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ROBUST ACTUATOR AND SENSOR FAULT ESTIMATION FOR FUZZY DELAY SINGULARLY PERTURBED SYSTEMS VIA PROPORTIONAL MULTIPLE-INTEGRAL OBSERVER

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Abstract. *This paper addresses the problem of fault estimation (FE) for a class of time delay fuzzy singular perturbed systems with actuator fault, sensor faults and external disturbances simultaneously. By taking the system fault as auxiliary disturbance signal, a proportional multiple-integral (PMI) FE observer under H_∞ constraint is constructed. Then, the less conservative sufficient conditions for the existence of observer are explicitly provided. The resulted estimator can guarantee that the error dynamic systems are asymptotically stable $\omega(t) = 0$ and satisfy H_∞ performance for sufficiently small perturbation parameter ε. Compared with the existing results, the gains of estimator are solved directly by a set of ε-independent LMIs, and the proposed estimator can better describe the shape and size of system faults. Meanwhile, the design scheme is with less conservative and a wilder application range. Finally, the simulation results show the effectiveness of the proposed approach.*
Keywords: Fault estimation, Singular perturbation, State time delays, Asymptotically stable, Proportional multiple-integral observer, Linear matrix inequalities (LMIs)

1. Introduction.

Recently, due to an increasing demand for higher safety and reliability, the fault estimation (FE) problem of a practical system has been an active field of research. Under the T-S fuzzy model framework, lots of research into FE for T-S fuzzy systems has been carried out and various methods have been proposed in [13, 16, 17]. It should be noted that most physical systems and processes inherently contain small perturbation parameters, which makes FE of T-S fuzzy systems become stiff and unwieldy. Moreover, the existence of small parameter brings forth ill-conditioning and high dimension problems in system analysis and synthesis. Therefore, many researchers have been seeking effective approaches to estimate singular perturbed system fault.

Time delays are frequently encountered in various engineering and communication systems, and a time delay in dynamical system is often a primary source of instability and performance degradation. As a result, there are some recent controller design results [31, 32] and FE results [3, 7, 8, 10, 15] for T-S fuzzy system with time delays. In [8], the adaptive fault estimation problem is studied for a class of T-S fuzzy stochastic Markovian jumping systems with time delays and nonlinear parameters. Based on the $(k-1)$th fault estimation information, a k-step fault estimation observer is proposed to estimate the actuator fault of time delay T-S fuzzy systems in [7]. In [15], a fuzzy descriptor learning

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observer is constructed to achieve simultaneous reconstruction of system states and actuator faults for T-S fuzzy descriptor systems with time delays. A fault estimation filtering is designed for discrete-time T-S fuzzy systems with low-frequency faults, which satisfy two finite frequency H_∞ performance indices simultaneously in [29]. In [10], sufficient conditions for observer based fault tolerant saturated control design for discrete-time T-S fuzzy systems with delay are developed. However, in practice, when the external disturbances are involved in nonlinear plant or the output of system is affected by the sensor fault and disturbance simultaneously, the existing method will fail. It means that the above-mentioned results can only deal with actuator fault or sensor fault individually. Simultaneous estimation of actuator fault and sensor fault under the condition that the external disturbances exist is more challenging and has not gotten big concern. In addition, although the influence of delay has been taken into account in the observer design process [7, 8, 15], it should be pointed out that the time-varying delay and singular perturbation parameter ε is not considered.

On the other hand, a kind of practical system model embraces complicated slow and fast dynamics. A state-space model has a small positive parameter ε multiplying some derivatives of the states. Since the fact that ill-conditioning problem will be caused by singular perturbation parameter ε is inevitable, it is more difficult to investigate fault estimation issue for singularly perturbed systems. By virtue of fast-slow decomposition approach, several results have been obtained in these areas. By using an observer-based residual generator, the sensor fault is detected in [2] but the fast subsystem is ignored. Results in [1] have some conservativeness arising from the absence of accurate fault information. In addition, actuator faults are reconstructed by a composite observer-based residual generator in [26] for linear singularly perturbed systems. In [25], robust fault estimation scheme is presented to estimate faults whose derivative is bounded for Lipschitz singularly perturbed systems, but only sensor fault is considered. In [33], a fuzzy adaptive observer is developed to achieve simultaneous estimations of actuator fault and sensor fault, but the actuator fault and system control input must satisfy matching condition. In [30], one less conservative delay-dependent sufficient condition for the existence of fault estimation observer is given to estimate actuator and sensor fault simultaneously; however, perturbation is not considered. For the above existing FE research results, it should be noted that the singularly perturbed systems are considered in the aforementioned papers, and FE problem mainly focuses on normal singular systems with no consideration for T-S fuzzy singular perturbed systems [3, 7, 8, 10, 15, 33], also with no time delays in [23-26].

Motivated by the above factors, this paper presents a new fault estimation scheme for a class of T-S fuzzy singular perturbed systems with state time delays. By taking the system fault as auxiliary disturbance signal, a proportional multiple-integral (PMI) fuzzy fault estimation observer under H_∞ performance constraint is constructed to achieve the estimation of system faults. In contrast to the existing results, the proposed approaches have the advantage that the gains of estimator are solved directly by a set of ε-independent LMIs, so the given methods are easy to implement and can be applied to both standard and nonstandard singularly perturbed systems. The main contributions of this paper are summarized as follows.

1) The obtained robust PMI observer-based fault estimator with H_∞ performance index can be synthesized to estimate actuator and sensor faults simultaneously when small perturbation parameter ε and external disturbances exist.

2) The resulted estimator can guarantee that the error dynamic systems are asymptotically stable $\omega(t) = 0$ and satisfy H_∞ performance for sufficiently small ε. Due to the fact that the information of k-order derivative of fault is considered, it is more effective to

estimate the time-varying fault, which is a more general type in actual systems. Finally, simulation results are presented to illustrate the effectiveness of the proposed approach.

The rest of this paper is organized as follows. The system description is presented in Section 2. In Section 3, we show the main results and provide some new sufficient conditions for the existence of robust fault estimation observer. In Section 4, several numerical examples are given to demonstrate the effectiveness and merits of the proposed methods. Finally, a brief conclusion is drawn in Section 5.

Notations: $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space; I denotes the identity matrix; the superscripts T and -1 stand for the matrix transpose and inverse, respectively; notation $X > 0$ ($X \geq 0$) means that matrix X is real symmetric positive definite (positive semi-definite); $\|\cdot\|$ is the spectral norm. All matrices are assumed to have compatible dimensions for algebraic operations. The symbol "$*$" stands for matrix block induced by symmetry. For any square matrix M, $Sym(M)$ is defined by $Sym(M) = M + M^T$.

2. **Problem Formulation.** Consider a nonlinear T-S fuzzy singularly perturbed system with the following r number of rules:

Plant rule i: **IF** $\xi_1(t)$ is M_{i1} and … and $\xi_p(t)$ is M_{ip} **THEN**

$$\begin{cases} E_\varepsilon \dot{x}(t) = A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i u(t) + B_{f_{ai}} f_a(t) + B_{di} d(t) \\ y(t) = C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_i u(t) + D_{f_{si}} f_s(t) + D_{di} d(t) \\ x(t) = \phi_i(t), \quad \forall t \in [-\tau, 0],\ i = 1, 2, \ldots, r \end{cases} \tag{1}$$

where $E_\varepsilon = diag\{I_{n_1}, \varepsilon I_{n_2}\}$, ε ($0 < \varepsilon < 1$) is a singular perturbation parameter, $x(t) \in R^{n_1+n_2}$ is the state vector, $u(t) \in R^q$ is the control input and $y(t) \in R^l$ represents the system output vector. M_{ij} ($i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, p$) are fuzzy sets, $d(t) \in R^m$ is the exogenous disturbance input that belongs to $L_2[0, \infty)$, $f_a(t) \in R^q$ and $f_s(t) \in R^p$ represent the possible actuator and sensor fault respectively. A_i, $A_{\tau i}$, B_i, $B_{f_{ai}}$, B_{di}, C_i, $C_{\tau i}$, D_i, $D_{f_{si}}$ and D_{di} are constant real matrices of appropriate dimensions. Without loss of generality, it is assumed the pairs (A_i, C_i) are observable, where $i = 1, 2, \ldots, r$ and r is the number of IF-THEN rules. And $\xi_1(t), \ldots, \xi_p(t)$ are the premise variables, $\phi_i(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau, 0]$. In this paper it is also assumed that the premise variables do not depend on the input variables. $\tau(t)$ is the time-varying delay, and we will consider the case $\tau(t)$ is a differentiable function satisfying for all $t \geq 0$, $0 \leq \tau(t) \leq \tau$, $\dot{\tau}(t) \leq \tau_D$, where τ and τ_D are constants.

For convenience of notations, in the sequel, we denote

$$A(t) = \sum_{i=1}^{r} \mu_i(\xi) A_i, \quad A_\tau(t) = \sum_{i=1}^{r} \mu_i(\xi) A_{\tau i}, \quad B(t) = \sum_{i=1}^{r} \mu_i(\xi) B_i, \quad B_d(t) = \sum_{i=1}^{r} \mu_i(\xi) B_{di}$$

$$D(t) = \sum_{i=1}^{r} \mu_i(\xi) D_i, \quad C(t) = \sum_{i=1}^{r} \mu_i(\xi) C_i, \quad C_\tau(t) = \sum_{i=1}^{r} \mu_i(\xi) C_{\tau i}, \quad D_d(t) = \sum_{i=1}^{r} \mu_i(\xi) D_{di}$$

where $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_p(t))$, $\xi_i(t)$ are the premise variables. And $\mu_i(\xi(t)) = \beta_i(\xi(t)) / \sum_{j=1}^{r} \beta_j(\xi(t))$, $\beta_i(\xi(t)) = \Pi_{i=1}^{p} M_{ij}(\xi(t))$, where $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij}. It is easy to find that $\mu_i(\xi(t))$ satisfies $\mu_i(\xi(t)) \geq 0$, $\sum_{j=1}^{r} \mu_j(\xi(t)) = 1$ for any $\xi(t)$.

Then, by fuzzy blending, the overall system model with disturbance input, actuator and sensor faults is given by

$$\begin{cases} E_\varepsilon \dot{x}(t) = A(t)x(t) + A_\tau(t)x(t-\tau(t)) + B(t)u(t) + B_{fa}(t)f(t) + B_d(t)d(t) \\ y(t) = C(t)x(t) + C_\tau(t)x(t-\tau(t)) + D(t)u(t) + D_{fs}(t)f(t) + D_d(t)d(t) \\ x(t) = \phi(t), \quad t \in [-\tau, 0], \end{cases} \tag{2}$$

where $B_{fa}(t) = [\sum_{i=1}^{r} \mu_i(\xi(t))B_{f_{ai}} \quad 0]$, $D_{fs}(t) = [0 \quad \sum_{i=1}^{r} \mu_i(\xi(t))D_{f_{si}}]$, $f(t) = \left[f_a^T(t) \; f_s^T(t)\right]^T$. Here, the faults $f(t)$ are assumed as the time-varying signals whose k-order time derivatives are bounded, that is $\dot{f}(t) = f_1(t), \dot{f}_1(t) = f_2(t), \dot{f}_2(t) = f_3(t), \ldots, \dot{f}_{k-1}(t) = f_k(t), f_k(t) = 0$.

It should be mentioned that, in practice, when external disturbances are involved in nonlinear plant or the output of systems is affected by sensor fault and disturbances simultaneously, the existing results in [8, 15, 25, 32] will fail. On the other hand, we knew that by the fuzzy membership functions, the relevant T-S fuzzy model can give a feasible framework to express the nonlinear plant by a series of local linear sub-models. So the considered system (2) has lots of applications in the fields of fault detection of robot manipulator systems, sampled-data control and passivity analysis of delayed complex dynamic network systems. Based on the above considerations, how to design the robust fault estimator for T-S fuzzy systems (2) becomes complicated but meaningful, which will be researched in our work.

Assumption 2.1. *Assume that actuator fault $f_a(t)$ and sensor fault $f_s(t)$ along with its k-order corresponding derivative, as well as the external disturbance $d(t)$ satisfy $f_a(t) \in L_2[0,\infty)$, $\dot{f}_{aj}(t) \in L_2[0,\infty)$, $f_s(t) \in L_2[0,\infty)$, $\dot{f}_{sj}(t) \in L_2[0,\infty)$, $d(t) \in L_2[0,\infty)$, $(j = 0, 1, 2, \ldots, k-1)$.*

Remark 2.1. *In general, Assumption 2.1 is a normal one since for most practical systems, the energy of actuator fault or sensor fault is boundary. Moreover, once a system failure occurs, then they remain somehow constant, which implies their derivatives are energy-bounded, and similar assumptions can be found in* [35, 36].

Remark 2.2. *For T-S nonlinear system description* (2)*, we can see that a more general fuzzy system with state time delays is considered in this paper, including small singular perturbation parameter ε, possible exogenous disturbance input, actuator and sensor fault simultaneously. If there occur no perturbation and state time delay, i.e., $\varepsilon = 1$, $\tau(t) = 0$, then* (2) *reduces to the existing one in* [4]*. Further, if, $\tau(t) \neq 0$,* (2) *can be transformed to the one in* [7]*. When perturbation parameter $\varepsilon \neq 1$ is small enough, this kind of systems embraces complicated slow and fast dynamics and is difficult to design fault estimator.*

3. Main Results.

In the following, we are ready to express our main results and provide some new sufficient conditions for the existence of robust fault estimation observer for system (2).

3.1. Observer design with ε-dependent.

In order to estimate faults, the following proportional multiple-integral fault estimation observer is constructed:

$$\begin{cases} E_\varepsilon \dot{\hat{x}}(t) = A(t)\hat{x}(t) + A_\tau(t)\hat{x}(t-\tau(t)) + B(t)u(t) + B_{f_a}(t)\hat{f}(t) \\ \qquad\qquad - L_P(t)\left(\hat{y}(t) - y(t)\right) \\ \hat{y}(t) = C(t)\hat{x}(t) + C_\tau(t)\hat{x}(t-\tau(t)) + D(t)u(t) + D_{f_s}(t)\hat{f}(t) \\ \dot{\hat{f}}(t) = -F_I(t)(\hat{y}(t) - y(t)) + \hat{f}_1(t), \\ \dot{\hat{f}}_1(t) = -F_I^1(t)(\hat{y}(t) - y(t)) + \hat{f}_2(t) \\ \vdots \\ \dot{\hat{f}}_j(t) = -F_I^j(t)(\hat{y}(t) - y(t)) + \hat{f}_{j+1}(t), \ldots, \dot{\hat{f}}_k(t) = 0, \quad (j = 1, 2, \ldots, k-1) \end{cases} \tag{3}$$

where $\hat{x}(t) \in R^n$ is the observer state, $\hat{y}(t) \in R^l$ is the observer output, and $\hat{f}(t) \in R^{q+p}$ is an estimate of fault $f(t)$. The objective of observer-based estimator design scheme is to obtain the appropriate dimension gain matrices $L_P(t) \in R^{n\times l}$, $F_I^j(t) \in R^{q\times l}$, where $L_P(t) = \sum_{i=1}^r \mu_i(\xi(t))L_{Pi}$, $F_I^j(t) = \sum_{i=1}^r \mu_i(\xi(t))F_i^j$, and estimate fault despite the presence of disturbance, perturbation parameter ε and time-varying state delay.

By defining $e_x(t) = \hat{x}(t) - x(t)$, $e_y(t) = \hat{y}(t) - y(t)$, $e_f(t) = \hat{f}(t) - f(t)$, we can obtain that

$$\begin{aligned} E_\varepsilon \dot{e}_x(t) &= (A(t) - L_p(t)C(t))e_x(t) + (A_\tau(t) - L_p(t)C_\tau(t))e_x(t-\tau(t)) \\ &\quad + (B_{f_a}(t) - L_p(t)D_{f_s}(t))e_f(t) - (B_d(t) - L_p(t)D_d(t))d(t) \end{aligned}$$

and

$$\begin{cases} \dot{e}_f(t) = -F_I(t)(C(t)e_x(t) + C_\tau(t)e_x(t-\tau(t)) + D_{f_s}(t)e_f(t) + D_d(t)d(t)) + e_{f_1}(t) \\ \dot{e}_{f_1}(t) = -F_I^1(t)(C(t)e_x(t) + C_\tau(t)e_x(t-\tau(t)) + D_{f_s}(t)e_f(t) + D_d(t)d(t)) + e_{f_2}(t) \\ \vdots \\ \dot{e}_{f_j}(t) = -F_I^j(t)(C(t)e_x(t) + C_\tau(t)e_x(t-\tau(t)) + D_{f_s}(t)e_f(t) + D_d(t)d(t)) + e_{f_{j+1}}(t) \\ \vdots \\ \dot{e}_{f_{k-1}}(t) = -F_I^{(k-1)}(t)(C(t)e_x(t) + C_\tau(t)e_x(t-\tau(t)) + D_{f_s}(t)e_f(t) + D_d(t)d(t)) + e_{f_k}(t) \end{cases}$$

where $j = 1, 2, \ldots, k-1$ is the time derivative order of the system fault $f(t)$, and then by denoting

$$\begin{aligned} e^T(t) &= \left[e_x^T(t), e_f^T(t), e_{f_1}^T(t), \ldots, e_{f_{(k-1)}}^T(t)\right] \\ \omega^T(t) &= \left[d^T(t), \dot{f}^T(t), \dot{f}_1^T(t), \ldots, \dot{f}_{k-1}^T(t)\right], \end{aligned}$$

the error dynamic systems are deduced from (2) and (3) as follows:

$$\begin{cases} E(\varepsilon)\dot{e}(t) = \left[\bar{A}(t) - \bar{L}(t)\bar{C}(t)\right]e(t) + \left[\bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\right]e(t-\tau(t)) \\ \qquad\qquad + \left[\bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)\right]\omega(t) \\ e_y(t) = \bar{C}(t)e(t) + \bar{C}_\tau(t)e(t-\tau(t)) - \bar{D}_d(t)\omega(t) \end{cases} \tag{4}$$

where

$$\bar{A}(t) = \begin{bmatrix} A(t) & B_{f_a}(t) & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & \cdots & 0 \\ 0 & 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{A}_\tau(t) = \begin{bmatrix} A_\tau(t) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\bar{L}(t) = \begin{bmatrix} L_p(t) \\ F_I(t) \\ F_I^1(t) \\ \vdots \\ F_I^{k-1}(t) \end{bmatrix}, \quad \bar{B}_d(t) = \begin{bmatrix} B_d(t) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$E(\varepsilon)=\begin{bmatrix} E_\varepsilon & 0 & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I \end{bmatrix}, \quad \bar{C}(t)=\begin{bmatrix} C(t) & D_{f_s}(t) & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\bar{C}_\tau(t)=\begin{bmatrix} C_\tau(t) & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{D}_d(t)=\begin{bmatrix} D_d(t) & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Remark 3.1. *In general, sliding mode observer based fault estimation requires the preliminary knowledge of the upper bound of $f(t)$ in* [18, 19] *and the fault estimation filter is designed under the assumption $f(t) \in L_2[0,\infty)$. However, in many practical systems, there is a transient period during which the fault establishes itself, after which, there remains more or less constant, meaning that the derivatives of the faults are energy-bounded, i.e., $\dot{f}(t) \in L_2[0,\infty)$. Therefore, the assumption that $\dot{f}(t) \in L_2[0,\infty)$ is satisfied in this paper, which is more general than those assumption used in aforementioned design methods.*

From error dynamics (4), we can see that the matrix $\bar{L}(t)$ contains the two matrices $L_P(t)$, $F_I^j(t)$ that have to be designed. Therefore, a necessary condition for the existence of fuzzy fault estimation observer is that the pairs $\left(\bar{A}_i, \bar{C}_i\right)$ are observable, and then the proposed robust FE observer design is converted to the problem of seeking the gain matrix $\bar{L}(t)$ such that

(i) the error dynamic system (4) with time-varying state delay is asymptotically stable ($\omega(t)=0$);

(ii) the following performance is satisfied:

$$\int_0^L \|e_f(t)\|^2 dt \le \gamma^2 \int_0^L \|\omega(t)\|^2 dt \tag{5}$$

for $L>0$ and $\omega(t) \in L_2[0,\infty)$ under zero initial conditions.

3.2. **Observer existence conditions with ε-independent.** For simplicity, we introduce the following vectors:

$$\zeta^T(t)=\begin{bmatrix} e^T(t) & e^T(t-\tau(t)) & e^T(t-\tau) & \omega^T(t) \end{bmatrix}$$

$$\Gamma(t)=\begin{bmatrix} \bar{A}(t)-\bar{L}(t)\bar{C}(t) & \bar{A}_\tau(t)-\bar{L}(t)\bar{C}_\tau(t) & 0 & \bar{L}(t)\bar{D}_d(t)-\bar{B}_d(t) \end{bmatrix}$$

Then, the state equation of error dynamics (4) can be rewritten as $E(\varepsilon)\dot{e}(t)=\Gamma(t)\zeta(t)$. Next, a fuzzy-augmented fault estimation PMI observer design method under H_∞ performance is proposed to achieve robust fault estimation.

Theorem 3.1. *Consider system* (4)*, for the given $\gamma>0$ and positive scalars τ, τ_D, there exists a sufficiently small $\varepsilon^*>0$ such that for any $\varepsilon \in (0,\varepsilon^*]$, the error dynamic system* (4) *is asymptotically stable (with $\omega(t)=0$) while satisfying a prescribed H_∞ performance* (5)*, if there exist matrices $P=\begin{bmatrix} P_{11} & 0 & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}$, ($P_{11}$, P_{22}, P_{33} are symmetric matrices), $Q_1>0$, $Q_2>0$, $R>0$, $\bar{L}(t)$ and free weighting matrices M, N such that the following inequalities hold:*

$$\begin{bmatrix} \tilde{\Omega}(t) & \tau M \\ * & -\tau R \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} \tilde{\Omega}(t) & \tau N \\ * & -\tau R \end{bmatrix} < 0 \tag{7}$$

where $\tilde{\Omega}(t) = \Omega(t) + \tau\Gamma(t)^T R\Gamma(t) - \Phi - \Phi^T$

$$\Omega(t) = \begin{bmatrix} \Theta_{11}(t) & \Theta_{12}(t) & 0 & \Theta_{14}(t) \\ * & -(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Theta_{11}(t) = Sym\left(P^T\left(\bar{A}(t) - \bar{L}(t)\bar{C}(t)\right)\right) + Q_1 + Q_2 + \bar{I}_{k(q+p)}\bar{I}^T_{k(q+p)}$$

$$\Theta_{12}(t) = P^T\left(\bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\right), \quad \Theta_{14}(t) = P^T\left(\bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)\right)$$

$$\Phi = \begin{bmatrix} M & -M+N & -N & 0 \end{bmatrix}, \quad \bar{I}^T_{k(q+p)} = \begin{bmatrix} 0 & I_{k(q+p)} \end{bmatrix}$$

Proof: Here, we can see that the error dynamic (4) contains the singular perturbation parameter E_ε in $E(\varepsilon)$, where $E_\varepsilon = diag\{I_{n_1}, \varepsilon I_{n_2}\}$ means that the error dynamic embraces the slow and fast error states, in order to facilitate us to better analyze system stability, $E(\varepsilon)$ can be divided as follows:

$$E(\varepsilon) = \begin{bmatrix} \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix} & 0 & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & \cdots & 0 \\ 0 & 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix} & 0 \\ 0 & I_{(k\times q)} \end{bmatrix}$$

$$= \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & \varepsilon I_{n_2} & 0 \\ 0 & 0 & I_{(k\times q)} \end{bmatrix}$$

Corresponding to the matrix $E(\varepsilon)$ above-mentioned, we define

$$P(\varepsilon) = P + \varepsilon P_0 = \begin{bmatrix} P_{11} & 0 & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix} + \varepsilon \begin{bmatrix} 0 & P_{21}^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} P_{11} & \varepsilon P_{21}^T & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}$$

where $P_{21} \in R^{n_2\times n_1}$, and $P_{11} \in R^{n_1\times n_1} > 0$, $P_{22} \in R^{n_2\times n_2} > 0$, $P_{33} \in R^{kq\times kq} > 0$ are symmetric positive-definite matrices. Then based on the above definition form about $E(\varepsilon)$ and $P(\varepsilon)$, it can be obtained that there exists a sufficiently small parameter ε_0^*, for $\forall\varepsilon \in (0, \varepsilon_0^*]$, an invertible matrix $P_\varepsilon = \begin{bmatrix} P_{11} & \varepsilon P_{21}^T \\ P_{21} & P_{22} \end{bmatrix}$ can be defined, where $P_{11} = P_{11}^T > 0$, $P_{22} = P_{22}^T > 0$. For $\forall\varepsilon \in (0, \varepsilon_1^*]$, if there exists a scalar $\varepsilon_1^* > 0$ such that $P_{11} - \varepsilon P_{21}^T P_{22}^{-1} P_{21} > 0$, then according to Schur complement theorem, we have

$$E^T(\varepsilon)P(\varepsilon) = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & \varepsilon I_{n_2} & 0 \\ 0 & 0 & I_q \end{bmatrix} \begin{bmatrix} P_{11} & \varepsilon P_{21}^T & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix} = \begin{bmatrix} P_{11} & \varepsilon P_{21}^T & 0 \\ \varepsilon P_{21} & \varepsilon P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix} > 0$$

For $\forall\varepsilon \in (0, \varepsilon_1^*]$, it is trivial that

$$E^T(\varepsilon)P(\varepsilon) = P^T(\varepsilon)E(\varepsilon) \tag{8}$$

Then the Lyapunov-Krasovskii functional candidate is constructed as follows:

$$V(t) = e^T(t)E(\varepsilon)P(\varepsilon)e(t) + \int_{t-\tau(t)}^{t} e^T(s)Q_1e(s)ds + \int_{t-\tau}^{t} e^T(s)Q_2e(s)ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)dsd\theta \tag{9}$$

where $Q_1 > 0$, $Q_2 > 0$, $R > 0$. And the time derivatives of $V(t)$, along the trajectories of the error dynamic systems (4) satisfy

$$\begin{aligned}
\dot{V}(t) = {} & e^T(t)\left[Sym\left(P^T(\varepsilon)\left(\bar{A}(t) - \bar{L}(t)\bar{C}(t)\right)\right) + Q_1 + Q_2\right]e(t) \\
& + 2e^T(t)P^T(\varepsilon)\left(\bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\right)e(t-\tau(t)) \\
& + 2e^T(t)P^T(\varepsilon)\left(\bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)\right)\omega(t) - (1-\dot{\tau}(t))\,e^T(t-\tau(t))Q_1e(t-\tau(t)) \\
& - e^T(t-\tau)Q_2e(t-\tau) + \tau\dot{e}^T(t)E(\varepsilon)RE(\varepsilon)\dot{e}(t) - \int_{t-\tau}^{t}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds
\end{aligned}$$

Denoting $\beta^T(t,s) = \left[\begin{array}{cc} \zeta^T(t) & (E(\varepsilon)\dot{e}(s))^T \end{array}\right]$, we obtain

$$\begin{aligned}
& \dot{V}(t) + e_f^T(t)e_f(t) - \gamma^2\omega^T(t)\omega(t) \\
= {} & e^T(t)\left[Sym\left(P^T(\varepsilon)\left(\bar{A}(t) - \bar{L}(t)\bar{C}(t)\right)\right) + Q_1 + Q_2 + \bar{I}_{k(q+p)}\bar{I}_{k(q+p)}^T\right]e(t) \\
& + 2e^T(t)P^T(\varepsilon)\left(\bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\right)e(t-\tau(t)) + 2e^T(t)P^T(\varepsilon)\left(\bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)\right)\omega(t) \\
& - (1-\dot{\tau}(t))e^T(t-\tau(t))Q_1e(t-\tau(t)) + \tau\zeta^T(t)\Gamma^T(t)R\Gamma(t)\zeta(t) \\
& - e^T(t-\tau)Q_2e(t-\tau) - \int_{t-\tau}^{t}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds - \gamma^2\omega^T(t)\omega(t) \\
& - 2\zeta^T(t)M\left[e(t) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{e}(s)ds\right] \\
& - 2\zeta^T(t)N\left[e(t-\tau(t)) - e(t-\tau) - \int_{t-\tau}^{t-\tau(t)}\dot{e}(s)ds\right]
\end{aligned}$$

Here, the free weighting matrices M, N are introduced in order to deal with the integral term $\int_{t-\tau}^{t}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds$ effectively to reduce the result conservatism. By decomposing the integral term $\int_{t-\tau}^{t}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds$ into $\int_{t-\tau(t)}^{t}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds$ and $\int_{t-\tau}^{t-\tau(t)}\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)ds$ and the change of only one matrix inequality relation $\dot{\tau}(t) < \tau_D$, we can obtain that

$$\begin{aligned}
& \dot{V}(t) + e_f^T(t)e_f(t) - \gamma^2\omega^T(t)\omega(t) \\
\le {} & \frac{1}{\tau}\int_{t-\tau(t)}^{t}\left[\zeta^T(t)\Omega_\varepsilon(t)\zeta(t) + 2\tau\zeta^T(t)M\dot{e}(s) - \tau\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)\right]ds \\
& + \frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)}\left[\zeta^T(t)\Omega_\varepsilon(t)\zeta(t) + 2\tau\zeta^T(t)N\dot{e}(s) - \tau\dot{e}^T(s)E(\varepsilon)RE(\varepsilon)\dot{e}(s)\right]ds \\
= {} & \frac{1}{\tau}\int_{t-\tau(t)}^{t}\beta^T(t,s)\left[\begin{bmatrix} \tilde{\Omega}(t) & \tau M \\ * & -\tau R \end{bmatrix} + \begin{bmatrix} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{bmatrix}\right]\beta(t,s)ds \\
& + \frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)}\beta^T(t,s)\left[\begin{bmatrix} \tilde{\Omega}(t) & \tau N \\ * & -\tau R \end{bmatrix} + \begin{bmatrix} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{bmatrix}\right]\beta(t,s)ds
\end{aligned}$$

where

$$\zeta^T(t) = \left[\begin{array}{cccc} e^T(t) & e^T(t-\tau(t)) & e^T(t-\tau) & \omega^T(t) \end{array}\right], \quad \Omega_\varepsilon(t) = \tilde{\Omega}(t) + \varepsilon\tilde{\Omega}_0(t)$$
$$\tilde{\Omega}(t) = \Omega(t) + \tau\Gamma(t)^T R\Gamma(t) - \Phi - \Phi^T, \quad \Phi = \left[\begin{array}{cccc} M & (-M+N) & -N & 0 \end{array}\right]$$

with

$$\Omega(t) = \left[\begin{array}{cccc} \Theta_{11}(t) & \Theta_{12}(t) & 0 & \Theta_{14}(t) \\ * & -(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{array}\right]$$
$$\Omega_0(t) = \left[\begin{array}{cccc} Sym\left(P_0^T\left(\bar{A}(t) - \bar{L}(t)\bar{C}(t)\right)\right) & P_0^T\left(\bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\right) & 0 & P_0^T\left(\bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)\right) \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{array}\right]$$

For the perturbation parameter integral term, it is obvious that

$$\int_{t-\tau(t)}^{t} \beta^T \left[\begin{array}{cc} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{array}\right] \beta ds + \int_{t-\tau}^{t-\tau(t)} \beta^T \left[\begin{array}{cc} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{array}\right] \beta ds$$
$$= \int_{t-\tau}^{t} \beta^T \left[\begin{array}{cc} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{array}\right] \beta ds$$

so we can obtain that

$$\begin{aligned} &\dot{V}(t) + e_f^T(t)e_f(t) - \gamma^2\omega^T(t)\omega(t) \\ &< \frac{1}{\tau}\int_{t-\tau(t)}^{t} \beta^T(t,s) \left[\begin{array}{cc} \tilde{\Omega}(t) & \tau M \\ * & -\tau R \end{array}\right] \beta(t,s)ds \\ &\quad + \frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)} \beta^T(t,s) \left[\begin{array}{cc} \tilde{\Omega}(t) & \tau N \\ * & -\tau R \end{array}\right] \beta(t,s)ds \\ &\quad + \frac{1}{\tau}\int_{t-\tau}^{t} \beta^T(t,s) \left[\begin{array}{cc} \varepsilon\tilde{\Omega}_0(t) & 0 \\ 0 & o(\varepsilon) \end{array}\right] \beta(t,s)ds \\ &= \frac{1}{\tau}\left[\int_{t-\tau(t)}^{t} \beta^T(t,s) \left[\begin{array}{cc} \tilde{\Omega}(t) & \tau M \\ * & -\tau R \end{array}\right] \beta(t,s)ds \right. \\ &\quad \left. + \int_{t-\tau}^{t-\tau(t)} \beta^T(t,s) \left[\begin{array}{cc} \tilde{\Omega}(t) & \tau N \\ * & -\tau R \end{array}\right] \beta(t,s)ds \right] + O(\varepsilon) \end{aligned}$$

Then according to the results in [27, 28], we know that there exists ε_2^*, for any $\varepsilon \in (0, \varepsilon_2^*]$, Inequalities (6) and (7) hold, one has $\dot{V}(t) + e_f^T(t)e_f(t) - \gamma^2\omega^T(t)\omega(t) < 0$. Therefore, by $V(L) \geq 0$ and $V(0) = 0$ under zero initial conditions, we can conclude that $\exists\varepsilon^* > 0$, where $\varepsilon^* = \min\{\varepsilon_0, \varepsilon_1, \varepsilon_2\}$, for any $\varepsilon \in (0, \varepsilon^*]$, Inequality (5) holds for all $L > 0$ and any nonzero $\omega(t) \in L_2[0, \infty)$.

In addition, when $\omega(t) = 0$, by choosing the same Lyapunov function as (9) and following the similar line in the earlier deduction under conditions (6) and (7), we can easily obtain that the time derivative of $V(t)$ along the solution of error dynamics (4) with $\omega(t) = 0$ satisfies $\dot{V}(t) < 0$, which indicates the asymptotic stability of systems (4). This completes the proof.

Remark 3.2. *In the process of analyzing stability of error dynamic system and giving stability analysis results, the derivative of Lyapunov functional candidate is always dealt with* *matrix inequalities approach. However, the integral term* $\int_{t-\tau}^{t} \dot{e}^T(s)R\dot{e}(s)ds$ *was ignored in* [20-22] *and some useful negative integral term* $\int_{t-\tau}^{t-\tau(t)} \dot{e}^T(s)R\dot{e}(s)ds$ *was directly deleted in* [23, 24]*. In this paper, in order to further reduce the conservatism of conclusion, only one identical matrix equality about* $\dot{\tau}(t) < \tau_D$ *is used and other derivation processes are equivalent transformations. Meanwhile, the full information about* $\tau(t)$ *and free weight matrix method is considered in LKF, so the less conservatives results than the existing ones in* [20, 24] *can be obtained.*

Remark 3.3. *From the above analysis process, we can see that the augment error dynamic systems (4) are effectively constructed to make the gain matrix* $L_p(t)$ *and* $F_I^j(t)$ $j = 0, 1, \ldots, k-1$ *to be designed in one matrix variable* $\bar{L}(t)$*. In addition, based on the configuration of* $E(\varepsilon)$ *and* $P(\varepsilon)$*, the fault estimator design scheme can be transformed into* ε*-independent type, and then all the obtained conditions above-mentioned can be formulated in the form of LMSs, which could be solved easily by toolbox to overcome ill-conditioned solution problem.*

Remark 3.4. *It should be mentioned that the results in Theorem 3.1 cannot be applied directly to dealing with the practical problem, because the obtained conditions are not linear matrix inequality due to the fact that* $P^T\bar{L}(t)$ *and* $\tau\Gamma(t)^T R\Gamma(t)$ *exist. Based on Theorem 3.1 and matrix inequality method, we transformed the conditions in Theorem 3.1 into Theorem 3.2 in terms of LMIs, which can be solved efficiently by using existing solvers such as LMI toolbox in the Matlab software.*

Theorem 3.2. *Consider system* (4)*, for the given positive scalars* τ*,* τ_D*,* δ *and* $\gamma > 0$*, if there exist matrices* $P = \begin{bmatrix} P_{11} & 0 & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}$ *(*P_{11}*,* P_{22}*,* P_{33} *are symmetric positive matrices),* $Q_1 > 0$*,* $Q_2 > 0$*,* $R > 0$*,* Y_i *and free weighting matrices* $M_i^T = \begin{bmatrix} M_{1i}^T & M_{2i}^T & M_{3i}^T & M_{4i}^T \end{bmatrix}$*,* $N_i^T = \begin{bmatrix} N_{1i}^T & N_{2i}^T & N_{3i}^T & N_{4i}^T \end{bmatrix}$ $(i = 1, 2, \ldots, r)$ *such that the following inequalities hold*

$$\Xi_{ii} < 0 \quad i = 1, 2, \ldots, r \tag{10}$$

$$\Xi_{ij} + \Xi_{ji} \le 0 \quad 1 \le i < j \le r \tag{11}$$

$$\Pi_{ii} < 0 \quad i = 1, 2, \ldots, r \tag{12}$$

$$\Pi_{ij} + \Pi_{ji} \le 0 \quad 1 \le i < j \le r \tag{13}$$

where

$$\Xi_{ij} = \begin{bmatrix} \bar{\Omega}_{ij} & \tau M_i & \sqrt{\tau}\bar{\Gamma}_{ij}^T \\ * & -\tau R & 0 \\ * & * & -2\delta p + \delta^2 R \end{bmatrix}$$

$$\Pi_{ij} = \begin{bmatrix} \bar{\Omega}_{ij} & \tau N_i & \sqrt{\tau}\bar{\Gamma}_{ij}^T \\ * & -\tau R & 0 \\ * & * & -2\delta p + \delta^2 R \end{bmatrix}$$

where

$$\bar{\Omega}_{ij} = \Omega_{ij} - \Phi_i - \Phi_i^T$$

$$\Omega_{ij} = \begin{bmatrix} \Theta_{1ij} & P\bar{A}_{\tau i} - Y_i\bar{C}_{\tau j} & 0 & Y_i\bar{D}_{dj} - P\bar{B}_{di} \\ * & -(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Theta_{1ij} = P\bar{A}_i - Y_i\bar{C}_j + \left[P\bar{A}_i - Y_i\bar{C}_j\right]^T + Q_1 + Q_2 + \bar{I}_{k(q+p)}\bar{I}^T_{k(q+p)}$$
$$\Phi_i = \left[\begin{array}{cccc} M_i & -M_i + N_i & -N_i & 0 \end{array}\right], \quad \bar{I}^T_{k(q+p)} = \left[\begin{array}{cc} 0 & I_{k(q+p)} \end{array}\right]$$
$$\bar{\Gamma}_{ij} = \left[\begin{array}{cccc} P\bar{A}_i - Y_i\bar{C}_j & P\bar{A}_{\tau i} - Y_i\bar{C}_{\tau j} & 0 & Y_i\bar{D}_{dj} - P\bar{B}_{di} \end{array}\right]$$

Then there exists a sufficiently small $\varepsilon^* > 0$ such that for any $\varepsilon \in (0, \varepsilon^*]$, the error dynamic systems (4) *are asymptotically stable (with $\omega(t) = 0$) while satisfying a prescribed H_∞ performance* (5), *and the observer gain matrices can be obtained as follows:*

$$\bar{L}_i = \left[\begin{array}{c} L_{P_i} \\ F_{I_i} \\ F^1_{I_i} \\ \vdots \\ F^{k-1}_{I_i} \end{array}\right] = \left(P^T\right)^{-1} Y_i$$

Proof: It follows from the fact $(\delta R - P)R^{-1}(\delta R - P) \geq 0$ that $-PR^{-1}P \leq -2\delta P + \delta^2 R$, for any scalar δ, we can see that (6) and (7) hold if the following inequalities hold

$$\left[\begin{array}{ccc} \bar{\Omega}(t) & \tau M & \sqrt{\tau}\Gamma(t)^T P \\ * & -\tau R & 0 \\ * & * & -2\delta p + \delta^2 R \end{array}\right] < 0$$

$$\left[\begin{array}{ccc} \bar{\Omega}(t) & \tau N & \sqrt{\tau}\Gamma(t)^T P \\ * & -\tau R & 0 \\ * & * & -2\delta p + \delta^2 R \end{array}\right] < 0$$

where

$$\bar{\Omega}(t) = \Omega(t) - \Phi - \Phi^T$$

with

$$\Omega(t) = \left[\begin{array}{cccc} \Theta_{11}(t) & \Theta_{12}(t) & 0 & \Theta_{14}(t) \\ * & -(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{array}\right]$$
$$\Theta_{11}(t) = Sym\left(P^T\bar{A}(t) - P^T\bar{L}(t)\bar{C}(t)\right) + Q_1 + Q_2 + \bar{I}_{k(q+p)}\bar{I}^T_{k(q+p)}$$
$$\Theta_{12}(t) = P^T\bar{A}_\tau(t) - P^T\bar{L}(t)\bar{C}_\tau(t), \quad \Theta_{14}(t) = P^T\bar{L}(t)\bar{D}_d(t) - P^T\bar{B}_d(t)$$
$$\Phi = \left[\begin{array}{cccc} M & -M + N & -N & 0 \end{array}\right], \quad \bar{I}^T_{k(q+p)} = \left[\begin{array}{cc} 0 & I_{k(q+p)} \end{array}\right]$$

Then, by Theorem 3.1 and with the changes of variables as $Y(t) = P^T\bar{L}(t)$, we can see that if the conditions (10)-(12) hold, it is true that there exists a sufficiently small $\varepsilon^* > 0$, where $\varepsilon^* = \min\{\varepsilon_0, \varepsilon_1, \varepsilon_2\}$, such that for any $\varepsilon \in (0, \varepsilon^*]$,

$$\sum_{i=1}^{r} \mu_i^2(\xi)\Xi_{ii} + \sum_{i=1}^{r}\sum_{i<j}^{r} \mu_i(\xi(t))\mu_j(\xi)(\Xi_{ij} + \Xi_{ji}) + o(\varepsilon) < 0 \tag{14}$$

$$\sum_{i=1}^{r} \mu_i^2(\xi)\Pi_{ii} + \sum_{i=1}^{r}\sum_{i<j}^{r} \mu_i(\xi)\mu_j(\xi(t))(\Pi_{ij} + \Pi_{ji}) + o(\varepsilon) < 0 \tag{15}$$

which means the inequality conditions (10)-(12) are sufficient for (14) and (15). This completes the proof.

Remark 3.5. *In contrast to the existing results, the proposed approach do not decompose the original systems into the slow and fast subsystems, so the designed observer-based estimator for considered systems without assumption that A_{22} is nonsingular for the fast subsystems in* [1, 2, 27]. *Besides, the gains of estimator to be designed are solved directly by a set of ε-independent LMIs; therefore, the proposed results have the broader scope of application than the traditional ones.*

Remark 3.6. *PMI based fault estimator considers the derivative of $f(t)$, when $f(t)$ is time-varying fault signal, the introduction of term $\hat{f}_j(t)$ $(j = 1, 2, \ldots, k-1)$ can enhance the speed and accuracy of fault estimation compared with constant fault. The following example will better illustrate this feature and the effectiveness of the proposed method.*

4. **Numerical Examples.** We provide two illustrative examples with simulation results to demonstrate the applicability and the effectiveness of the proposed design method.

4.1. **Example 1.** The dynamic equation of a flexible joint inverted pendulum device is given as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = I_2^{-1}\left(mgl\sin x_1(t) - \beta_s\varepsilon^2 x_3(t) - \beta_d\varepsilon x_4(t)\right) \\ \varepsilon\dot{x}_3(t) = x_4(t) \\ \varepsilon\dot{x}_4(t) = I_2^{-1}mgl\sin x_1(t) - I_p^{-1}\beta_s\varepsilon^2 x_3(t) - I_p^{-1}\beta_d\varepsilon x_4(t) - I_1^{-1}u(t) + I_1^{-1}\alpha\psi(t) \end{cases} \tag{16}$$

where $x_1(t) = \theta_2(t)$, $x_2(t) = \dot{\theta}_2(t)$, $x_3(t) = \varepsilon^{-2}(\theta_2(t) - \theta_1(t))$, $x_4(t) = \varepsilon^{-1}\left(\dot{\theta}_2(t) - \dot{\theta}_1(t)\right)$, ε is the perturbation parameter, $I_p = I_1 \cdot I_2(I_1 + I_2)^{-1}$, and $\theta_1(t)$ denotes the angle (rad) of the pendulum from the vertical, $\theta_2(t)$ denotes the angle (rad) of the rotor from the vertical, I_1 is the moment of inertia (kgm^2) of the rotor, I_2 is the moment of inertia (kgm^2) of the pendulum, m is the mass (kg) of the pendulum, l is the length (m) from the center of mass of the pendulum round its center of mass. The parameters of the plant are given as $g = 9.8$ m/s^2, $m = 1$ kg, $l = 1$ m, $I_1 = 1$ kgm^2, $I_2 = 5$ kgm^2, $\beta_s = 30000$ Nm, $\beta_d = 300$ Nms, and $\alpha = 0.5$. The objective here is to estimate the possible fault of inverted pendulum system under disturbance input. Assuming $-\pi < x_1(t) < \pi$, the

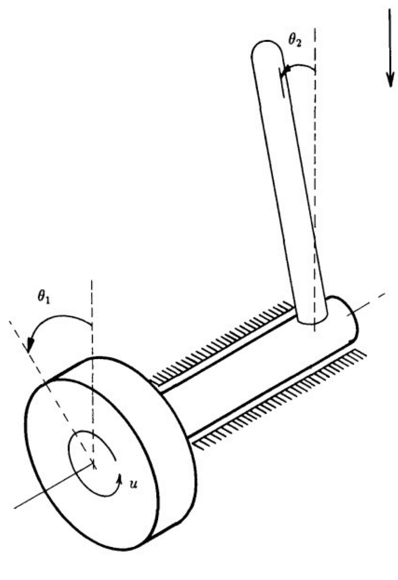


FIGURE 1. Flexible joint inverted pendulum device

nonlinear system (16) can be described by the following fuzzy singularly perturbed model with fault and disturbance.

Rule 1: **IF** $x_1(t)$ is about 0, **THEN**

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= A_1 x(t) + A_{\tau 1} x(t-\tau(t)) + B_1 u(t) + B_{f_{a1}} f_a(t) + B_{d1} d(t) \\ y(t) &= C_1 x(t) + C_{\tau 1} x(t-\tau(t)) + D_1 u(t) + D_{f_{s1}} f_s(t) + D_{d1} d(t) \end{aligned}$$

Rule 2: **IF** $x_1(t)$ is about $\pm\pi$, **THEN**

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= A_2 x(t) + A_{\tau 2} x(t-\tau(t)) + B_2 u(t) + B_{f_{a2}} f_a(t) + B_{d2} d(t) \\ y(t) &= C_2 x(t) + C_{\tau 2} x(t-\tau(t)) + D_2 u(t) + D_{f_{s2}} f_s(t) + D_{d2} d(t) \end{aligned}$$

where $E_\varepsilon = diag\{1, 1, \varepsilon, \varepsilon\}$ and $x = [x_1(t), x_2(t), x_3(t), x_4(t)]$,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.96 & 0 & -0.4 & -0.6 \\ 0 & 0 & 0 & 1 \\ 1.96 & 0 & -2.4 & -3.6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad B_{f_{a1}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.4 & -0.6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2.4 & -3.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad B_{f_{a2}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

and $C_1 = C_2 = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$, $A_{\tau i} = (1-a)A_i$ with $a = 0.2$, $C_{\tau 1} = C_{\tau 2} = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}$, $D_i = 1$, $D_{f_{si}} = 0.1$, $D_{di} = 0.2$ for $(i = 1, 2)$. For the state time-varying delay $\tau(t)$, we assume that $\tau = 0.1$, $\tau_D = 0.05$. Then we set $\delta = 10$, H_∞ performance index $\gamma = 2.5$, by solving the conditions (10)-(13) in Theorem 3.2 based on the LMIs approach of Matlab toolbox, one obtains the following feasible solution P:

$$1.0e+03 * \begin{bmatrix} 0.0398 & -0.0283 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0283 & 0.0362 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0172 & 0.0021 & 0.0132 & 0.0107 & 0 & 0 & 0 & 0 \\ -0.0137 & -0.0037 & 0.0107 & 0.0156 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0018 & -0.0067 & 0.0088 & 0.0324 \\ 0 & 0 & 0 & 0 & -0.0067 & 0.0411 & -0.1073 & -0.1362 \\ 0 & 0 & 0 & 0 & 0.0088 & -0.1073 & 0.5014 & -0.5046 \\ 0 & 0 & 0 & 0 & 0.0324 & -0.1362 & -0.5046 & 8.9348 \end{bmatrix}$$

and the observer gains matrices are

$$L_{P1} = \begin{bmatrix} 8.8440 \\ 9.2824 \\ 8.7416 \\ 9.1149 \end{bmatrix}, \quad F_{I1} = \begin{bmatrix} 68.4854 \\ 22.6227 \end{bmatrix}, \quad F_{I1}^1 = \begin{bmatrix} 3.9719 \\ 0.3207 \end{bmatrix}$$

and

$$L_{P2} = \begin{bmatrix} 8.2765 \\ 8.1145 \\ 8.0005 \\ 7.8583 \end{bmatrix}, \quad F_{I2} = \begin{bmatrix} 69.5441 \\ 23.0101 \end{bmatrix}, \quad F_{I2}^1 = \begin{bmatrix} 4.0360 \\ 0.3265 \end{bmatrix}$$

In order to further illustrate the effectiveness of the proposed approach, we choose different parameters to calculate the observer gain matrix and present more results, such as when setting $g = 9.8$ m/s^2, $m = 2$ kg, $l = 1$ m, $I_1 = 2$ kgm^2, $I_2 = 10$ kgm^2, $\beta_s = 30000$

Nm, $\beta_d = 300$ Nms, and $\alpha = 0.5$, the matrix of fuzzy singularly perturbed model can be obtained as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.96 & 0 & -0.2 & -0.3 \\ 0 & 0 & 0 & 1 \\ 1.96 & 0 & -1.2 & -1.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.2 & -0.3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.2 & -1.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

And by solving the conditions in Theorem 3.2 based on the LMIs approach, the following feasible solution can be obtained:

$$L_{P1} = \begin{bmatrix} 9.0443 \\ 9.3885 \\ 8.3716 \\ 9.6936 \end{bmatrix}, \quad \begin{bmatrix} F_{I1} \\ F_{I1}^1 \end{bmatrix} = \begin{bmatrix} 55.8940 \\ 17.9801 \\ 3.0967 \\ 0.2433 \end{bmatrix}$$

$$L_{P2} = \begin{bmatrix} 8.2997 \\ 8.1891 \\ 7.7509 \\ 8.2144 \end{bmatrix}, \quad \begin{bmatrix} F_{I2} \\ F_{I2}^1 \end{bmatrix} = \begin{bmatrix} 56.8992 \\ 18.3321 \\ 3.1525 \\ 0.2483 \end{bmatrix}$$

Here, in order to facilitate simulation, we choose membership functions for Rules 1 and 2 are $\mu_1(x_1(t)) = \sin(x_1(t))/x_1(t)$ when $x_1(t) \neq 0$, otherwise, $\mu_1(x_1(t)) = 1$ when $x_1(t) = 0$, $\mu_2(x(t)) = 1 - \mu_1(x(t))$, and considering the following actuator and sensor fault respectively:

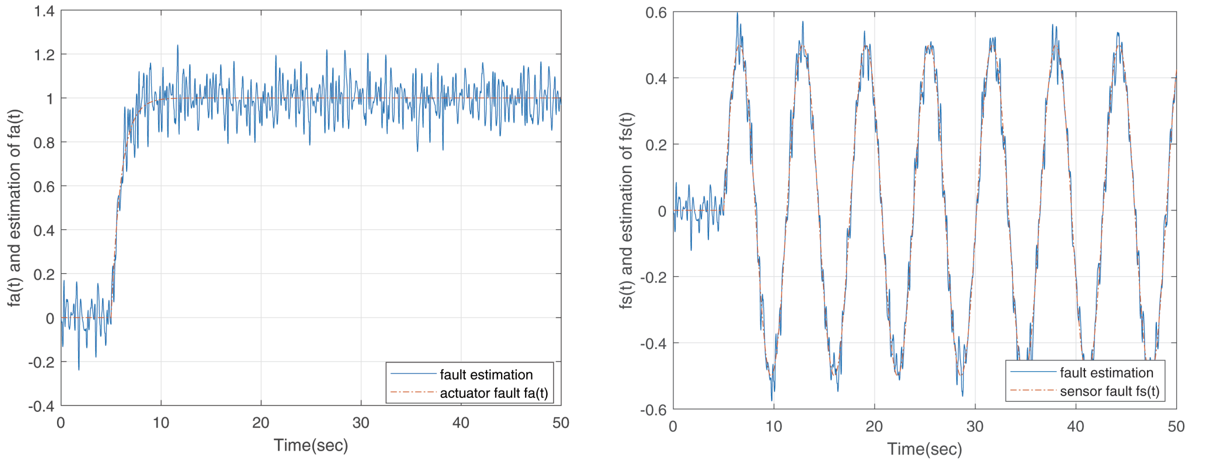
$$f_a(t) = \begin{cases} 0 & 0 \le t < 5 \\ \left(1 - e^{-(t-5)}\right) & 5 \le t \le 50 \end{cases} \qquad f_s(t) = \begin{cases} 0 & 0 \le t < 5 \\ 0.5\sin(t) & 5 \le t \le 50 \end{cases}$$

and the disturbance $d(t)$ is band-limited white noise with power 0.001 and sampling time 0.1 s.

By using the first obtained observer gain matrix, Figures 2 and 3 illustrate the simulation result of the proposed robust fault estimation with $\varepsilon = 0.5$. Therein, the actual actuator and sensor fault are depicted by red dashed line, and the fault estimation is represented by the blue line one. As shown in Figures 2(a) and 2(b), it is obvious that the robust fault estimation observer has a good performance to estimate the constant $f_a(t)$ and time-varying fault $f_s(t)$. And when the system disturbance exists, Figures 3(a) and 3(b) show that the variation of system state error and fault estimate error is limited in a small range, which guarantee the H_∞ stability of error dynamic and accuracy of fault estimation.

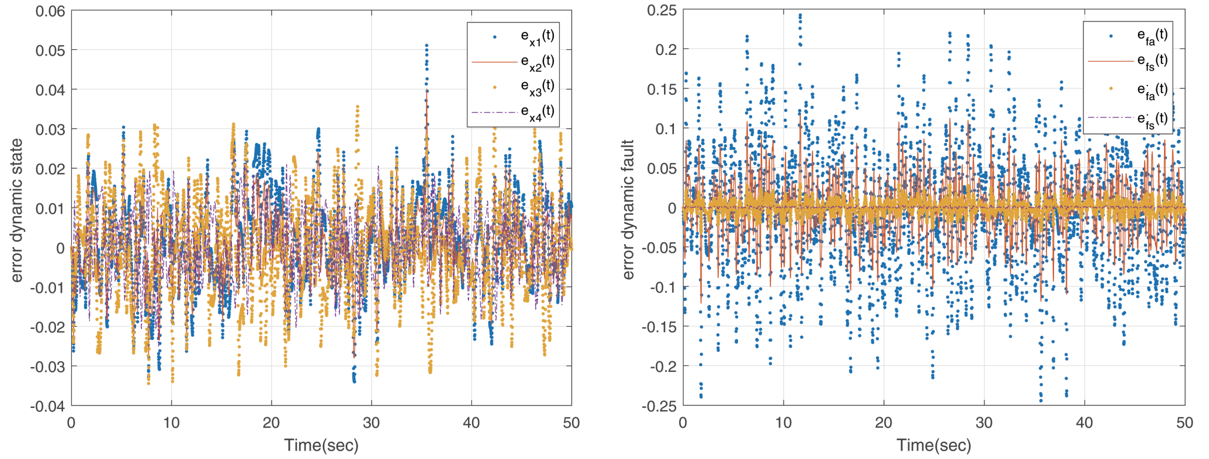
On the other hand, this paper mainly deals with the singular perturbed system with ε parameter disturbance, in order to further illustrate the effectiveness of the proposed method, we choose the different value of ε to investigate the estimation result in relationships with ε.

1) By setting $\varepsilon = 0.05$, Figures 4 and 5 show the estimation result based on the obtained fault estimation observer gains. It is seen from the design procedures and simulation results that, when the perturbation parameter ε changes from large $\varepsilon = 0.5$ to small $\varepsilon = 0.05$, the accuracy of the estimation is getting better. This implies that fault estimation can still be completed when ε is small enough.



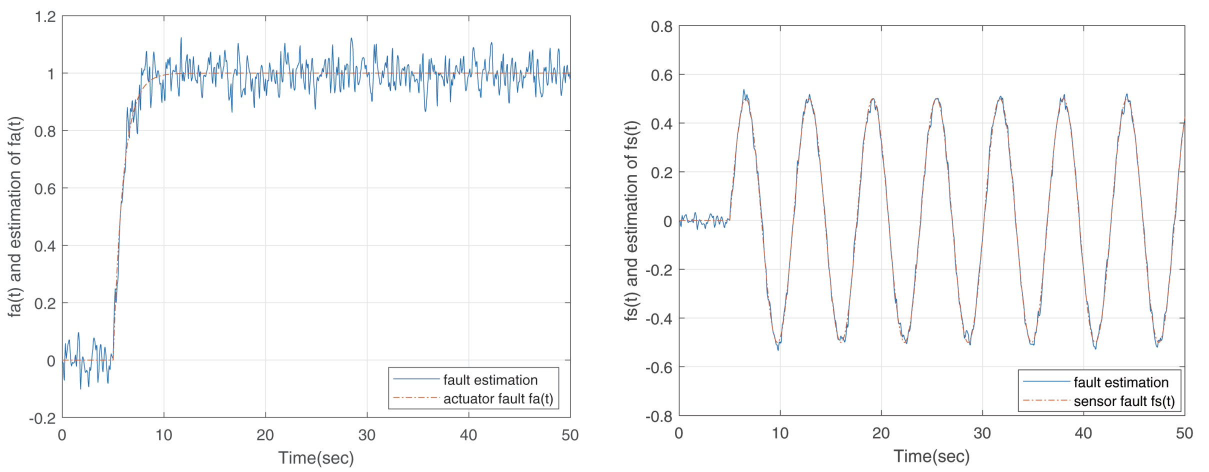
(a) $\varepsilon = 0.5$ $f_a(t)$ estimation (b) $\varepsilon = 0.5$ $f_s(t)$ estimation

FIGURE 2. $\varepsilon = 0.5$ actuator and sensor fault estimation



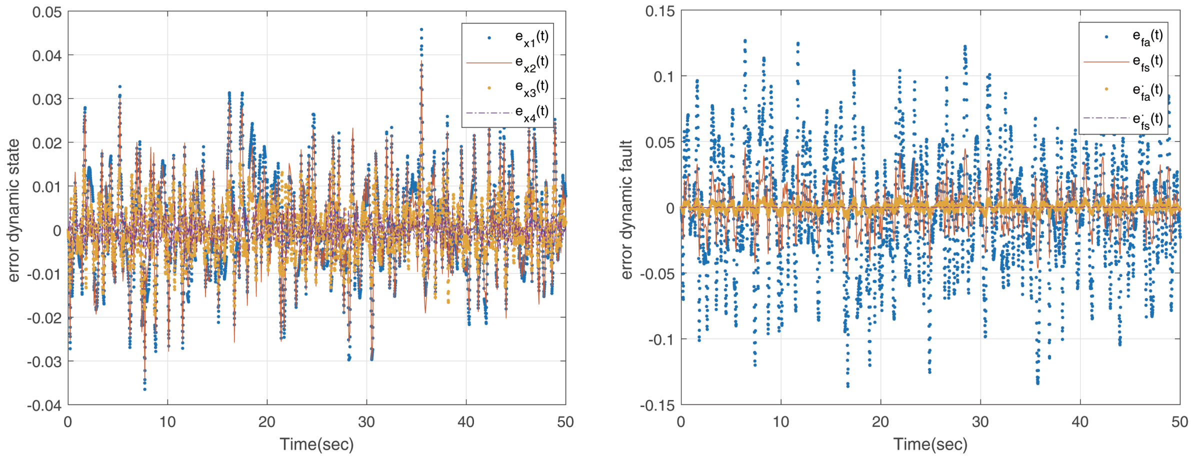
(a) $\varepsilon = 0.5$ state error dynamic (b) $\varepsilon = 0.5$ fault error dynamic

FIGURE 3. $\varepsilon = 0.5$ state and fault error dynamic response



(a) $\varepsilon = 0.05$ $f_a(t)$ estimation (b) $\varepsilon = 0.05$ $f_s(t)$ estimation

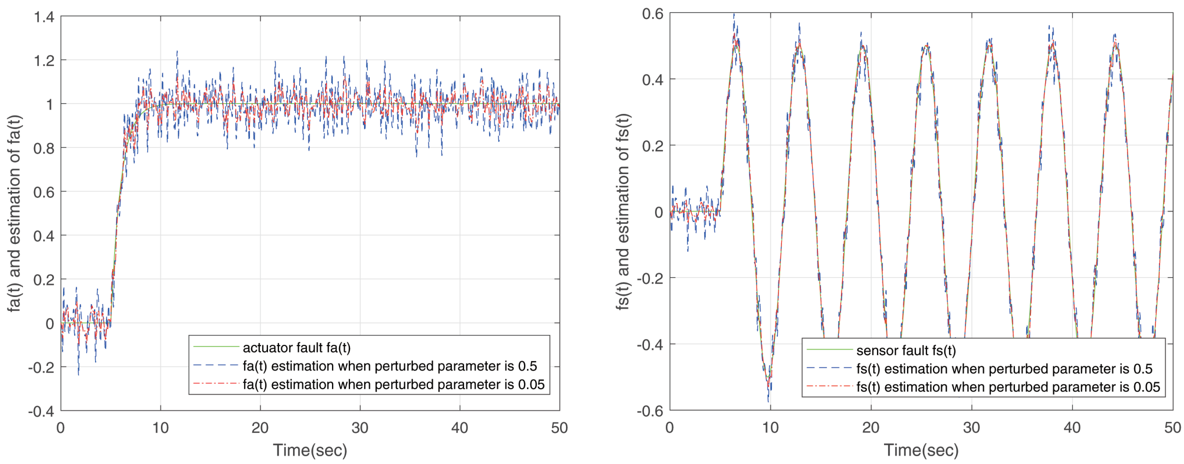
FIGURE 4. $\varepsilon = 0.05$ actuator and sensor fault estimation



(a) $\varepsilon = 0.05$ state error dynamic (b) $\varepsilon = 0.05$ fault error dynamic

FIGURE 5. $\varepsilon = 0.05$ state and fault error dynamic response

2) In Figures 6(a) and 6(b), FE results with $\varepsilon = 0.5$ and $\varepsilon = 0.05$ are shown respectively, we can see that whether the constant or time-varying fault, when parameter ε is small enough, the proposed method has the fast convergence speed and the high accuracy.



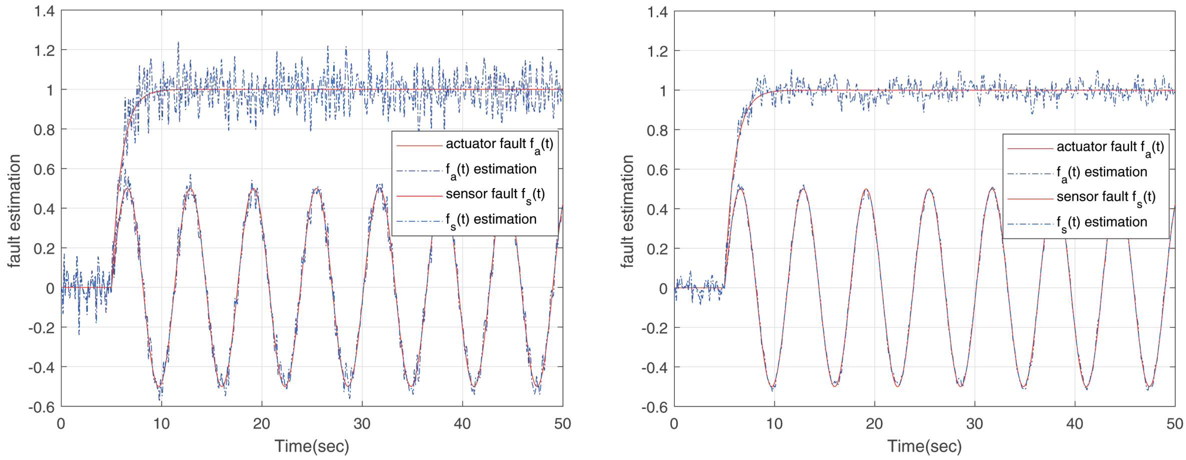
(a) $f_a(t)$ estimation $\varepsilon = 0.5$, $\varepsilon = 0.05$ (b) $f_s(t)$ estimation $\varepsilon = 0.5$, $\varepsilon = 0.05$

FIGURE 6. Actuator and sensor fault estimation comparison

3) Due to the adoption of proportional multiple-integral observer to estimate system fault, the derivatives of fault signals are effective considered in the process of constructing fault estimator, so from Figures 7(a) and 7(b) it is obvious that the method is more suitable for time-varying fault compared with constant fault, and result in wider application.

In other words, our method proposed in this paper can not only estimate the faults when perturbation and disturbance exist, but also reduce the conservativeness of the obtained result.

4.2. **Example 2.** For comparison purpose, next we consider a computer simulated truck-trailer system borrowed from [32]. The system $x_1(t)$ is perturbed by time-delay and the



(a) $f_a(t)$ and $f_s(t)$ estimation $\varepsilon = 0.5$

(b) $f_a(t)$ and $f_s(t)$ estimation $\varepsilon = 0.05$

FIGURE 7. Actuator and sensor fault estimation comparison

delayed model is given as

$$\begin{cases} \dot{x}_1(t) = -a\dfrac{v\bar{t}}{Lt_0}x_1(t) - (1-a)\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau(t)) + \dfrac{v\bar{t}}{lt_0}u(t) \\ \dot{x}_2(t) = a\dfrac{v\bar{t}}{Lt_0}x_1(t) + (1-a)\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau(t)) \\ \dot{x}_3(t) = \dfrac{v\bar{t}}{t_0}\sin\left[x_2(t) + a\dfrac{v\bar{t}}{2L}x_1(t) + (1-a)\dfrac{v\bar{t}}{2L}x_1(t-\tau(t))\right] \end{cases} \tag{17}$$

The constant a is the retarded coefficient, which satisfies the conditions: $a \in [0, 1]$. The limits 1 and 0 correspond to no delay term and to a completed delay term, respectively. In this example, the model parameters are given as $a = 0.7$, $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$. Then, we use the following fuzzy models to design the fuzzy fault observer:

Rule 1: **IF** $\xi_1(t) = x_2(t) + a\frac{v\bar{t}}{2L}x_1(t) + (1-a)\frac{v\bar{t}}{2L}x_1(t-\tau)$ is about 0, **THEN**

$$\begin{cases} \dot{x}(t) = A_1x(t) + A_{\tau 1}x(t-\tau(t)) + B_1u(t) \\ y(t) = C_1x(t) \end{cases} \tag{18}$$

Rule 2: **IF** $\xi_1(t) = x_2(t) + a\frac{v\bar{t}}{2L}x_1(t) + (1-a)\frac{v\bar{t}}{2L}x_1(t-\tau)$ is about π or $-\pi$, **THEN**

$$\begin{cases} \dot{x}(t) = A_2x(t) + A_{\tau 2}x(t-\tau(t)) + B_2u(t) \\ y(t) = C_2x(t) \end{cases} \tag{19}$$

Thus, the delay model with fault $f(t)$ and disturbance $d(t)$ is given by T-S fuzzy systems as follows

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{2}\mu_i(\xi(t))[A_ix(t) + A_{\tau i}x(t-\tau(t)) + B_iu(t) + B_{fi}f(t) + B_{di}d(t)] \\ y(t) = \displaystyle\sum_{i=1}^{2}\mu_i(\xi(t))C_ix(t) \end{cases} \tag{20}$$

where $x(t) = \left[\begin{array}{ccc} x_1(t) & x_2(t) & x_3(t) \end{array}\right]^T$ and

$$A_1 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{\tau 1} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{dv^2\bar{t}^2}{2Lt_0} & \dfrac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{\tau 2} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}$$

Here, we assume that $B_i = B_{fi}$ $(i = 1, 2)$ due to the fact that the actuator faults usually occur in the input channel, $C_1 = C_2 = \left[\begin{array}{ccc} 0.5 & 0.5 & 0.5 \end{array}\right]$ and the disturbance distribution matrices are $B_{d1} = B_{d2} = \left[\begin{array}{ccc} 0.05 & 0.05 & 0.05 \end{array}\right]^T$. For simulation purpose, we choose membership functions for Rules 1 and 2 are $\mu_1(\xi(t)) = 1/(1+\exp(x_1(t)+0.5))$, $\mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$ with initial condition $\left[\begin{array}{ccc} 0.5\pi & 0.75\pi & -5 \end{array}\right]^T$, and set $d = 10 * t_0/\pi$.

It should be noted that the above mentioned time delay fuzzy system in [32] is a special case that we considered in this paper, which do not have the small singular perturbation parameter ε and only actuator fault considered under different fuzzy rules. Here, we consider a more practical situation that the output is affected by the sensor fault and external disturbances. It is assumed that $C_{\tau i} = (1-a)C_{\tau i}$, $D_{f_{si}} = D_{di} = 1.5$, which means that $y(t) = \sum_{i=1}^{2} \mu_i(\xi(t))[C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_{f_{si}} f_s(t) + D_{di} d(t)]$. The time-varying delay acting on system state is given by $0 < \tau(t) < \tau = 0.5$, $\tau_D = 10$, then by computing matrix Inequalities (10)-(13) in Theorem 3.2 based on Matlab LMIs toolbox, when $\delta = 10$, $\gamma = 5$ one obtains a feasible solution:

$$L_{P1} = \begin{bmatrix} -24.3243 \\ 8.4446 \\ -44.9258 \end{bmatrix}, \quad F_{I1} = \begin{bmatrix} 23.8673 \\ 7.4062 \end{bmatrix}, \quad F_{I1}^1 = \begin{bmatrix} 1.2679 \\ 0.1182 \end{bmatrix}$$

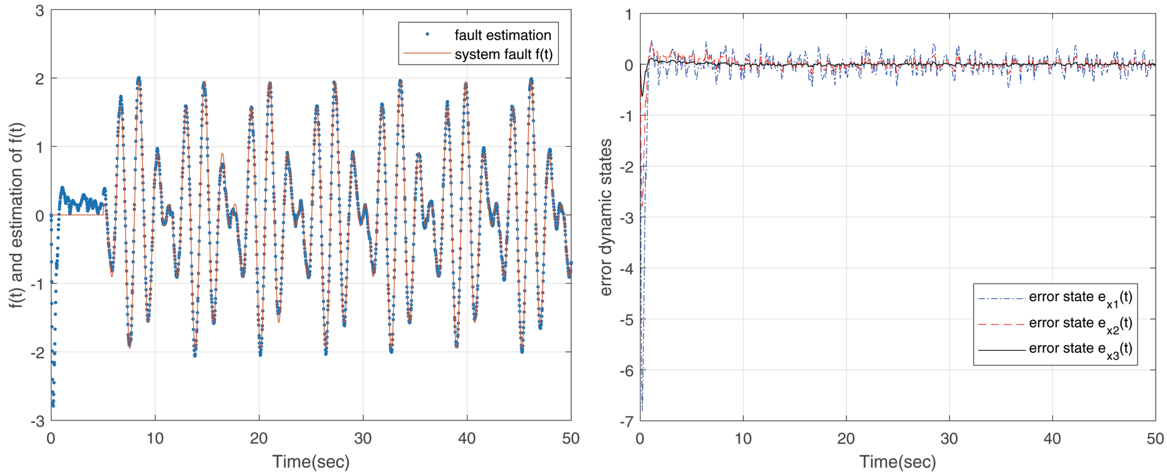
$$L_{P2} = \begin{bmatrix} -23.7953 \\ 8.2424 \\ -43.7820 \end{bmatrix}, \quad F_{I2} = \begin{bmatrix} 23.5514 \\ 7.3090 \end{bmatrix}, \quad F_{I2}^1 = \begin{bmatrix} 1.2514 \\ 0.1166 \end{bmatrix}$$

Then, a time-varying fault $f_a(t) = f_s(t) = f(t)$ is simulated as

$$f(t) = \begin{cases} 0 & 0 \le t < 5 \\ \sin(3t-9) + \sin(4t-12) & 5 \le t \le 20 \end{cases}$$

Figure 8(a) illustrates the fault estimation simulation results. It is obvious that despite the initial error exists, the fault is estimated with satisfactory accuracy and rapidity with time. It follows from Figure 8(b) that the error states $e_{x_1}(t)$, $e_{x_2}(t)$, $e_{x_3}(t)$ fluctuate in a limited range when the external disturbances $d(t) \neq 0$.

Remark 4.1. *It is worth mentioning that the work in* [32] *has considered no small singular perturbation parameter ε and sensor faults exist case under different fuzzy rules. However, this is not the situation always because in practice, the output of system will be affected by sensor faults and external disturbances. Considering this characteristic of system, the*



(a) Fault $f(t)$ estimation (b) Error dynamic states response

FIGURE 8. Actuator and sensor fault estimation comparion

FE strategy in [32] *is not valid. From Figure 8(a) we can see that, the proposed method can accurately realize fault estimation under disturbance input despite there is error in the initial stage of fault estimation.*

Remark 4.2. *In contrast to the existing results* [32]*, the proposed approaches have the advantage that the gains of estimator are solved directly by a set of ε-independent LMIs, so the given methods are easy to implement and can be applied to both standard and non-standard singularly perturbed systems. It means that whether the perturbation parameter ε and sensor fault exist or not, the fault estimator always works to give the fault estimation information.*

5. **Conclusion.** In this paper, the problem of robust fault estimation has been studied for a class of time delay T-S fuzzy singular perturbation systems with external disturbances. By selecting the appropriate Lyapunov-Krasovskii functional and applying LMIs techniques, the PMI observer is designed to simultaneously estimate actuator and sensor faults by attenuation of the disturbance influence. The existence conditions of observer-based estimator are provided and proved. Two numerical examples are given to demonstrate the effectiveness of the developed techniques. It should be noted that the approach presented in this paper requires that the FE observer has the same membership functions as the plants model, and dose not consider the interval-valued fuzzy systems. In addition, how to solve the fault-tolerant control problem under such conditions becomes particularly important. We will consider these problems in our future work.

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REFERENCES

[1] A. Tellili, M. Abdelkrim and M. Benrejeb, Reliable H_∞ control of multiple time scales singularly perturbed systems with sensor failure, *Int. J. Control*, vol.80, no.5, pp.659-665, 2007.

[2] A. Tellili, M. N. Abdelkrim and M. Benrejeb, Model-based fault diagnosis of two-time scales singularly perturbed systems, *International Symposium on Control, Communications and Signal Processing*, pp.819-822, 2004.

[3] K. Zhang, B. Jiang and P. Shi, A new approach to observer-based fault-tolerant controller design for Takagi-Sugeno fuzzy systems with state delay, *Circuits Syst. Signal Processing*, vol.28, pp.679-697, 2009.

[4] K. Zhang, B. Jiang and M. Staroswiecki, Dynamic output feedback fault tolerant controller design for Takagi-Sugeno fuzzy systems with actuator faults, *IEEE Trans. Fuzzy Syst.*, vol.18, pp.194-201, 2010.

[5] Z. Gao, X. Shi and S. X. Ding, Fuzzy state-disturbance observer design for T-S fuzzy systems with application to sensor fault estimation, *IEEE Trans. Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol.38, pp.875-880, 2008.

[6] C. Sun, F. Wang and X. He, Robust fault estimation for Takagi-Sugeno nonlinear systems with time-varying state delay, *Circuits Syst. Signal Processing*, vol.34, pp.641-661, 2015.

[7] S. Huang and G. Yang, Fault tolerant controller design for T-S fuzzy systems with time-varying delay and actuator faults: A k-step fault-estimation approach, *IEEE Trans. Fuzzy Syst.*, vol.22, pp.1526-1540, 2014.

[8] S. He, Fault estimation for T-S fuzzy Markovian jumping systems based on the adaptive observer, *Int. J. Control Autom.*, vol.12, pp.977-985, 2014.

[9] J. Yoneyama, Output feedback control for fuzzy systems with immeasurable premise variables, *Proc. of IEEE Conference on Fuzzy Systems*, Jeju Island, Korea, pp.802-807, 2009.

[10] A. Benzaouia, A. EI-Hajjaji, A. Hmamedc and R. Oubaha, Fault tolerant saturated control for T-S fuzzy discrete-time systems with delays, *Nonlinear Analysis: Hybrid Systems*, vol.18, pp.60-71, 2015.

[11] H. Bao and J. Cao, Delay-distribution-dependent state estimation for discrete-time stochastic neural networks with random delay, *Neural Netw.*, vol.24, pp.19-28, 2011.

[12] P. Liu, New results on stability analysis for time-varying delays systems with non-linear perturbations, *ISA Trans.*, vol.52, pp.318-325, 2013.

[13] B. Marx, D. Koenig and J. Ragot, Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis, *IET Control Theory Appl.*, vol.1, pp.1487-1495, 2007.

[14] Q. Han, Absolute stability of time-delay systems with sector-bounded nonlinearity, *Automatica*, vol.41, pp.2171-2176, 2005.

[15] Q. Jia, W. Chen, Y. Zhang and H. Li, Fault reconstruction and fault-tolerant control via learning observers in Takagi-Sugeno fuzzy descriptor systems with time delays, *IEEE Trans. Ind. Electron.*, vol.62, pp.3885-3895, 2015.

[16] H. Ghorbel, A. El-Hajjaji, M. Souissi and M. Chaabane, Fault-tolerant trajectory tracking control for Takagi-Sugeno systems with unmeasurable premise variables: Descriptor approach, *Circuits Syst. Signal Processing*, vol.33, no.6, pp.1763-1781, 2014.

[17] D. Rotondo, M. Witczak, V. Puig, F. Nejjari and M. Pazera, Robust unknown input observer for state and fault estimation in discrete-time Takagi-Sugeno systems, *Int. J. Syst. Sci.*, vol.47, no.14, pp.3409-3424, 2016.

[18] S. K. Nguang, P. Shi and S. X. Ding, Delay-dependent fault estimation for uncertain time-delay nonlinear systems: An LMI approach, *Int. J. Robust Nonlinear Control*, vol.16, no.18, pp.913-933, 2006.

[19] H. Choi, LMI-based nonlinear fuzzy observer-controller design for uncertain MIMO nonlinear systems, *IEEE Trans. Fuzzy Syst.*, vol.15, no.5, pp.956-971, 2007.

[20] S. Huang, X. He and N. Zhang, New results on H_∞ filter design for nonlinear systems with time delay via T-S fuzzy models, *IEEE Trans. Fuzzy Syst.*, vol.19, no.1, pp.193-199, 2011.

[21] C. Lin, Q. Wang, T. Lee and B. Chen, H_∞ filter design for nonlinear systems with time-delay through T-S fuzzy model approach, *IEEE Trans. Fuzzy Syst.*, vol.16, no.3, pp.739-745, 2008.

[22] C. Lien and K. Yu, Robust control for Takagi-Sugeno fuzzy systems with time-varying state and input delays, *Chaos Solitons and Fractals*, vol.35, pp.1003-1008, 2008.

[23] Y. Su, B. Chen, C. Lin and H. Zhang, A new fuzzy H_∞ filter design for nonlinear continuous-time dynamic systems with time-varying delays, *Fuzzy Sets Syst.*, vol.160, pp.3539-3549, 2009.

[24] J. Zhang, Y. Xiao and R. Tao, New results on H_∞ filtering for fuzzy time-delay systems, *IEEE Trans. Fuzzy Syst.*, vol.17, no.1, pp.128-137, 2009.

[25] D. Liu, Y. Yang and Y. Zhang, Robust fault estimaion for singularly perturbed systems with Lipschitz nonlinearity, *J. Franklin Inst.*, vol.353, pp.876-890, 2016.

[26] F. Gong and K. Khorasani, Fault diagnosis of linear singularly perturbed systems, *IEEE Conference on Decision and Control*, pp.2415-2420, 2005.

[27] E. Fridman, Effects of small of delays on stability of singularly perturbed systems, *Automatica*, vol.38, no.5, pp.897-902, 2002.

[28] J. Chen, Y. Sun and H. Min, New results on static output feedback H_∞ control for fuzzy singularly perturbed systems: A linear matrix inequality approach, *Int. J. Robust Nonlinear Control*, vol.23, pp.681-694, 2013.

[29] D. Ding, X. Du and X. Xie, Fault estimation filter design for discretetime Takagi-Sugeno fuzzy systems, *IET Control Theory Appl.*, vol.10, no.18, pp.2456-2465, 2016.

[30] F. You, S. Chen, X. Zhang and N. Chen, Robust fault estimation for Takagi-Sugeno fuzzy systems with state time-varying delay, *Int. J. Adapt. Control Signal Processing*, pp.1-10, 2019.

[31] T. Chen, A. Babanin, A. Muhammad, B. Chapron and J. CYChen, Modified fuzzy delay-dependent control criterion for systems, *ICIC Express Letters, Part B: Applications*, vol.10, no.1, pp.25-29, 2019.

[32] H. Li, F. You, F. Wang and S. Guan, Robust fast adaptive fault estimation and tolerant control for T-S fuzzy systems with interval time-varying delay, *Int. J. Syst. Sci.*, vol.48, no.8, 2017.

[33] X. Li, D. Lu, G. Zeng, J. Liu and W. Zhang, Integrated fault estimation and non-fragile fault-tolerant control design for uncertain Takagi-Sugeno fuzzy systems with actuator fault and sensor fault, *IET Control Theory Appl.*, vol.11, no.10, pp.1542-1553, 2017.

[34] X. H. Li, H. Karimi, Y. Wang, D. Lu and S. Guo, Robust fault estimation and fault-tolerant control for Markovian jump systems with general uncertain transition rates, *J. Franklin. Inst.*, vol.355, pp.3508-3540, 2018.

[35] J. Lan and R. J. Patton, Integrated design of fault-tolerant control for nonlinear systems based on fault estimation and T-S fuzzy modeling, *IEEE Trans. Fuzzy Syst.*, vol.25, pp.1141-1153, 2017.

[36] Y. F. Mu, H. G. Zhang, K. Zhang and H. Ren, Integrated design of robust fault estimation and fault-tolerant control against simultaneous actuator and sensor faults, *Asian J. Control*, pp.1-10, 2019.