International Journal of Innovative
Computing, Information and Control
Volume **17**, Number **3**, June **2021**

ICIC International ©2021 ISSN 1349-4198
pp. 1007–1017

CONSIDERING DYNAMIC SIMILARITY OF MEASURED VALUES FOR MULTI-SENSOR DATA FUSION

Yuanlong Yue, Yu Zhang and Xin Zuo

Department of Automation
China University of Petroleum
No. 18, Fuxue Road, Changping District, Beijing 100049, P. R. China
{ yueyuanlong; zuox }@cup.edu.cn; zhangyuu808@163.com

Received November 2020; revised March 2021

Abstract. *Multi-sensor state fusion and measurement fusion focus on the minimum of fusion variance and the sensor that suffers from less input noise will contribute more to the fusion result. However, if the measurement of the sensor which includes systematic error obviously discriminates against that of the others, it should play a non-significant role in the fusion process despite lower variance. Therefore, this paper proposes a new data fusion method, called Dynamic Weighted Fusion (DWF). Differing from the existing minimum variance fusion methods, it tunes the weighted coefficients calculated in terms of optimal variance through the dynamic similarity degree among all the sensors' measurement values that reflect the real-time contextual information. DWF fusion coefficients varying along with the sample time are non-constants. The sensor with lower input noise may be placed in a non-essential position if its output value differs from the other one. The proposed approach can be readily extended to output voting occurring in safety instrument systems. We show through simulations that DWF generally outperforms previous fusion methods.*

Keywords: Data fusion, Similarity, Contextual information, Dynamic weighted coefficients

1. **Introduction.** Information fusion or multi-sensor data fusion, which concerns the problem of integrating data from multiple and distinct sensors in order to achieve more accurate and specific inferences than those available by processing data from a single sensor, has been widely applied to many fields, such as military, target tracking, GPS positioning, and the process of control [1-5]. Data from diverse sensors are combined using techniques related to several disciplines: signal processing, statistics, artificial intelligence, pattern recognition, and information theory. The fusion strategy based on Kalman filter is one of the most significant methods and has been deeply investigated in different communities [6-8]. If state transition matrix and input noise vector equal identity matrix and zero vector respectively, then Kalman filter is reduced to least-square unbiased estimation [9]. This paper is limited to the multi-sensor fusion estimation.

For Kalman-filtering-based fusion, there are two different types of methods to cope with the measure, i.e., centralized (or measurement) and distributed (or weighted) fusion methods, depending on whether raw data are used directly for fusion center or not [10]. The centralized fusion method can give the globally optimal state estimation by directly combining local measurement data in an augment equation. The centralized fusion method generally involves minimal information loss in linear minimum mean square error, for all measured sensor data are communicated to the central site processing. However, it may result in disadvantages that 1) high dimension measurement vector and matrices require a

DOI: 10.24507/ijicic.17.03.1007

larger computational burden on inverse matrices and high data rates for communication; 2) the fusion may suffer from poor accuracy and stability if some measurements have some delay or even missing. The weighted fusion method includes the state-vector and measurement weighted fusion methods [11]. The weighted measurement fusion method weighs the local sensor measurements to obtain a weighted measurement fusion equation, which accompanies the state equation to obtain a final weighted measurement fusion Kalman filter by using a single estimator. The state-vector weighted fusion method uses a group of estimators to obtain an individual sensor-based estimate. In the state-vector fusion, only a smaller computational burden is required because of the lower dimension of the fused measurement vector. Furthermore, it readily leads to fault detection and isolation.

It is well known that, under the linear-Gaussian assumption, the distributed fusion has global optimality, i.e., it is algebraically equivalent to centralized fusion if measurement noises are uncorrected across for all the sensors. Various distributed fusion algorithms have been reported over the last decades. Carlson [12] developed a federated architecture applicable to distributed sensor systems with parallel processing capabilities. In [13], the functional equivalence between the weighted measurement fusion method and the centralized fusion method was proved using the information filter method and the method of computing the inverse of the partitioned matrix. Kim [14] proposed an optimal fusion filter under the assumption of normal distribution based on the maximum likelihood sense for systems with multiple sensors and assumed the process noise to be independent of measurement noise. An optimal distributed fusion steady-state Kalman in [15] was proposed for multi-sensor systems with colored measurement noises and different local dynamic models. Weighted measurement fusion fractional-order Kalman filter was presented under the independent white noises situation in [16]. Optimal sequential Kalman filtering was presented for discrete time-varying linear control systems with cross-correlated measurement noises in [17]. It is known that the existing methods are concerned with unbiased estimation and fusion strategies are limited to the minimum covariance framework. However, we have to face the problem that some sensors with smaller variance may be not reliable because their measurement data contaminated by systematic error resulting from the working environment is seriously distinct from that of the others in the fusion system.

Up to now, the issue of multi-sensor data fusion including the contextual information has not been fully investigated and remains to be improved. When the sensor is initially used, it can be considered that the accuracy of its measurement data is the same, but the performance index of the sensor declines over time, which eventually leads to a difference in its real-time data measurement. So we need to consider the contextual information contained in the real-time data. We develop an efficient computational method for recalculating the optimal weighted assignment strategy by introducing the similarity between two different sensors into the fusion process. The degree of similarity reflects distinction to some extent. The new data fusion method, called Dynamic Weighted Fusion (DWF), combines the inherent variance of each sensor and contextual information. The results of mathematic analysis and simulation show that the proposed fusion method is more effective and reasonable than the existing fusion method.

The remainder of this paper is organized as follows. In Section 2 the problem formulation is presented. A distributed fusion estimator with independent noise is proposed in Section 3 and the equivalence between centralized fusion estimator and distributed fusion estimator is proved. The dynamic reliability is researched in Section 4, and Dynamic Weighted Fusion Estimation (DWFE) is presented in Section 5. Section 6 gives the application to the single variable measurement based on the multi-sensor system. The conclusions are presented in Section 7.

2. **Problem Formulation.** Consider the multi-sensor linear discrete stochastic system with the same measurement matrix

$$y_i(k) = Hx(k) + v_i(k), \quad i = 1, \ldots, N, \tag{1}$$

where k is the discrete-time, and i denotes the ith sensor. $x(k) \in R^n$ is the unknown parameter vector to be estimated, $y_i(k) \in R^m$ is the observed data, H is the $m_i \times n$ known measurement matrix (we always assume that $m_i > n$), and $v_i(k) \in R^m$ is a zero-mean Gaussian noise vector. Given the data $y_i(k)$, we seek a fusion estimator $\hat{x}(k)$ of x that is close to x in some sense. This estimation problem arises in a large variety of areas in science and engineering, e.g., communication, signal processing and process control. In the following, I and 0 denote the identity matrix and zero matrix with compatible dimensions, respectively.

Because $v_i(k) \in R^m$ approximately obeys the distribution of Gaussian white noise, the following assumptions are made.

Assumption 2.1. *$v_i(k)$ and $v_j(k)$ are mutually independent white noises with zero mean, i.e.,*

$$E\left\{\begin{bmatrix} v_i(t) \\ v_j(t) \end{bmatrix}\begin{bmatrix} v_i^T(k), & v_j^T(k) \end{bmatrix}\right\} = \begin{bmatrix} R_i & \\ & R_j \end{bmatrix}\delta_{tk}, \tag{2}$$

where E denotes the mathematical expectation, the superscript T denotes the transpose, and δ_{tk} is the Kronecker delta function.

Assumption 2.2. *$v_i(k)$, $i = 1, \ldots, N$ and $x(t)$ are mutually independent, i.e.,*

$$E\left[v_i(k)x_j^T(t)\right] = 0. \tag{3}$$

The problem is based on the above information, using distributed fusion estimations, presenting an efficient computational method for recalculating the optimal weighted assignment strategy by introducing the similarity between two different sensors into the fusion process.

3. **Two Kinds of Fusion Estimations.** Centralized fusion estimation and distributed fusion estimation are two commonly used fusion methods, and centralized fusion estimation has problems such as heavy computational burden [18]. Centralized fusion estimation has theoretical significance and distributed fusion estimation is more in line with practical applications. Real-time data is updated iteratively over time. It is proved that their equivalence reduces the computational burden to a certain extent.

3.1. **Centralized fusion estimation.** Introducing an augmented measurement vector $y^{(C)}(k)$, then we combine all measurement equations to obtain a centralized measurement fusion equation as

$$y^{(C)}(k) = H^{(C)}x(k) + v^{(C)}(k), \tag{4}$$

with the definitions

$$y^{(C)}(k) = \left[y_1^T(k), y_2^T(k), \ldots, y_N^T(k)\right]^T, \tag{5}$$

$$H^{(C)}(k) = \left[H_1^T(k), H_2^T(k), \ldots, H_N^T(k)\right]^T, \tag{6}$$

$$v^{(C)}(k) = diag\left[v_1^T(k), v_2^T(k), \ldots, v_N^T(k)\right]^T \tag{7}$$

and the variance matrix is given as follows

$$R^{(C)} = diag\left[R_1^T, R_2^T, \ldots, R_N^T\right]^T. \tag{8}$$

For the centralized fusion estimation system (4), applying Gaussian-Markov estimation, we can obtain a centralized fusion estimator; for convenience, time k is omitted from Equation (9) to Equation (23)

$$\hat{x}^{(C)} = \left[H^{(C)^T} R^{(C)^{-1}} H^{(C)}\right]^{-1} H^{(C)^T} R^{(C)^{-1}} y^{(C)}. \tag{9}$$

The centralized fusion method has the minimized estimator error variance matrix as

$$R\left(\hat{x}^{(C)}\right) = \left[H^{(C)^T} R^{(C)^{-1}} H^{(C)}\right]^{-1}. \tag{10}$$

It is globally optimal in the sense that its precision is higher than that of each local estimator.

3.2. Distributed fusion estimation.

Lemma 3.1. *Let $\hat{x}_i$ be the unbiased estimators of n dimension stochastic vector x. Let the estimation errors be $\tilde{x}_i = \hat{x}_i - x$, $i = 1, \ldots, N$. Assuming that $\tilde{x}_i$ and $\tilde{x}_j$ $(i \neq j)$, are uncorrelated, i.e., $E\left[x_i x_j^T\right] = 0$ and $E\left[x_i x_i^T\right] = P_i$, then the optimally distributed fusion (i.e., linear minimum variance) estimator [19] with matrix weights is given as*

$$\hat{x}_0 = A_1\hat{x}_1 + A_2\hat{x}_2 + \cdots + A_N\hat{x}_N, \tag{11}$$

where the optimal matrix weights A_i, $i = 1, \ldots, N$ are computed as follows

$$A_i = \left[\sum_{i=1}^{N} P_i^{-1}\right]^{-1} P_i^{-1} \tag{12}$$

and its error variance matrix is given by

$$P_0 = \left[\sum_{i=1}^{N} P_i^{-1}\right]^{-1}. \tag{13}$$

Based on Gaussian-Markov Theorem and Equation (1), we have N unbiased estimators of the n dimension stochastic vector x denoted by $\hat{x}_i$, $i = 1, \ldots, N$

$$\hat{x}_i = \left[H_i^T R_i^{-1} H_i\right]^{-1} H_i^T R_i^{-1} y_i, \tag{14}$$

$$R(\tilde{x}_i) = \left[H_i^T R_i^{-1} H_i\right]^{-1}. \tag{15}$$

Directly applying Lemma 3.1, we have the distributed fusion estimator denoted by

$$\hat{x}^{(D)} = W_1\hat{x}_1 + W_2\hat{x}_2 + \cdots + W_N\hat{x}_N, \tag{16}$$

where the optimal matrix weights W_i, $i = 1, \ldots, N$ are computed as follows

$$W_i = \left[\sum_{i=1}^{N} R^{-1}(\tilde{x}_i)\right]^{-1} R^{-1}(\tilde{x}_i) \tag{17}$$

and its error variance matrix is given by

$$R\left(\hat{x}^{(D)}\right) = \left[\sum_{i=1}^{N} R^{-1}(\tilde{x}_i)\right]^{-1}. \tag{18}$$

Comparing Equations (16)-(18) with Equations (4)-(10), we note that the treatment in the two measurement fusion methods is quite different. However, we find that there exists a form of functional equivalence between the two methods, proved by the following theorem.

Theorem 3.1. *If the N sensors are used for data fusion, with different and independent noise characteristics, then the distributed fusion estimation is functionally equivalent to the centralized fusion estimation.*

Proof: Equation (10) can be reformed as

$$R\left(\hat{x}^{(C)}\right)=\left\{\left[H_1^T,\ldots,H_N^T\right]diag\left(R_i^{-1}\right)\begin{bmatrix}H_1\\ \vdots\\ H_N\end{bmatrix}\right\}^{-1}=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}. \tag{19}$$

Combining Equation (15) and (18), we can get

$$R\left(\hat{x}^{(D)}\right)=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}. \tag{20}$$

It is easy to find that the two fusion methods have the same fusion variance and

$$\hat{x}^{(C)}=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}\sum_{i=1}^{N}H_i^TR_i^{-1}y_i, \tag{21}$$

$$\begin{aligned}\hat{x}^{(D)}&=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}\sum_{i=1}^{N}R^{-1}(\tilde{x}_i)\hat{x}_i\\&=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}\sum_{i=1}^{N}R^{-1}(\tilde{x}_i)\left[H_i^TR_i^{-1}H_i\right]^{-1}H_i^TR_i^{-1}y_i.\end{aligned} \tag{22}$$

Considering Equation (15), Equation (22) can be simplified as

$$\hat{x}^{(D)}=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}\sum_{i=1}^{N}R^{-1}(\tilde{x}_i)\hat{x}_i=\left[\sum_{i=1}^{N}H_i^TR_i^{-1}H_i\right]^{-1}\sum_{i=1}^{N}H_i^TR_i^{-1}y_i. \tag{23}$$

Comparing with Equation (21), we immediately know that $\hat{x}^{(D)}$ is equal to $\hat{x}^{(C)}$.

The distributed fusion estimation avoids the inverse of the higher dimension matrix. The functional equivalence between $\hat{x}^{(D)}$ and $\hat{x}^{(C)}$ implies a functional equivalence between the two measurement fusion methods based on standard fusion estimation.

Analyzing Equations (16) and (17), it is clear that the optimal matrix weight W_i is only dependent on the variance $R(\tilde{x}_i)$, which can be regarded as prior static reliability of the sensor [21]. However, the static evaluation does not take account of the change of the sensor reliability in varying environments. Because environmental uncertainty and opposite disturbance may cause the sensors to degrade or fail, it must be able to dynamically monitor and assess them in the multi-sensor fusion system. Otherwise, the data with large variation will affect the result devastatingly and decrease the performance of the fusion system. So the information contained in the actual values $\hat{x}_1$ should be extracted to determine its influence on the fusion process, and can be regarded as dynamic reliability, which is used to evaluate the ability of the sensor to understand a dynamic working environment [21]. For instance, one sensor has higher static reliability (less variance), its contribution to the fusion result may be degraded for lower dynamic reliability. To deal with this problem, similarity applied to measuring the degree of consensus among a group of sensors is introduced in this paper.

4. **Similarity for Quantifying Dynamic Reliability.** Intuitively, for a given sensor, the concepts of distance and similarity are related to the other sensor in an inverse way, i.e., the less the distance between the sensor readings and that of the others, the greater the similarity degree of this sensor [22].

Definition 4.1. *For time k, the similarity measure between $\hat{x}_i^l$ and $\hat{x}_j^l$, denoted by $s_{ij}^l(k)$, can be obtained from the distance measure as*

$$s_{ij}^l(k) = \exp\left(-a\left|\hat{x}_i^l(k) - \hat{x}_j^l(k)\right|^a\right), \quad a > 0, \tag{24}$$

where l, $l = 1, \ldots, n$ denotes the lth element of the given sensor's estimation value.

$s_{ij}^l(k)$ is a quantifying index describing the similarity degree between the two sensors. When $\left|\hat{x}_i^l(k) - \hat{x}_j^l(k)\right| = 0$, then $s_{ij}^l(k)$ is 1, which implies the two sensors have the highest similarity degree; otherwise, $s_{ij}^l(k)$ is always less than 1. a is a tuning parameter.

Now, we can construct a similarity matrix for the lth element with identity diagonal elements denoted by $SM^l(k)$ given time k

$$SM^l(k) = \begin{bmatrix} 1 & s_{12}^l(k) & \cdots & s_{1N}^l(k) \\ s_{21}^l(k) & 1 & \cdots & s_{2N}^l(k) \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1}^l(k) & s_{N2}^l(k) & \cdots & 1 \end{bmatrix}. \tag{25}$$

Not that $\sum_{j=1}^N s_{ij}^l(k)$ represents the total similarity degree for the ith sensor and satisfies the properties of symmetry, reflexivity and transitively as similarity relationship introduced by Zadeh [23]. If the value of $\sum_{j=1}^N s_{ij}^l(k)$ is larger, it indicates that the measurement value of the ith sensor is nearer to those of the other sensors. Otherwise, it deviates from the others' measurement values, and its dynamic reliability should be lower. We define the total similarity degree for the lth element of the ith sensor, $i = 1, \ldots, N$ as

$$SM_i^l(k) = \sum_{j=1}^N s_{ij}^l(k). \tag{26}$$

$SM_i^l(k)$ illustrates the ith sensor compatibility with the others, so the dynamic reliability associated with a sensor is directly related to the compatibility. Now, update the total similarity degree up to a normalizing relative one indicating the relatively dynamic reliability. According to Equations (25) and (26), the normalizing relative similarity can be obtained as

$$R_i^l(k) = \frac{SM_i^l(k)}{\sum_{i=1}^N SM_i^l(k)}. \tag{27}$$

$R_i^l(k)$ only represents the dynamic reliability of given time k. However, the reliability may be lower during all the observation time, even though the normalizing total similarity is higher given time k. The dynamic reliability of the sensor should be shown by the consistent similarity along all the samples time.

Definition 4.2. *Up to time k, the consistency normalizing relative similarity for the lth element of the ith sensor is defined as follows*

$$\overline{R}_i^l(k) = \frac{\sum_{t=1}^k R_i^l(t)}{k}. \tag{28}$$

$\overline{R}_i^l(k)$ decreases the effect caused by measurement value fluctuation on the dynamic reliability. For instance, the reading dramatically changes for a certain time, so the $R_i^l(k)$ will be great small. In this situation, $\overline{R}_i^l(k)$ can resist this unnormal phenomenon. The recursive method for $\overline{R}_i^l(k)$ is described as follows to save the computational expenditure

$$\overline{R}_i^l(k) = \frac{k-1}{k}\overline{R}_i^l(k-1) + \frac{1}{k}R_i^l(k). \tag{29}$$

Now, we can construct a dynamic similarity matrix for sensor i given time k

$$R_i(k) = \begin{bmatrix} \overline{R}_i^1(k) & 0 & \cdots & 0 \\ 0 & \overline{R}_i^2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \overline{R}_i^n(k) \end{bmatrix}. \tag{30}$$

The matrix $\overline{R}_i^l(k)$ including the dynamic reliability information on the estimation $\hat{x}_i$ will be used to obtain adjusted fusion matrix composing variances and readings information. $R_i(k)$, $i = 1, \ldots, N$ satisfy the normalization, i.e.,

$$\sum_{i=1}^{N} R_i(k) = I. \tag{31}$$

The online data of the sensor fusion process is dynamic, and the dynamic similarity matrix between different sensors can be obtained based on the above information. In order to obtain the best fusion results, this paper considers a new fusion method including static and dynamic reliability.

5. **Novel Fusion Method Including Static and Dynamic Reliability.** Once the degree of similarity is obtained, the problem of how to incorporate it into the fusion process may arise. It stands to reason to believe that the sensor with smaller variance and larger dynamic reliability should contribute more to the fusion estimation result. The variance function and the dynamic reliability can be quantified by the optimal matrix weights W_i and dynamic similarity matrix $R_i(k)$, respectively. Now, combining W_i and $R_i(k)$, we obtain a novel fusion process embodying static and dynamic reliability information. Time k is omitted in the following. The new weighted matrix for the sensor i is defined as

$$W_i^{(R)} = R_i \times W_i. \tag{32}$$

The coefficient $W_i^{(R)}$ means that the role of $\hat{x}_i$ in the fusion result is proportional to the dynamic reliability and its inherent character expressed by variance. Normalize new weighted matrix

$$\overline{W}_i^{(R)} = R_i \times W_i \times \left[\sum_{i}^{N} R_i \times W_i \right]^{-1}. \tag{33}$$

Considering Equation (16), we can get the novel fusion method as follows

$$\hat{x}^{(R)} = \overline{W}_1^{(R)} \hat{x}_1 + \overline{W}_2^{(R)} \hat{x}_2 + \cdots + \overline{W}_N^{(R)} \hat{x}_N. \tag{34}$$

6. **Case Study.** We take the single variable measurement suffered from noise as an example to illustrate the new fusion method proposed in this paper. Assuming there are two measurement systems, each measurement system comprises three sensors, and all the sensors have the same observation model given by

$$y_i(k) = x(k) + v_i(k), \quad i = 1, 2, 3. \tag{35}$$

Model (35) is widely used in process measurements, such as temperature, pressure, and flow rate measure. We assume v_i, $i = 1, 2, 3$ is independent normal observed noise with zero mean and known variance σ_i^2. If the variance is unknown, it can be estimated appealing to the complete algorithm of least square estimations.

$$\hat{\sigma}_i^2 = \frac{[y_i - H\hat{x}_i(k_o)]^T [y_i - H\hat{x}_i(k_o)]}{k_o - 1}, \tag{36}$$

where k_o is the number of samples and $\hat{x}_i(k_o)$ is the estimation value of the complete algorithm of LS. y_i and H are defined as follows with the corresponding dimension.

$$y_i = [y_i(1), \ldots, y_i(k_o)]^T, \quad H = [1, \ldots, 1]^T \tag{37}$$

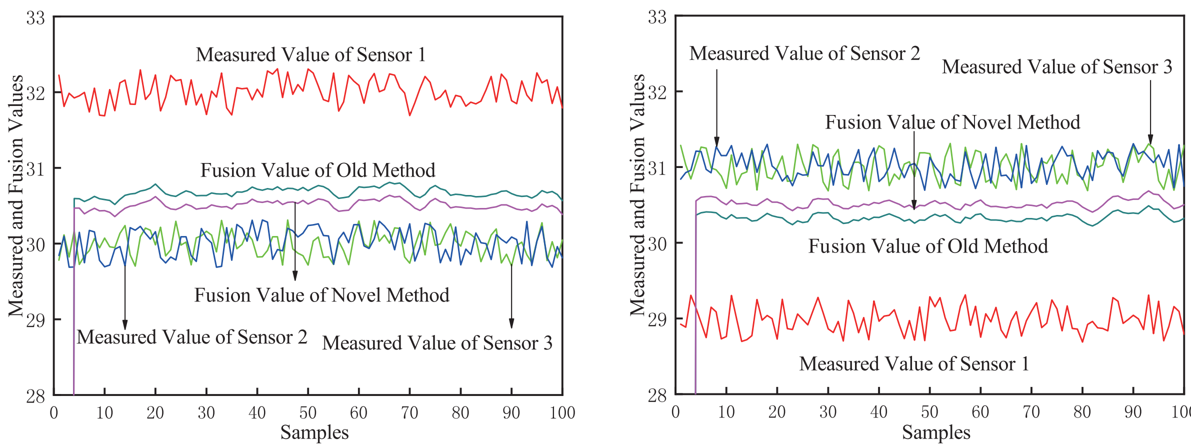
After getting the variance information, the simulation experiment can be carried out. Assuming the true value of x and the variance of the three sensors are known. The simulation measurement data is generated by the model (35). The observed data applied to fusion is updated by the moving horizon method, and the fusion estimator starts when getting 4 measurement data. Assumed moving step is 1 and sample size k is 100.

Note that if all the sensors have the same variance, the fusion method based on static reliability is equivalent to the average method on measure value. In this situation, we can immediately obtain the optimally weighted coefficient W_i based on Equation (17) before multi-sensor data fusion. So Equation (16) does not read information reflecting the environment changing in a round-about way.

Suppose the true values of x in the two measurement systems are 30 and 31, the simulation experiment includes two parts: 1) assuming all the sensors have the same variance 0.1, and the measurement value of the first sensor is obviously larger or smaller than the other two sensors readings; 2) the first sensor variance is lessened to be 0.05, and that of the others remains the value 0.1. The readings of the three sensors have the same scope. The results are compared with Equation (16) in terms of absolute error. The simulation results are shown as Figure 1 to Figure 4.

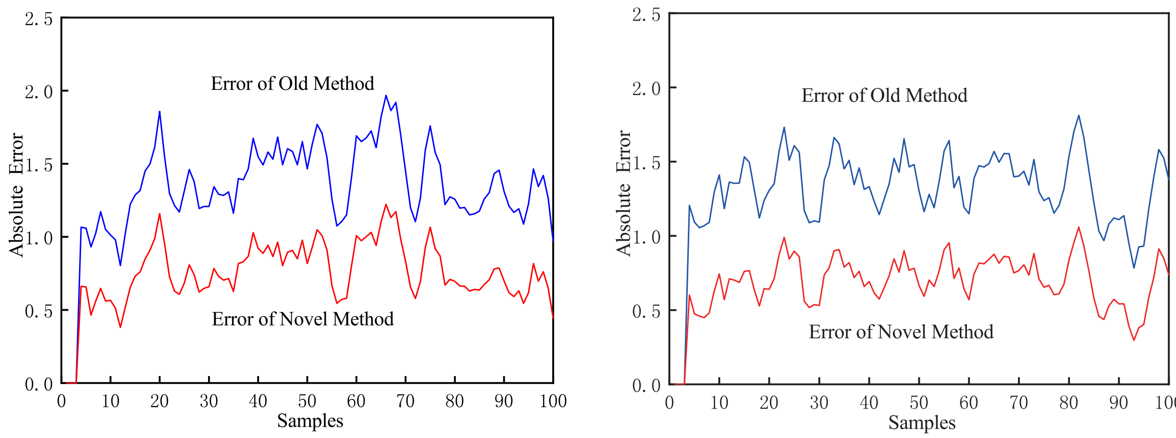
6.1. **Part one.** From Figure 1(a), we know that the value of the novel fusion method is less than that of the old method, and is closer to sensor 2 and sensor 3. From Figure 1(b), we know that the value of the novel fusion method is greater than that of the old method, and is also closer to sensor 2 and sensor 3. Comparing Figure 1(a) with Figure 1(b), this phenomenon implies that the function of sensor 1 with un-normal measured value is reduced in the fusion results. That is to say, the sensor with the higher similarity plays the main role in the fusion process. Besides, it can be found that the fusion result of the novel method is closer to the true value than the old method.

Figure 2 shows that the fusion method combining dynamic measurement information is superior to the one obtained by static variance characters in absolute error sense, because



(a) When the true value of x is 30 (b) When the true value of x is 31

FIGURE 1. The comparison curves of fusion results between novel and old fusion methods when all the sensors have the same variance 0.1

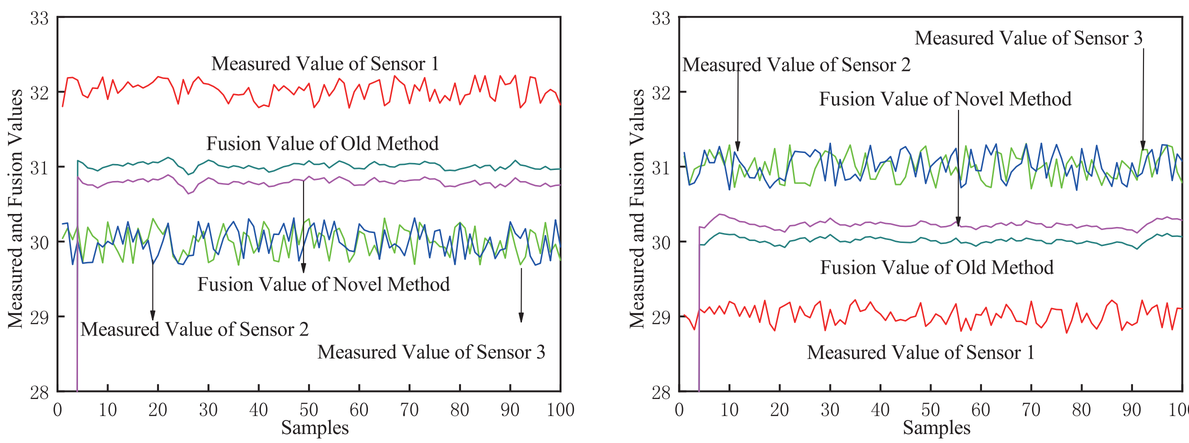


(a) When the true value of x is 30 (b) When the true value of x is 31

FIGURE 2. The comparison curves of errors between novel and old fusion methods when all the sensors have the same variance 0.1

the contribution of contextual disturbance on the fusion results is diminished by the tuned fusion matrix in Equation (34).

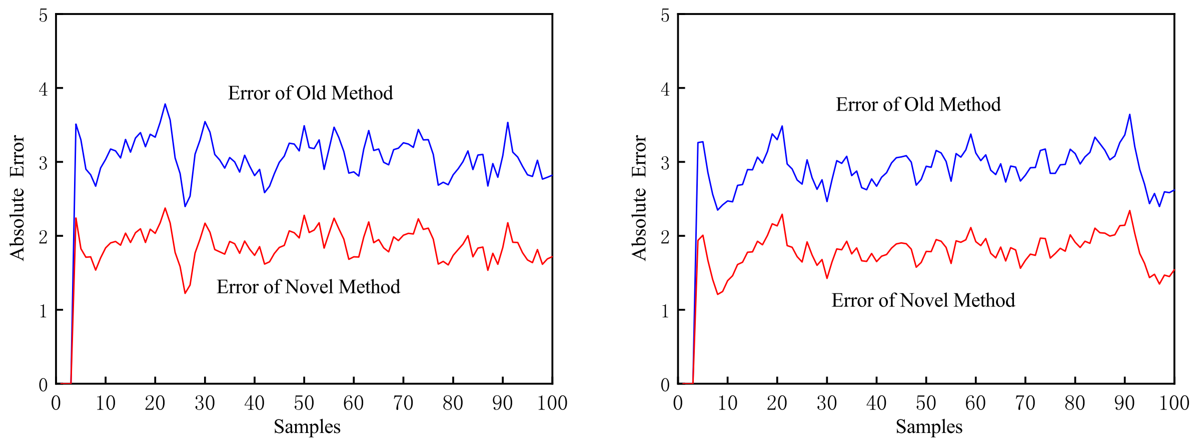
6.2. **Part two.** Comparing Figure 3 with Figure 1, we can know that although the variance of sensor 1 is reduced, the fusion result of Figure 3 deviates from the true value more than the fusion result of Figure 1. So we can obtain that the proposed method satisfies the conclusion that the less the variance of one sensor, the more the contribution of the sensor to the fusion results. However, the new fusion method can further adjust the output of the multi-sensor measurement system to be near to the sensors that possess the coincident tendency of readings.



(a) When the true value of x is 30 (b) When the true value of x is 31

FIGURE 3. The comparison curves of fusion results between novel and old fusion methods when the variance of sensor 1 is 0.05 and that of the others is 0.1

Figure 4 shows that the absolute error is larger than that of Figure 2 because the variance of sensor 1 is less and its contribution to the fusion results is enlarged. In this case, environment information plays a more significant role in the properties of the fusion method, and the alternative fusion method on the reduction of systematic error of contextual disturbance is required to be proposed. At the same time, it can also be seen from



(a) When the true value of x is 30 (b) When the true value of x is 31

FIGURE 4. The comparison curves of errors between novel and old fusion methods when the variance of sensor 1 is 0.05 and that of the others is 0.1

Figure 4 that the fusion method combining dynamic measurement information proposed in this paper is superior to the one obtained by static variance characters in absolute error sense and is more suitable for practical applications.

7. **Conclusions.** The classical multi-sensor data fusion method is optimal in the minimum mean square sense. However, the precision of the fusion results is constant when all the variance is known, even if the estimation of the variance cannot exclude the contextual disturbance from the output of the measurement system either. All these drawbacks result from that the variance is the static reliability depending on the physical properties before application, so the static fusion weighted coefficient can be regarded as a sensor performance index or prior knowledge for any application environment. Intuitively, when the sensor is used in different situations or at different stages, its reliability will change. In order to deal with these questions, the dynamic reliability index proposed in this paper is used to evaluate environmental information. Quantifying dynamic reliability is described through the similarity among the group of sensors. Then the novel fusion process including the dynamic reliability is discussed. The contextual information is combined into the fusion process. So it can degrade the absolute error, and enhance the fusion precision by adjusting fusion coefficients dynamically.

Acknowledgment. This work was supported by the Engineering Product Development Project of the Shallow Water Directional Control Valve (No. CCL2020RCPS0093RCN) and Science Foundation of China University of Petroleum Beijing (No. 2462020YXZZ023). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] K. Bahador, K. Alaa, O. K. Fakhreddine and N. R. Saiedeh, Multisensor data fusion: A review of the state-of-the-art, *Information Fusion*, vol.14, pp.28-44, 2013.
[2] Y. Omae, M. Kobayashi, K. Sakai, T. Akiduki, A. Shionoya and H. Takahashi, Detection of swimming stroke start timing by deep learning from an inertial sensor, *ICIC Express Letters, Part B: Applications*, vol.11, no.3, pp.245-251, 2020.
[3] E. F. Nour-Eddin, L. Henry and K. Ajeesh, Data fusion in intelligent transportation systems: Progress and challenges – A survey, *Information Fusion*, vol.12, pp.4-10, 2011.

[4] L. H. David and L. James, An introduction to multisensor data fusion, *Proc. of the IEEE*, vol.85, no.1, pp.6-23, 1997.
[5] Z. G. Feng, K. L. Teo, N. U. Ahmed, Y. Zhao and W. Y. Yan, Optimal fusion of sensor data for discrete Kalman filtering, *Dynamic Systems and Applications*, vol.16, pp.393-406, 2007.
[6] Y. T. Luo, Y. M. Zhu, D. D. Luo, J. Zhou, E. B. Song and D. H. Wang, Globally optimal multisensor distributed random parameter matrices Kalman filtering fusion with applications, *Sensors*, pp.8086-8103, 2008.
[7] J. B. Gao and C. J. Harris, Some remarks on Kalman filters for the multisensor fusion, *Information Fusion*, vol.3, pp.191-201, 2002.
[8] S. L. Sun, Multi-sensor optimal fusion fixed-interval Kalman smoothers, *Information Fusion*, vol.9, pp.293-299, 2008.
[9] Z. Paul and M. Howard, *Fundamentals of Kalman Filtering: A Practical Approach*, 3rd Edition, American Institute of Aeronautics and Astronautics, Inc. 2009.
[10] X. R. Li, Y. M. Zhu, J. Wang and C. Z. Han, Optimal linear estimation fusion – Part I: Unified fusion rules, *IEEE Transactions on Information Theory*, vol.49, no.9, pp.2192-2208, 2003.
[11] Q. Gan and C. J. Harris, Comparison of two measurement fusion methods for Kalman-filter-based multisensor data fusion, *IEEE Transactions on Aerospace and Electronic Systems*, vol.37, no.1, pp.273-280, 2001.
[12] N. A. Carlson, Federated square root filter distributed parallel processes, *IEEE Transactions on Aerospace and Electronic Systems*, vol.26, no.3, pp.517-525, 1990.
[13] C. J. Harris and Q. Gan, State estimation and multi-sensor data fusion using data-based neurofuzzy local linearisation process models, *Information Fusion*, vol.2, pp.17-29, 2001.
[14] K. H. Kim, Development of track to track fusion algorithm, *American Control Conference*, pp.1037-1041, 1994.
[15] Z. L. Deng, Y. Gao, L. Mao, Y. Li and G. Hao, New approach information fusion steady-state Kalman filtering, *Automatica*, vol.41, pp.1695-1707, 2005.
[16] X. J. Sun and G. M. Yan, Weighted measurement fusion fractional order Kalman filter, *Procedia Engineering*, vol.29, pp.207-215, 2012.
[17] C. B. Wen, Y. A. Cai, C. L. Wen and X. M. Xu, Optimal sequential Kalman filtering with cross-correlated measurement noises, *Aerospace Science and Technology*, 2012.
[18] B. Xu and A. A. Razzaqi, Placement of surface sensors for cooperative localization of submerged autonomous underwater vehicles, *International Journal of Innovative Computing, Information and Control*, vol.16, no.4, pp.1477-1484, 2020.
[19] S. L. Sun and Z. L. Deng, Multi-sensor optimal information fusion Kalman filter, *Automatica*, vol.40, pp.1017-1023, 2004.
[20] H. W. Guo, W. K. Shi and Y. Deng, Evaluating sensor reliability in classification problems based on evidence theory, *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, vol.36, no.5, pp.970-981, 2006.
[21] E. Zanaj, E. Gambi, B. Zanaj and D. Disha, Customizable hierarchical wireless sensor networks based on genetic algorithm, *International Journal of Innovative Computing, Information and Control*, vol.16, no.5, pp.1623-1638, 2020.
[22] L. Sun and X. Liu, Algorithm of signal source localization via sensor networks, *ICIC Express Letters*, vol.12, no.5, pp.441-447, 2018.
[23] L. A. Zedeh, Similarity relations and fuzzy orderings, *Information Sciences*, vol.3, no.2, pp.177-200, 1971.