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OPTIMAL OFFLINE MPC DESIGN: OUTPUT FEEDBACK

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Abstract. *The paper deals with a problem of model predictive controller design, with a unique structure of output feedback (observer-computed state vector) and optimal offline design. The obtained controller ensures the stability of the closed-loop system with a guaranteed cost for the given prediction horizon. The model predictive control strategy is based on calculating the offline output feedback controller gain and designing the online plant observer to compute the system's next state for predicted outputs and inputs. Finally, a numerical example illustrates the effectiveness of the proposed method.*
Keywords: Predictive control, Output feedback, Model based control

1. **Introduction.** Model predictive control (MPC) is desirable for discrete-time dynamic systems. MPC is one of the most popular control methods at the moment. The predictive algorithm computes the control variable by minimizing the quadratic cost function, while considering the expected value of future errors along the given prediction horizon. Besides the classical PID control, predictive control is declared as the second most accepted practical algorithm nowadays. It is mainly due to its ability to take all constraints directly into the sample account. In MPC, the future control inputs are computed at each sampling time by solving the optimization problem [1-11,16-18]. By using the discrete-time process model, the future behavior of the process output is predicted. On the base of control errors, the set of future input signals is calculated on-line at each step by minimizing the quadratic cost function, while assuming that there are constraints on the control variables, as well as on the controlled ones. According to the receding horizon predictive control strategy, only the first input variable of the sequence calculated at time t is applied. Other input variables are used to calculate predictive output variables, this procedure being repeated at each instant. Therefore, assuming the presence of accuracy of the plant model, it is a necessary condition for MPC of any real system. The MPC approach requires solving the constrained optimization problem in each sampling period (which is significant) and limits this algorithm's practical applicability to relatively slow dynamics. The main criticism related to MPC could be used for slow dynamics to bring the following design of the tube-based output feedback MPC [12]. The explicit MPC [1] for all the types of constraints leads to an inappropriately high number of the control regions. For the case of a second-order system, the resulting control algorithm is in more than 189 regions. In this paper, by using a sequential model predictive controller design procedure, all calculations of a control sequence minimizing an objective function with the given constraints on the model predictive controller are set up offline sequentially and only the predictive controller inputs are calculated online. Due to the definition of the

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Bellman-Lyapunov function, the stability robustness and performance of the closed-loop system are guaranteed [13].

The paper is organized as follows. Section 2 presents the preliminaries and problem formulation. Section 3 brings the main results of the optimal offline and output feedback sequential MPC controller design procedure, and Section 4 shows the effectiveness of the proposed method using a numerical example. The predictive controller design has been performed using YALMIP with PENBMI solver [14]. The paper is closed by concluding remarks in Section 5.

The following notation is used in this paper. A symmetric matrix $P_k = P_k^T > 0$ ($P_k \geq 0$) denotes the positive definiteness (semi-definiteness) of the matrix. Given two symmetric matrices P_k, Q_k, the inequality $P_k > Q_k$ indicates that $P_k - Q_k > 0$. In predictive control, the notation $x(t+k)$ will be used to define k-steps ahead-prediction of the system variable x at the time t under specified initial state and input scenario. Notation $y(t+k|t)$ for $k = 1, 2, \ldots$, will be used to define the expected value of predicted output with available information at instant t. I denotes the identity matrix of corresponding dimensions.

2. **Problem Formulation and Preliminaries.** Consider a time-invariant linear discrete-time system that is defined in the state-space domain

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are state, control and output variables of the system respectively, A, B, C are known constant matrices of corresponding dimensions, and (A, B) is a controllable pair.

For the first step of sequential predictive controller design procedure, the cost function to be minimized is

$$\begin{aligned} J_1 &= \sum_{t=0}^{\infty} J_1(t) \\ J_1(t) &= x(t)^T Q_1 x(t) + u(t)^T R_1 u(t) \end{aligned} \tag{2}$$

where $Q_1 \geq 0$, $R_1 > 0$ are weighting matrices of the respective dimensions.

This paper aims to design a model predictive controller for the given control N_u and prediction N horizons ($N_u = N$). The control algorithm is in the form

$$u(t+k-1) = \sum_{i=1}^{k+1} F_{ki} y(t+i-1) \tag{3}$$

where $F_{ki} \in R^{m \times l}$ and k is the given prediction horizon. The matrix F_{ki} is output (state) feedback gain matrix to be determined by minimization of the cost function (2). For sequential design in the first step ($k = 1$), one starts with the following control algorithm

$$u(t) = F_{11} y(t) + F_{12} y(t+1) \tag{4}$$

or

$$u(t) = (I - F_{12}CB)^{-1}(F_{11}C + F_{12}CA)x(t) = K_1 x(t) \tag{5}$$

where F_{11} and F_{12} are output (state) feedback gain matrices to be determined in such a way that the cost function (2) is optimal with respect to system variables.

Substitution of the control algorithm (4) to (1) for the first step of design procedure gives

$$x(t+1) = Ax(t) + B(F_{11}Cx(t) + F_{12}Cx(t+1)) \tag{6}$$

which leads to

$$\begin{aligned} M_1x(t+1) - A_{c1}x(t) &= 0 \\ x(t+1) = M_1^{-1}A_{c1}x(t) &= D_1x(t) \end{aligned} \tag{7}$$

with

$$\begin{aligned} M_1 &= I - BF_{12}C \\ A_{c1} &= A + BF_{11}C \end{aligned}$$

The first step design procedure requires calculation of the controller gain matrices F_{11}, F_{12} by minimizing the cost function (2). Considering the above results, let us introduce the following Bellman-Lyapunov equation.

Lemma 2.1. *Consider the system* (1) *with control algorithm* (4)*. The control algorithm* (4) *is the guaranteed cost control law for the closed-loop system if and only if there exists Lyapunov function* $V_1(t) = x(t)^TP_1x(t)$ *such that for its first difference* $\Delta V_1(t)$ *holds*

$$B_{e1} = \min_{u(t)}\{\Delta V_1(t) + J_1(t) < 0\} \tag{8}$$

Note that for chosen designer Lyapunov function iff might be reduced to if.

3. **Sequential Model Predictive Controller Design.**

In this section, the new MPC design procedure is obtained in the form of sequential mode. At each step of MPC design procedure $k = 1, 2, \ldots, N$ under the given quadratic cost function constraints and Bellman-Lyapunov function, the predictive controller gains are obtained. The obtained results are guaranteed the optimal value of the closed-loop system performance and stability for $k = 1, 2, \ldots, N$. The detail of output feedback MPC sequential design procedure is given as follows.

3.1. **First step.**

In this section, the new BMI design procedure is given, in order to calculate the gain controllers F_{11}, F_{12}. Let Lyapunov function in (8) be $V_1(t) = x(t)^TP_1x(t)$, and its first difference is given as

$$\Delta V_1(t) = \begin{bmatrix} x(t+1)^T & x(t)^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & -P_1 \end{bmatrix} \begin{bmatrix} x(t+1) \\ x(t) \end{bmatrix} \tag{9}$$

If (9) with the closed-loop system (7) is negative definite, the closed-loop system will be stable.

Let us assume that the Lyapunov matrix from system matrices can be split as follows

$$\begin{bmatrix} x(t+1)^T & x(t)^T \end{bmatrix} \begin{bmatrix} 2N_{11}^T \\ 2N_{21}^T \end{bmatrix} \begin{bmatrix} M_1 & -A_{c1} \end{bmatrix} \begin{bmatrix} x(t+1) \\ x(t) \end{bmatrix} = 0 \tag{10}$$

where two auxiliary matrices $N_{11}, N_{21} \in R^{n\times n}$ are included. For the sake of achieving of the required performance (2), the following formula will be obtained

$$\begin{aligned} J_1(t) &= \begin{bmatrix} x(t+1)^T & x(t)^T \end{bmatrix} \begin{bmatrix} C^TF_{12}^TR_1F_{12}C & C^TF_{12}^TR_1F_{11}C \\ C^TF_{11}^TR_1F_{12}C & Q_1 + C^TF_{11}^TR_1F_{11}C \end{bmatrix} \begin{bmatrix} x(t+1) \\ x(t) \end{bmatrix} \\ v_1(t)^T &= \begin{bmatrix} x(t+1)^T & x(t)^T \end{bmatrix} \end{aligned} \tag{11}$$

where (4) is substituted to (2) and (11) is obtained after small manipulation.

Summarizing (9) and (10) with the obtained results and substituting (11) to (8) lead to

$$B_{e1} = v_1(t)^TW_1v_1(t) < 0 \tag{12}$$

where $W_1 = \{w_{1ij}\}_{2\times 2}$ and

$$\begin{aligned} w_{111} &= P_1 + N_{11}^T M_1 + M_1^T N_{11} + C^T F_{12}^T R_1 F_{12} C \\ w_{112} &= -N_{11}^T A_{c1} + M_1^T N_{21} + C^T F_{12}^T R_1 F_{11} C \\ w_{122} &= -P_1 - N_{21}^T A_{c1} - A_{c1}^T N_{21} + Q_1 + C^T F_{11}^T R_1 F_{11} C \end{aligned}$$

Note that the following condition should apply for the sake of obtaining optimal controller parameters: the trace of matrix P_1 must be the minimum value. From (12) the results are obtained for the controller parameter (4) for $k = 1$.

3.2. **Second step.** The second step ($k = 2$) of design procedure leads to

$$x(t+2) = Ax(t+1) + Bu(t+1) \tag{13}$$

From (3) will be obtained

$$\begin{aligned} u(t+1) &= F_{21}y(t) + F_{22}y(t+1) + F_{23}y(t+2) \\ &= F_{21}Cx(t) + F_{22}CD_1x(t) + F_{23}Cx(t+2) \end{aligned} \tag{14}$$

Then, the closed-loop system is

$$x(t+2) = AD_1x(t) + B(F_{21}Cx(t) + F_{22}CD_1x(t) + F_{23}Cx(t+2)) \tag{15}$$

or

$$M_2x(t+2) - A_{c2}x(t) = 0$$

where

$$\begin{aligned} M_2 &= I - BF_{23}C \\ A_{c2} &= AD_1 + B(F_{21}C + F_{22}CD_1) \end{aligned}$$

The quadratic cost function for the second step is

$$J_2(t) = x(t)^T Q_2 x(t) + u(t+1)^T R_2 u(t+1)$$

Substituting control algorithm from (14) will be obtained

$$J_2(t) = \begin{bmatrix} x(t+2)^T & x(t)^T \end{bmatrix} \begin{bmatrix} P_{e21}^T R_2 P_{e21} & P_{e21}^T R_2 P_{e22} \\ P_{e22}^T R_2 P_{e21} & Q_2 + P_{e22}^T R_2 P_{e22} \end{bmatrix} \begin{bmatrix} x(t+2) \\ x(t) \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} P_{e21} &= F_{23}C \\ P_{e22} &= F_{21}C + F_{22}CD_1 \end{aligned}$$

Using the same argument as for (9), (10) leads to

$$v_2(t)^T = \begin{bmatrix} x(t+2)^T & x(t)^T \end{bmatrix}$$

$$v_2(t)^T \begin{bmatrix} 2N_{12}^T \\ 2N_{22}^T \end{bmatrix} \begin{bmatrix} M_2 & -A_{c2} \end{bmatrix} v_2(t) = 0 \tag{17}$$

and the first difference of V_2 can be written as

$$\Delta V_2(t) = v_2(t)^T \begin{bmatrix} P_2 & 0 \\ 0 & -P_2 \end{bmatrix} v_2(t) = 0 \tag{18}$$

When substituting the obtained results to the Bellman-Lyapunov equation for the second step, the following inequality is obtained

$$B_{e2} = v_2(t)^T W_2 v_2(t) < 0 \tag{19}$$

where $W_2 = \{w_{2ij}\}_{2\times 2}$ and

$$\begin{aligned} w_{211} &= P_2 + N_{12}^T M_2 + M_2^T N_{12} + P_{e21}^T R_2 P_{e21} \\ w_{212} &= -N_{12}^T A_{c2} + M_2^T N_{22} + P_{e21}^T R_2 P_{e22} \\ w_{222} &= Q_2 + P_{e22}^T R_2 P_{e22} - D_1^T P_2 D_1 \end{aligned}$$

The following matrices are calculated in the second step: three control gain matrices F_{21}, F_{22}, F_{23}, matrices N_{12}, N_{22} and $P_2 > 0$.

The following results are obtained for the control law

$$u(t+1) = (I - F_{23}CB)^{-1}(F_{21}C + F_{22}CD_1 + F_{23}CAD_1)x(t) = K_2 x(t) \tag{20}$$

and the closed-loop system is

$$x(t+2) = (AD_1 + BK_2)x(t) = D_2 x(t)$$

For the next developments we assume that input and output prediction horizon is $k = 1, 2, \ldots, N$. Using the same idea as for the first and second steps, the following results for the quadratic cost function are obtained:

$$\begin{aligned} J_k(t) &= x(t)^T Q_k x(t) + u(t+k-1)^T R_k u(t+k-1) \\ v_k(t) &= \begin{bmatrix} x(t+k) \\ x(t) \end{bmatrix} \end{aligned}$$

or

$$J_k(t) = v_k(t)^T \begin{bmatrix} P_{ek1}^T R_k P_{ek1} & P_{ek1}^T R_k P_{ek2} \\ P_{ek2}^T R_k P_{ek1} & Q_k + P_{ek2}^T R_k P_{ek2} \end{bmatrix} v_k(t) \tag{21}$$

where

$$\begin{aligned} P_{ek1} &= F_{kk+1} C \\ P_{ek2} &= \sum_{l=1}^{k} F_{kl} C D_{l-1}, \quad D_0 = I \end{aligned}$$

The control algorithm is

$$u(t+k-1) = \begin{bmatrix} P_{ek1} & P_{ek2} \end{bmatrix} v_k(t) \tag{22}$$

and the closed-loop system is in the form

$$M_k x(t+k) - A_{ck} x(t) = 0 \tag{23}$$

where

$$\begin{aligned} M_k &= I - BP_{ek1} \\ A_{ck} &= AD_{k-1} + BP_{ek2} \\ D_k &= M_k^{-1} A_{ck} \end{aligned}$$

The Lyapunov function for k steps and the first difference of Lyapunov function are

$$\begin{aligned} V_k(t) &= x(t+k-1)^T P_k x(t+k-1) \\ \Delta V_k(t) &= x(t+k)^T P_k x(t+k) - x(t+k-1)^T P_k x(t+k-1) \end{aligned}$$

$$\Delta V_k(t) = v_k(t)^T \begin{bmatrix} P_k & 0 \\ 0 & -D_{k-1} P_k D_{k-1} \end{bmatrix} v_k(t) \tag{24}$$

Now, new auxiliary matrices N_{1k}, N_{2k} will be introduced for the same argument as follows:

$$v_k(t)^T \begin{bmatrix} 2N_{1k}^T \\ 2N_{2k}^T \end{bmatrix} \begin{bmatrix} M_k & -A_{ck} \end{bmatrix} v_k(t) = 0 \tag{25}$$

After substituting all obtained results to the Bellman-Lyapunov equation, the following results will be obtained for the k step design procedure:

$$B_{ek} = v_k(t)^T W_k v_k(t) < 0 \tag{26}$$

where $W_k = \{w_{kij}\}_{2\times 2}$ and

$$\begin{aligned} w_{k11} &= P_k + N_{1k}^T M_k + M_k^T N_{1k} + P_{ek1}^T R_k P_{ek1} \\ w_{k12} &= -N_{1k}^T A_{ck} + M_k^T N_{2k} + P_{ek1}^T R_k P_{ek2} \\ w_{k22} &= -N_{2k}^T A_{ck} - A_{ck}^T N_{2k} - D_{k-1}^T P_k D_{k-1} \end{aligned}$$

Note that the following condition should apply for the sake of obtaining optimal controller parameters: $trace(P_k) \to \min$.

Equations (16), (21), (26) imply that if matrices D_k (for $k = 1, 2, \ldots, N$) are stable, then the closed-loop system with predictive control algorithm (3) ensures meeting of the following conditions:

- quadratic stability of the closed-loop system;
- the cost is guaranteed;
- the minimal value of the cost functions is defined for each step $k = 1, 2, \ldots, N$.

Due to control strategy, at each step Bellman-Lyapunov function is used to obtain the optimal MPC controller with output feedback. The optimal MPC controller might ensure some level of the closed-loop system's robustness properties, which proves the following example. Using the control algorithm (3) for each $k = 1, 2, \ldots, N$, it is possible to calculate offline the all controller gain matrices from the original size of the system.

4. **Case Study and Simulation Results.** The application considered involves an isothermal reactor in which the Van Vusse reaction kinetic scheme is carried out [15]. In the following analysis, A is the feed product arriving at the reactor, B is the desired product, C and D are unwanted byproducts

$$\begin{aligned} A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ 2A \xrightarrow{k_3} D \end{aligned} \tag{27}$$

From the perspective of design, the objective is to make k_2 and k_3 small in comparison to k_1, namely by appropriate choice of catalyst and reaction conditions. The concentration of B in the product may be controlled by manipulating of the inlet flow rate and/or the reaction temperature. The educt flow contains only cyclopentadiene in low concentration, C_{Af}. Assuming constant density and an ideal residence time distribution within the reactor, the mass balance equations for the relevant concentrations of cyclopentadiene and of the desired product cyclopentanol, C_A and C_B, are as follows:

$$\begin{aligned} \dot{C}_A &= -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A) \\ \dot{C}_B &= k_1 C_A - k_2 C_B + \frac{F}{V} C_B \\ y &= C_B \end{aligned} \tag{28}$$

Kinetic parameters of the chemical reactor are given in Table 1. This example has been considered by a number of researchers as a benchmark problem for evaluating the nonlinear process control algorithm.

TABLE 1. Kinetic parameters

k_1	50 h^{-1}
k_2	100 h^{-1}
k_3	10 l mol^{-1} h^{-1}
C_{Af}	10 mol l^{-1}
V	1 l

By normalizing the process variables around the following operating point and substituting the values for the physical constants, the process model becomes

$$\begin{aligned}\dot{x}_1(t) &= -50x_1(t) - 10x_1^2(t) + u(10 - x_1(t))\\ \dot{x}_2(t) &= 50x_1(t) - 100x_2(t) + u(-x_2(t))\\ y(t) &= x_2(t)\end{aligned}$$

where the deviation variable for the concentration of component A is denoted by x_1, the concentration of component B at the outlet of the reactor and in the interior by x_2, and the inlet flow rate by u. Using Taylor's first order approximation in environs of the operating point: $C_{B0} = 1$ mol/l, $F = 25$ l/h the following form of a transfer function is obtained

$$G(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} = \frac{-s + 500}{s^2 + 250s + 15625} \tag{29}$$

For a sampling time of 0.01 h, the following discrete-time state-space equations are obtained by discretizing the continuous-time equations of the system

$$\begin{aligned}A &= \begin{bmatrix}-0.0716 & -0.3497\\ 0.3667 & 0.6446\end{bmatrix}\\ B &= \begin{bmatrix}0.0057\\ 0.0058\end{bmatrix}\\ C &= \begin{bmatrix}-0.5 & 0.9766\end{bmatrix}\end{aligned}$$

Taking parameters $\rho = 1000$ ($P_k < \rho I$), prediction and control horizons $N = 10$, $N_u = 10$, performance matrices $R_k = 0.1I$, $Q_k = 10I$, (where $k = 1, 2, \ldots, N$), the following results are obtained using the proposed sequential design approach (3). The model is used for a sequential design approach to obtain gain matrices so that robust stability and guaranteed cost are ensured for the respective closed loop. Approach constraints on system variables can be also implemented in the proposed design. The effectiveness of the proposed design procedure is illustrated in the example. Figure 1 shows the scheme of sequential MPC, where w, u, y, $\hat{x}$ are the reference signal, controller, measured output, estimated state variables, respectively. The numerical solution has been carried out by

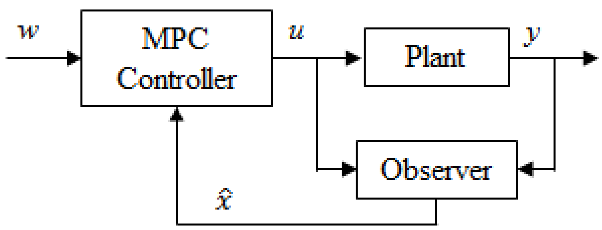

FIGURE 1. The structure of sequential MPC

TABLE 2. Results for the sequential MPC

k	F_{kl}	Eig (CL)
1	$F_{11} = -0.0332$; $F_{12} = 0.0062$	$0.2865 \pm 0.0138i$
2	$F_{21} = -0.0331$; $F_{22} = -0.0437$; $F_{23} = -0.0447$	$0.0817 \pm 0.0143i$
3	$F_{31} = 0.0068$; $F_{32} = 0.0214$; $F_{33} = 0.0161$; $F_{34} = 0.0102$	$0.0234 \pm 0.0018i$
4	$F_{41} = 0.0058$; $F_{42} = 0.0084$; $F_{43} = 0.0081$; $F_{44} = 0.0082$; $F_{45} = 0.0102$	0.0046; 0.0089
5	$F_{51} = -0.0017$; $F_{52} = 0.0002$; $F_{53} = -0.0041$; $F_{54} = -0.0132$; $F_{55} = -0.0129$; $F_{56} = 0.0087$	$0.0019 \pm 0.0001i$
6	$F_{61} = 0.0043$; $F_{62} = 0.0067$; $F_{63} = 0.0026$; $F_{64} = -0.0061$; $F_{65} = -0.0059$; $F_{66} = -0.0069$; $F_{67} = 0.0109$	-0.0001; 0.0012
7	$F_{71} = -0.0078$; $F_{72} = -2.4976$; $F_{73} = -0.0098$; $F_{74} = -0.0181$; $F_{75} = -0.0212$; $F_{76} = -0.0152$; $F_{77} = -0.0125$; $F_{78} = -0.0276$	-0.0009; -0.0161
8	$F_{81} = -0.0218$; $F_{82} = -2.4942$; $F_{83} = -0.0223$; $F_{84} = -0.0283$; $F_{85} = -0.0311$; $F_{86} = -0.0281$; $F_{87} = -0.0270$; $F_{88} = -0.0261$; $F_{89} = -0.0288$	-0.0002; -0.0265
9	$F_{91} = -0.0090$; $F_{92} = -2.4778$; $F_{93} = -0.0077$; $F_{94} = -0.0138$; $F_{95} = -0.0150$; $F_{96} = -0.0141$; $F_{97} = -0.0137$; $F_{98} = -0.0134$; $F_{99} = -0.0075$; $F_{910} = -0.0018$	-0.0001; -0.0306
10	$F_{101} = 0.0011$; $F_{102} = -2.4767$; $F_{103} = 0.0034$; $F_{104} = -0.0037$; $F_{105} = -0.0047$; $F_{106} = -0.0043$; $F_{107} = -0.0042$; $F_{108} = -0.0041$; $F_{109} = 0.0025$; $F_{1010} = -0.0040$; $F_{1011} = 0.0077$	-0.0001; -0.0319

MATLAB using YALMIP [14]. The simulations have been done using Simulink (in MATLAB). Summarized in Table 2, the results are obtained for different values of prediction.

The calculation of eigenvalues for the closed-loop (Eig (CL) in Table 2) for optimal and sequential MPC confirms that the closed-loop system is stable. Simulation results (Figure 2 and Figure 3) using sequential MPC are compared with standard MPC. In the simulations w, C_B, F are the reference signal, measured output, controller output, respectively. The solid lines denote the closed-loop system with the proposed sequential MPC algorithm. The dotted lines represent the closed-loop system with the classic MPC algorithm. The standard MPC has the same predictive control parameters (R_k, Q_k, N) and also the quadratic cost function J_k (21) as the sequential approach.

Robustness. Model uncertainties of the described reactor (28) follow from several facts. At first, there are three parameters, which can change their values around the nominal values as it is shown in Table 3.

TABLE 3. Uncertain parameters of the reactor

Variable	Minimal value	Maximal value
k_1 [h^{-1}]	47.5	52.5
k_2 [h^{-1}]	95	105
k_3 [h^{-1}]	9.5	10.5

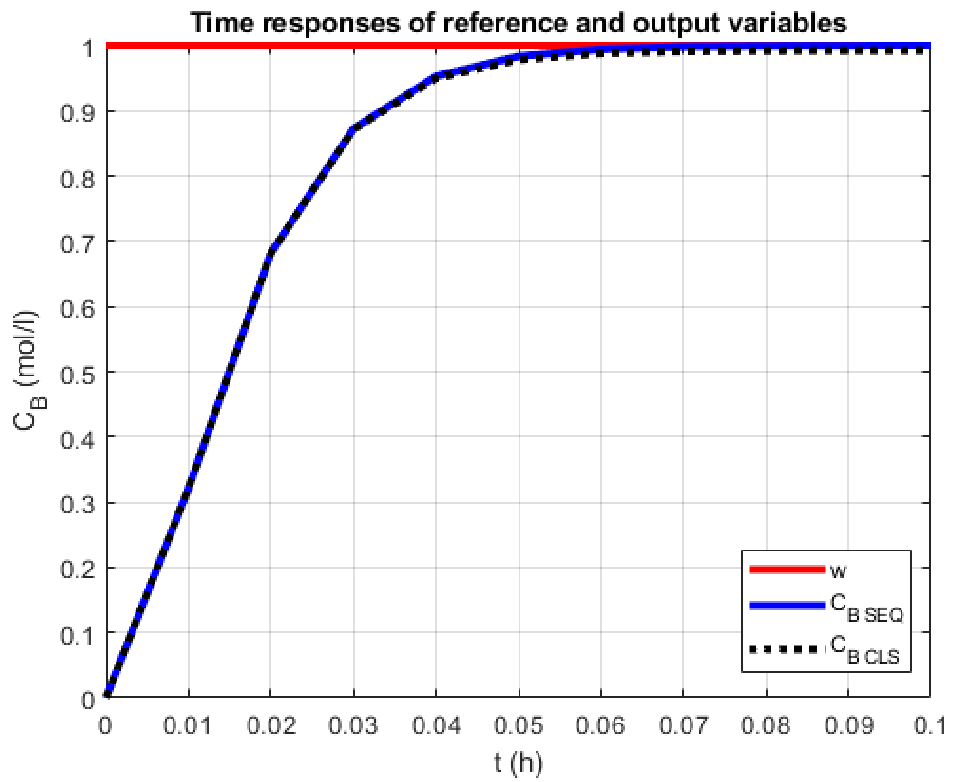

FIGURE 2. Simulation results for time responses of reference and output variables using sequential (SEQ) and standard (CLS) MPC

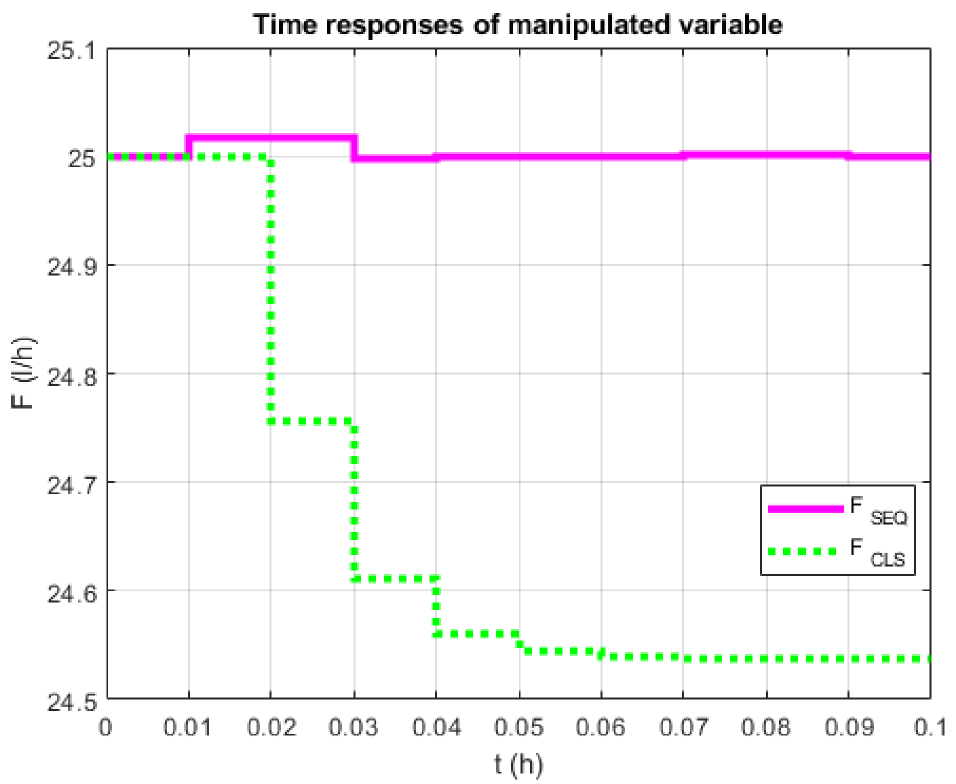

FIGURE 3. Simulation results for time responses of controller output using sequential (SEQ) and standard (CLS) MPC

So the parametric uncertainties are in our system. The controlled system also includes dynamic uncertainties. They correspond to the values of the uncertain parameters under operating conditions. They result in linearized models of the reactor with different values of matrix coefficients in the state space description.

The dynamic uncertainties also include the gap between linearized models and the original nonlinear model or the true physical system. Then for (29) the parameters are $b_1 = -1$, $b_0 \in \left[b_0^-, b_0^+\right] = [476.25, 523.75]$, $a_2 = 1$, $a_1 \in \left[a_1^-, a_1^+\right] = [240, 260]$, $a_0 \in \left[a_0^-, a_0^+\right] = [14400, 16900]$.

Then eight transfer functions are obtained (all possible combinations of minimal and maximal values of three parameters k_1, k_2, k_3 in Table 3). The optimal design procedure to calculate controller gains which are shown in Table 2 is applied for all transfer functions (30). Transfer functions are

$$
\begin{aligned}
k_{1_{\min}}, k_{2_{\min}}, k_{3_{\min}} &: \; G_1(s) = \frac{-s + 476.25}{s^2 + 240s + 14400} \\
k_{1_{\min}}, k_{2_{\min}}, k_{3_{\max}} &: \; G_2(s) = \frac{-s + 481.25}{s^2 + 245s + 15000} \\
k_{1_{\min}}, k_{2_{\max}}, k_{3_{\max}} &: \; G_3(s) = \frac{-s + 481.25}{s^2 + 255s + 16250} \\
k_{1_{\min}}, k_{2_{\max}}, k_{3_{\min}} &: \; G_4(s) = \frac{-s + 476.25}{s^2 + 250s + 15600} \\
k_{1_{\max}}, k_{2_{\min}}, k_{3_{\min}} &: \; G_5(s) = \frac{-s + 518.75}{s^2 + 245s + 15000} \\
k_{1_{\max}}, k_{2_{\min}}, k_{3_{\max}} &: \; G_6(s) = \frac{-s + 523.75}{s^2 + 250s + 15600} \\
k_{1_{\max}}, k_{2_{\max}}, k_{3_{\max}} &: \; G_7(s) = \frac{-s + 523.75}{s^2 + 260s + 16900} \\
k_{1_{\max}}, k_{2_{\max}}, k_{3_{\min}} &: \; G_8(s) = \frac{-s + 518.75}{s^2 + 255s + 16250}
\end{aligned}
\tag{30}
$$

The calculation of eigenvalues for the closed-loop (Eig (CL) for each transfer function (30) are shown in Table 4) for optimal and sequential MPC. The results of this design confirm that the closed-loop system is stable and robust.

5. **Conclusions.** This paper deals with the novel method for the MPC algorithm. The proposed optimal sequential approach with output feedback ensures the closed-loop stability guaranteed cost. Due to the optimal design procedure to calculate controller gains the robustness properties are ensured. All results are illustrated on the example (model of the isothermal reactor). The obtained results show the new proposed method's effectiveness and performance, as well as its comparability with standard MPC. The main advantage of the optimal sequential MPC is the offline computation of the optimal control gains and inputs for N steps ahead. In the standard MPC algorithm, a plan for N steps ahead is determined, and only the first element of the control output vector is applied. Then in the next step, the optimization procedure with new measurement is repeated. Standard MPC is time-consuming. The other disadvantage of standard MPC with using the cost function J_k (21) is the existence of a steady-state offset.

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TABLE 4. Eigenvalues for the closed-loop with transfer functions

k	$G_1(s)$	$G_2(s)$	$G_3(s)$	$G_4(s)$	$G_5(s)$	$G_6(s)$	$G_7(s)$	$G_8(s)$
1	0.2875 0.3151	0.2784 0.3094	0.2646 0.2946	0.2673 0.3065	0.2775 0.3104	0.2670 0.3069	0.2591 0.2861	0.2643 0.2949
2	0.0848 0.0969	0.0793 0.0938	0.0717 0.0849	0.0727 0.0925	0.0789 0.0942	0.0726 0.0927	0.0691 0.0797	0.0851 0.0715
3	0.0251 0.0297	0.0226 0.0284	0.0194 0.0245	0.0197 0.0279	0.0224 0.0285	0.0197 0.0280	0.0184 0.0222	0.0194 0.0245
4	0.0081 0.0085	0.0068 0.0083	0.0056 0.0067	0.0056 0.0082	0.0068 0.0083	0.0056 0.0082	0.0055 0.0055	0.0056 0.0067
5	0.0024 0.0026	0.0019 0.0025	0.0015 0.0019	0.0015 0.0025	0.0019 0.0025	0.0015 0.0025	0.0015 0.0015	0.0015 0.0019
6	0.0006 0.0008	0.0005 0.0007	0.0004 0.0006	0.0004 0.0007	0.0005 0.0008	0.0004 0.0007	0.0003 0.0004	0.0004 0.0006
7	−0.0011 −0.0302	−0.0009 −0.0297	−0.0006 −0.0282	−0.0008 −0.0287	−0.0009 −0.0356	−0.0008 −0.0314	−0.0005 −0.0297	−0.0006 −0.0303
8	−0.0003 −0.0485	−0.0002 −0.0471	−0.0002 0.0436	−0.0002 −0.0449	−0.0002 −0.0559	−0.0002 −0.0489	−0.0001 −0.0452	−0.0002 −0.0466
9	−0.00001 0.0565	−0.00004 −0.0545	−0.00002 −0.0498	−0.00001 −0.0517	−0.00001 −0.0646	−0.00001 −0.0562	0.00001 −0.0513	0.00002 −0.0532
10	−0.0001 −0.0595	−0.00005 0.0572	−0.00004 −0.0520	−0.00004 −0.0541	−0.00005 −0.0678	−0.00004 −0.0589	−0.00004 −0.0534	−0.00004 −0.0555

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