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A DEVICE-LIFE MODEL FOR RELIABILITY DEMONSTRATION TEST FOR A PRODUCT MADE UP OF A LARGE ARRAY OF ELECTRONIC DEVICES

Changmuk Kang

Department of Industrial and Information Systems Engineering
Soongsil University
369 Sangdo-ro, Dongjak-gu, Seoul 06978, Korea
ckang@soongsil.ac.kr

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Abstract. *This study considers a reliability demonstration test to estimate a failure-time distribution of a product, which is made up of a large array of electronic devices, such as a display made up of pixel devices, a solar panel made up of photo-voltaic cells, and a semiconductor memory chip made up of memory cells. As a product ages, its devices encounter failures individually, and finally the product malfunctions when the number of failed devices exceeds a certain threshold. This study proposes a device-life model that estimates a product-failure distribution from device-failure observations. Product-failure time is extrapolated from a device-failure distribution. Because device failures are much more frequently observed than product failures, a test can be finished earlier than a conventional test observing product failures. The proposed model is illustrated by an example of a dark spot defect of OLED (organic light-emitting diode) displays. In this numerical study, it estimates a distribution that closely follows an empirical distribution.*
Keywords: Reliability demonstration test, Device-life model, Large array of devices, Failure-time estimation, Frailty model

1. **Introduction.** This study considers a reliability demonstration test (RDT) to estimate a failure-time distribution of a product that is made up of a large array of electronic devices, such as a display panel made up of pixel devices, a solar panel made up of photo-voltaic cells, and a semiconductor memory chip made up of memory cells. As a product ages, its devices encounter failures individually. Whereas it is tolerable that a small portion of them fails, the whole product would fail when failed devices are more than a certain threshold. For example, an organic light-emitting diode (OLED) display suffers from the formation of dark spots, which are non-emissive pixel devices, and the growth of dark spots leads to total malfunction of a display panel [1].

A focus of RDT is to estimate the time until a product encounters a failure: a failure-time distribution. A conventional life test operates test samples for a certain period and estimates it from the observed failure time of the samples [2]. If some of the samples survive until the end of the test, they contribute to the estimation as censored data. It is unavailable, however, if most of the samples are censored. In the OLED case, a commercial display is usually required to guarantee over 30,000 hours of operation. None of such samples may fail during the usual test period of one or two thousand hours. Although a special test environment, e.g., high temperature and humidity, can be established for accelerating deterioration speed, it may still yield an insufficient number of failures, and it is not sure that the resulting estimation sustains for a decelerated environment.

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This study proposes a *device-life model*, which is a method to estimate a product-failure distribution from device-failure observations. Because device failures are more frequent than product failures, it may be possible to observe a sufficient number of failures for a relatively short test duration. A tester can extrapolate product-failure time by computing a quantile value that matches a product-failure threshold on the device-failure distribution.

There are two problems to solve to apply the device-life model. First, where a product has a large array of devices, a device-failure distribution is unidentifiable because of too many survived devices. For example, a 4K display panel has 3,840 $\times$ 2,160 $\times$ 3 (RGB) $\approx$ 24 millions of pixels. Theoretically, all of the devices in a product are a population to estimate a failure-time distribution of devices. Although a small portion of them have failed, other survived devices have censored failure times, which lead to almost an infinite expected value. To this end, this study introduces the concept of *failure-latent* (FL) devices and considers them as the total population of devices. The number of FL devices r is determined much smaller than the original number of devices to make the estimation identifiable. This study reveals that the product-failure time estimation is robust even if r is arbitrarily chosen.

Second, individual products are independent populations of devices, and they are heterogeneous in device reliability. This study takes the heterogeneity into account by adopting the *shared frailty model* [3]. This model assumes the hazard function of a device is a multiple of heterogeneous effect (frailty) of each product sample and a baseline hazard function. The device-failure distribution is estimated as a mixture of the frailty distribution and the baseline hazard function.

This study solves these two problems and provides a model to estimate a product-life distribution from short-period test data, in which no product fails at all. This model only requires a sufficient number of device failures that are frequently observed during a test. Thus, a tester can early terminate an RDT and save testing time related costs. The paper is organized as follows. Section 2 reviews the RDT literature including the degradation models. Section 3 describes the proposed model and estimation methods. The model is applied to an artificial test dataset for OLED dark spot defects in Section 4. Section 5 concludes this paper with limitations and future research directions.

2. **Literature Review.** The main purpose of the RDT is to estimate the failure-time distribution function of a whole product population from test samples. Meeker and Escobar [2] comprehensively review test planning and estimation methods. If exact failure times of all samples are given, it is rather simple to estimate distribution parameters and their confidence intervals [4, 5]. Unfortunately, most of RDT data is censored by test time (type I) or the number of failures (type II). A tester only knows that censored samples have survived until a certain moment rather than their exact failure time. Lawless's seminal work [6] provides a method for finding exact confidence intervals for log-location-scale distributions, e.g., Weibull and lognormal, with censored data. Wu [7] proposed general weighted moments estimator of the scale parameter for two-parameter exponential distributions.

Nevertheless, an RDT is extremely expensive and time-consuming ironically because commercial products are usually highly reliable. It usually takes a very long time to observe a sufficient number of failures to construct a reasonable confidence interval. When allowed test time is too short to observe sufficient failures, accelerated life tests (ALT) are used to demonstrate reliability. The ALT accelerates the failure time of a test sample by operating it in a high-stress environment like high temperature, humidity, and voltage [8]. The failure-time distribution estimated in an accelerated environment is extrapolated to a normal usage condition.

Another alternative is to model degradation of a key characteristic that may cause failure in the future. Failure time is predicted by when an estimated degradation path crosses a threshold that causes the total malfunction of a product. Degradation also can be tested in an accelerated environment. Such a test is called an accelerated degradation test (ADT). The ADT is widely used in practice to predict the reliability of electronic devices [9, 10]. Meeker et al. [11] describe how to model accelerated degradation data with a general degradation path and to estimate a failure-time distribution. A degradation model assumes unit-to-unit variability of degradation paths to take different failure time of individual products into account. Lu and Meeker [12] model the paths with random coefficients, and Bae and Kvam [13] extend it to nonlinear path forms. Other studies model degradation as a stochastic process of random increments rather than a parametric function [14, 15]. The stochastic process models also can incorporate random effects in their parameters [16, 17].

This study alternatively models the failure time of each device as an individual random variable. Failure-time distributions of devices are assumed homogeneous and independent within a product and heterogeneous between products. This assumption is implemented by the shared frailty model. The concept of frailty, which is random effect of an unobserved covariate, was proposed by Vaupel et al. [3]. Clayton [18] applies the concept to associating event times based on Cox's [19] regression model. In this model, a hazard function of device-failure time is separated into a baseline hazard and random frailty, which is different between products. Whereas this study utilizes the existing methods to estimate those distributions, it introduces an idea to resolve an unidentifiability problem arising from a large number of devices and provides a method to compute a distribution function of product-failure times triggered by the number of failed devices.

3. **Model.** Consider an RDT to estimate a cumulative distribution function $Q(t)$ of product-failure time T. The RDT is performed for n test products, each of which is equipped with M electronic devices, for S test periods. Some of the devices may fail as a product ages, and the product eventually malfunctions if more than u devices fail; a product-failure threshold is u. A device that has once failed can never be recovered or repaired. The test data gives us time point t_{ij} when a device j of product sample i fails. The total number of failed devices of product i is denoted by c_i. Assume that c_i is most likely less than u. Otherwise, if many product samples fail, $Q(t)$ can be directly estimated from the product-failure times. The proposed model first estimates a cumulative distribution function $F(t|z)$ of device-failure time variable τ, and extrapolates the distribution to compute the time when more than u devices fail. The underlying parameter z is assumed different between individual products taking account of their heterogeneity of reliability.

3.1. **Estimation of device-failure distribution.** Let us pick a sample product i whose product-specific parameter is z_i. In this section, will denote its device-failure distribution $F(t|z_i)$ is denoted by $F_i(t)$ for brevity. The distribution $F_i(t)$ is estimated by t_{ij}'s where $j = 1, \ldots, c_i$ and the information that $M - c_i$ devices survived until S. Where $F_i(t)$ has a parameter θ, its maximum likelihood estimator (MLE) θ^* is derived by (1).

$$\theta^* = \arg\max_{\theta} \prod_{j=1}^{c_i} f_i(t_{ij}|\theta) \cdot \bar{F}_i(S|\theta)^{(M-c_i)}, \tag{1}$$

where $f_i(t)$ is a derivative of $F_i(t)$, and $\bar{F}_i(t) = 1 - F_i(t)$. The first term is for devices that actually failed and the second term is for devices that survived until S.

A problem rises where M is too large. As mentioned above, a display panel has 24 million pixel devices. Then the second likelihood term goes to 0 for any θ; it is not identifiable. In order to resolve the problem, this study replaces M with the number of FL devices r. It is assumed that only a portion of the devices are latent to fail, so r is much less than M. Then Equation (1) becomes identifiable. This concept is physically meaningful because some failure is subject to a device's own condition. In the example of the dark spot defect, foreign particles intruded into a manufacturing process are suspected as its cause; only particle-intruded pixels are latent to fail.

The number r is usually unknown in practice. It is impossible to count dark-spot-latent OLEDs because intruded particles are too small (< 1 μm) to detect. This study suggests choosing an arbitrary number for r, instead. Although estimated parameter θ^* strongly depends on r, the estimated failure time of product i is fairly robust to how large it is. Let θ_r^* be an estimate of θ assuming r. Upon this estimation, product i is expected to fail at t such that $F_i(t|\theta_r^*) = u/r$. A more slowly growing, i.e., more reliable, $F_i(t|\theta_r^*)$ is estimated from a larger r value. At the same time, a larger r decreases the right-hand-side ratio and makes the failure time earlier. These compensating effects keep its failure time robust to any r.

This study numerically shows the robustness by estimation results on simulated data, as illustrated in Figure 1. For a single experiment, 1,000 device failure times are randomly generated from a Weibull distribution, and only the earliest 10 failures are chosen assuming other failures are censored by limited test time. The distribution parameters are estimated from this data varying r values. Where $T^*(r) = F_i^{-1}(u/r|\theta_r^*)$ is the extrapolated product failure time from the estimated parameter θ_r^*, a ratio of $T^*(r)/T^*(1{,}000)$ presents their similarity. Figure 1 shows distributions of the ratio with 1,000-times repetition of the experiments. It reveals that the failure time is on-average longer estimated for smaller r, but the difference is under 7% in the worst case. Moreover, the susceptibility to r exponentially diminishes as r grows. It also shows that shape parameter γ affects the robustness. An increasing failure rate distribution ($\gamma = 2$) is more robust than a decreasing failure rate distribution ($\gamma = 0.5$).

3.2. **Estimation of product-failure distribution.** First, derive product-failure distribution $Q(t)$ with device-failure distribution $F(t|z)$ and heterogeneity distribution $G(z)$

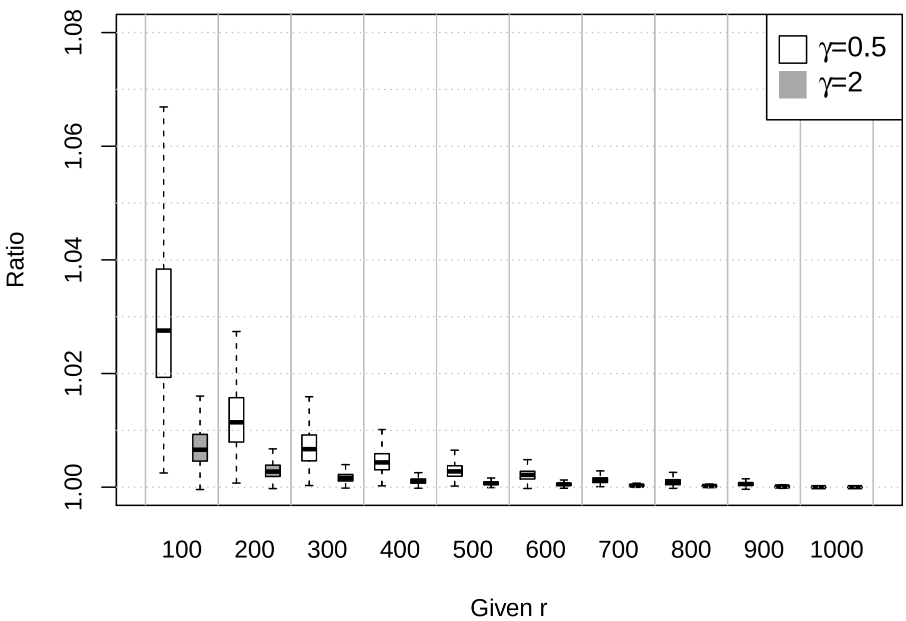

Figure 1. Distributions of the ratio of $T^*(r)/T^*(1{,}000)$

from which z is drawn. Function $Q(t)$ is the probability that a product fails earlier than t. A product fails when u devices fail out of r FL devices. Define a sample cumulative proportion of failed devices until t as $\hat{F}(t|z)$. Then probability of $P[T \leq t]$ is the probability that $\hat{F}(t|z)$ is larger than u/r. Then, $Q(t)$ is derived by Equations (2)-(4).

$$Q(t) = P[T \leq t] = \int_{\Omega} P[T \leq t|z] \cdot g(z)dz \tag{2}$$

$$= \int_{\Omega} P\left[\hat{F}(t|z) > \frac{u}{r}\right] \cdot g(z)dz \tag{3}$$

$$= \int_{\Omega} \bar{\Phi}\left(\frac{\sqrt{r}\{u/r - F(t|z)\}}{\sqrt{F(t|z)\bar{F}(t|z)}}\right) \cdot g(z)dz, \tag{4}$$

where Ω is the support of z, $g(z)$ is the density function of $G(z)$, and $\bar{\Phi}(x)$ is the complimentary cumulative probability of the standard normal distribution. Equations (3) and (4) hold because $\sqrt{r}\hat{F}(t|z)$ follows a normal distribution of mean $F(t|z)$ and variance $F(t|z)\bar{F}(t|z)$. A random variable $\hat{F}(t|z)$ is a sample mean of a Bernoulli random variable $I_{\{\tau \leq t|z\}}$ (whether a device of frailty z fails or not before t) for r devices. Where expectation and variance of $I_{\{\tau \leq t|z\}}$ are $F(t|z)$ and $F(t|z)\bar{F}(t|z)$, respectively, $\sqrt{r}\hat{F}(t|z)$ asymptotically follows the normal distribution by the central limit theorem.

To model $F(t|z)$, this study adopts the shared frailty survival model where z represents frailty of an individual product. The term frailty means an unobserved random effect shared by subjects with similar risks in the analysis of mortality rates [18]. In this study, devices are subjects, and their risks of failure are assumed similar within the same product. Where frailty is conditioned, the failure time of an individual device is assumed independently and identically distributed. The model forms the hazard function $\lambda(t|z) = f(t|z)/\bar{F}(t|z)$ as a multiple of frailty z and the baseline hazard function $\lambda_0(t)$; $\lambda(t|z) = z\lambda_0(t)$. The frailty z is drawn from a random variable $Z \sim G(z)$. Distribution $G(z)$ is also estimated from the test samples. The value of $Q(t)$ is numerically evaluated by the estimation on $G(z)$ and $\lambda_0(t)$.

This study considers $G(z)$ and $\lambda_0(t)$ as parametric distributions, in order to extrapolate future device failures. Their parameters are estimated by maximizing the likelihood of the data [20]. Let $\lambda_0(t)$ have a parameter vector $\boldsymbol{\theta}$ and $G(z)$ have variance σ^2. Its mean value is normalized to 1 since frailty multiplies the hazard. First, define a conditional likelihood of device-failure observations t_{ij}'s for given z. Substituting $y_{ij} = t_{ij}$, $\delta_{ij} = 1$ for $j = 1, \ldots, c_i$ and $y_{ij} = S$, $\delta_{ij} = 0$ for $j = c_i + 1, \ldots, r$, rewrite the likelihood of (1) to (5). The first term becomes a hazard function by dividing the density by the complementary cumulative probability. Then the conditional likelihood is given by (7).

$$L_i(\boldsymbol{\theta}|z) = \prod_{j=1}^{r} f_i(y_{ij}|\boldsymbol{\theta})^{\delta_{ij}} \bar{F}_i(y_{ij}|\boldsymbol{\theta})^{(1-\delta_{ij})} \tag{5}$$

$$= \prod_{j=1}^{r} \left\{\frac{f_i(y_{ij}|\boldsymbol{\theta})}{\bar{F}_i(y_{ij}|\boldsymbol{\theta})}\right\}^{\delta_{ij}} \bar{F}_i(y_{ij}|\boldsymbol{\theta}) \tag{6}$$

$$= \prod_{j=1}^{r} \{z\lambda_0(y_{ij}|\boldsymbol{\theta})\}^{\delta_{ij}} e^{-z\Lambda_0(y_{ij}|\boldsymbol{\theta})}, \tag{7}$$

where cumulative hazard function $\Lambda_0(t) = \int_0^x \lambda_0(x)dx$. The marginal likelihood function of the whole data is $L(\boldsymbol{\theta}, \sigma) = \prod_{i=1}^{n} \int_{\Omega} L_i(\boldsymbol{\theta}|z) \cdot g(z|\sigma)dz$. The likelihood function is derived for various forms of baseline hazard and frailty distributions in the literature. This study uses the `parfm` package in `R` that implements MLE of log-location-scale baseline hazard distributions and gamma, inverse Gaussian, and lognormal frailty distributions [21].

Estimation on $Q(t)$ is affected by sampling bias. This study computes its confidence intervals by a simulation-based bootstrap method [2]. The bootstrap method draws independent samples from an estimated failure-time distribution and makes another estimation for each sample. The estimated parameters from the newly drawn samples provide confidence intervals of the original estimate. Where $\boldsymbol{Q}_B$ is an empirical set of newly estimated $Q(t)$ values from the bootstrap samples, $100(1-\alpha)\%$ confidence interval of $Q^*(t)$ is a range between $\alpha/2$ and $(1-\alpha/2)$ quantiles of $\boldsymbol{Q}_B$.

4. **Illustrative Example.** The numerical example shows how to estimate product life distribution by the proposed device-life model. This study generates an artificial data set mimicking a dark-spot defect test for OLED displays. A display panel is an assembly of millions of pixels emitting light. A panel fails if more than $u = 15$ pixels turn into dead pixels (dark spots). The RDT is conducted for $S = 1{,}500$ hours in a certain accelerated environment. For $n = 20$ sample panels, frailty z_i, $i = 1, \dots, n$ is drawn from a lognormal distribution with $\mu = 0$ and $\sigma = 0.5$. Each panel i has 100 FL devices, and their failure time is generated from a hazard function $\lambda(t|z) = z_i\lambda_0(t)$, where $\lambda_0(t)$ is a Weibull hazard function with shape parameter $\gamma = 2.5$ and scale parameter $\alpha = \exp(8.5)$. Increases of dark spots are illustrated in Figure 2. Connected dots represent a single panel. The estimation procedure only uses pixel failure times earlier than 1,500 hours. Its fitness is evaluated by the full data of 3,000 hours.

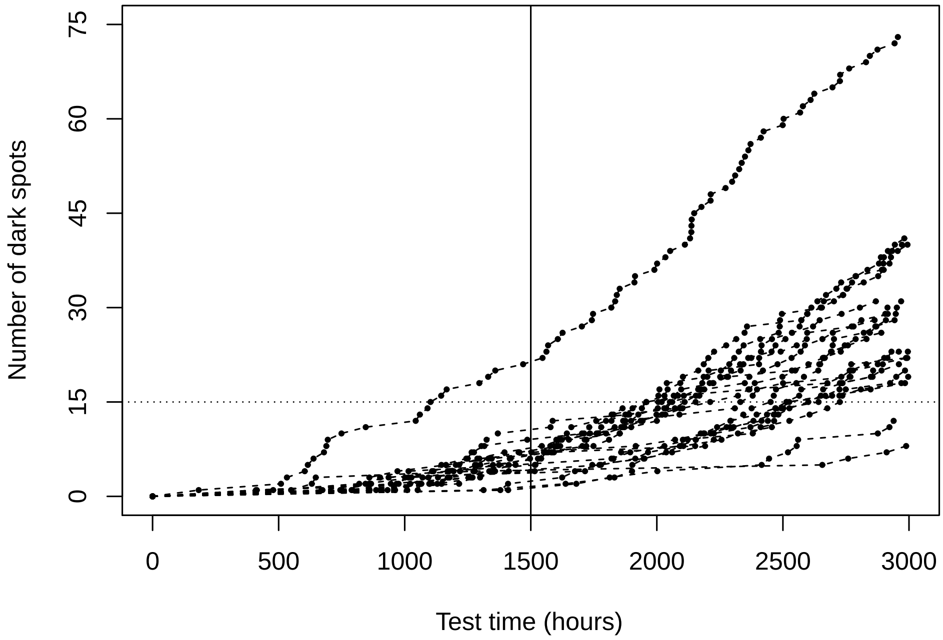

FIGURE 2. Increase of dark spots for 20 sample panels (artificial data)

The distribution parameters are estimated as $\sigma^* = 0.517$, $\alpha^* = \exp(8.399)$ and $\gamma^* = 2.705$ using `parfm` package in `R`. The number of FL devices r is assumed as 100 devices. The cumulative product-failure probability $Q(t)$ is numerically evaluated by (4). Figure 3 compares the estimated $Q(t)$ with the empirical and theoretical probabilities. The empirical one shows the time when each panel forms more than fifteen dark spots, and the theoretical one is evaluated from the true parameter values. The estimated curve is close to the empirical and theoretical curves. Confidence intervals are found by simulating

$B = 10{,}000$ bootstrap samples. Each sample k is constructed by simulating 100 dark spots of 20 panels from parameter values of σ^*, α^*, and γ^*. The probability $Q_k^*(t)$ is re-estimated for each sample. Among $\{Q_k^*(t)|k = 1, \ldots, 10{,}000\}$, the smallest 250th and 9,750th values are lower and upper bounds of the 95% confidence interval, respectively.

Figure 3 also shows $Q(t)$ estimated by a random-effect degradation path model. The degradation path is specified by a Weibull cumulative probability function with a normal random scale parameter. It greatly under-estimates failure probabilities. This example shows that the device-life model is a good alternative for the data that a degradation model poorly fits for.

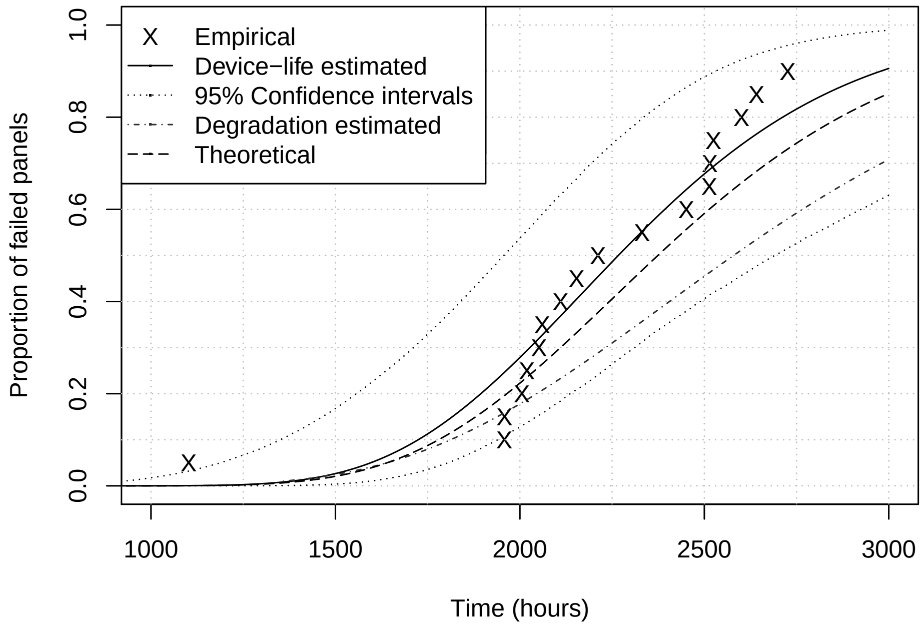

FIGURE 3. Empirical, estimated and theoretical $Q(t)$'s

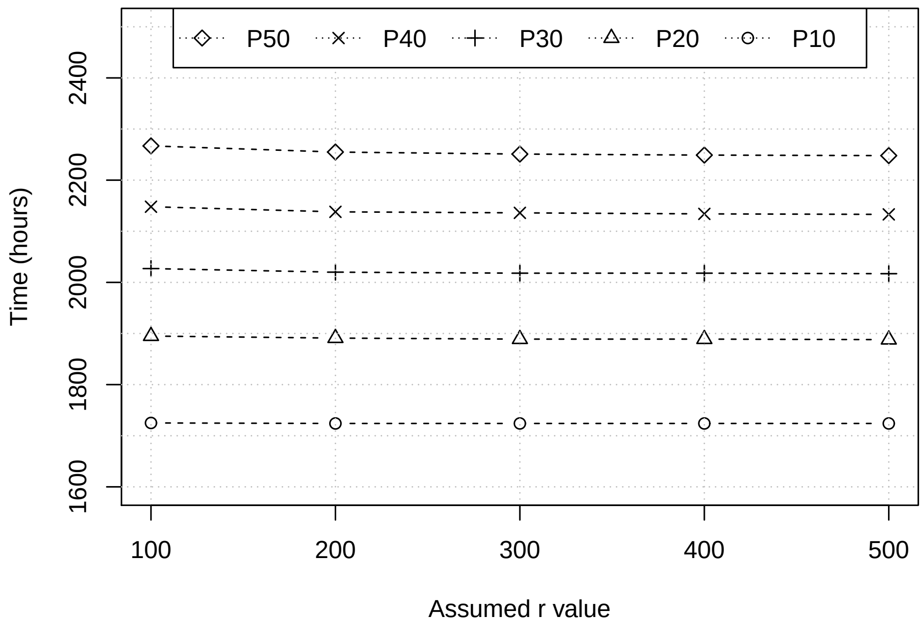

FIGURE 4. Estimated product-failure percentiles for different values of r

Finally, whereas the above estimation assumes $r = 100$, the result is robust to how large it is. Section 3.1 already has shown that the expected product-failure time is robust to r. Figure 4 confirms it by comparing estimated percentile values Px's, the time until which x percent of products have failed, for $r = 100$ to 500. They are almost the same.

5. **Conclusion.**

The RDT for estimating a failure-time distribution of products may require an extremely long time to observe a sufficient number of failures. It is not allowed for the timely introduction of a new product. This study proposes the device-life model that can reduce test time by observing failures of devices rather than products. Because devices much more frequently fail than products, shorter time is required to observe failures. It first estimates device-life distributions for heterogeneous products and extrapolates them to compute a product-life distribution. A tester can early terminate a test if a sufficient number of device have failed even if no product has failed at all. Although the existing degradation model also extrapolates product-failure time from a degrading measure, the proposed model provides a different specification for the data that any degradation model poorly fits for. The illustrative case in Section 4 shows such an example.

This model adopts the shared frailty model to accommodate heterogeneity between products. It first jointly estimates a baseline device-failure distribution and a frailty distribution, and next evaluates a cumulative probability function of product-failure time. The device-failure distribution is unidentifiable if a large number of devices are all included in a population to observe a failure. This study resolves this problem by introducing FL devices and shows that the product-failure distribution is robust to any given number of FL devices. The proposed model is widely applicable to any baseline and frailty distributions for which parametric estimators are known.

Whereas this study assumes a unified test environment, the model is easily extendable to accelerate test environments by adding deterministic hazard coefficients to the device-life distribution. In future research, competing failure modes could be modeled as frailty factors as well as the product heterogeneity.

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REFERENCES

[1] M. M. Azrain, M. R. Mansor, S. H. S. M. Fadzullah, G. Omar, D. Sivakumar, L. M. Lim and M. N. A. Nordin, Analysis of mechanisms responsible for the formation of dark spots in organic light emitting diodes (OLEDs): A review, *Synthetic Metals*, vol.235, pp.160-175, 2018.
[2] W. Q. Meeker and L. A. Escobar, *Statistical Methods for Reliability Data*, John Wiley & Sons, 1998.
[3] J. W. Vaupel, K. G. Manton and E. Stallard, The impact of heterogeneity in individual frailty on the dynamics of mortality, *Demography*, vol.16, no.3, pp.439-454, 1979.
[4] D. R. Thoman, L. J. Bain and C. E. Antler, Inferences on the parameters of the Weibull distribution, *Technometrics*, vol.11, no.3, pp.445-460, 1969.
[5] D. R. Thoman, L. J. Bain and C. E. Antler, Maximum likelihood estimation, exact confidence intervals for reliability, and tolerance limits in the Weibull distribution, *Technometrics*, vol.12, no.2, pp.363-371, 1970.
[6] J. F. Lawless, Confidence interval estimation for the Weibull and extreme value distributions, *Technometrics*, vol.20, no.4, pp.355-364, 1978.
[7] S.-F. Wu, Prediction interval of the future observations of the two-parameter exponential distribution under multiply type II censoring, *ICIC Express Letters*, vol.13, no.11, pp.1073-1077, 2019.
[8] L. A. Escobar and W. Q. Meeker, A review of accelerated test models, *Statistical Science*, vol.21, no.4, pp.552-577, 2006.

[9] S. Saxena, Y. Xing, D. Kwon and M. Pecht, Accelerated degradation model for C-rate loading of lithium-ion batteries, *International Journal of Electrical Power & Energy Systems*, vol.107, pp.438-445, 2019.

[10] F. Haghighi and S. J. Bae, Reliability estimation from linear degradation and failure time data with competing risks under a step-stress accelerated degradation test, *IEEE Trans. Reliability*, vol.64, no.3, pp.960-971, 2015.

[11] W. Q. Meeker, L. A. Escobar and C. J. Lu, Accelerated degradation tests: Modeling and analysis, *Technometrics*, vol.40, no.2, pp.89-99, 1998.

[12] C. J. Lu and W. O. Meeker, Using degradation measures to estimate a time-to-failure distribution, *Technometrics*, vol.35, no.2, pp.161-174, 1993.

[13] S. J. Bae and P. H. Kvam, A nonlinear random-coefficients model for degradation testing, *Technometrics*, vol.46, no.4, pp.460-469, 2004.

[14] N. D. Singpurwalla, Survival in dynamic environments, *Statistical Science*, vol.10, no.1, pp.86-103, 1995.

[15] J. Zhang, X. Ma and Y. Zhao, Reliability demonstration for long-life products based on hardened testing method and gamma process, *IEEE Access*, vol.5, pp.19322-19332, 2017.

[16] J. Lawless and M. Crowder, Covariates and random effects in a gamma process model with application to degradation and failure, *Lifetime Data Analysis*, vol.10, no.3, pp.213-227, 2004.

[17] S. Tang, X. Xu, C. Yu, X. Sun, H. Fan and X.-S. Si, Remaining useful life prediction with fusing failure time data and field degradation data with random effects, *IEEE Access*, vol.8, pp.11964-11978, 2019.

[18] D. G. Clayton, A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence, *Biometrika*, vol.65, no.1, pp.141-151, 1978.

[19] D. R. Cox, Regression models and life-tables, *Journal of the Royal Statistical Society: Series B (Methodological)*, vol.34, no.2, pp.187-202, 1972.

[20] A. Wienke, *Frailty Models in Survival Analysis*, CRC Press, 2010.

[21] M. Munda, F. Rotolo, C. Legrand et al., parfm: Parametric frailty models in R, *Journal of Statistical Software*, vol.51, no.11, pp.1-20, 2012.