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# ITERATIVE IDENTIFICATION AND MODEL PREDICTIVE CONTROL: THEORY AND APPLICATION

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Abstract. *Although system identification and model predictive control are two different research fields separately, one gap exists between these two subjects. To alleviate this gap between them, system identification and model predictive control are combined to be one iterative identification and model predictive control strategy in this paper, where the process of system identification and model predictive control will be carried out iteratively many times until convergence. Based on some priori information about the asymptotic analysis for predictor error identification, variance analysis corresponding to closed loop output response is derived to show the tracking performance for this proposed iterative strategy. From this derived variance analysis, some factors can be chosen to guarantee the perfect tracking performance, such as input spectrum, and noise filter. Furthermore, to extend this iterative strategy to more general cases in industry, one model predictive control based reference governor is studied to provide proportional-integral-differential (PID) controller. Finally, several simulation experiments about flight control for helicopter have been performed to demonstrate the effectiveness of our proposed theories.*
**Keywords:** System identification, Model predictive control, Variance analysis, Reference governor

1. **Introduction.** Inferring a dynamic model based on first principle laws of physics, biology, chemistry, economic, etc., requires detailed process knowledge from specialists, which might be even impossible to obtain if the required knowledge of first principles is missing. Such a modeling may also result in a highly complex mathematical description of the considered system with the need to perform dedicated experiments to estimate the model coefficients. An efficient alternative is to construct a mathematical model of the dynamic system on the basis of experimentally measured data. Such an approach is known as data driven modeling or so called system identification. Building models from observations and studying their properties is really what science is about. The models may be of more or less formal character, but they have the basic feature that they attempt to link observations together into some pattern. System identification is a well developed technology for estimating plant models from operational data, typically taken during dedicated plant testing/excitation. Data driven estimation and maintenance of dynamic models is considered as a key technology for realizing a higher level of autonomy of model based controllers, when maintaining economic optimal operation of the considered plant.

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As the goal of system identification strategy is to build a mathematical model of a dynamic system based on some initial information about the considered system and the measured data, collected from the experiment. The detailed processes of the system identification strategy consist of designing and conducting one identification experiment in order to collect the measurement data, selecting the structure of the dynamic system and specifying the unknown parameters to be identified and eventually fitting the model parameters to the obtained data. As a consequence, the quality of the obtained model is evaluated through the model validation procedure. Generally roughly speaking, system identification strategy is an iterative process and if the quality of the obtained model is not satisfactory, some or all of the listed phases can be repeated to yield one satisfied model. As system identification theory is sometimes considered mature field, with a wide and solid literature, then many tools or methods for system identification, as well as for control theory, stability analysis, have emerged in recent years. So in this present paper, the system identification strategy is combined or applied in model predictive control (MPC). Model predictive control has developed considerably over the past two decades, both within the research control community and in industry. This success can be attributed to the fact that model predictive control is perhaps the most general way of posing control problem in time domain. Model predictive control formulation integrates optimal control, stochastic control, control of processes with dead time, multi-variable control and future references when available. One important advantage of model predictive control is that because of the finite control horizon used, constraints and, in general nonlinear processes which are frequently found in industry, can be handled. The rationale behind model predictive control is the following: at each time step, an L2 or other alternative variation of the cost function is locally optimized over time to design the open loop controller as a function of time, only a small portion of which is actually applied to the considered system. Then the time horizon is shifted, and the process is repeated at a latter time step based on state feedback. Although model predictive control has been found to be quite a robust type of control in most reported applications, some new and very promising results allow one to think that this control technique will experience greater expansion within this community in near future. However, although lots of applications have been reported in both industry and research institutions, model predictive control has not yet reached in industry the popular.

However, the main problem on how to combine system identification strategy into model predictive control must be explained in detail. As it is a fact that the most important element in model predictive control is the prediction of the output value. After deriving the prediction of the output value by classical prediction error method and substituting it into the considered cost function, we take the derivative of the cost function with respect to the input value to obtain one optimal input. However, the problem of yielding the prediction of the output value is dependent of external noise, which is always assumed to be independent and identically distributed white noise. Due to the fact that white noise is an ideal case, it does not exist in engineering and in additional, deriving statistical properties of noise is often very difficult in practice, as it is usually not possible to measure the external noise directly. As linear system identification is a mature field, then the most common classical approaches can be used to perform the prediction of the output value, such as prediction error method, and maximum likelihood method, in case of the linear dynamic system. Although there is always a separation between system identification and model predictive control design, an alternative idea to obtain the output estimation for model predictive control is to regard the system identification process as a procedure to be designed by bearing the final control application. Such a rationale is known as identification for control, and has been addressed as model based control. It means firstly system identification strategy is used to derive the output estimation, and then the output estimation is substituted in the cost function for model predictive control. On the contrary to the model based control, the data driven control is widely studied in recent years, for example, in [1], the existing data driven control approaches are divided into on line and off line ones. In on line schemes, the controller is adjusted with each new data in closed loop environment. Examples of on line direct data driven control techniques are given for nonlinear system in [2]. The main advantage of on line data driven control is the ability to improve the control performance using the measured data; furthermore, machine learning and reinforcement learning are applied in data driven control technique to obtain some different optimal p-steps ahead prediction models [3]. However, lots of measured data are needed in data driven control and in analyzing control performance the number of measured data is always assumed to be infinity.

This assumption about the number of measured data is not realistic in engineering or industry, as in reality the number of measured data is finite, so the model based control approach is always used in engineering. Before designing the controllers in whatever open loop or closed loop system, system identification strategy is needed to identify the mathematical model for the considered plant firstly, and it means the obtained mathematical model about the considered plant is a basis for the next control step. A robust adaptive model predictive control approach for asymptotically stable, constrained linear time varying systems with multiple inputs and outputs is studied in [4], where the unknown but bounded noise is dealt with by set membership identification. For discrete time linear invariant systems with constraints on inputs and states, an algorithm to determine explicitly the state feedback control law is developed in [5], where the control law is piecewise linear and continuous for both the finite horizon problem. A method based on conceptual tools of predictive control is described for solving set-point tracking problems wherein point wise in time input and state inequality constraints are presented in [6]. A learning model predictive control for iterative task is presented in [7], where the controller is reference force and is able to improve its performance by learning from previous iterations. The concept of learning model predictive control means the machine learning strategy is applied to obtaining the mathematical model for the next model predictive control. [9] studies symmetry in linear model predictive control, and some properties of both model predictive control symmetries are also studied by using a group theory formalism. After a mathematical description of the considered plant is provided, a tube based model predictive control approach is a powerful tool for control design in case of unknown systems, such as unknown noise and un-modeled terms [10], where the robustness against unstructural uncertainty is also analyzed in detail. Recently the author studies model predictive control based on zonotope parameter identification, i.e., the unknown parameters from the mathematical model for the considered plant are identified by set membership identification [11], where the unknown and bounded noise is known to be in one priori zonotope. Generally model based control approach is divided as two steps, one is system identification for the considered plant, and the other step is control design for the optimal input.

From our above descriptions, after modeling the considered plant by system identification, then the process of system identification is finished and the process of control design is started. And the control performance depends on the mathematical model for the considered plant closely, i.e., the accuracy of the mathematical model will affect the latter control performance. To relax this dependence, in this paper, we not only introduce system identification theory into the popular model predictive control strategy, but also propose iterative identification and model predictive control approach. The iteration means during the whole model predictive control, the process of system identification will not run at only once, i.e., the processes of system identification and model predictive control will be carried out iteratively many times. This presently popular iteration identification and control is proposed in [12], which considers the direct minimization of both a one step and a multi step minimum variance regulation criterion. However, not any other reference on iterative identification and control is referred except that reference. In our studying model predictive control, we find that the most element of model predictive control is obtain the output estimation, used in the cost function. After system identification strategy is used to identify the output estimation, the accuracy of this output estimation will not achieve our desired goal, so this poor accuracy will affect the latter control performance. Then idea of iterative identification and model predictive control is needed to alleviate this dependence. It tells us the processes of system identification and model predictive control exist simultaneously, and each of them will not be terminated until the control performance is achieved to the priori given index, such as tracking performance, unbiased, and minimum variance. After iterative identification and model predictive control approach is proposed in detail, the variance analysis of the output response is derived to measure the quality of control performance. From our derived variance expression, we can observe how the designed controller and input spectrum determine the control performance. To extend this proposed iterative identification and model predictive control strategy, a concept of reference governor based on model predictive control is formulated to provide PID controllers [13], which is widely used in industry to modify the user given reference, such that the quality and safety of a closed loop system is greatly improved. Generally the contributions of this paper are formulated as follows. Due to the fact that model predictive control is a special model based control approach, system identification and model predictive control are combined in this paper to formulate this iterative identification and model predictive control, which is applied in identifying plant and designing controller iteratively. To measure the control performance, variance analysis about the output estimation is also derived. In order to apply this iterative identification and model predictive control in industry, not only limited in theory, reference governor based on model predictive control is formulated here to explain the commonly used PID controller.

The paper is organized as follows. In Section 2, the problem formulation is addressed, and the structure of the considered closed loop system is introduced. Iterative identification and model predictive control approach is applied to identifying the unknown plant and design closed loop controller iteratively in Section 3. Variance analysis on the output estimation is derived in Section 4, where the controller performance is affected by some factors, such as input spectrum, estimated model, noise spectrum, and designed controller. In Section 5, reference governor based on model predictive control is formulated to execute the PID controller. A simulation example on how to apply the iterative identification and model predictive control into flight control for one helicopter is introduced in Section 6. Section 7 ends the paper with final conclusion and points out the next subject of our ongoing research.

## 2. **Problem Formulation.**

Linear time invariant process is very simple, and theory on it is very mature. Moreover, after linearizing other nonlinear system, then nonlinear system can be reduced to one linear time invariant process. So in this paper, we assume the plant is a linear time invariant process, denoted by a rational transfer function form $G(z)$, and $G(z)$ is unknown. Throughout the closed loop experimental process, a sequence of input-output measured data for plant $G(z)$ are collected. The input-output relation is described as follows.

$$y(t) = G(z)u(t) + d(t) \tag{1}$$

where $z$ is a time shift operator, i.e., $zu(t) = u(t-1)$. $G(z)$ is one transfer function of the unknown plant, $u(t)$ is the measured input, $y(t)$ is the measured output corresponding to the plant $G(z)$, and $d(t)$ is the external noise. When $d(t)$ in Equation (1) is unknown but has known bound, we regard it as the uncertainty associated with $d(t)$ as additive noise because of the way it enters the input-output relation in Equation (1).

Consider the following simple closed loop system in Figure 1, the input-output relations in the whole closed loop system are written as follows.

$$\begin{cases} y(t) = G(z)u(t) + d(t) \\ u(t) = K(z)[r(t) - y(t)] \end{cases} \tag{2}$$

where $r(t)$ is the excited signal, and $K(z)$ is one unknown controller, which is designed by model predictive control. External noise $d(t)$ may be colored or white noise, without loss of generality, here we consider $d(t)$ as one colored noise. Then after introducing one stable, minimum phase filter $H(z)$, external noise $d(t)$ can be obtained by filtering one white noise $e(t)$ with mean zero and variance $\lambda$ through our introduced stable, minimum phase filter $H(z)$. In case of white noise, i.e., $d(t) = e(t)$, then filter $H(z)$ is equal to 1, so it means that $d(t) = H(z)e(t)$.

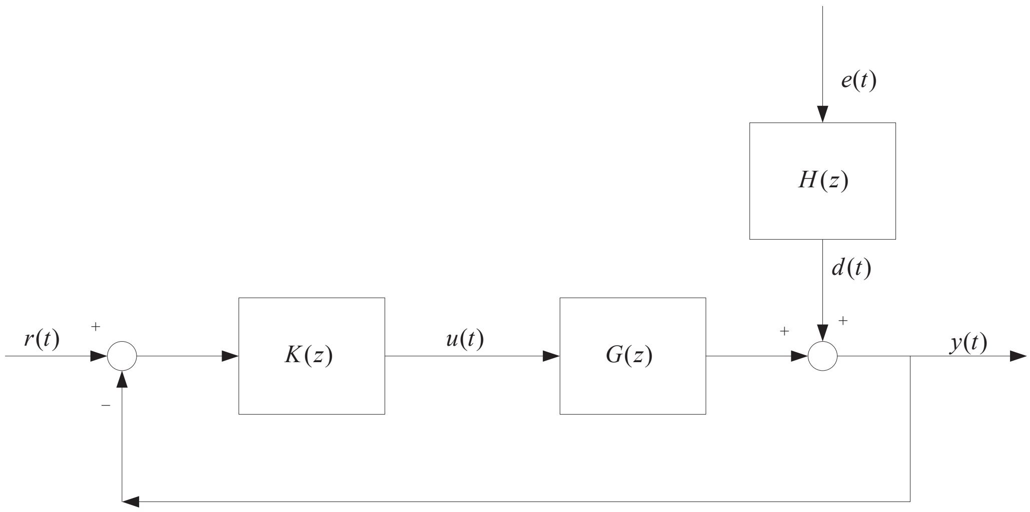


FIGURE 1. Closed loop system

Continuing to do some computations on Equation (1), we get

$$y(t) = G(z)u(t) + H(z)e(t) = \frac{G(z)K(z)}{1+G(z)K(z)}r(t) + \frac{H(z)}{1+G(z)K(z)}e(t) \tag{3}$$

Given one sequence of input-output measured data.

$$Z^N = \{u(1), y(1)\ldots u(N), y(N)\} = \{u(t), y(t)\}_{t=1}^N$$

where $N$ is the number of input-output measured data.

Introduce one unknown parameterized vector $\theta$ in closed loop system, its parameterized form is given as

$$y(t) = \frac{G(z,\theta)K(z)}{1+G(z,\theta)K(z)}r(t) + \frac{H(z,\theta)}{1+G(z,\theta)K(z)}e(t) \tag{4}$$

where $\theta$ denotes the unknown parameter vector, which includes some unknown parameters from $G(z)$ and $H(z)$, i.e., it exists in the parameterized plant model $G(z,\theta)$ and noise filter $H(z,\theta)$, respectively. The goals of this paper are to identify the unknown parameter vector $\hat{\theta}_N$ from one collected input-output data set $Z^N = \{u(1), y(1) \ldots u(N), y(N)\} = \{u(t), y(t)\}_{t=1}^N$, and then apply the identified model $\left\{G\left(z, \hat{\theta}\right), H\left(z, \hat{\theta}\right)\right\}$ to designing the controller $K(z)$ by using model predictive control approach.

## 3. Iterative Identification and Model Predictive Control.

In this section, iterative identification and model predictive control is proposed to identify the unknown parameter vector and design predictive controller.

### 3.1. System identification.

According to parameterized Equation (4), the prediction for output or output prediction $\hat{y}(t, \theta)$ can be calculated as the following one step ahead prediction, i.e.,

$$\begin{aligned}\hat{y}(t, \theta) &= \frac{1 + G(z, \theta)K(z)}{H(z, \theta)} \times \frac{G(z, \theta)K(z)}{1 + G(z, \theta)K(z)} r(t) + \left[1 - \frac{1 + G(z, \theta)K(z)}{H(z, \theta)}\right] y(t) \\ &= \frac{G(z, \theta)K(z)}{H(z, \theta)} r(t) + \frac{H(z, \theta) - 1 - G(z, \theta)K(z)}{H(z, \theta)} y(t) \end{aligned} \tag{5}$$

Computing the one step ahead prediction error or residual $\epsilon(t, \theta)$, it becomes

$$\begin{aligned}\epsilon(t, \theta) &= y(t) - \hat{y}(t, \theta) = y(t) - \frac{G(z, \theta)K(z)}{H(z, \theta)} r(t) - \frac{H(z, \theta) - 1 - G(z, \theta)K(z)}{H(z, \theta)} y(t) \\ &= \frac{1 + G(z, \theta)K(z)}{H(z, \theta)} \left[y(t) - \frac{G(z, \theta)K(z)}{1 + G(z, \theta)K(z)} r(t)\right] \end{aligned} \tag{6}$$

Above two Equations (5) and (6) are called as one step ahead prediction and one step ahead prediction error or residual respectively. Their detailed forms and physical meanings are from the theory of system identification, and they are the basis for the classical least squares identification method.

In the standard prediction error identification, when using the input-output data set $Z^N = \{u(t), y(t)\}_{t=1}^N$, with the number of data $N$, then unknown parameter vector $\theta$ is identified by

$$\hat{\theta}_N = \arg\min_\theta V_1\left(\theta, Z^N\right) = \arg\min_\theta \frac{1}{N} \sum_{t=1}^{N} \epsilon^2(t, \theta) \tag{7}$$

The above identification process is known as the standard prediction error identification. After unknown parameter vector $\theta$ is identified as its parameter estimation $\hat{\theta}_N$, then the model for unknown plant and the noise filter are given as $\left\{\hat{G}\left(z, \hat{\theta}_N\right), \hat{H}\left(z, \hat{\theta}_N\right)\right\}$. Then our identified models are used for the next model predictive control approach.

### 3.2. Model predictive control.

The goal of model predictive control is to control the closed loop system in order to track a desired output reference and reject disturbances from $t = 0$ up to some finite time step $N$, where this time step $N$ can be arbitrary large. In model predictive control framework, the control performance criterion could take the following form.

$$V_2(K(z)) = \frac{1}{N} \sum_{t=1}^{N} \left[(y(t) - y_{des}(t))^2 + \gamma u(t)^2\right] \tag{8}$$

where $y_{des}(t)$ is the desired output reference, $y(t)$ and $u(t)$ are the output signal and input signal, respectively. $\gamma$ is a positive weighting factor that reflects the respective importance given to the tracking error and the control effort. Generally the above contents in Section 3 are incremental to the literature on system identification and model predictive control.

These above parts are used to our later analysis on combining system identification and model predictive control, i.e., iterative strategy is introduced in them.

The criterion function (8) cannot be minimized because the first one depends explicitly on the unknown $G(z)$ and $H(z)$, while the second term depends on $G(z)$ and $H(z)$ through the closed loop system that links $r(t)$, $u(t)$ and $y(t)$. Instead controller $K(z)$ can be designed on the basis of estimations $\left\{\hat{G}\left(z,\hat{\theta}_N\right),\hat{H}\left(z,\hat{\theta}_N\right)\right\}$, which are obtained from measured data by system identification.

Replacing $y(t)$ and $u(t)$ by their parameterized forms to obtain that

$$V_2(K(z),\theta)=\frac{1}{N}\sum_{t=1}^{N}\left[(y(t,\theta)-y_{des}(t))^2+\gamma u(t,\theta)^2\right] \tag{9}$$

where parameterized models $\{y(t,\theta),u(t,\theta)\}$ are given as follows.

$$\begin{cases} y(t,\theta)=\dfrac{G(z,\theta)K(z)}{1+G(z,\theta)K(z)}r(t)+\dfrac{H(z,\theta)}{1+G(z,\theta)K(z)}e(t) \\ u(t,\theta)=\dfrac{K(z)}{1+G(z,\theta)K(z)}r(t)-\dfrac{K(z)H(z,\theta)}{1+G(z,\theta)K(z)}e(t) \end{cases} \tag{10}$$

Substituting Equation (10) into (9), we have

$$\begin{aligned} V_2(K(z),\theta)=&\frac{1}{N}\sum_{t=1}^{N}\left[\left(\frac{G(z,\theta)K(z)}{1+G(z,\theta)K(z)}r(t)+\frac{H(z,\theta)}{1+G(z,\theta)K(z)}e(t)-y_{des}(t)\right)^2\right.\\ &\left.+\gamma\left[\frac{K(z)}{1+G(z,\theta)K(z)}r(t)-\frac{K(z)H(z,\theta)}{1+G(z,\theta)K(z)}e(t)\right]^2\right] \end{aligned} \tag{11}$$

After simple but tedious calculations on the first term and using Parseval's theorem, we have

$$\begin{aligned} &\frac{1}{N}\sum_{t=1}^{N}\left[\left(\frac{G(z,\theta)K(z)}{1+G(z,\theta)K(z)}r(t)+\frac{H(z,\theta)}{1+G(z,\theta)K(z)}e(t)-y_{des}(t)\right)^2\right]\\ =&\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\frac{|G(z,\theta)|^2|K(z)|^2}{|1+G(z,\theta)K(z)|^2}\phi_r(w)+\frac{|H(z,\theta)|^2}{|1+G(z,\theta)K(z)|^2}\lambda\right.\\ &\left.-\frac{2G(z,\theta)K(z)}{1+G(z,\theta)K(z)}\phi_{y_{des}r}(w)\right\}dw \end{aligned} \tag{12}$$

In this expression $\phi_r(w)$ and $\phi_{y_{des}r}(w)$ are the spectrum and cross spectrum of the signals $r(t)$, and $y_{des}(t)$. Also we use the following relations.

$$\begin{cases} Ed(t)=EH(z)e(t)=0;\ Er(t)d(t)=H(z)Er(t)e(t)=0 \\ \phi_d(w)=|H(z)|^2\phi_e(w)=\lambda|H(z)|^2 \end{cases}$$

Similarly doing some manipulations on the second term of Equation (11), it holds that

$$\begin{aligned} &\frac{1}{N}\sum_{t=1}^{N}\left[\frac{K(z)}{1+G(z,\theta)K(z)}r(t)-\frac{K(z)H(z,\theta)}{1+G(z,\theta)K(z)}e(t)\right]^2\\ =&\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|K(z)|^2(\phi_r(w)+\lambda|H(z)|^2)}{|1+G(z,\theta)K(z)|^2}dw \end{aligned} \tag{13}$$

Add Equations (12) and (13) to get

$$V_2(K(z),\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\frac{|G(z,\theta)|^2|K(z)|^2}{|1+G(z,\theta)K(z)|^2}\phi_r(w) + \frac{|H(z,\theta)|^2}{|1+G(z,\theta)K(z)|^2}\lambda - \frac{2G(z,\theta)K(z)}{1+G(z,\theta)K(z)}\phi_{y_{des}r}(w) + \frac{|K(z)|^2(\phi_r(w)+\lambda|H(z)|^2)}{|1+G(z,\theta)K(z)|^2}\right\}dw \quad (14)$$

Due to the fact that unknown parameter vector $\theta$ and controller $K(z)$ are all unknown, and the processes of system identification and model predictive control correspond to two minimization problems, we need to rewrite these two criterion functions as $V_1(K(z),\theta)$ and $V_2(K(z),\theta)$.

3.3. **Iterative identification and model predictive control.** Ideally, the problem of system identification could be reformulated as a parameter estimation problem as follows.

$$\hat{\theta}_N = \arg\min_{\theta} V_1(K(z),\theta)$$

and the problem of predictive controller could be also reformulated as a minimization problem as follows.

$$\hat{K}(z) = \arg\min_{K(z)} V_2(K(z),\theta)$$

The direct minimizations of such two global identification and control performance criterions over a set of restricted complexity models are typical intractable, but we can attack these two problems by performing a succession of local identification steps and local predictive control design steps in an iterative way. The basic steps corresponding to our iterative identification and model predictive control are formulated as follows.

Step 1: Identify an open loop model $\hat{G}_0\left(z,\hat{\theta}_0\right)$ and noise filter $\hat{H}_0\left(z,\hat{\theta}_0\right)$, from input-output data, and design a controller $\hat{K}_0$, that stabilizes both the true plant $G$ and the identified model $\hat{G}_0$. Apply this controller to the plant and collect new input-output data.

Step 2: Using the closed loop measured data on the plant while the initial controller $\hat{K}_0$ operates, identify a new model $\hat{G}\left(z,\hat{\theta}_1\right)$ and noise filter $\hat{H}\left(z,\hat{\theta}_1\right)$ by minimizing a local identification criterion, i.e., $\hat{\theta}_1$ is identified by

$$\hat{\theta}_1 = \arg\min_{\theta} V_1\left(\hat{K}_0,\theta,\hat{\theta}_0\right)$$

Step 3: Using these new model and noise filter $\left\{\hat{G}\left(z,\hat{\theta}_1\right),\hat{H}\left(z,\hat{\theta}_1\right)\right\}$, design a new predictive controller $\hat{K}_1$ that stabilizes both $G(z)$ and $\left\{\hat{G}\left(z,\hat{\theta}_1\right)\right\}$, by minimizing a local control design criterion, i.e., predictive controller $\hat{K}_1$ is designed by

$$\hat{K}_1 = \arg\min_K V_2\left(K,\hat{K}_0,\hat{\theta}_1\right)$$

...

Step $i$: Using the new model and noise filter $\left\{\hat{G}\left(z,\hat{\theta}_i\right),\hat{H}\left(z,\hat{\theta}_i\right)\right\}$, design a new predictive controller $\hat{K}_i$ that stabilizes both $G(z)$ and $\left\{\hat{G}\left(z,\hat{\theta}_i\right)\right\}$, by minimizing a local control design criterion. Apply this controller to the plant and collect new input-output data.

...

Repeat: Repeat step 2 and step 3, replacing $i$ by $i+1$.

Generally, here iterative identification and model predictive control approach is applied to identifying the unknown plant and designing closed loop controller iteratively.

4. **Variance Analysis.** From above steps in iterative identification and model predictive control, the predictive controller $\hat{K}(z)$ is dependent of identified model and noise filter $\left\{\hat{G}\left(z,\hat{\theta}\right),\hat{H}\left(z,\hat{\theta}\right)\right\}$, i.e., predictive controller can be abbreviated as $\hat{K}(z) = K\left(\hat{G},\hat{H}\right)$. Then the control performance depends on the accuracy of the identified model $\left\{\hat{G}\left(z,\hat{\theta}\right),\hat{H}\left(z,\hat{\theta}\right)\right\}$. To show the control performance about the predictive controller $\hat{K}(z)$, one performance index must be chosen to analyze. As the goal of model predictive control is to guarantee the closed loop system to tracking a desired output reference, here for analysis purpose, the tracking performance is chosen as a performance index. For convenience, firstly we introduce some priori information about the asymptotic analysis about the identified model and noise filter $\left\{\hat{G}\left(z,\hat{\theta}\right),\hat{H}\left(z,\hat{\theta}\right)\right\}$.

4.1. **Asymptotic analysis.** To simplify notation, we stack the model $G(e^{jw})$ and noise filter $H(e^{jw})$ in frequency domain as follows.

$$T\left(e^{jw}\right) = \begin{bmatrix} G\left(e^{jw}\right) \\ H\left(e^{jw}\right) \end{bmatrix}$$

and its parameterized form is

$$\hat{T}\left(e^{jw},\theta\right) = \begin{bmatrix} \hat{G}\left(e^{jw},\theta\right) \\ \hat{H}\left(e^{jw},\theta\right) \end{bmatrix}$$

**Comment:** When the power spectrums corresponding to the excitation signal are continuous in interval $[0,\pi]$, and the number of measured input-output data approaches to infinity, then the following asymptotical expression holds.

$$\hat{T}\left(e^{jw},\theta\right) \to T\left(e^{jw}\right),\ N \to \infty$$

Furthermore, the variance about $\hat{T}(e^{jw},\theta)$ is one Gaussian distribution, and its variance matrix is that

$$Cov\left[\hat{T}\left(e^{jw},\theta\right)\right] \approx \frac{n}{N}\phi_d(w)\begin{bmatrix} \phi_u(w) & \phi_{ue}(w) \\ \phi_{eu}(w) & \lambda \end{bmatrix}^{-1} \tag{15}$$

where $\phi_u(w)$ is the input spectrum, and $\phi_{ue}(w)$ is the cross spectrum between $u(t)$ and white noise $e(t)$. $\phi_d(w)$ is the noise spectrum, $n$ is the model order and $N$ is the same as the number of measured data.

In the framework of feedback effect, control input $u(t)$ can be rewritten as

$$u(t) = K(z)[r(t) - y(t)] = K(z)S(z)[r(t) - H(z)e(t)] \tag{16}$$

where $S(z)$ is the sensitivity function, i.e.,

$$S(z) = \frac{1}{1 + G(z)K(z)}$$

If excitation signal $r(t)$ is independent of white noise $e(t)$, then input spectrum $\phi_u(w)$ is divided as two terms.

$$\phi_u(w) = \phi_u^r(w) + \phi_u^e(w) = |K(z)|^2|S(z)|^2\phi_r(w) + |K(z)|^2|S(z)|^2|H(z)|^2\lambda \tag{17}$$

where the first term is from excitation signal $r(t)$, and the second term comes from white noise $e(t)$.

Use relation $\phi_d(w) = |H(z)|^2\lambda$ to get

$$\phi_{ue}(w) = -K(z)S(z)H(z)\lambda \tag{18}$$

Substitute Equations (17) and (18) into variance matrix (15) to obtain

$$\begin{aligned} Cov\left[\hat{T}\left(e^{jw},\theta\right)\right] &\approx \frac{n}{N}\phi_d(w)\begin{bmatrix} \phi_u(w) & \phi_{ue}(w) \\ \phi_{eu}(w) & \lambda \end{bmatrix}^{-1} \\ &= \frac{n}{N}\frac{\phi_d(w)}{\lambda\phi_u(w)-|\phi_{ue}(w)|^2}\begin{bmatrix} \lambda & -\phi_{ue}(w) \\ -\phi_{eu}(w) & \phi_u(w) \end{bmatrix} \\ &= \frac{n}{N}\frac{|H(z)|^2}{|K(z)|^2|S(z)|^2\phi_r(w)}\begin{bmatrix} \lambda & -\phi_{ue}(w) \\ -\phi_{eu}(w) & \phi_u(w) \end{bmatrix} \end{aligned} \tag{19}$$

Expanding the above variance matrix to get the variance of the identified model $\hat{G}(e^{jw},\theta)$, i.e.,

$$Var\left[\hat{G}\left(e^{jw},\theta\right)\right] \approx \frac{n}{N}\frac{|H(z)|^2\lambda}{|K(z)|^2|S(z)|^2\phi_r(w)} = \frac{n}{N}\frac{\phi_d(w)}{|K(z)|^2|S(z)|^2\phi_r(w)} \tag{20}$$

similarly based on Equation (3), we have

$$y(t) = \frac{G(z)K(z)}{1+G(z)K(z)}r(t) + \frac{H(z)}{1+G(z)K(z)}e(t) = G(z)K(z)S(z)r(t) + H(z)S(z)e(t)$$

Then the output spectrum is that

$$\phi_y(w) = |G(z)|^2\phi_u^r(w) + |S(z)|^2\phi_d(w) = |G(z)|^2|K(z)|^2|S(z)|^2\phi_r(w) + |S(z)|^2\phi_d(w) \tag{21}$$

Variance matrix (19) is important for the next variance analysis about output response.

4.2. **Variance analysis for predictive controller.** Rewrite the output response of the closed loop system here again.

$$y(t) = \frac{G(z)K(z)}{1+G(z)K(z)}r(t) + \frac{H(z)}{1+G(z)K(z)}e(t)$$

From the above iterative identification and model predictive control, predictive control $K(z)$ depends on the estimations $\hat{G}(z,\theta)$ and $\hat{H}(z,\theta)$, which correspond to their model and noise filter. Using these estimations in output response, then

$$y(t) = \frac{G(z)\hat{K}(z)}{1+G(z)\hat{K}(z)}r(t) + \frac{H(z)}{1+G(z)\hat{K}(z)}e(t) \tag{22}$$

where predictive controller $\hat{K}(z) = K\left(\hat{G},\hat{H}\right)$.

Our variance analysis for predictive control is to measure the variance of the output response, i.e., $E\left\{\|y(t)-y_0(t)\|^2\right\}$, where $y_0(t)$ is the true output response and $y(t)$ is the output response, obtained by predictive controller $\hat{K}(z)$. Suppose two estimations $\hat{G}(z,\theta)$ and $\hat{H}(z,\theta)$ are close enough to their true values $G(z,\theta)$ and $H(z,\theta)$, but perturbations still exist between estimations and true values, i.e.,

$$\hat{G} = G + \Delta G;\ \hat{H} = H + \Delta H;\ \hat{K} = K + \Delta K$$

For the sake of brevity, variable $z$ is neglected in the following derivations.

Consider the following approximate formulas.

$$\begin{aligned} \left(1+G\hat{K}\right)^{-1} &= (1+GK+G\Delta K)^{-1} = \left(1+(1+GK)^{-1}G\Delta K\right)^{-1}(1+GK)^{-1} \\ &\approx (1-SG\Delta K)S = S - SG\Delta KS \end{aligned} \tag{23}$$

$$\begin{aligned} \left(1+G\hat{K}\right)^{-1}G\hat{K} &\approx (1-SG\Delta K)SG(K+\Delta K) \\ &\approx T - SG\Delta KT + SG\Delta K = T + SG\Delta KS \end{aligned} \tag{24}$$

where we use some relations in deriving Equations (23) and (24)

$$(1-\Delta)^{-1} \approx 1-\Delta;\ S=(1+GK)^{-1};\ T=(1+GK)^{-1}GK=1-S=1-(1+GK)^{-1}$$

Based on Equations (23) and (24), then Equation (22) is rewritten as

$$y(t)=(T+SG\Delta KS)r(t)+(S-SG\Delta KS)He(t)\approx y_0(t)+SG\Delta KS(r(t)-He(t)) \quad (25)$$

where true output response $y_0(t)$ is defined as

$$y_0(t)=Tr(t)+SHe(t)$$

The true output response $y_0(t)$ is obtained by true model and noise filter $\{G(z),H(z)\}$.

From Equation (25), the control performance is given as follows

$$E\left\{\|y(t)-y_0(t)\|^2\right\}=E\left\{\|SG\Delta KS(r(t)-He(t))\|^2\right\} \quad (26)$$

Doing some manipulations from the practical perspective and using Parseval's theorem again, then it holds

$$\begin{aligned}&E\left\{\|y(t)-y_0(t)\|^2\right\}\\ &=E\left\{\|SG\Delta KS(r(t)-He(t))\|^2\right\}=E\left\{E\|SG\Delta KS(r(t)-He(t))\|^2|\Delta K\right\}\\ &=E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}SG\Delta KS[\phi_r(w)+\phi_d(w)]S^T\Delta K^TG^TS^T\right\}dw\end{aligned} \quad (27)$$

For ease of analysis, the middle term is decomposed as

$$\phi_r(w)+\phi_d(w)=RR^T$$

Use vectorization operation to get

$$\begin{aligned}&E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}SG\Delta KS[\phi_r(w)+\phi_d(w)]S^T\Delta K^TG^TS^T\right\}dw\\ &=E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}SG\Delta KSRR^TS^T\Delta K^TG^TS^T\right\}dw=E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\|vec\{SG\Delta KSR\}\|_2^2\right\}\end{aligned} \quad (28)$$

Introducing Kronecker product $\otimes$ to compute Equation (28), then it holds that

$$\begin{aligned}&E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\|vec\{SG\Delta KSR\}\|_2^2\right\}\\ &=E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|\left[R^T\otimes S^TSG\right]vec\{\Delta K\}\right\|_2^2\right\}=E\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}tr\left[\left(SRR^T\otimes G^TS^TSG\right)P_k\right]\right\}\\ &=\frac{1}{2\pi}\int_{-\pi}^{\pi}tr\left[\left(S[\phi_r(w)+\phi_d(w)]S^T\otimes G^TS^TSG\right)P_k\right]\end{aligned}$$

$$P_k=cov\left\{vec\left(\hat{K}\right)\right\} \quad (29)$$

When to compute $P_k$, due to $\hat{K}=K\left(\hat{G},\hat{H}\right)$, then

$$\Delta K\approx\frac{\partial vecK}{\partial vecG}\Delta G+\frac{\partial vecK}{\partial vecH}\Delta H=\begin{bmatrix}\frac{\partial vecK}{\partial vecG} & \frac{\partial vecK}{\partial vecH}\end{bmatrix}\begin{bmatrix}\Delta G\\ \Delta H\end{bmatrix} \quad (30)$$

It means $P_k$ is that

$$P_k=\begin{bmatrix}\frac{\partial vecK}{\partial vecG} & \frac{\partial vecK}{\partial vecH}\end{bmatrix}P_{[\hat{G},\hat{H}]}\begin{bmatrix}\left(\frac{\partial vecK}{\partial vecG}\right)^T\\ \left(\frac{\partial vecK}{\partial vecH}\right)^T\end{bmatrix} \quad (31)$$

combining Equations (19) and (31) to give that

$$\begin{aligned} P_k &= \left[\frac{\partial vecK}{\partial vecG}\,\frac{\partial vecK}{\partial vecH}\right]\frac{n}{N}\frac{|H(z)|^2}{|K(z)|^2|S(z)|^2\phi_r(w)}\begin{bmatrix}\lambda & -\phi_{ue}(w)\\ -\phi_{eu}(w) & \phi_u(w)\end{bmatrix}\begin{bmatrix}\left(\dfrac{\partial vecK}{\partial vecG}\right)^T\\ \left(\dfrac{\partial vecK}{\partial vecH}\right)^T\end{bmatrix}\\ &= \frac{n}{N}\frac{|H(z)|^2}{|K(z)|^2|S(z)|^2\phi_r(w)}\left[\lambda\frac{\partial vecK}{\partial vecG}\left(\frac{\partial vecK}{\partial vecG}\right)^T + \phi_u(w)\frac{\partial vecK}{\partial vecH}\left(\frac{\partial vecK}{\partial vecH}\right)^T\right]\\ &\quad + M(\phi_{eu}(w)) \end{aligned} \tag{32}$$

where the third term $M(\phi_{eu}(w))$ includes something about cross-spectrum $\phi_{eu}(w)$.

Use the expression of input spectrum $\phi_u(w)$ (17) to give

$$\begin{aligned} & E\left\{\|y(t)-y_0(t)\|^2\right\}\\ \approx\; & \frac{n}{2\pi N}\int_{-\pi}^{\pi} tr\left\{\left(S[\phi_r+\phi_d]S^T\otimes G^TS^TSG\right)|H|^2\frac{\phi_d}{\phi_u}\frac{\partial vecK}{\partial vecG}\left(\frac{\partial vecK}{\partial vecG}\right)^T\right\}dw\\ & + \frac{n}{2\pi N}\int_{-\pi}^{\pi} tr\left\{\left(S[\phi_r+\phi_d]S^T\otimes G^TS^TSG\right)|H|^2\frac{\partial vecK}{\partial vecH}\left(\frac{\partial vecK}{\partial vecH}\right)^T\right\}dw \end{aligned} \tag{33}$$

From above variance analysis for the output response, we see which variables affect the variance greatly. Furthermore, due to the fact that input spectrum $\phi_u(w)$ only exists in the first term, we can choose one optimal input spectrum to guarantee the above variance converges to its minimum value. This process of choosing optimal spectrum is known as the optimal input signal design for closed loop system.

5. **Model Predictive Control Based Reference Governor.** The concept of model predictive control based reference governor supplies setpoints for primary controller to be an optimization based reference governor. In order to apply our proposed iterative identification and model predictive control in industry, the reference governor is introduced to give a supplement for above analysis. The reference governor is a control strategy whose objective is to modify the user given reference $r(t)$ in order to improve the quality and safety of a closed loop system, which consists of a primary controller. Here consider a situation where the plant is controlled primary by a set of PID controller, as PID controller is very common in industry. This section is devoted to such closed loop systems with a plant and an inner controller in a form of a PID controller. Such model predictive control based reference governor is seen in Figure 2, where the transfer function of the plant is described as

$$G(z) = \frac{Y(z)}{U(z)} \tag{34}$$

where $U(z)$ and $Y(z)$ represent the Laplace transform of input variable and output variable, respectively in $z$ domain. For such a plant there exists a PID controller in the following transfer function form.

$$R(z) = \frac{U(z)}{E(z)} \tag{35}$$

Similarly $E(z)$ is the Laplace transform of the error signal $e(t) = w(t) - y(t)$.

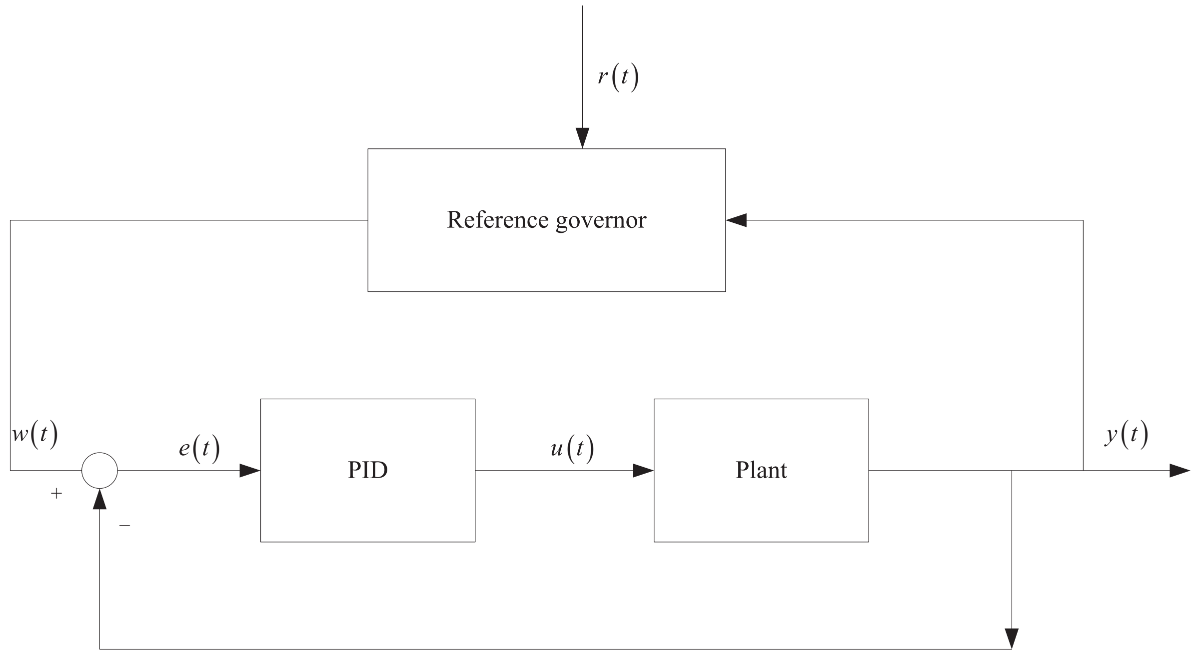


FIGURE 2. Model predictive control based reference governor

Applying the transfer function algebra, the transfer function corresponding to the closed loop system is given by

$$G_{cl}(z) = \frac{Y(z)}{W(z)} = \frac{G(z)R(z)}{1 + G(z)R(z)}$$

In model predictive control design, the numerator and the denominator of $G_{cl}(z)$ can be written as a polynomial of $z$, i.e.,

$$G_{cl}(z) = \frac{G(z)R(z)}{1 + G(z)R(z)} = \frac{\sum_{j=0}^{m} b_j z^{-j}}{\sum_{i=0}^{n+1} a_i z^{-i}} \tag{36}$$

where $m$ and $n$ are the orders for the numerator and the denominator polynomials.

Rewrite above transfer function (36) into a recursive form as follows

$$y(t+1) = \frac{1}{a_0}\left(-\sum_{i=1}^{n} a_i y(t-i+1) + \sum_{j=0}^{m} b_j w(t-j+1)\right) \tag{37}$$

Then Equation (37) denotes exactly the output estimation in model predictive control, i.e., model predictive control based reference governor has following form.

$$\begin{aligned}
&\min_{u_1 \cdot u_N} \sum_{t=1}^{N} L_t(y(t), u(t), w(t)) \\
&\text{subject to } y(t+1) = \frac{1}{a_0}\left(-\sum_{i=1}^{n} a_i y(t-i+1) + \sum_{j=0}^{m} b_j w(t-j+1)\right) \\
&y_{\min} \le y(t) \le y_{\max};\ \eta_0 = \eta(t)
\end{aligned} \tag{38}$$

with the objective function

$$\begin{aligned}
L_t(y(t), u(t), w(t)) = {}& \|Q_1(r(t) - y(t))\|_2 + \|Q_2(r(t) - w(t))\|_2 \\
&+ \|Q_3(w(t) - y(t))\|_2 + \|Q_4(w(t) - w(t-1))\|_2
\end{aligned} \tag{39}$$

and

$$\eta = \left[ \begin{array}{ccccccccc} y(t) & \cdots & y(t-n) & w(t-1) & \cdots & w(t-n) & r(1) & \cdots & r(N) \end{array} \right]^T \tag{40}$$

where $Q_1$, $Q_2$, $Q_3$, $Q_4$ are four positive definite matrices.

**Comment:** The first term in objective function penalizes the difference between the measured output and the user defined reference. The second term penalizes the enforce convergence of the shaper reference. The third term provides that the output prediction or output estimation converges to the shaped reference. The last term dampens the fluctuations in the evolution of the shaped reference.

According to the problem of solving the optimization problem (38), many classical optimization algorithms can be applied here, for example, Newton method, gradient descent method, exact primal dual first order algorithm, distributed subgradient method, and Lagrange multiplier method.

6. **Simulation Example.** Here the example of helicopters hover is used to prove the effectiveness of our proposed iterative identification and model predictive control strategy. The structure of our considered helicopter is seen in Figure 3. In controlling the helicopter system, one six degree of freedom motion is regarded as a high precision servo device. The higher requirements for position following control are those. 1) The expected position tracking cannot overshoot, and the dynamic response process would be fast or smooth. 2) To ensure the tracking accuracy, the position tracking is required to show a small steady state error.


FIGURE 3. The structure of helicopter

In the servo control, we apply proportional and integral (PI) controller. Due to the influence of external noise, differential controller is not often used here. To increase the frequency response of the position loop, a series of advanced correction or compound correction control strategies can be employed.

After designing the controller for the speed loop, many parts are combined to be the plant in the position loop, such as the controller, the power amplifier, the DC torque motor and the platform load. This part is equivalent to a link of the position loop. The simplified structure diagram for the position loop is referred to Figure 4.

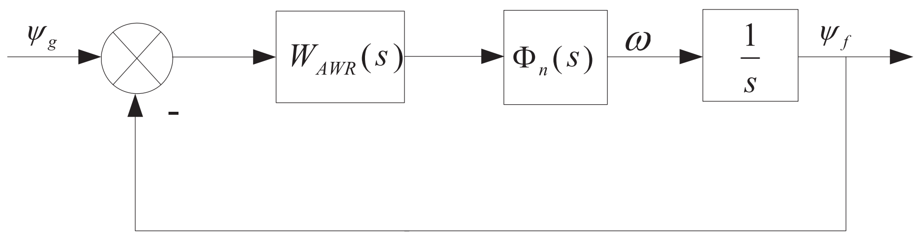


FIGURE 4. Simplified structure for position loop

In Figure 4, $W_{AWR}(s)$ is one regulator for the position loop, $\phi_n(s)$ is the equivalent transfer function for velocity loop, and

$$\phi_n(s) = \frac{3.259s + 135.9}{0.0000113s^3 + 0.01053s^2 + 3.467s + 136.9}$$

In Figure 4, a physical integral link exists in the control loop. From the principle of the control, the integral link of the position mode cannot be used in the position loop to ensure the stability for the entire position loop. In fact, due to the nonlinear factors, such as static friction in the servo loop, only the proportional controller will guarantee the steady state accuracy for the position loop. To reduce the influence coming from nonlinear factors, it is necessary to add a multi-integral controller in a small range for the position loop, and it means one integral controller is added to consider the finite output and limited angle, which is seen in Figure 5.

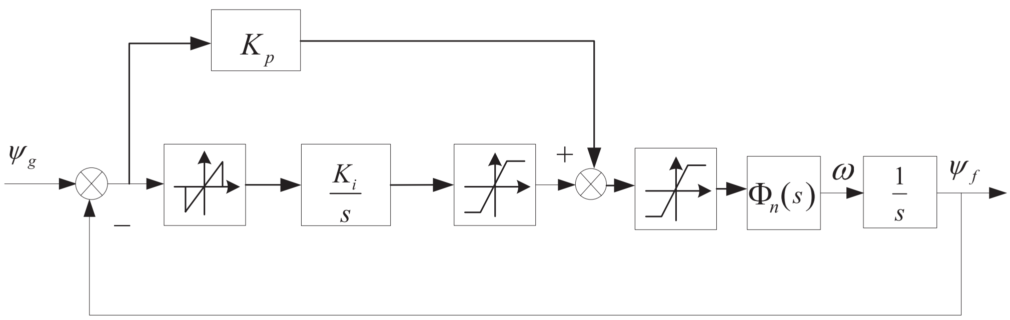


FIGURE 5. Multi-integral controller

The controller parameters are tuned around the stable boundary by Matlab, the tuned parameters are given as $K_P = 100$; $K_I = 2.57$, and the closed loop transfer function of the position following system is as follows after PID correction.

$$\phi(s) = \frac{3.846s^3 + 486.3s^2 + 13600s + 349.3}{0.0000113s^4 + 0.01053s^3 + 3.467s^2 + 136.9s}$$

In order to verify the performance index of the position following system with PID controller under different input signals, here the step signal and sinusoidal signal with different frequencies are chosen as the input signals for the entire closed loop system.

1) Step signal

The input signal is chosen as step signal, i.e., $r(t) = 1(t)$, and then the simulated system output and error are obtained in Figure 6.

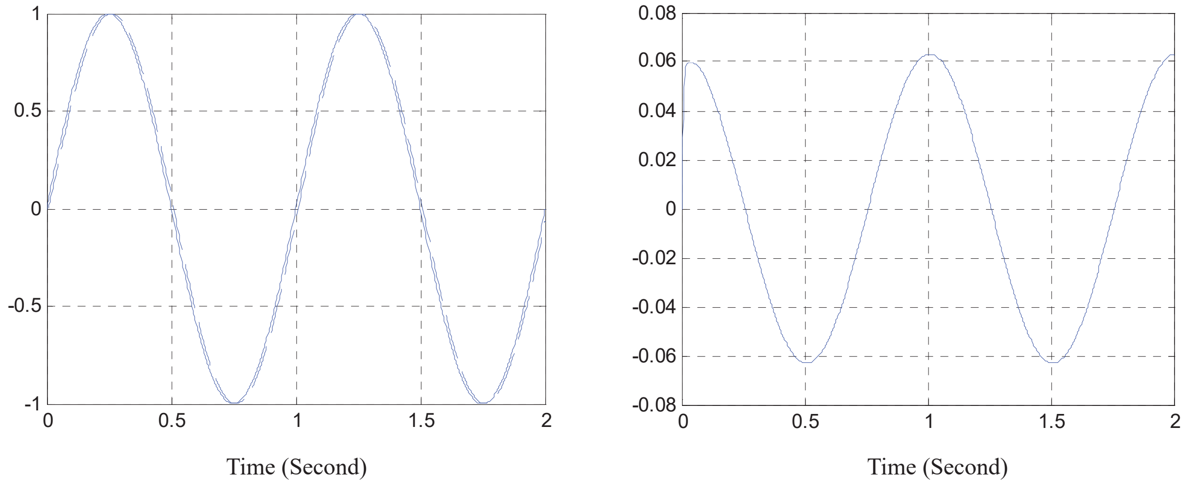


FIGURE 6. Simulated system output and error with step signal

2) Sinusoidal signal

Sinusoidal signals with frequency 1HZ and 3HZ are selected as the input signals for the position following system, then also the simulated system output and error are obtained in Figure 7.

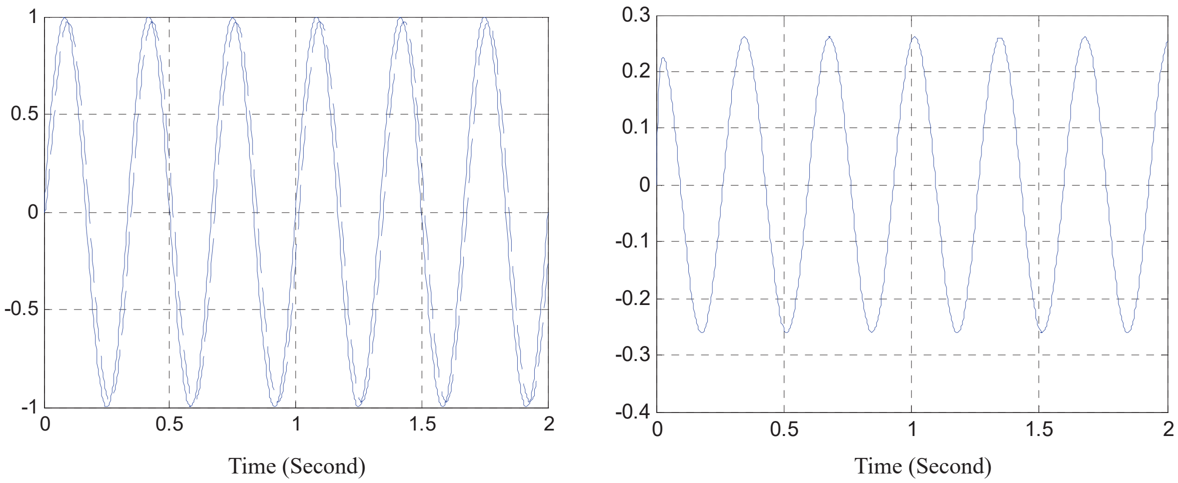


FIGURE 7. Simulated system output and error with sinusoidal signal

According to the system response curves and error curves, when input signal is a unit step signal, the adjustment time of the system is less, the output has no overshoot, and the steady state error is less, which corresponds to the position system time domain index. When the frequency of the sinusoidal signal is low, the closed loop system can accurately track the input signal. As the frequency of the sinusoidal signal increases, the tracking of the input signal by the closed loop system has a certain hysteresis, and the top of the signal will appear flat. Therefore, the classical PID controller of the position following system can realize accurate tracking of low frequency input signals, and there is a large position tracking error for tracking high frequency signals. Furthermore, we see the fact that many nonlinear factors and coupling errors caused by the model are neglected when building the motor model, and the effects of nonlinear factors such as multiple frictional moments of the mechanism are neglected in the simulation experiment. These neglected nonlinear factors must be considered in our further research.

## 7. Conclusion.

In this paper, system identification strategy and model predictive control are combined to form one iterative identification and model predictive control approach, which can guarantee control performance not depending on the identified model greatly. To show the tracking performance for model predictive control, variance analysis corresponding to the closed loop output response is derived in detail by our own mathematical derivation. When to extend this iterative identification and model predictive control to more general case in industry, reference governor based on model predictive control is also formulated to provide PID controller. Due to the fact that system identification is a basis for next control design and input signal design is one first element during the whole system identification, future research will focus on studying optimal signal for model predictive control.

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