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ADAPTIVE FUZZY SYSTEM COMPENSATION BASED MODEL-FREE CONTROL FOR STEER-BY-WIRE SYSTEMS WITH UNCERTAINTY

Zezheng Wang

School of Automation Science and Electrical Engineering
Beihang University
No. 37, Xueyuan Road, Haidian District, Beijing 100191, P. R. China
18813111735@163.com

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Abstract. *For steer-by-wire (SbW) systems, the most traditional model-based control methods have been conducted in the practical application under the assumptions that the friction torque and self-aligning torque are known in advance. However, it is difficult to measure and model the friction torque and self-aligning torque. In this paper, a composite control scheme including proportion integration differentiation (PID) controller and fuzzy logic system (FLS) is proposed so that the above assumptions are relaxed. Firstly, an adaptive FLS is adopted to approximate the unknown dynamics of SbW system so that the friction torque and self-aligning torque are no longer needed in the control design. Secondly, the classic PID controller combined with the intelligent compensation is proposed to achieve the stable steering tracking control. To support the theoretical analysis, the system tracking error is proved to be uniformly ultimately bounded by Lyapunov stability technique. Finally, numerical simulations and hardware-in-loop (HiL) experiments show the rationality and superiority of the proposed method.*

Keywords: Steer-by-wire system, Adaptive fuzzy modeling, Control compensation, Hardware-in-loop

1. **Introduction.** As one of the most advanced automobile technologies, the unmanned ground vehicle technique has attracted increasing attention from the industrial communities over the past two decades. The steer-by-wire (SbW) technique is an essential component in the development of automatic drive. Compared with the conventional steering system, the SbW system has two distinct characteristics: 1) the mechanical linkage between the steering wheel and the front-wheels is no longer required; 2) an additional steering motor is applied to regulating the front-wheels steering angle. SbW system is closely related to the traffic safety of unmanned vehicles, so the researches on controlling the steering angle of front-wheels have been attracted great attention [1-5].

The researches on the modeling and control of the SbW system have become a popular topic and many successes have been obtained in recent years. Specifically, based on several assumptions, the two-degree-of-freedom nonlinear model of the SbW system has been established in [1-4]. In subsequent studies, a variety of model-based control methods have been proposed for SbW to achieve the trajectory tracking control of the front-wheels steering angle, such as proportional-derivative (PD) control [2,3], model predictive control [4], H_∞ control [5], and sliding mode control [6-10]. It should be noted that when there exists a large modeling error, the above model-based control methods are not available.

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Moreover, the robust control methods [5-10] have complex structures and excessive parameters, which make it difficult to tune the control parameters, and thus, they are not suitable to be applied in the practical application.

The general proportional-integral-derivative (PID) controller has been widely used in the process control technology for many decades [11-15]. Meanwhile, the PID controller is a key part of control loop and it has the significantly untapped capability, which offers the simplest and efficient solution to many practical control problems. Hence, in many real industrial applications, the PID controller is still widely used even though lots of new control methods have been proposed. Motivated by the above fact, we concentrate on the exploitation of PID control. However, it is hard for PID control to achieve satisfied tracking performance when larger uncertainties and strong nonlinearities exist in the system. Therefore, for the desired result, it is necessary to combine with the feedforward/feedback methods and adaptive technologies in the design of PID controller.

Adaptive control techniques include the system identification and controller design. Over the past few decades, there are significant researches on adaptive control schemes for linear and nonlinear systems [16-19]. Many practical results have shown the advantage of adaptive control techniques. However, the conventional adaptive control theory was developed based on mathematical models and parameter estimation algorithms. Therefore, it is hard to perform tracking control tasks when the large uncertainties and strong nonlinearities exist. A remarkable characteristic of the existing adaptive control technique is that system design does not take advantage of the domain knowledge from operators or workers. Recently, in many practical industrial tasks, traditional approaches of mathematical modeling are not easy to offer satisfactory performance because the nonlinear dynamics, unmodeled dynamics, parameter uncertainties and varying external disturbances make the real systems more complex. Fuzzy logic system (FLS) has attracted a lot of attention in the engineering community due to its universal approximation performance. In particular, an adaptive fuzzy system has received considerable attention because it can be used to deal with the problems of complex industrial process modeling and control [20-23]. To this end, fuzzy modeling and adaptive control techniques have been successfully applied to complex systems, where traditional approaches rarely achieve satisfactory results due to the nonlinearity, uncertainty and lack of domain knowledge.

As for this, this paper proposes a novel fuzzy compensation-based PID for an uncertain SbW system, which combines the on-line approximation ability of the FLS. It is worth noting that this controller does not need the known parameters of the SbW system model and nonlinear dynamics. The unknown dynamics can be online approximated by the designed adaptive mechanism, so as to achieve the fuzzy modeling and adaptive control of the SbW system. The main contributions of this paper can be concluded as the following.

- The nonlinearities including friction torque and self-aligning torque of SbW system are approximated by the adaptive FLS. Compared with the existing researches on S-bW systems, a priori knowledge of the system dynamics can be avoid in the controller design.
- Different from recent complex adaptive control methods, a PID control combined with fuzzy compensation is proposed in this paper. Because the controller has a simple structure and fewer designed parameters, it is convenient to realize the parameter tuning and steering control in the practical application.

The structure of this paper is organized as follows. The dynamics model of the SbW system and the FLS are briefly described in Section 2. In Section 3, a novel FLS-based PID control and the corresponding stability analysis are given. The numerical simulations results of the hardware-in-loop test (HiL) and its corresponding analysis are given in Section 4. Section 5 is the concluding remarks.

Notations: Throughout this paper, $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of $m \times n$ matrices. $|\cdot|$ refers to the absolute value and $\|\cdot\|$ denotes the standard Euclidean vector norm.

2. **Dynamics Model and Preliminaries.** The SbW system is mainly composed of the steering motor and motor driver, reducer, gearbox, etc. Figure 1 shows the mechanical

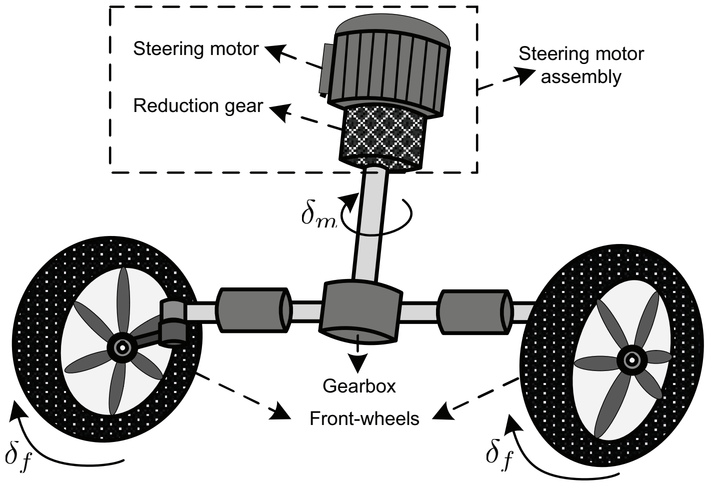


FIGURE 1. Schematic diagram of the SbW system

TABLE 1. Variables of the SbW system

Symbol	Model variable	Unit
J_m	Moments of inertia of the front wheels	[kg·m^2]
B_m	Viscous friction of the front wheels	[Nms/rad]
J_f	Moments of inertia of the front wheels	[kg·m^2]
B_f	Viscous friction of the front wheels	[Nms/rad]
J_e	Moments of inertia of the front wheels	[kg·m^2]
τ_m	Steering motor torque	[Nm]
τ_{12}	Motor lumped torque perturbation	[Nm]
τ_e	Self-aligning torque	[Nm]
δ_f	Front-wheels steering angle	[Rad]
δ_m	Motor assembly steering angle	[Nm]
μ	Steering motor assembly angle/front-wheels angle	[]
φ	Yaw rate	[rad/s]
Y	Lateral displacement	[m]
M	Vehicle Mass	[kg]
I	Polar moment of inertia	[kg·m^2]
v_X	Longitudinal velocity	[m/s]
C_f	Front-wheels cornering stiffness	[N/rad]
C_r	Rear-wheels cornering stiffness	[N/rad]
l_f	Distance from mass center to front axle	[m]
l_r	Distance from mass center to rear axle	[m]
t_m	Front-wheels mechanical trail	[m]
t_p	Front-wheels pneumatic trail	[m]

architecture of the SbW system and Table 1 is given to clarify each of the components in Figure 1.

2.1. **Dynamic model of the SbW system.** According to [1-6], the dynamic model of the steering motor can be established as

$$J_m\ddot{\delta}_m + B_m\dot{\delta}_m + \tau_{12} = \tau_m. \tag{1}$$

The rotation of the front-wheels around their vertical axes can be modeled as in [2,5], that is

$$J_f\ddot{\delta}_f + \tau_f + \tau_e = \tau_s \tag{2}$$

in which τ_f represents the damping and friction effects, which can be expressed as

$$\tau_f = F_c\text{sign}\left(\dot{\delta}_f\right) + B_f\dot{\delta}_f. \tag{3}$$

The self-aligning torque τ_e is given by

$$\tau_e = C_f(t_m + t_p)\left(\frac{\dot{Y} + l_f\dot{\varphi}}{v_X} - \delta_f\right) \tag{4}$$

where lateral motion Y and the yaw angle φ can be obtained from the two-degree-of-freedom model established based on the following assumptions [3,6]:

1) the longitudinal vehicle speed v_X is assumed to be constant.
2) $\cos\delta_f = 1$.

The two-degree-of-freedom model can be simplified as

$$\begin{aligned}\ddot{Y} &= \left(\frac{C_f + C_r}{Mv_X}\right)\dot{Y} + \left(\frac{l_fC_f - l_rC_r}{Mv_X} - v_X\right)\dot{\varphi} - \frac{C_f}{M}\delta_f \\ \ddot{\varphi} &= \left(\frac{l_fC_f - l_rC_r}{Iv_X}\right)\dot{Y} + \left(\frac{l_f^2C_f + l_r^2C_r}{Iv_X}\right)\dot{\varphi} - \frac{l_fC_f}{I}\delta_f.\end{aligned} \tag{5}$$

Remark 2.1. *Please note that the yaw motion of the vehicle in (5) is derived under the assumption that the tire slip angle is less than 4^o. Based on this assumption, the nonlinear self-aligning torque τ_e can be approximated as (4) in the linear region. Thereby, the simplified dynamics model (4) is limited to describe the actual self-aligning torque well under a wide range of vehicle conditions.*

Moreover, the transmission ratio between the steering motor and the front-wheels is

$$\frac{\delta_f}{\delta_m} = \frac{\dot{\delta}_f}{\dot{\delta}_m} = \frac{\ddot{\delta}_f}{\ddot{\delta}_m} = \frac{\tau_{12}}{\tau_s} = \frac{1}{\mu} \tag{6}$$

which together with (1)-(5) yields

$$J_e\ddot{\delta}_f + \mathcal{N}\left(\delta_f, \dot{\delta}_f\right) = \mu\tau_m \tag{7}$$

where $\mathcal{N}\left(\delta_f, \dot{\delta}_f\right) := \mu^2B_m\dot{\delta}_f + \tau_f + \tau_e$ and $J_e := J_f + \mu^2J_m$. Note that δ_f, $\dot{\delta}_f$, and $\ddot{\delta}_f$ represent the angular position, the velocity and the acceleration of the front-wheels, respectively.

For brevity, the dynamics of the SbW system (7) can be rewritten as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_0(x) + gu\end{aligned} \tag{8}$$

where $x = [x_1, x_2]^T := \left[\delta_f, \dot{\delta}_f\right]^T \in \mathbb{R}^2$, u is the control input which is represented as τ_m in (7), $f_0(x) := -\mathcal{N}(x)/J_e : \mathbb{R}^2 \to \mathbb{R}$ is an uncertain nonlinear function, and $g := \kappa/J_e$ is the parameter which is positive but unknown.

Control Objective: As shown in Figure 2, the main purpose of this paper is to design the PID control with fuzzy compensation for uncertain SbW system, such that the front-wheels steering angle x_1 can follow its reference signal y_d.

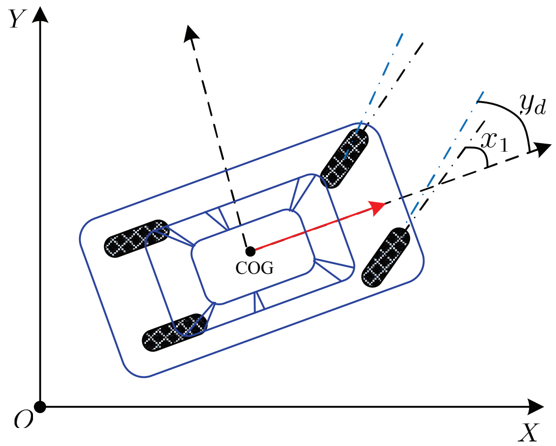


FIGURE 2. Control objective

2.2. **Fuzzy logic system.** Normally, the FLS mainly contains four parts: fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. The fuzzifier is considered as a mapping from the state space to the fuzzy sets. The fuzzy rule base consists of several linguistic rules based on the expert experience, such that

$$R_j\text{: If } x_1 \text{ is } F_1^j \text{ and } x_2 \text{ is } F_2^j \cdots \text{ and } x_r \text{ is } F_r^j, \text{ then } y_j \text{ is } G_j$$

where $x = (x_1, x_2, \ldots, x_r)^T \in \mathbb{R}^r$ is the input vector of FLS, and $y(x) \in \mathbb{R}$ is the output result of FLS. F_i^j $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m)$ and G_j represent fuzzy sets. The estimated output of FLS can be described as

$$y(x) = \frac{\sum_{j=1}^{m} \bar{y}^j \prod_{i=1}^{r} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{m} \prod_{i=1}^{r} \mu_{F_i^j}(x_i)} \tag{9}$$

where $\bar{y}^j$ is the center point at μ_{G^j} achieving its maximum value $\mu_{G^j}(\bar{y}^j) = 1$.

Define the basis function vector as $\theta = [\bar{y}^1, \ldots, \bar{y}^m]^T$ and the regressive vector is defined as $\xi(x) = [\xi^1(x), \ldots, \xi^m(x)]^T$ with

$$\xi^j(x) = \frac{\prod_{i=1}^{r} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{m} \prod_{i=1}^{r} \mu_{F_i^j}(x_i)}. \tag{10}$$

Thus, the output y can be rewritten as

$$y(x) = \theta^T \xi(x). \tag{11}$$

The following lemma is indispensable in the design of PID control and stability analysis of the control system.

Lemma 2.1. [16]*: For any continuous function $f(x)$: $\Omega \to \mathbb{R}$ such that $\Omega \subseteq \mathbb{R}$ is a compact set and an arbitrary small constant ϖ, there exists an FLS such that*

$$\sup_{x \in \Omega} \left| \hat{f}^*(x) - \theta^T \xi(x) \right| \le \varpi \tag{12}$$

in which $\hat{f}^*(x) = \theta^{*\mathrm{T}}\xi(x)$ *and* θ^* *is the optimal estimated parameter vector, which is defined as*

$$\theta^* = \arg\min\left(\sup_{x\in\Omega}\left|\hat{f}^*(x) - \theta^T\xi(x)\right|\right). \tag{13}$$

3. Main Results.

3.1. FLS-based PID control design. For convenience, we first define the tracking error and error vector of the system (8), that is

$$e_d = y_d - x_1 \tag{14}$$

$$e = [e_1, e_2, e_3]^T = \left[\int e_d \mathrm{d}t, e_d, \dot{e}_d\right]^T \tag{15}$$

in which y_d is the reference steering angle of the front-wheels. Note that y_d has always been considered a known smooth function in the study of SbW system [8-11]. Combining with (8), (14) and (15), one gets

$$\dot{e} = Ae - B\left(f(x) + u - \ddot{y}_d\right) \tag{16}$$

with $A = [0,1,0;0,0,1;0,0,0] \in \mathbb{R}^{3\times3}$, $B = [0,0,g]^T \in \mathbb{R}^{3\times1}$ and $f(x) = f_0(x)/g$. Then, based on the aforementioned dynamics, our control objective can be equivalent to making $\|e\|$ converge to the neighborhood of the origin when there exists unknown nonlinear function $f(x)$ in dynamics (16).

Considering the dynimics (16) with model uncertainties, as shown in Figure 3, the following FlS-based PID control is constructed in this paper, that is

$$u = k_p e_d + k_i \int e_d \mathrm{d}t + k_d \dot{e}_d + \ddot{y}_d - \hat{f}(x) \tag{17}$$

where k_p, k_i and k_d are designed as positive parameters and satisfy that $s^3 + k_d s^2 + k_p s + k_i$ is Hurwitz. Moreover, $\hat{f}(x)$ is the approximation result of FLS to the uncertain nonlinear function $f(x)$, i.e.,

$$\hat{f}(x) = \theta^T\xi(x). \tag{18}$$

The updated laws of θ is designed as

$$\dot{\theta} = -\frac{e^T P B_c \xi(x)}{\|e\| + \gamma} - \sigma\theta \tag{19}$$

in which $B_c = [0,0,1]^T \in \mathbb{R}^{3\times1}$, $P \in \mathbb{R}^{3\times3}$ is a positive-definite solution of the Lyapunov equation $PA_c + A_c^T P = -Q$ with $A = [0,1,0;0,0,1;-k_i,-k_p,-k_d] \in \mathbb{R}^{3\times3}$ and $Q = Q^T > 0$. γ and σ are positive design parameters.

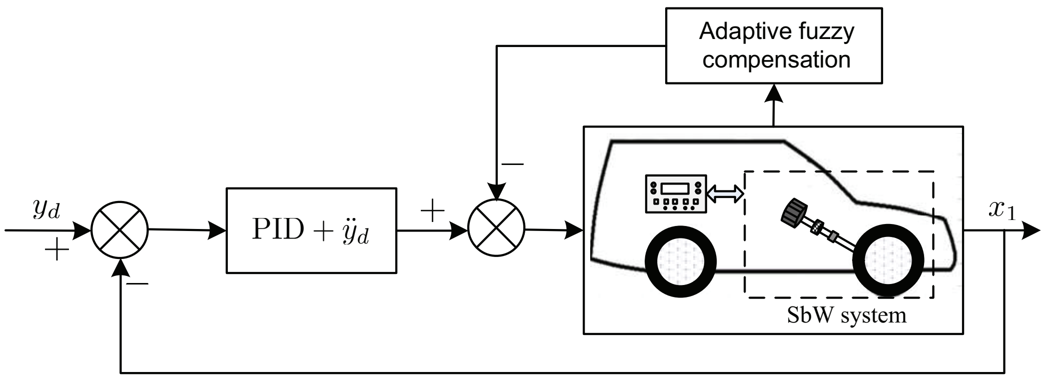


Figure 3. Controller structure diagram

3.2. **Main conclusions and analysis.** The following lemma gives the modeling performance of adaptive FLS for the uncertain nonlinearity.

Lemma 3.1. *Considering the updating laws* (19)*, the estimation error of θ has an unknown upper bound, i.e., there exists unknown constant $\bar{c}$ such that $\left\|\theta - \hat{\theta}\right\| \le \bar{c}$ for $t \ge 0$, with*

$$\bar{c} := \|\theta^*\| + \max\left(\frac{\lambda_{\max}(P)}{\sigma}, \|\theta(0)\|\right) \tag{20}$$

in which $\lambda_{\max}(P)$ is the maximum eigenvalue of matrix P, and $\theta(0)$ is the initial value of vector θ.

Proof: Consider the Lyapunov function as

$$V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T\tilde{\theta} \tag{21}$$

with $\tilde{\theta} = \theta - \theta^*$. Combining the fact that $\dot{\tilde{\theta}} = \dot{\theta}$, the time derivative of (21) along with (19) can be obtained

$$\begin{aligned} \dot{V}_{\tilde{\theta}} &= \tilde{\theta}^T\left(-\frac{e^TPB_c\xi(x)}{\|e\|+\gamma} - \sigma\theta\right) = -\sigma\tilde{\theta}^T\left(\tilde{\theta}+\theta^*\right) - \frac{e^TPB_c\tilde{\theta}^T\xi(x)}{\|e\|+\gamma} \\ &\le -\sigma\left\|\tilde{\theta}\right\|^2 + \sigma\left\|\tilde{\theta}\right\|\|\theta^*\| + \frac{\|e\|\|PB_c\|\|\xi(x)\|}{\|e\|+\gamma}\left\|\tilde{\theta}\right\|. \end{aligned} \tag{22}$$

Using $\|\xi(x)\| \le 1$ and $\|e\|/(\|e\|+\gamma) \le 1$, one can get

$$\begin{aligned} \dot{V}_{\tilde{\theta}} &\le -\sigma\left\|\tilde{\theta}\right\|^2 + \left(\sigma\|\theta^*\| + \lambda_{\max}(P)\right)\left\|\tilde{\theta}\right\| \\ &= -\sigma\left\|\tilde{\theta}\right\|\left[\left\|\tilde{\theta}\right\| - \left(\|\theta^*\| + \frac{\lambda_{\max}(P)}{\sigma}\right)\right] \end{aligned} \tag{23}$$

where $\lambda_{\max}(P)$ is the maximum eigenvalue of matrix P. It can be seen from (23) that $V_{\tilde{\theta}}$ is monotonically decreasing outside the set $\left\{\left|\tilde{\theta}\right| : \left|\tilde{\theta}\right| \le \|\theta^*\| + \lambda_{\max}(P)/\sigma\right\}$. This together with $\left\|\tilde{\theta}(0)\right\| \le \|\theta^*\| + \|\theta(0)\|$ yields

$$\left\|\tilde{\theta}\right\| \le \|\theta^*\| + \max\left(\frac{\lambda_{\max}(P)}{\sigma}, \|\theta(0)\|\right), \quad \forall t \ge 0. \tag{24}$$

From (24), we can see that $\left\|\tilde{\theta}\right\|$ is always bounded. The proof is complete. □

The following theorem is given the tracking performance of the PID control combined with fuzzy compensation.

Theorem 3.1. *Considering the uncertain system* (8)*, the proposed FLS-based PID control* (17) *can ensure tracking error of x_1 ultimately converge to an adjustable neighborhood of origin.*

Proof: Consider the Lyapunov function as

$$V_e = \frac{1}{2}e^TPe. \tag{25}$$

Substituting (17) into (16), one can get

$$\begin{aligned} \dot{e} &= A_ce + B\left(\hat{f}(x) - f(x)\right) \\ &= A_ce + B\left(\theta^T\xi(x) - \theta^{*T}\xi(x) - \omega\right) \end{aligned}$$

$$= A_c e + B\left(\tilde{\theta}^T \xi(x) - \omega\right) \tag{26}$$

with $\tilde{\theta} = \theta - \theta^*$ and $\omega = f(x) - \theta^{*T}\xi(x)$. Thus, the time derivative of (25) along with (26) can be obtained

$$\begin{aligned}\dot{V}_e &= \frac{1}{2}e^T P\left[A_c e + B\left(\tilde{\theta}^T \xi(x) - \omega\right)\right] \\ &\quad + \frac{1}{2}\left[A_c e + B\left(\tilde{\theta}^T \xi(x) - \omega\right)\right]^T Pe \\ &= \frac{1}{2}e^T (PA_c + A_c P)e + e^T PB\left(\tilde{\theta}^T \xi(x) - \omega\right).\end{aligned} \tag{27}$$

Using Lyapunov function $PA_c + A_c P = -Q$, one can get

$$\dot{V}_e = -\frac{1}{2}\lambda_{\min}(Q)\|e\|^2 + \lambda_{\max}(P)\left(\left\|\tilde{\theta}\right\| + \omega\right) \tag{28}$$

where the $\lambda_{\max}(P)$ and $\lambda_{\min}(Q)$ are the maximum eigenvalue of matrix P and the minimum eigenvalue of matrix Q, respectively. Combining with (28) and the conclusion of Lemma 2.1 and Lemma 3.1, one gets

$$\begin{aligned}\dot{V}_e &\le -\frac{1}{2}\lambda_{\min}(Q)\|e\|^2 + \lambda_{\max}(P)\left(\bar{c} + \varpi\right) \\ &\le -\frac{1}{2}\|e\|\left[\|e\| - \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\left(\bar{c} + \varpi\right)\right].\end{aligned} \tag{29}$$

It can be seen from (29) that V_e is monotonically decreasing outside the set $\{\|e\| : \|e\| \le \lambda_{\min}(Q)\left(\bar{c} + \varpi\right)/\lambda_{\max}(P)\}$. Thus, one can find that $\|e\|$ ultimately converges to the adjustable set $\{\|e\| : \|e\| \le \lambda_{\min}(Q)\left(\bar{c} + \varpi\right)/\lambda_{\max}(P)\}$. The proof is complete. □

Remark 3.1. *To make the process of proof more clear, the overall boundedness proof is divided into the approximation error and the tracking error. Lemma 3.1 gives the modeling performance of adaptive FLS for the uncertain nonlinearity. It can be noted that the approximation error of uncertain nonlinearity is bounded and adjustable. Meanwhile, the corresponding conclusion proved in Lemma 3.1 can be utilized in the proof of Theorem 3.1. Theorem 3.1 gives the tracking performance of the PID control combined fuzzy compensation, from which we can find that the tracking error of SbW system can asymptotically converge to the neighborhood of the origin despite of the uncertain nonlinearity existing.*

4. **Simulation and Experiment.**

4.1. **Numerical simulation.**

Step 1. Parameter choice for simulation model.

The parameters of (8) are chosen as $\mu = 18$, $J_{eq} = 4.934$ kg·m^2. $B_m = 0.018$ Nms/rad, $F_s = 2.68$ Nm. $I = 1300$ kg·m^2, $t_p = 0.023$ m, $t_m = 0.016$ m, $M = 2000$ kg, $l_f = 1.2$ m, $l_r = 1.05$ m, $C_f = C_r = -12000$ N/rad, $v_X = 10$ m/s. For convenience of simulation, the reference signal selected here is $y_d(t) = 0.4\sin(0.4t)$.

Step 2. Parameter choice for FLS-based PID controller.

After normalization of x_i $(i = 1, 2)$, the following membership functions are given as

$$\begin{aligned}\mu_{F_i^1}(x_i) &= \exp\left(-|x_i|^2/2\right) \\ \mu_{F_i^2}(x_i) &= \exp\left(-|x_i - 0.5|^2/2\right) \\ \mu_{F_i^3}(x_i) &= \exp\left(-|x_i - 1|^2/2\right).\end{aligned} \tag{30}$$

Select the parameters of controller (17) as

$$k_p = 240,\ k_i = 400,\ \text{and}\ k_d = 5 \tag{31}$$

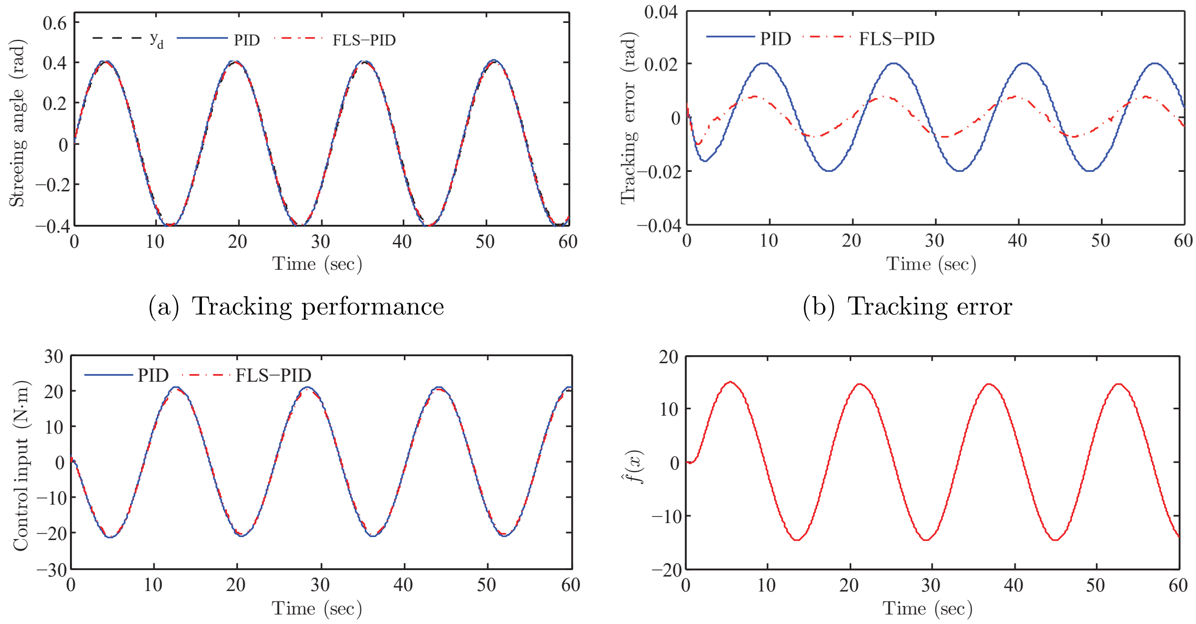
which make the matrix A_c stable. Selecting $Q = diag\,(10^7, 10^3, 10)$ and combined with the Lyapunov equation $PA_c + A_c^T P = -Q$, we get

$$P = 10^4 \times \begin{bmatrix} 315.8471 & 9.4600 & 1.2500 \\ 9.4600 & 0.8741 & 0.0396 \\ 1.2500 & 0.0396 & 0.0080 \end{bmatrix}.$$

Moreover, the initial conditions of the integrators are specified as $x_1(0) = x_2(0) = 0$ and $\theta = [0, 0, 0]^T$. The MATLAB command "ode23" is used to simulate the overall control system with step size 0.01.

Step 3. Comparison of control algorithms.

In order to verify the superiority of FLS-based PID controller designed in this paper, the desigend FLS-based PID controller is compared with traditional PID controller. To be fair, the same parameters (31) are used in traditional PID controller.



(a) Tracking performance (b) Tracking error

(c) Control input (d) Estimation result

Figure 4. Comparison results of FLS-PID controller and PID controller in simulation

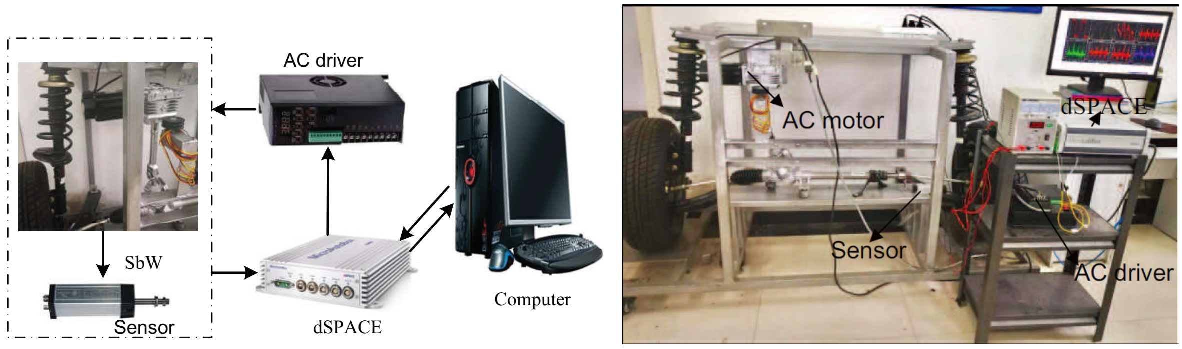
Step 4. Simulation results.

The simulation comparison results of the SbW system between the classic PID controller and the FLS-based PID controller are shown in Figure 4. Specifically, it can be seen from Figures 4(a) and 4(b) that the classical PID control can realize the steering control of SbW system after simple parameter tuning; however, the FLS-based PID controller significantly improves the control performance. Figure 4(c) shows the control input of SbW system under different control methods, from which we can find that the FLS can compensate the uncertainties for PID controller to some extent when the steering angle is larger at 12 seconds, 20 seconds, 28 seconds, 35 seconds, etc. Figure 4(d) shows the compensation effect of the FLS on PID control at the time points of 12 seconds, 20 seconds, 28 seconds, 35 seconds, etc. The above results show the obvious robustness of FLS under a varying reference angle.

4.2. **HiL test.**

Step 1. Establish the HiL test bench.

The HiL test platform of the SbW system is shown in Figure 5. In this platform, the controller (dSPACE-ds1202) is used as the control unit of the SbW system and the servo motor driver (XiNJE DS2-20P7) is used for driving the steering motor (XiNJE MS80ST-M02430B-20P7). The linear sensor (KTR11-10) fixed on the steering arm measures the steering angle of the front-wheels. A computer is applied to displaying the experimental results of the HiL test on-line and storing the experimental data of the HiL test.



(a) Schematic diagram of the test platform (b) Physical diagram of the test platform

FIGURE 5. The HiL test platform of SbW system

Step 2. Parameter choice for FLS-based PID control.

After normalization of x_i $(i = 1, 2)$, the following membership functions are given as (30). Select the parameter of controller (17) as

$$k_p = 160, \; k_i = 220, \text{ and } k_d = 5 \tag{32}$$

which make the matrix A_c stable. Select $Q = diag\left(10^5, 10^2, 10\right)$ and combine with the Lyapunov equation $PA_c + A_c^T P = -Q$. Thus, we can get

$$P = 10^2 \times \begin{bmatrix} 390.3088 & 18.8981 & 2.2727 \\ 18.8981 & 3.8131 & 0.1212 \\ 2.2727 & 0.1212 & 0.0342 \end{bmatrix}.$$

Moreover, the initial conditions of the integrators are specified as $\theta = [0, 0, 0]^T$ and the reference signal is selected as $y_d(t) = 0.4 \sin(0.4t)$.

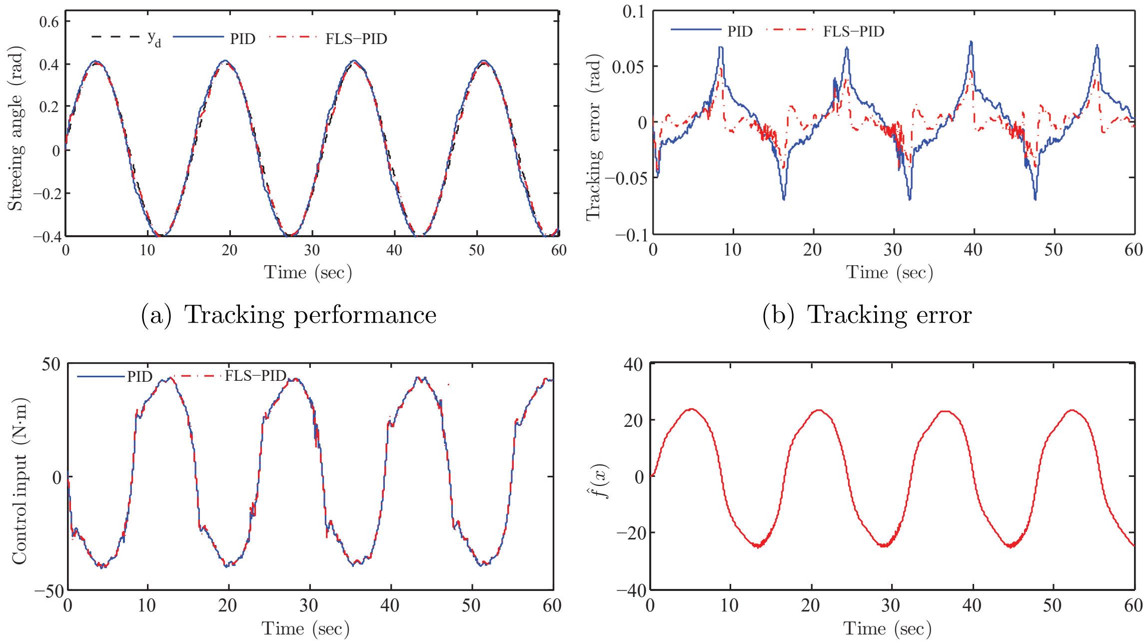
Step 3. Comparison of control algorithms.

In order to verify the superiority of FLS-based PID controller designed in this paper, the desigend FLS-based PID controller is compared with traditional PID controller. To be fair, the same parameters (32) are used in traditional PID controller.

Step 4. Experimental results.

The HiL experimental results of two controllers are shown in Figure 6. Figure 6(a) is the tracking performance. Figure 6(b) is the tracking error. Figure 6(c) is the control input and Figure 6(d) is the estimated result of the unknown function $f(x)$. The experimental results show that the proposed FLS-based PID controller is superior to the traditional PID controller in SbW system control and we can find that the FLS controller has a good practicability.

In the existing researches on SbW systems, the design of the controller requires that the friction torque and self-aligning torque model are known in advance, which is limited in practical application. In this study, the friction torque model and self-aligning torque model are regarded as lumped uncertainty and it is approximated by an FLS. Therefore, the corresponding assumption of the known friction torque model and self-aligning torque model can be relaxed.



(a) Tracking performance (b) Tracking error

(c) Control input (d) Estimation result

Figure 6. Comparison results of FLS-PID controller and PID controller in HiL tests

5. **Conclusions.** This paper presents an FLS-based PID control scheme for controlling of uncertain nonlinear SbW system. In our control algorithm, the adaptive fuzzy system term employs the tracking error to compensate the PID controller. Mathematically, we prove the stability of the closed-loop system and show that the outputs of the SbW system can follow the reference signals rapidly. Finally, the contrastive simulation and HiL test results have demonstrated favorable performances of our proposed controller. Our main shortage of this paper is the lack of road experiments with a real vehicle. In the future work, we will build a real vehicle platform of the SbW system to verify and improve the control method.

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