International Journal of Innovative Computing, Information and Control
Volume **17**, Number **2**, April **2021**

ICIC International ©2021 ISSN 1349-4198
pp. **687–700**

TENT CHAOS AND NONLINEAR CONVERGENCE FACTOR WHALE OPTIMIZATION ALGORITHM

Huanlong Zhang[1], Youmei Pan[1], Jie Zhang[1], Kejie Dai[2,*] and Yuxing Feng[1]

[1]School of Electrical and Information Engineering
Zhengzhou University of Light Industry
No. 5, Dongfeng Road, Zhengzhou 450000, P. R. China
{ zzuli407; quhn1234 }@163.com; { 1823979887; 2458685991 }@qq.com

[2]Electrical and Mechanical Engineering
Pingdingshan College
Southern Section of Weilai Road, Xincheng District, Pingdingshan 467000, P. R. China
*Corresponding author: daikejie@pdsu.edu.cn

Received October 2020; revised February 2021

Abstract. *To solve the problems of slow convergence speed and difficulty in balancing exploration and development in whale optimization algorithm (WOA), a TWOA algorithm integrating Tent chaos and nonlinear convergence factors is proposed. First, the whale population in WOA algorithm uses random walk, which causes the uneven distribution of whale individuals in the exploration stage, and by adding the Tent chaotic mapping strategy and taking advantage of the ergodicity of Tent chaotic mapping, the whale population is distributed more evenly in the random walk stage, which enhances the global search ability of the algorithm. Second, because WOA algorithm adopts linear convergence factor, it cannot effectively solve the balance problem of balancing exploration and development, and by changing linear convergence factor into nonlinear convergence factor, the optimization accuracy of the algorithm can be improved. Finally, by using 10 standard test functions, the problems of unimodal functions and multimodal functions were tested. The results show that the whale optimization algorithm with Tent chaotic map and nonlinear convergence factor is superior to the original algorithm in both mean and standard deviation. From the convergence curve, it can be observed that the convergence speed and convergence accuracy are obviously improved.*

Keywords: Whale optimization algorithm, Tent chaotic map, Nonlinear convergence factor, Intelligent optimization algorithm, Global optimization

1. **Introduction.** In recent years, computer technology has shown a rapid development trend. In order to solve complex problems such as nonlinearity, global optimization, local optimization and combinatorial optimization, many optimization algorithms are constantly emerging [1]. Optimization algorithms, which can find the optimal solution under certain conditions, are usually used to deal with the optimization problem. Common optimization algorithms include mountain climbing, simulated annealing, genetic algorithm, etc., [2]. Because of its novel algorithm and mechanism, it has been widely used in image processing, pattern recognition, signal processing and so on. In addition to common optimization algorithms, a new meta-heuristic algorithm has gained wide attention in the scientific community, especially in solving many complex optimization problems [3, 4]. Of course, many researchers have also improved the existing algorithms to obtain better convergence performance [5, 6].

DOI: 10.24507/ijicic.17.02.687

Whale optimization algorithm (WOA) is a new meta-heuristic swarm intelligence optimization algorithm proposed by Mirjalili and Lewis in 2016 [7]. And it has gained wide attention in the scientific community in recent years, especially in solving many complex optimization problems [8]. The main advantage of the WOA algorithm is to simulate the hunting process of humpback whale by using the best search agent with randomness, and to simulate the bubble net attacking process by using the spiral strategy and use the spiral strategy to simulate the humpback whale's bubble net attack process. This mechanism makes WOA algorithm different from other optimization algorithms. However, the WOA algorithm also has certain defects, which are mainly manifested in the following two aspects: 1) The adaptive parameters of the WOA algorithm rely on random distribution, which leads to uneven distribution; 2) It is easy to fall into the local optimal solution [9, 10, 11, 12].

In order to solve the problems of traditional WOA algorithms that are easy to fall into local optimal solutions and explore and fail balancing exploration and development, Wu [13] proposed a reverse learning strategy to solve the problems of traditional WOA algorithm, such as easy falling into local optimal solutions. The reverse learning strategy is integrated into WOA algorithm to enhance population diversity and avoid local optimal solutions. Huang et al. [14] proposed to use chaotic dynamic weighting factors to avoid the algorithm falling into local optimal solution, so as to improve the convergence accuracy of the algorithm. In order to avoid falling into local optimal, Wu and Mou [15] added adaptive weight to update the whale position, and then added random difference mutation strategy to update the whale position again. Chu et al. [16] proposed an adaptive weight strategy, and then introduced simulated annealing algorithm to accept poor solutions with a certain probability, so as to enhance the algorithm's global optimization ability and jump out of the local optimal solution. Shang et al. [17] improved the algorithm by using random inertia weight and non-uniform mutation strategy, so that the algorithm could jump out of the local optimal solution. Kong et al. [18] adjusted the weight of adaptation according to the changes of whale population. In order to avoid falling into the local optimum, an adaptive search strategy is designed to improve the ability to jump out of the local optimum. Zhang and Wang [20] designed a non-linear adaptive weight strategy and set two different models respectively in the exploration and development stages, so that the search agent can explore the search space adaptively. Ding et al. [19] proposed a chaotic inertial weight strategy, and introduced chaos into WOA in the iterative process, which improved the exploration ability of the algorithm without damaging its development ability.

Although the improved strategy of WOA algorithm has been greatly improved in avoiding falling into local optimal solution, it still needs to be improved in terms of convergence speed and solution accuracy. In this paper, although the improved strategy of WOA algorithm has been greatly improved in avoiding falling into local optimal solution, it still needs to be improved in terms of convergence speed and solution accuracy [21]. The randomness, ergodicity and regularity of the chaotic system can be used to improve the optimization algorithm, and the chaotic mapping method can easily help to jump out of the local optimum [22, 23].

In most swarm intelligence optimization algorithms, there is a balance between exploration and development, which enhances the optimization ability of the algorithm that is enhanced by coordinating the balance between them [24]. If the relationship between exploration and development is not well balanced, there will be some problems. Investing a lot of time in the exploration will affect the local accurate search, yet not having enough time to search for the target area in the early phase of exploration will lead to a local optimal solution. Therefore, instead of a linear convergence factor, this paper uses a nonlinear convergence factor, which makes the algorithm search slowly in the early stage of iteration to enhance the global search ability of the algorithm and converge fast convergence in the late iteration to search for the target quickly. It will balance the ability between global exploration and local development and improve the optimization accuracy of the algorithm. Finally, through the simulation of 10 benchmark test functions, the results show that the above two improved strategies can significantly improve the convergence speed and optimization accuracy of the algorithm.

2. **Whale Optimization Algorithm.**

WOA algorithm is inspired by whales' unique bubble-net feeding behavior. In nature, whales search for prey through random walks, and when they locate prey, they attack the prey by shrinking spiral to form a bubble net. By simulating this behavior, the basic WOA consists of three main stages: the stage of encircling prey, the stage of bubble net attack, and the stage of searching for prey.

2.1. **Encircling prey stage.**

In the process of hunting for prey, whales may be uncertain about the exact position of prey. So it is necessary for whales to communicate constantly. It is assumed that the position of the whale position closest to the prey currently is the optimal whale position, Other whales in the population close in on the optimal individual, so that the whole whale population moves to the whale closest to the prey to keep the whales close to the prey. In WOA algorithm, assuming that the whale population size is N and the search space is d dimension, then the spatial position of the i-th whale individual can be expressed as: $X_i = \left(x_i^1, x_i^2, \ldots, x_i^d\right)$, $i = 1, 2, \ldots, N$. Its location is updated as follows:

$$D = |C \cdot X_p(t) - X(t)| \tag{1}$$

$$X(t+1) = X_p(t) - A \cdot D \tag{2}$$

where D is the distance vector between the current optimal solution and the whale search individual; t is the number of current iterations; $X(t)$ is individual position vector; $X_p(t)$ is the prey position vector (the current optimal solution); A and C are coefficient vectors that control the distance D between the current optimal solution and the search individual. The mathematical models of A and C are as follows.

$$A = 2a \cdot r_1 - a \tag{3}$$

$$C = 2 \cdot r_2 \tag{4}$$

where r_1 and r_2 are the random vectors in $[0, 1]$, and a set of numbers will be generated randomly in each iteration; a is the convergence factor, whose value decreases linearly from 2 to 0 with the increase of iteration times, and the convergence factor formula is

$$a(t) = 2 - \frac{2t}{Max\_iter} \tag{5}$$

Max_iter is the maximum number of iterations.

2.2. **Bubble network attack stage.**

There are two ways in bubble net attack stage: shrinking encircling and spiral attacking. The shrinking encircling mechanism is realized by reducing the value of convergence factor a, which makes the linear transformation from 2 to 0 by Formula (5). The value range of coefficient vector A is $[-a, a]$. When $|A| < 1$, the current whale individual approaches the target prey from the original position for local optimal search and encircles the prey according to Formula (2). A spiral equation is used to simulate the spiral motion of the whale between the position of the whale and prey. The mathematical model is

$$X(t+1) = D' \cdot e^{bl} \cdot \cos(2\pi l) + X_p(t) \tag{6}$$

where $D' = |X_p(t) - X(t)|$, $X_p(t)$ is the distance between the whale and the optimal position currently, b is the constant coefficient used to define the logarithmic spiral shape, and l is the random number between $[-1, 1]$.

In WOA algorithm, shrinking encircling and spiral updating are carried out simultaneously. In order to simulate this behavior, probability P is introduced $P \in [0, 1]$, and its mathematical model is

$$X(t+1) = \begin{cases} X_p(t) - A \cdot D, & (p < 0.5) \\ D \cdot e^{bl} \cdot \cos(2\pi l) + X_p(t), & (p \geq 0.5) \end{cases} \tag{7}$$

2.3. **Stage of searching for prey.** When $|A| \geq 1$, whales no longer choose to update in the direction of prey, but choose an individual whale as the global optimal solution randomly, so as to enhance the global searching ability of the algorithm. The mathematical model is as follows:

$$D = |C \cdot X_{rand}(t) - X(t)| \tag{8}$$

$$X(t+1) = X_{rand}(t) - A \cdot D \tag{9}$$

where X_{rand} is the position vector of the whale individual randomly selected from the current population.

In the search process, in Formula (9), the basic WOA adopts the random walk method to ensure the global search, but this method will make the sample distribution in the solution space uneven, which cannot guarantee the convergence performance of the algorithm. In addition, basic WOA uses linear convergence factors to balance exploration and development performance, which does not reflect actual optimization problems. Therefore, this paper needs to make further improvement to enhance the convergence performance of the algorithm.

3. **Improved Whale Optimization Algorithm.** Aiming at solving the problems of slow convergence rate and difficulty in balancing exploration and development of standard whale optimization algorithm, a TWOA optimization algorithm, which introduces two improvement strategies, was proposed. The specific process is as follows.

3.1. **Exploration mechanism based on Tent chaos.** When $|A| \geq 1$, whales randomly choose individual whale for random search of the population. However, the random walk method will make uneven distribution in the solution space, resulting in poor diversity and unable to carry out global exploration effectively, thus reducing the search efficiency of the algorithm.

In recent years, chaos mechanism has been widely applied to meta-heuristic algorithms. Applying chaos to optimization is a relatively novel optimization algorithm [25], which is characterized by uncertainty, long-term unpredictability, the existence of random irregular motion, uniform distribution and good correlation. Moreover, its dynamic characteristics are helpful for the optimization algorithm to explore the search space in a more detailed and comprehensive way. Therefore, chaos mechanism is added to the algorithm in this paper. Logistic mapping and Tent chaotic mapping are commonly used chaos. The research results show that: Tent chaotic mapping can greatly improve the performance of the algorithm in all chaotic maps [26]. Therefore, in this paper, Tent chaotic map is selected to replace random search. To compare the global search performance of random search strategy and Tent chaotic mapping, the distribution diagram of the two strategies is given in Figure 1.

Figure 1 shows the distribution of the random search strategy and the Tent chaotic mapping in the solution space. There are 200 red dots and 200 blue dots respectively. Red dots represent the random distribution of whale population in solution space, while

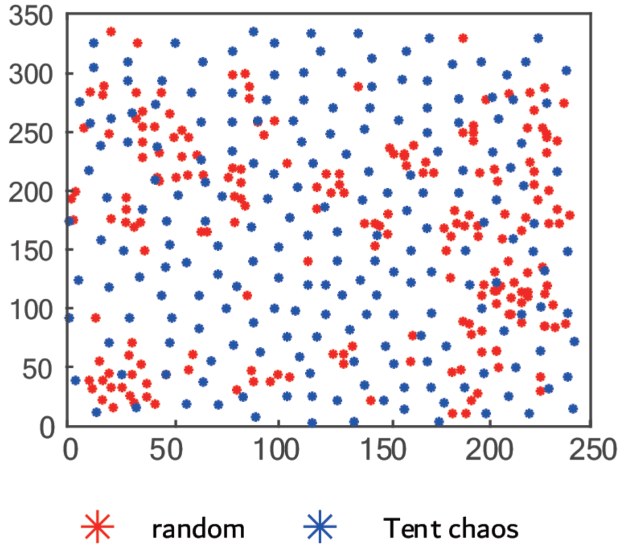

FIGURE 1. (color online) Distribution of stochastic and Tent chaotic strategies

blue dots represent the distribution of whale population in solution space after adding Tent chaos strategy. It can be seen from the figure that the distribution of blue dots is more uniform and wider than that of red dots. Because of the ergodicity and randomness of the Tent chaotic mapping mechanism, blue dots can distribute more evenly in the space with randomness. However, the distribution of red dots is chaotic under the simple random distribution mechanism. Therefore, the algorithm has stronger global search ability after adding Tent chaotic mapping mechanism.

The randomness and ergodicity of chaos make the overall search faster, which plays a crucial role in accelerating the convergence of the algorithm [27]. In the whale optimization algorithm with random components, the randomness of whale wandering is realized through probability distribution [28], so it is easy to cause uneven distribution in space. Therefore, after adding chaos mechanism, the algorithm performs better in jumping out of the local optimal solution [29, 30]. So the inclusion of the Tent chaotic mapping in this paper will enhance the global search capability of the algorithm. The specific expression is as follows:

$$\omega(t+1) = \begin{cases} 2\omega(t), & 0 < \omega(t) < 0.5 \\ 2(1-\omega(t)), & 0.5 \le \omega(t) < 1 \end{cases} \tag{10}$$

Improved location update:

$$X(t+1) = \omega(t) \cdot X_{rand} - A \cdot D \tag{11}$$

3.2. **Nonlinear convergence factor of equilibrium exploration and development.** In the intelligent optimization algorithm of group iteration, it is very important to balance the global exploration and local development reasonably. Moreover, the balance between the two ensures that the algorithm global optimal can be achieved. If the balance between the two is not good, the convergence rate will be slow or premature in the iterative process [31, 32]. In WOA algorithm, the balance between global exploration and local development is determined by parameter A, which is determined by the convergence factor a in Equation (3). However, the convergence factor a changes linearly in the iterative process, which will make it easy for the algorithm to lose the balance between searching in early iteration and exploiting in late iteration. It will greatly reduce the optimization ability of the algorithm. In order to solve this problem, a nonlinear convergence factor is introduced in this paper. A large a is adopted in the early iteration of the algorithm, so that whale individuals can have better exploration ability and avoid falling into the local optimal solution. In the late iteration of the algorithm, a small a value enables the algorithm to have strong local development ability and improves the convergence accuracy of the algorithm. The specific expression is as follows:

$$a = 2 - 2\left(\frac{\tan\left(\frac{t}{Max\_iter}\right)}{\tan 1}\right)^{\mu} \tag{12}$$

where Max_iter is the maximum number of iterations, t is the current number of iterations, and μ is the control constant coefficient. The value of μ determines the balance between early exploration and late development of this algorithm. To determine the value of μ, convergence curves of different values of μ are given in Figure 2, as shown below.

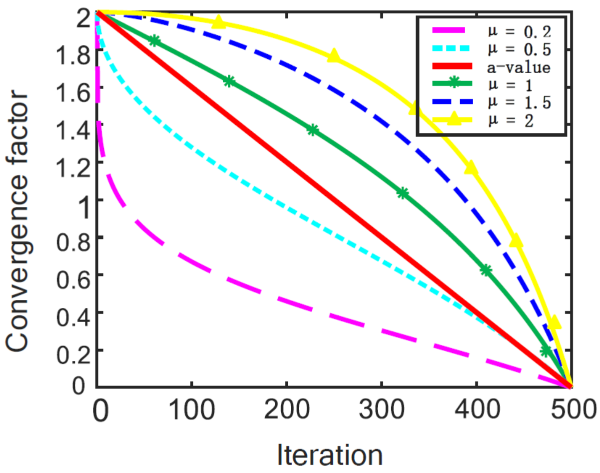

FIGURE 2. Convergence curves of different values

In Equation (12), the reasonable value of μ should ensure that in the early stage of the algorithm, with the increase of iteration times, the nonlinear convergence factor shows a slow downward trend, which is beneficial for whale population to search globally and lock the target area. However, in the late iteration, it should be ensured that the nonlinear convergence factor shows a rapid downward trend, to enable the whale population to quickly search for the target prey. Different μ values in the graph correspond to different curves. When $\mu = 0.2$ and $\mu = 0.5$, the algorithm converges rapidly in the early iteration and slowly converges in the late iteration. This goes against ensuring better exploration and development performance. When $\mu = 1$, $\mu = 1.5$ and $\mu = 2$, the algorithm converges slowly at the early stage of iteration and quickly at the late stage. At this point, the algorithm is guaranteed to have good global exploration ability in the early stage of search and obvious local development ability in the late stage of search. So $\mu > 1$ should be set. It can be seen from the figure that the larger the value of μ is, the greater the difference between the convergence speed of the algorithm in the early iteration and the late iteration is. In order to balance the convergence speed between the early and late iteration, $\mu = 1.5$ is selected in this paper.

3.3. **Steps to improve the algorithm.** For TWOA optimization algorithm, firstly, initialize the whale population, calculate the fitness value of all whale individuals, and record the global optimal solution. Secondly, replace a with nonlinear convergence factor to balance the exploration and development, add Tent chaotic mapping in the random walk phase and update the location according to the three stages of whale predation.

Finally, calculate the fitness value of the individual after position updated, update the current optimal individual and its position and then enter the next iteration. The steps of TWOA optimization algorithm are as follows:

1) Initialize the parameter. Set the population size N, the maximum number of iterations Max_iter, the spatial dimension d, and the whale position initialization $\{x_i, i = 1, 2, \ldots, N\}$;

2) Calculate the fitness value $\{f(x_i), i = 1, 2, \ldots, N\}$ of each individual whale and record the global optimal solution;

3) Use the nonlinear convergence factor in Formula (12) to replace the linear factor in Formula (5) of traditional WOA to calculate the value of a, and update the values of parameters A, C, l, P and $\omega(t)$;

4) If $p < 0.5$ and $|A| < 1$, update the position of the individual whale according to Equation (1);

5) If $p < 0.5$ and $|A| \geq 1$, select a whale (X_{rand}) randomly from the population, and the improved Formula (11) instead of the traditional Formula (9) to update the position of individual whales;

6) If $p \geq 0.5$, update the current position of the whale individual according to Equation (6);

7) Output the current optimal solution if the set maximum number of iterations is reached; otherwise, return to step 3).

4. **Simulation Experiment and Analysis.**

4.1. **Experimental setup.**

Select 10 different types of benchmark test functions from the 23 test functions used in [1] for testing. Among the 10 test functions, $f1$-$f5$ are unimodal functions with only one global minimum to test the convergence speed of the algorithm. $f6$-$f10$ are multimodal functions with a large number of local extremum points in the domain to test the ability of jumping out of the local optimal solution of the algorithm. All experiments in this paper are conducted under the condition of MATLAB R2019a. The details of each function are listed in Table 1.

4.2. **Parameter analysis.**

Parameter setting: It is well known that the number of population will also affect the performance of the algorithm. Too small population will lead to premature convergence of the algorithm, and large population can maintain or improve the optimization precision, but it will increase the time. Finally, consider the whale population is set to 30. To verify the impact of iteration times on the algorithm, WOA algorithm was run with different iteration times. The Max_iter values used in the experiment were 300, 500 and 1000 respectively. A detailed analysis of the iterative process is shown in Table 2. It can be seen from Table 2 that when Max_iter is 300, only get good results in $f1$ and $f2$. While when the number of iterations is 1000, the optimal value is the same as the optimal value when the number of iterations is 500. Therefore, Max_iter is selected as 500 in this paper.

In Table 1, firstly, use the unimodal function to test the convergence speed and convergence accuracy of the algorithm, then the multimodal function to test the global exploration ability of the algorithm, and finally, the mean value and standard deviation of the optimal solution to measure the performance of the algorithm.

4.3. **TWOA performance analysis.**

In order to verify the performance of each improved strategy on WOA algorithm, the population size was set as 30 and the maximum number of iterations was set as 500. Four algorithms, WOA, WOA1 (Tent chaotic mapping), WOA2 (nonlinear convergence factor) and TWOA (nonlinear convergence factor)

Table 1. Unconstrained test functions

Function	Dim	Range	Optimum value
$f1(x) = \sum_{i=1}^{D} x_i^2$	30	$[-100, 100]$	0
$f2(x) = \sum_{i=1}^{D} \lvert x_i\rvert + \prod_{i=1}^{D} \lvert x_i\rvert$	30	$[-10, 10]$	0
$f3(x) = \sum_{i=1}^{D} \left(\sum_{j-1}^{i} x_j\right)^2$	30	$[-100, 100]$	0
$f4(x) = \max_i \{\lvert x_i\rvert, 1 \leq i \leq n\}$	30	$[-100, 100]$	0
$f5(x) = \sum_{i=1}^{D} i x_i^4 + rand(0, 1)$	30	$[-1.28, 1.28]$	0
$f6(x) = -20 \exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + 20 + e$	30	$[-600, 600]$	0
$f7(x) = \frac{\pi}{D}\left\{10\sin(\pi y_1) + \sum_{i=1}^{D}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_D - 1)^2\right\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$	30	$[-32, 32]$	0
$f8(x) = \left(\frac{1}{500} + \sum_{i=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	2	$[-65, 65]$	1
$f9(x) = \sum_{i=1}^{D}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	$[-5, 5]$	0.00030
$f10(x) = \left[1 + (x_1 + x_2 + 1)^2\left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right)\right] \times \left[30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)\right]$	2	$[-2, 2]$	3

were respectively run. Convergence curves of different improvement strategies are shown in Figure 3.

The convergence curves under different mechanisms are shown in Figure 3. Both of the strategies improve the WOA algorithm to varying degress. For unimodal functions, the convergence speed of the algorithm testing on $f1$, $f2$, $f3$ and $f4$ with Tent search strategy and nonlinear convergence factors added improved significantly, as the number of iterations increases. The convergence speed of adding a single improved strategy to function $f5$ was better than that of traditional WOA algorithm, and the convergence speed of TWOA algorithm is almost the same as that of traditional WOA algorithm. For multimodal functions, the convergence accuracy of TWOA algorithm testing on functions $f6$, $f7$, $f8$, $f10$ can all converge to the optimal value. As for function $f9$, the convergence accuracy of TWOA did not reach the optimal value, but it was better than the

TABLE 2. Comparison of WOA algorithm results under different iterations

Functions	Iterations	Optimal values
$f1$	300	1.3928e-48
	500	2.9218e-84
	1000	1.009e-161
$f2$	300	2.4344e-27
	500	1.7075e-53
	1000	2.2642e-109
$f3$	300	88142.2152
	500	42671.041
	1000	11030.4558
$f4$	300	36.337
	500	1.0146
	1000	4.9505
$f5$	300	0.0077
	500	0.0026
	1000	0.0057
$f6$	300	7.9936e-15
	500	4.4409e-15
	1000	4.4409e-15
$f7$	300	0.0310
	500	0.0099
	1000	0.0147
$f8$	300	0.99801
	500	0.998
	1000	0.998
$f9$	300	0.0006
	500	0.0003
	1000	0.0005
$f10$	300	3.0008
	500	3
	1000	3

traditional WOA algorithm. Compared with WOA, WOA1 and WOA2, TWOA had a faster convergence rate among the 10 test functions. It shows that the search efficiency of TWOA algorithm was obviously improved after the Tent chaotic search strategy and the nonlinear convergence factor strategy were added.

In order to verify the performance of TWOA algorithm, WOA, grasshopper optimization algorithm (GOA) [33] and TWOA were selected. to run independently for 30 times. The mean value and standard deviation of the 30 experimental results were calculated, and the test results were shown in Table 3.

From Table 3, it can be found that the average values of TWOA were superior to GOA and WOA except for $f8$ and $f9$. The better average value indicates that the convergence of the algorithm is better and the convergence precision is higher. The standard deviation reflects the deviation degree between the experimental results and the mean value, and the smaller the standard deviation is, the smaller the deviation degree of the experimental data is. The standard deviation of TWOA algorithm in the table was generally lower than

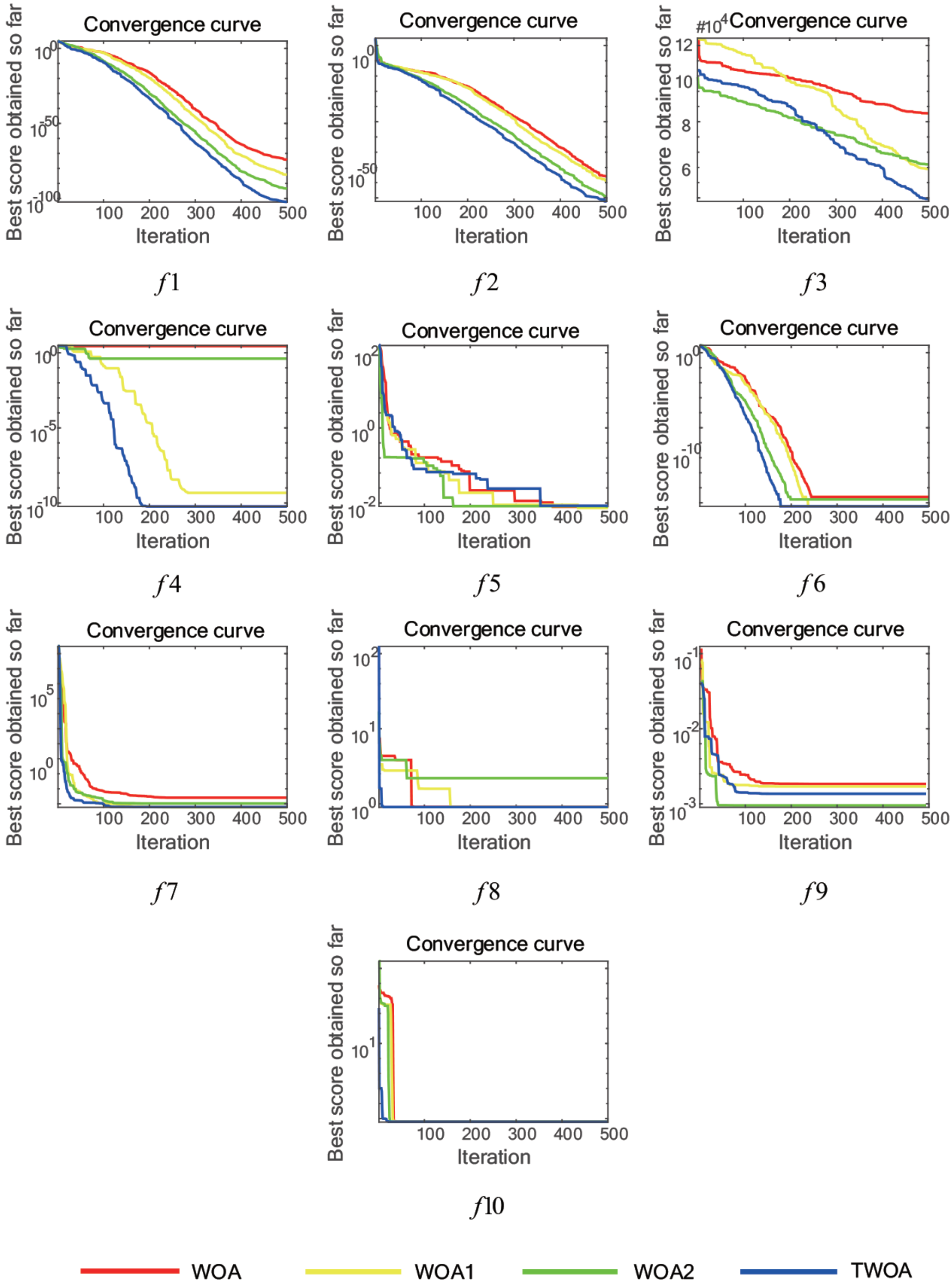

FIGURE 3. Convergence of the two improved strategies for 10 benchmark functions

TABLE 3. Comparison of optimization results of three algorithms

Function	Result	WOA	GOA	TWOA
$f1$	Ave	1.2346e-72	74.2031	2.6351e-90
	Std	4.6164e-72	84.6386	7.1462e-90
$f2$	Ave	7.3226e-52	6.8798e+03	1.5221e-60
	Std	3.0146e-51	3.4194e+04	3.5889e-60
$f3$	Ave	0.0013	1.8326e-07	3.8086e-13
	Std	0.0041	3.4219e-07	2.0280e-12
$f4$	Ave	47.3576	5.8004	2.0845e-08
	Std	28.4235	2.9460	4.8520e-08
$f5$	Ave	0.0039	0.4955	0.0012
	Std	0.0034	0.1818	0.0015
$f6$	Ave	4.0856e-15	7.88501	2.4277e-15
	Std	1.9132e-15	6.2720	1.9851e-15
$f7$	Ave	0.0041	4.7046	0.0017
	Std	0.0038	2.2513	0.0029
$f8$	Ave	3.4524	0.9980	1.2298
	Std	3.3779	5.2723e-16	0.4921
$f9$	Ave	7.5150e-04	0.0042	5.9840e-04
	Std	5.2350e-04	0.0066	1.9358e-04
$f10$	Ave	3.0000	3.0000	3.0000
	Std	4.8986e-05	6.6541e-12	0.0012

the standard deviation of the other two algorithms, indicating that TWOA algorithm had higher stability.

In order to observe the performance of TWOA algorithm, the convergence curves of three different algorithms under test functions are presented in Figure 4. The three algorithms are standard WOA algorithm, GOA algorithm and TWOA algorithm respectively.

It can be seen from the three curves given in Figure 4 that the convergence speed as well as the convergence accuracy testing on unimodal functions $f1$, $f2$, $f3$, $f4$ and $f5$ of the TWOA algorithm were better than that of WOA and GOA, and the convergence accuracy is higher, which indicates that the algorithm is improved to some extent. As for the multimodal functions $f6$, $f7$, $f9$, the convergence accuracy of the traditional WOA algorithm fails to reach the optimal value, while the improved TWOA algorithm can achieve the optimal value in convergence accuracy. For function $f8$, the convergence accuracy is improved compared with traditional WOA algorithm. To sum up, the improved whale optimization algorithm (TWOA) has better stability and faster convergence rate.

4.4. **Discussion.** The research of WOA algorithm is still in the immature stage, and there is no complex iterative process. There are still many problems to be further explored. Although the Tent chaotic map strategy and the nonlinear convergence factor strategy have played an optimized role in the traditional WOA algorithm to some extent, it can be seen from the data in Table 3 that the convergence accuracy needs to be further improved.

5. **Conclusion.** In order to improve the shortcomings of the standard WOA algorithm such as slow convergence rate and low convergence accuracy, this paper proposes a TWOA algorithm. By introducing the Tent chaos mapping, the global searching ability of the algorithm was effectively improved due to the ergodicity chaos, and the premature convergence of the algorithm was also improved. The nonlinear convergence factor was introduced to balance the global search and local development better, which improved the

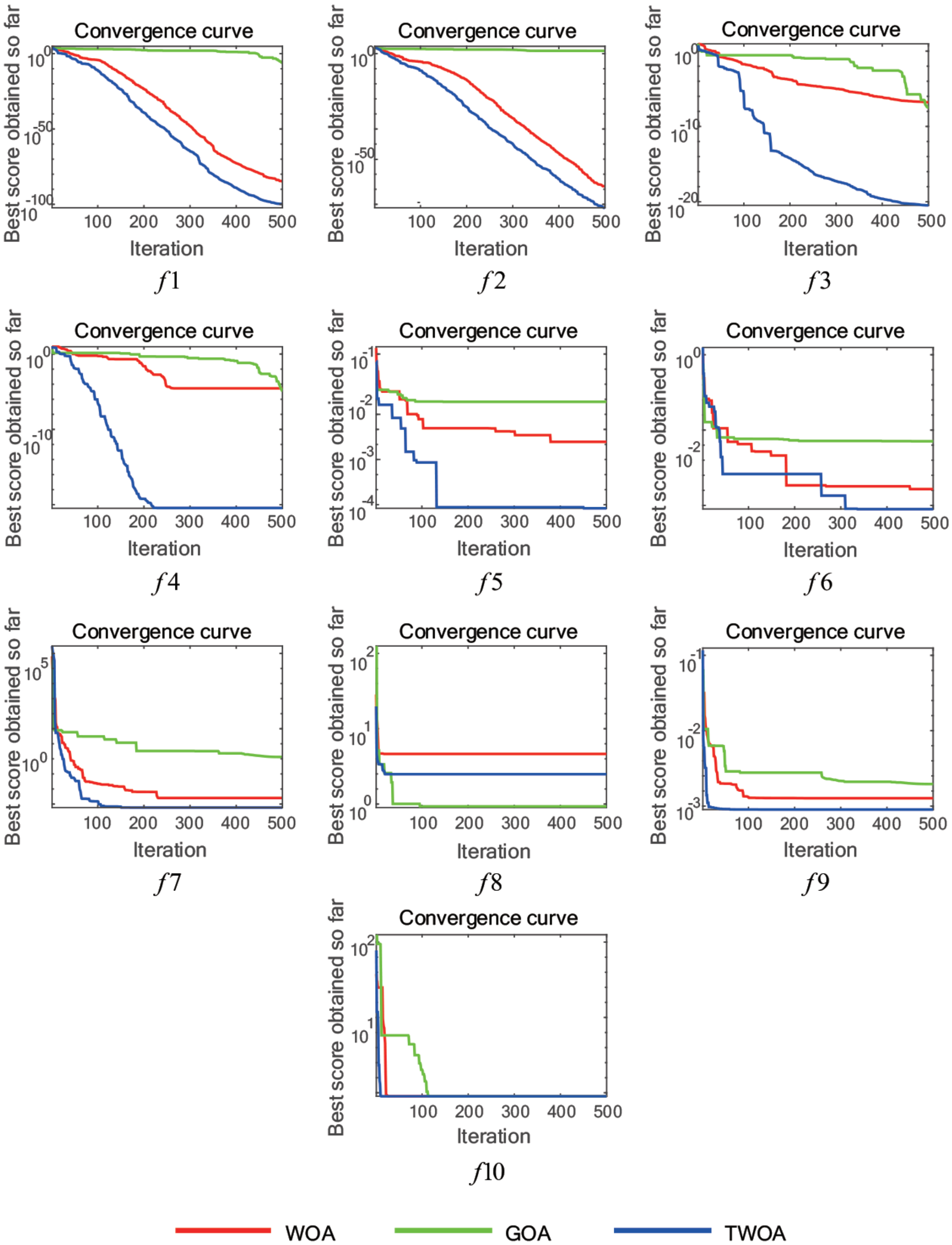

FIGURE 4. Convergence of 3 algorithms on 10 benchmark functions

optimization precision of the algorithm. Finally, through experiments on 10 test functions, the results show that the improved TWOA algorithm has a great improvement in the convergence speed and the balance between algorithm exploration and development. The next step is to improve the WOA algorithm, so as to better solve the balance between exploration and development and improve the algorithm optimization ability. It is also an important work to apply whale optimization algorithm to multi-objective optimization and constraint optimization.

Acknowledgments. This work was supported by the National Natural Science Foundation of China under Grant (61873246, 61672471, 61702462), Henan province science and technology research project (182102210607, 192102210108), sponsored by Program for Science & Technology Innovation Talents in Universities of Henan Province (21HASTIT028), sponsored by Natural Science Foundation of Henan (202300410495).

REFERENCES

[1] S. G. Ismail, D. Ashraf and H. A. Ella, A new chaotic whale optimization algorithm for features selection, *Journal of Classification*, vol.35, pp.300-344, 2018.
[2] K. Liu, L. L. Zhao and H. Wang, Whale optimization algorithm based on elite opposition-based and crisscross optimization, *Journal of Chinese Mini-Micro Computer Systems*, vol.41, no.10, pp.2092-2097, 2020.
[3] Y. L. Li, S. Q. Wang, Q. R. Chen et al., Comparative study of several new swarm intelligence optimization algorithms, *Computer Engineering and Applications*, pp.1-23, 2020.
[4] J. Y. Yu, N. J. Gao and H. Li, Whale optimization algorithm based on nonlinear cognitive factor and local disturbance, *Computer Engineering and Design*, vol.40, no.10, pp.2861-2866, 2019.
[5] R. Shan, L. Jia, J. Zhao and W. Liu, A new hybrid conjugate gradient method for optimization problems and its global convergence, *ICIC Express Letters*, vol.13, no.8, pp.681-687, 2019.
[6] A. Got, A. Moussaoui and D. Zouache, A guided population archive whale optimization algorithm for solving multiobjective optimization problems, *Expert Systems with Applications*, vol.141, no.3, 2020.
[7] S. Mirjalili and A. Lewis, The whale optimization algorithm, *Advances in Engineering Software*, vol.95, pp.51-67, 2016.
[8] H. Chen, C. J. Yang, A. A. Heidari and X. H. Zhao, An efficient double adaptive random spare reinforced whale optimization algorithm, *Expert Systems with Applications*, vol.154, 2020.
[9] D. Olive, M. A. E. Aziz and A. E. Hassanien, Parameter estimation of photovoltaic cells using an improved chaotic whale optimization algorithm, *Applied Energy*, vol.200, pp.141-154, 2017.
[10] L. F. Yue, R. N. Yang, Y. J. Zhang et al., Improved moth fluttering optimization algorithm for Tent chaos and simulated annealing, *Journal of Harbin Institute of Technology*, vol.51, no.5, pp.146-154, 2019.
[11] W. Y. Guo, T. Liu, F. Dai et al., An improved whale optimization algorithm for forecasting water resources demand, *Applied Soft Computing*, vol.86, 2019.
[12] Y. Li, T. Han, B. Han et al., Whale optimization algorithm with chaos strategy and weight factor, *Journal of Physics: Conference Series*, vol.1213, no.3, 2019.
[13] C. Z. Wu, An improved whale optimization algorithm, *Modern Computer*, no.14, pp.8-13, 2019.
[14] H. X. Huang, G. Y. Zhang, S. Y. Chen et al., WOA based on chaotic weight and elite guidance, *Transducer and Microsystem Technologies*, vol.39, no.5, pp.113-116, 2020.
[15] Z. Q. Wu and Y. M. Mou, Improved whale optimization algorithm, *Application Research of Computers*, vol.37, no.12, 2020.
[16] D. L. Chu, H. Chen and X. G. Wang, Whale optimization algorithm based on adaptive weight and simulated annealing, *Acta Electronica Sinica*, vol.47, no.5, pp.992-999, 2019.
[17] M. Shang, Z. P. Wan, J. W. Cao et al., Logistics path optimization based on improved whale optimization algorithm, *Mathematics in Practice and Theory*, no.15, pp.210-218, 2019.
[18] Z. Kong, Q. F. Yang, J. Zhao et al., Adaptive adjustment of weights and search strategies-based whale optimization algorithm, *Journal of Northeastern University*, vol.41, no.1, pp.35-43, 2020.
[19] H. Ding, Z. Y. Wu and L. C. Zhao, Whale optimization algorithm based on nonlinear convergence factor and chaotic inertial weight, *Concurrency and Computation Practice and Experience*, vol.32, 2020.
[20] J. Zhang and J. S. Wang, Improved whale optimization algorithm based on nonlinear adaptive weight and golden sine operator, *IEEE Access*, vol.1, no.8, pp.77013-77048, 2020.
[21] X. Guan, Mixed flower pollinate algorithm based on chaotic mapping, *Modern Information Technology*, vol.3, no.10, pp.5-8, 2019.

[22] J. H. Wang, L. Zhang, C. Shi et al., Whale optimization algorithm based on chaotic search strategy, *Control and Decision*, vol.34, no.9, pp.1893-1900, 2019.
[23] W. Long, T. B. Wu and B. Tang, Whale optimization algorithm with nonlinearly variable convergence factor, *Journal of Lanzhou University of Technology*, vol.43, no.6, pp.102-107, 2017.
[24] M. Wang and H. Chen, Chaotic multi-swarm whale optimizer boosted support vector machine for medical diagnosis, *Applied Soft Computing*, 2019.
[25] H. M. Mohammed, S. U. Umar and T. A. Rashid, A systematic and meta-analysis survey of whale optimization algorithm, *Computational Intelligence and Neuroscience*, vol.2019, 2019.
[26] Z.-M. Gao, J. Zhao, Y.-R. Hu et al., The improved Harris hawk optimization algorithm with the Tent map, *The 3rd International Conference on Electronic Information Technology and Computer Engineering (EITCE)*, pp.336-339, 2019.
[27] C. Paul, P. K. Roy and V. Mukherjee, Chaotic whale optimization algorithm for optimal solution of combined heat and power economic dispatch problem incorporating wind, *Renewable Energy Focus*, 2020.
[28] G. Kaur and S. Arora, Chaotic whale optimization algorithm, *Journal of Computational Design and Engineering*, vol.5, no.3, pp.275-284, 2018.
[29] Z. J. Teng, J. L. Lv, L. W. Guo et al., An improved hybrid grey wolf optimization algorithm based on Tent mapping, *Journal of Harbin Institute of Technology*, vol.50, no.11, pp.40-49, 2018.
[30] Z. Zhang, S. H. Rao and S. J. Zhang, A novel fruit fly optimization algorithm with chaotic step, *Computer Engineering and Science*, vol.42, no.4, pp.755-760, 2020.
[31] M. Zhong, W. Long, B. Xu et al., Whale optimization algorithm with nonlinear control parameter, *MATEC Web of Conferences*, vol.139, 2017.
[32] H. Zhang, Z. Gao, X. Ma et al., Hybridizing teaching-learning-based optimization with adaptive grasshopper optimization algorithm for abrupt motion tracking, *IEEE Access*, 2019.
[33] S. Saremi, S. Mirjalili and A. Lewis, Grasshopper optimisation algorithm: Theory and application, *Advances in Engineering Software*, vol.105, pp.30-47, 2017.