

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

TABLE 7. Volatility of Table 6

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

Table 8. Testrun3-1

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180

Table 9. Variance of Table 8

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.33	3	3	0	0	0
K5	1.33	1.33	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.33	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0

Table 10. Testrun3-2

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	*16	*16	*16	*17	*17	*16	*16
K6	16	18	18	18	18	18	18
K7	20	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	25	25	25	25	25	25
Total	168	165	164	163	166	165	164

Table 11. Volatility of Table 10 (K5(*): Preprocess)

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*0	*0	*0.33	*0.33	*0	*0
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	2	2	2.33	2	2	2.33
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	1.67	1.67	1.67	1.67	1.67	1.67