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EVALUATION OF PRODUCTION FLOW SYSTEM UTILIZING EXPECTED HIGH VOLUME EFFECTIVE RATE

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Abstract. *The relationship between production density and lead time (throughput) has not yet been clearly established. Based on these results, the expected high volume effective rate is an evaluation index of whether or not the target lead time can be met due to the volatility of workers. The premise of this idea is that lead time uncertainty correlates with production volume uncertainty. We also report that the profit evaluation at the final time of the system is described by using Black-Scholes equation based on a stochastic differential equation with lognormal type. As shown in the numerical example, the expected effective rate can be obtained if the actual appropriate production amount is known. After that, the value can be used to determine whether the production system is working effectively. There is no other research that utilizes the expected high volume effective rate for evaluation of production processes. Finally, we present a numerical example of an expected high volume effective rate and a profit evaluation based on actual data.*
Keywords: Lead time, Stochastic differential equation, Synchronization of the processes, Expected high volume effective rate, Fluctuation

1. **Introduction.** A motive of the present research that has led to promote such research during many years of experience in general industrial machine control equipment manufacturing business is as follows. There was no physical discussion about production processes that had been accumulated through equipment manufacturing business [1, 2, 3, 4, 7].

As a way to proceed with a general production business, a given control equipment is ordered from a customer, then manufactured in a manner classified into a number of production elements, and a finished product is delivered to the customer. The feature of the present paper is in a point that production elements in manufacturing processes are treated as stochastic production operation. In particular, in order to analyze a manufacturing process as a stochastic process, we have introduced an idea of a production level corresponding to an energy level being discussed in physics. A valence electron transits to a conducting state due to a rise in potential (transition of a manufacturing process), and lowers an energy level by radiating energy with time. On this occasion, radiated energy is made to correspond to a phenomenon to produce business return. When the Fermi level of a valence electron is high, a conduction electron density is increased, and a positive hole density reduces. Similarly, if operations from the order entry of a product to completion of the manufacturing processes proceed without delay, high return can be obtained.

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Increase of conduction electrons corresponds to increase of production density, and decrease of positive hole density corresponds to increase of return. As state transition of a valence electron is being analyzed stochastically, it is often also in the manufacturing industry that order entry delays, and deviation arises in planned manufacturing operations. Occurrence of such unexpected event will be handled as a stochastic event.

We present the stochastic model of the production density distribution. The relationship between production density and lead time (throughput) has not yet been clearly established. The changes in lead time cause fluctuations in production density. We build a production propagation model based on a stochastic theory by considering lead time as a medium. We propose two stochastic differential equations as a mathematical model. One is related to production density, and the other relates to lead time. A major feature of this paper is that the production density is a functional as a variable with time and a lead time as a variable. We further consider through stochastic analysis that lead time is strongly related to production density. Then, on the basis of the concept of continuity approximation, we build a mathematical model that considers production density. This idea is based on the diffusion approximation of a production process, which was used as a deterministic model in our previous research. In a company, it is important to determine a rational throughput rate for continuation of production under an incomplete information state. In the present research, aiming at rationally performing start date management in the manufacturing industry, a mathematical model of throughput is formulated based on data, and a mathematical structure of start date management is made clear to some extent.

We show that the productivity of the system can be evaluated relatively easily by utilizing the expected high volume effective rate in this study. In this paper, we consider a model that is conscious of the stage of the production flow system. We report that the entire production flow system is modeled by a stochastic differential equation. The fluctuation of the lead time that occurs in the system greatly depends on the probability of the human worker. As a result, it will affect the total production volume and cost. The expected high volume effective rate is a relatively simple method that is effective for system evaluation. As shown in the numerical example, the expected effective rate can be obtained if the actual appropriate production amount is known. After that, the value can be used to determine whether the production system is working effectively.

Next, conventionally, it is well known that for a fluctuation in physics, there is research related current fluctuation in an electric circuit. This is an important theme in mesoscopic physics, and theoretical studies have been conducted since the early 1990s. In general, noise (fluctuation) refers to the amount of variation in the measured values around the average value obtained in this way, that is, the "dispersion" of the measured values [13, 14]. In our previous research, we reported that a delay in the production process is equivalent to a "fluctuation" in physical phenomenon. For example, there is the deviation from the thermal equilibrium state to fluctuations in physics. The propagation of fluctuation (volatility) in each stage delays the entire process. We have mathematically analyzed this phenomenon and assessed whether volatility is encountered during manufacturing [7, 15]. The many concern that occurs in the supply chain is a major problem facing production efficiency and business profitability. A stochastic partial bilinear differential equation with time delay was derived for outlet processes. The supply chain was modeled by considering as time delay [7].

In this paper, we present that the stable regions of nonlinearity of the production process correspond to regions of phase transition [4]. On the basis of actual rate of return data, using an electrical circuit theory known as multimode vibration theory, we demonstrate that the factor causing reductions in production is rate of return variations of work. We introduce a potential field that corresponds to an electromagnetic field for analyzing the production process and apply multimode vibration theory to the potential field. The present analysis has been conducted for increasing the rate of return.

The previous interesting research applying Fluid mechanics that is the trial production of a new concept vertical take-off and landing rotorcraft of flexible kite wing attached multicopter gets a lot of attention [17].

From the evaluation of Testrun1, the effect of dispersion can be seen sparsely. The production of Testrun2 and Testrun3-1/3-2 is progressing within the range of work variations. In other words, it retains time propulsion symmetry. In addition, Testrun3-1/3-2 can obtain the same product according to space propulsion symmetry, even if it is carried out from different stages.

The proposals in this paper are summarized as follows.

1) The manufacturing process is most appropriately described using a diffusion equation.
2) We formulate the production model similarly to the propagation of heat in physics. The production process is modeled mathematically using a type of continuous partial differential equation consisting of temporal and spatial variables. The fluctuations generated by the upstream process are propagated to the downstream process, and their effects are magnified. The production flow process is thus modeled through a stochastic differential equation.
3) Two stochastic differential equations are suggested as a mathematical model. One concerns the density of production and the other concerns the lead time. A major feature of this paper is that the production density is a function as a variable with time and a lead time as a variable. In addition, we consider, by means of a stochastic analysis, that the lead time is strongly linked to the density of production.
4) The expected high volume effective rate is a relatively simple and effective way to evaluate the system. As the numerical example shows, the expected high volume effective rate can be obtained if the actual quantity of appropriate output is known. Then the value can be used to determine whether the production system is running effectively.

2. **Production Firm of a Small or Medium Enterprise and Production System.** The rate of production density passing through the stochastic production process causes fluctuations due to changes in workers. We present a partial differential stochastic equation to produce density for a mathematical model of a stochastic production method with fluctuations. As a result, the loss function is calculated assuming that the lead time, which is the same concept as the throughput, has a normal distribution in a stationary state. The expected high volume effective rate is determined as the inverse of the loss function. This value is more easily treated as a systematic valuation.

This is not a special system; it is a "Make-to-order system with version control". The make-to-order system is a system that allows necessary manufacturing after taking orders from customers, resulting in "volatility" depending on the delivery date and lead time. In addition, "volatility" also occurs within the timeframe based on the content of manufacturing products (production equipment).

However, the efficient use of production forecast information on orders can remove a number of "variations", but it will be difficult to completely eliminate variations. In other words, the "volatility" of monthly cash flows affects the rate of return of these companies. Production management systems, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, is called as "product number management system" and is widely used. All productions are controlled by numbered product and instructions are given for each numbered product. Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial number in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks. Consequently, prudent management of the production time or date cannot eliminate "volatility" in manufacturing (production).

The company in this study is the "supplier" in Figure 1 and "factory" here. Companies assume that N (number of) suppliers exist; however, this study focuses on one company since no data is published for the rest of the company ($N - 1$). Then a manufacturing process which is called a production flow process is shown in Figure 2. The production process, which produces small quantities of a wide variety of products, goes through several stages of the production process. In Figure 2, the processes consist of six stages. Every S1-S6 step of the manufacturing process produces materials.

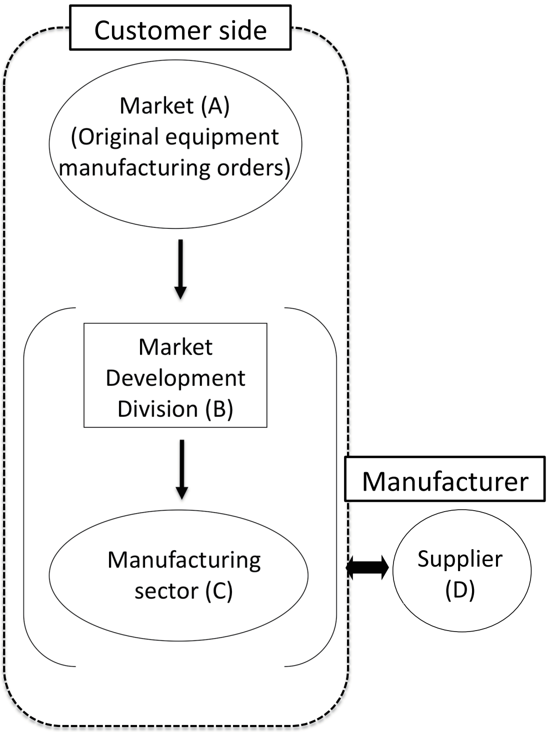

Figure 1. Business structure of company of research target

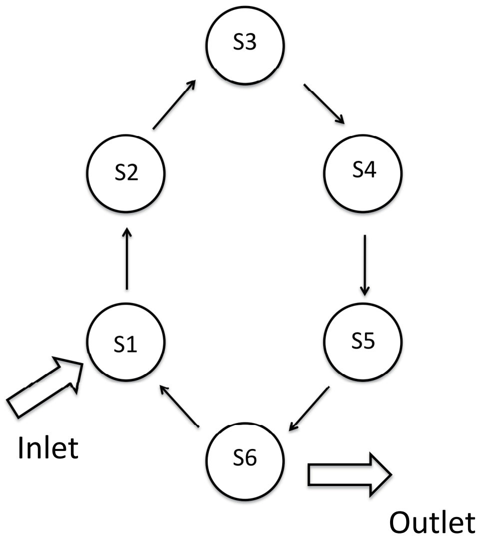

Figure 2. Production flow process

S1 to S6 perform work stages 1 to 6 on the production line in Figure 2. They are S1-S6 in Tables 6, 8, 10 and 12 in Appendix A. K1-K9 in the table are nine workers. Figure 2 represents a manufacturing process called a production flow system, which is a manufacturing method used in the production of control equipment. The production flow system, which in this case has six stages, is commercialized by the production of material in steps S1-S6 of the manufacturing process.

The direction of the deflection is the direction of the production flow. With this system, production materials are supplied from the inlet and the final product will be shipped from the outlet.

Assumption 2.1. *The production structure is nonlinear.*

Assumption 2.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system).*

- Reasonability of Assumption 2.1. Assumption 2.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the rate of return generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on demand, it is considered to have a non-linear structure. Since the value of such a commodity is dependent on the rate of return, its production structure is non-linear. As a result, Assumption 2.1 reflects the realistic production structure and is somewhat valid.
- Reasonability of Assumption 2.2. Assumption 2.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 2.2 is reasonable because production starts from S1. For more detailed analysis, please refer to our Appendix A.

3. **Diffusion Equation of the Production Process and Stochastic Modeling of the Production Process with Fluctuation.** Chapter 3 describes a mathematical model that uses the one-dimensional diffusion equation of the production flow system. Moreover, when the production flow system has fluctuations, the modeling taking account of stochastic characteristics must be described. Fluctuations here refer to changes in workers, insufficient inventory of parts, delays in transporting parts, and so on.

3.1. **Diffusion equation of the production process.** From Figure 3, the production process model, which is connected into a dimension, is described. The production process is indicated by moving the production units from one process (node) to the next. This production flow is equal to the transmission rate, which is defined as the data flow rate between connected nodes in communications engineering. Therefore, we frame the production model in a way similar to the propagation of heat in physics. Thus, the production process is modeled mathematically with the help of a type of continuous diffusion of partial differential equation composed of temporal and spatial variables [1].

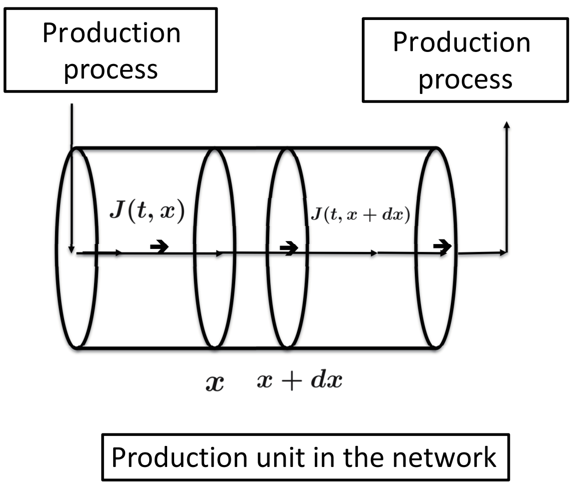

FIGURE 3. Network inter-process division of worker

By defining the network capacity (the available static production volume) at R in an inter-process network (production field, equivalent to a stochastic field), we get the following:

$$[J(t,x)dt - J(t,x+dx)dt]R = [S(t+dt,x) - S(t,x)]Rdx \tag{1}$$

where J is the production flow and S is the production density. $t \in [0,T]$, $x \in [0,L] \equiv \Omega$, $S(0,x) = S_0(x)$, $B_x S(t,x)|_{x=\partial\Omega}$. $B_x S(t,x)|_{x=\partial\Omega}$ indicates the boundary value.

In this model, the production flow shows the displacement of production processes in the direction of production density. In other words, the production density by output is:

Definition 3.1. *Production density per unit production*

$$J = -D\frac{\partial S}{\partial x} \tag{2}$$

where D is a diffusion coefficient.

From Equation (1), we obtain

$$-\frac{\partial J}{\partial x} = \frac{\partial S}{\partial t} \tag{3}$$

From Equations (2) and (3), we obtain

$$\frac{\partial S(t,x)}{\partial t} = D\frac{\partial^2 S(t,x)}{\partial x^2} \tag{4}$$

This equation corresponds to the diffusion equation derived from the condition for minimizing free energy in a productive field [1, 10]. Process linkages can be treated as diffuse product propagation (see Figure 3) [1, 12]. As shown in Figure 4, x represents the production elements that constitute a unit production and varies $x \to x'$ at $[t+dt]$. In other words, unit production varies with the excitation of the external force and forms the basis of revenue generation (an increase in potential energy). For example, in the transition from $S(t,x) \to S(t',x')$, the production density, which is the accumulated external force, increases. Connections between production processes are called "joints".

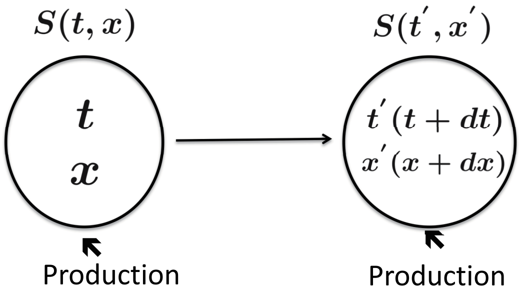

FIGURE 4. Unit of production by changing the excitation force

In the general idea of production flow, we define the joint propagation model at multiple stages in the production process and the potential energy in the production field. Thereafter, we can construct a control system, which increases the rate of return, by calculating the gradient function in the autonomous distributed system. The gradient function is described in the next opportunity.

$$\frac{\partial S}{\partial t} + \Delta(v \cdot S) = \frac{1}{2}\Delta\left(D^2 S\right) + \lambda \tag{5}$$

where λ denotes a forced external force function and v denotes a production propagation speed. Here, λ is omitted here. Δ represents the Laplacian $\partial^2/\partial x^2$.

3.2. **Stochastic modeling of production process with fluctuation.** We assume that S represents a production density with a fluctuation, and v also causes a fluctuation in throughput. Production is therefore proportional to the gradient of production density.

Definition 3.2. *Mathematical model of each stage*

$$dx(t) = \left\{a(t,x)dt + c(t,x)d\tilde{B}(t)\right\} + D(t,x)dB(t) \tag{6}$$

where $\tilde{B}$ and B denote an independent Brownian motion. c refers to a fluctuation term that follows the stochastic difference equation. The first term on the right side of Equation (6) indicates the flux of the medium, and the second term is broadcasting fluctuation. Additionally, $a(t,\cdot)$ indicates the average lead time and $c(t,x)d\tilde{B}(t)$ indicates the fluctuation around processes [7, 15].

We present the stochastic approach to a production process using the production density equation [1, 8], that is, the fluctuation is induced by the stochastic feature of the lead time function. In this case, a stochastic analysis is applied to evaluating how production density is constrained.

From Equation (5), a distribution of production density varies with the increase in production density. $S(t,x)$ meets a Fokker-Plank equation as shown below [3, 4, 5, 6]. The Fokker-Plank equation is Equation (7) in statistical mechanics that does not have a term of $n \geqq 3$ in the Kramer-Moyal equation.

$$\frac{\partial S(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D^2(t,x)S(t,x)\right\} - \frac{\partial}{\partial x}\{a(t,x)S(t,x)\} \tag{7}$$

where $x(t)$ satisfies Equation (6).

$S(t,x)$ represents a production density with a fluctuation, and v also causes a fluctuation in throughput. So production is proportional to the slope of production density. Equation (7) is generally bound by Equation (6). According to Okazaki's analysis, we get the following [6]:

$$\begin{aligned}\partial S(t,x) = {} & \left[\frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D^2(t,x) + c^2(t,x)\right\}S(t,x) - \frac{\partial}{\partial x}(a(t,x)S(t,x))\right]dt \\ & + \frac{\partial}{\partial x}\left\{c(t,x)S(t,x)\right\}\partial\tilde{B}(t)\end{aligned} \tag{8}$$

where $D^2(t,x) + c^2(t,x)$ indicates a trend, $a(t,x)S(t,x)$ indicates a fluctuation in stages and $c(t,x)S(t,x)$ indicates also a fluctuation of lead time. $S(t,x)$ indicates a production density and is derived as follows [11]:

$$S(t, I_h^x) = \int_0^t P\left(\tau, x_0; t, I_h^x\right) S(\tau, x_0)d\tau \tag{9}$$

where $I_h^x \equiv [x, x+h]$ and $S\left(t, I_h^x\right)$ is the production density.

From Equation (9), the distribution of production density varies as production density increases.

Definition 3.3. *Trend function of a production density distribution*

$$m(t,x) = E[S(t,x)] \tag{10}$$

According to Equation (5), $m(t,x)$ is derived as follows:

$$\frac{\partial}{\partial t}m(t,x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\left\{D^2(t,x) + c^2(t,x)\right\}m(t,x)\right] - \{a(t,x)m(t,x)\} \tag{11}$$

where the dispersion covariance of a production density $\chi\left(t,x,x^{'}\right)$ is defined as follows.

Definition 3.4. *Dispersion covariance of a production density* $\chi\left(t,x,x^{'}\right)$

$$\chi\left[t,x,x^{'}\right] = E\left[S(t,x)\cdot S\left(t,x^{'}\right)\right], \quad t \in R, \quad x^{'} \in R \tag{12}$$

where R denotes Euclidean space.

From Equation (10), we obtain as follows:

$$Cov.\left[S(t,x)\cdot S\left(t,x^{'}\right)\right]=\chi\left(t,x,x^{'}\right)-m(t,x)\cdot m\left(t,x^{'}\right) \tag{13}$$

According to a stochastic process theory, the following equation holds.

$$\begin{aligned} d\left\{S(t,x)\cdot S\left(t,x^{'}\right)\right\} &= S(t,x)\cdot dS\left(t,x^{'}\right)+S\left(t,x^{'}\right)\cdot dS(t,x) \\ &\quad +\frac{1}{2}\cdot 2\cdot d< S(\bullet,x),S\left(\bullet,x^{'}\right)>_t \end{aligned} \tag{14}$$

$$\begin{aligned} \chi\left[t,x,x^{'}\right] &= \left[\frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D^2(t,x)+c^2(t,x)\right\}S\left(t,x^{'}\right)\right. \\ &\quad \left.-\frac{\partial}{\partial x^{'}}\left\{a\left(t,x^{'}\right)S\left(t,x^{'}\right)\right\}\right]dt \\ &\quad +S\left(t,x^{'}\right)\left[\frac{1}{2}\frac{\partial^2}{\partial x^{'2}}\left\{D^2(t,x)+c^2(t,x)\right\}S(t,x)\right. \\ &\quad \left.-\frac{\partial}{\partial x}\{a(t,x)S(t,x)\}\right]dt \\ &\quad +\frac{\partial}{\partial x}\{c(t,x)S(t,x)\}\frac{\partial}{\partial x^{'}}\left\{c\left(t,x^{'}\right)S\left(t,x^{'}\right)\right\} \\ &\quad +S(t,x)\frac{\partial}{\partial x^{'}}\left\{c\left(t,x^{'}\right)S\left(t,x^{'}\right)\right\}d\tilde{B}(t) \\ &\quad +S\left(t,x^{'}\right)\frac{\partial}{\partial x}\{c(t,x)S(t,x)\}d\tilde{B}(t) \end{aligned} \tag{15}$$

Then, we obtain the dispersion covariance of a production density between stages as follows by taking the average value.

$$\begin{aligned} \frac{\partial}{\partial t}\chi\left[t,x,x^{'}\right] &= \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\left(D^2(t,x)+c^2(t,x)\right)\chi\left(t,x,x^{'}\right)\right] \\ &\quad +\frac{1}{2}\frac{\partial^2}{\partial x^{'2}}\left[D^2\left(t,x^{'}\right)+c^2\left(t,x^{'}\right)\right]\chi\left(t,x,x^{'}\right) \\ &\quad -\frac{\partial}{\partial x}\left\{a(t,x)\chi\left(t,x,x^{'}\right)\right\}-\frac{\partial}{\partial x^{'}}\left\{a\left(t,x^{'}\right)\chi\left(t,x,x^{'}\right)\right\} \end{aligned} \tag{16}$$

where $a(t,x)>0$ and $c(t,x)>0$.

Definition 3.5. *Correlation function of lead time function between stages*

$$dx^{i+1}(t)=\left\{a\left(t,x^{i+1}\right)dt+\int_R c\left(t,x^i,x^{i+1}\right)\tilde{B}\left(dt,dx^{i+1}\right)\right\}+b\left(t,x^{i+1}(t)\right)dB^i(t) \tag{17}$$

The production density distribution satisfies as follows based on Equation (17):

$$\begin{aligned} dS(t,x) &= \left[\frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{b^2(t,x)+\int_R c^2(t,x,z)dzS(t,x)\right\}-\frac{\partial}{\partial x}\left\{a(t,x)S(t,x)\right\}\right]dt \\ &\quad +\int_R\frac{\partial}{\partial x}\left\{c\left(t,x,x^{'}\right)S(t,x)\right\}\tilde{B}\left(dt,dx^{'}\right) \end{aligned} \tag{18}$$

4. **Trend Function of Production Density Distribution and Auto-Correlation Function at One Stage.** Chapter 4 describes the production density for the production stage in the stationary state. Furthermore, it is demonstrated that the auto-correlation function at one stage is the same as the trend function.

The following is a sample numerical parameter: $a > 0$ and $c > 0$ are constant parameters. Let $S(0, x) = \delta(x)$, which denotes as follows:

$$\delta(x) = \lim_{\sigma \to 0} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right), \quad x \in R \tag{19}$$

The condition of these parameters $a > 0$ and $c > 0$ represent that a production density exists between any stages.

Then, according to Equation (11), we obtain as follows:

$$\frac{\partial}{\partial t} m(t, x) = \frac{1}{2}\left(r\frac{\partial^2}{\partial x^2}\right) - am(t, x) \tag{20}$$

According to Equation (11), we obtain as follows:

$$\frac{\partial \chi\left(t, x, x'\right)}{\partial t} = \frac{1}{2}\left\{\frac{\partial^2 \chi\left(t, x, x'\right)}{\partial x^2} + \frac{\partial^2 \chi\left(t, x, x'\right)}{\partial x'^2}\right\} - a\left\{\frac{\partial \chi\left(t, x, x'\right)}{\partial x} + \frac{\partial \chi\left(t, x, x'\right)}{\partial x'}\right\} \tag{21}$$

From Equation (20), we obtain as follows:

$$m(t, x) = \frac{1}{\sqrt{2\pi rt}} \exp\left(-\frac{(x - at)^2}{2r^2t^2}\right) \tag{22}$$

Similarly, according to Equation (21), we obtain as follows:

$$\begin{aligned} \chi\left(t, x, x'\right) = \; & \frac{1}{2\pi\left(r^2 - c^4\right)t} \exp\left(-\frac{1}{2(r^2 - c^4)t}\right. \\ & \left. \times \left\{r(x - at)^2 - 2c^2(x - at)\left(x' - at\right) + r\left(x' - at\right)^2\right\}\right) \end{aligned} \tag{23}$$

where $r = D^2 + c^2$.

From Equation (23), the numerical data of correlation function can be calculated for x and x' of production density.

$$\begin{aligned} dS(t, x) = \; & \frac{1}{2}\left[\left\{D^2 + \int_R c^2\left(t, x, x'\right)\right\}\frac{\partial^2 S(t, x)}{\partial x^2} - a\frac{\partial S(t, x)}{\partial x}\right] dt \\ & + \frac{\partial}{\partial x}\left[\int_R c\left(t, x, x'\right) S(t, x) \tilde{B}\left(dt, dx'\right)\right] \end{aligned} \tag{24}$$

where $\tilde{B}\left(dt, dx'\right)$ denotes any of the k interval $F_1 = I_1 \times J_1, F_2 = I_2 \times J_2, \ldots, F_k = I_k \times J_k \subset R^2$. $(B(F_1), B(F_2), \ldots, B(F_k))'$ in $\tilde{B}\left(dt, dx'\right)$ denote a k-dimensional normal distribution with average zero. However, from Equation (8) in case of a single Brownian motion, we obtain the following:

$$\partial S(t, x) = \frac{1}{2}\left[\left(D^2 + c^2\right)\frac{\partial^2 S(t, x)}{\partial x^2}\right]\partial t - a\frac{\partial}{\partial x}S(t, x)\partial t + c\frac{\partial}{\partial x}S(t, x)\tilde{B}(t) \tag{25}$$

The above calculation states that the trend of a fluctuation in the distribution of the production density is a normal distribution of Equation (22). For a single Brownian motion, the trend indicates a partial derivative equation from Equation (25). In other words, the movement of the trend is affected by the coefficient c, which is caused in a lead time fluctuation.

For the lead time distribution, we obtain from Equation (6) the following:

$$dx(t) = \left\{adt + cd\tilde{B}(t)\right\} + DdB(t) \tag{26}$$

When Equation (26) is derived, the stochastic model of the distribution of the productive density is derived from Equation (25).

For simplicity, suppose $\tilde{B}(t) \approx B(t)$, we can rewrite Equation (26) in the following manner:

$$dx(t) = adt + (c + D)dB(t) \tag{27}$$

From Green's theorem, Equation (25) can be rewritten as follows:

$$dS(t) = \lambda S(t) + \sigma(x)S(t)dB(t) \tag{28}$$

where $\sigma(x)$ is a volatility.

Equation (28) refers to a state-dependent stochastic differential equation (log-normal type). In Figure 5, lead time fluctuations are strongly dependent on production density; they represent a phenomenon of mutual fluctuation. The characters "A" and "B" in Figure 5 represent respectively a lead time fluctuation and a fluctuation in production density. The mutual fluctuation between a lead time and production density represents the fluctuations in the actual data of Testruns 1 to 3 in Appendix A. The work time indicated by the circle in the table indicates that the reference time is over in Tables 6, 8 and 10 of Appendix A.

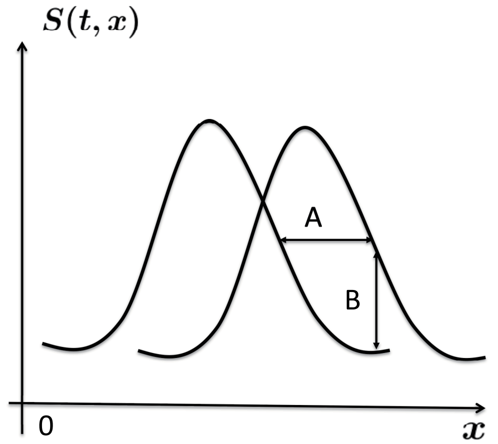

FIGURE 5. Influence of lead time fluctuation

Definition 4.1. *Probability density function from production density distribution in stationary* $S_p(x : \mu_x, \sigma_x)$

where μ_x and σ_x represent a trend coefficient and a volatility depending on x respectively.

As a result, the expectation $F_E(x; \mu_x, \sigma_x)$ is derived as follows:

$$F_E(x; \mu_x, \sigma_x) = \int_R g(x)dF_p(x; \mu_x, \sigma_x), \quad \frac{dF_p(x; \mu_x, \sigma_x)}{dx} = S_p(x; \mu_x, \sigma_x) \tag{29}$$

Similarly, fluctuating production density influences the expected total production volume. Second, the stochastic distribution model of production density is denoted as follows for a single Brownian movement:

$$dx(t) = a(t, x) + c(t, x)dB(t) \tag{30}$$

$$\partial S(t, x) = \frac{1}{2}\left\{\left(D^2 + c^2\right)\frac{\partial^2 S(t, x)}{\partial x^2} - a(t, x)\frac{\partial S(t, x)}{\partial x}\right\} + c(t, x)\frac{\partial S(t, x)}{\partial x}dB(t) \tag{31}$$

We can calculate an auto-correlation function of stages at $x = x'$ as follows:

$$\begin{aligned}\xi(t,x) &= \frac{1}{2\pi(r^2-c^4)t}\exp\left[-\frac{r(x-at)^2-c^2(x-at)^2}{(r^2-c^4)t}\right]\\ &= \frac{1}{2\pi(r^2-c^4)t}\exp\left[\frac{(r-c^2)(x-at)^2}{(r^2-c^4)t}\right]\end{aligned} \tag{32}$$

5. **Lead Time Analysis Using a Lead Time Function.** The lead time function $f(y)$ is assumed as a normal probability density function so that we can calculate the lead time using a continuous expected value calculation as shown in Figure 7.

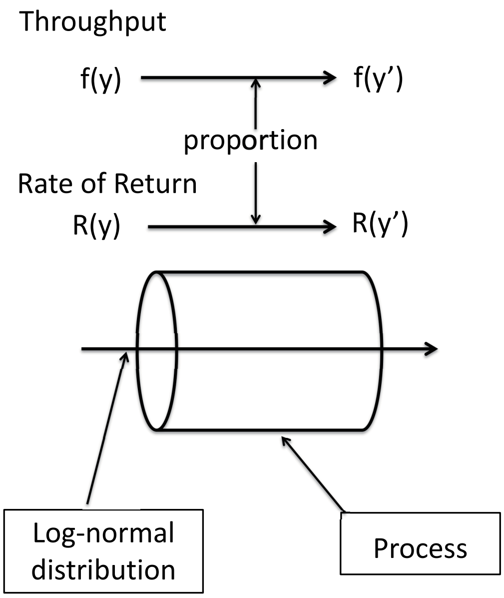

FIGURE 6. Throughput fluctuation in a process distribution amount

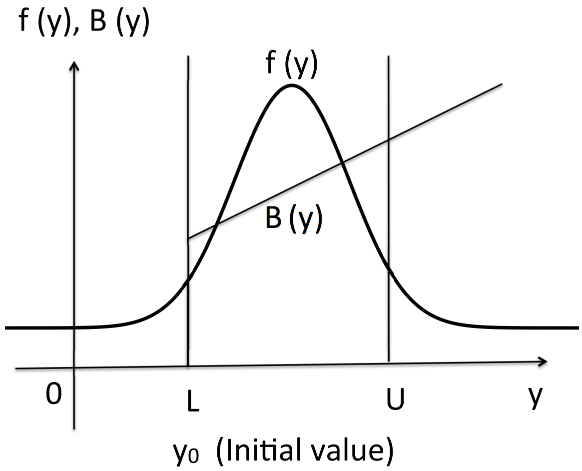

FIGURE 7. Lead time function $f(y)$ and loss function $B(y)$

Assumption 5.1. *The lead time function of a probability density function with normal distribution in stationary.*

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\} \tag{33}$$

where μ is an average value, σ is a volatility.

Now, leave F as a cash flow and leave C_0 as a fixed cost and now, $C_0 = 0$, and we calculate a continuous expected loss value F.

$$\begin{aligned}F &= \int_{-\infty}^{\infty} f(y)B(y)dy + C_0\\ &= \int_{-\infty}^{n_L} B(y)f(y)dy + \int_{n_L}^{n_U} B(y)f(y)dy + \int_{n_U}^{\infty} B(y)f(y)dy + C_0\end{aligned} \tag{34}$$

where

$$B(y) = py + q, \quad p \geq 0 \tag{35}$$

where p and q are constant parameters. n_U is a maximum lead time.

When $y < L$, production activities are not in progress. When $y > U$, the amount ordered is greater than the physical limits of production. Therefore, we have to reduce the demand, and the problem becomes $n_L \le y \le n_U$.

$$F = \int_{n_L}^{n_U} (py + q) f(y) dy \tag{36}$$

In general, the higher the lead time for a given product, the lower the throughput. Equation (36) is as follows:

$$\begin{aligned} F &= \int_L^{\infty} py \cdot f(y)dy + \int_L^{\infty} q(U) \cdot f(y)dy \\ &= p(n_U - n_L)\left(\Phi(n_U) - \Phi(n_L)\right) + q\left(\Phi(n_U) - \Phi(n_L)\right) \\ &= p(1-k)n_U\left(\Phi(n_U) - \Phi(n_L)\right) + q\left(\Phi(n_U) - \Phi(n_L)\right) \end{aligned} \tag{37}$$

where $k = n_L/n_U$. The current high volume effective rate of G for increased production is defined as follows.

Definition 5.1.

$$G = 1 - F \tag{38}$$

where F is the continuous expected loss value.

5.1. **Cash flow analysis.** Here, the profit at the final time is described. It is assumed that the production density passing through the production flow system process is defined by a stochastic differential equation with lognormal type from our previous research [13]. Therefore, it can be said that it is realistic to assume that a cash flow will also be the same lognormal distribution. Consequently, a $C(t)$ cash flow model is defined as follows.

Definition 5.2. *Definition of a cash flow model*

$$\frac{dC(t)}{dt} = \mu C(t)dt + \sigma C(t)dW^C(t) \tag{39}$$

where $C(0)$ is an initial expense considered to be required at the time of manufacture, the left side is a monthly rate of return, and a rate of return varies depending on the expected value μ_s and $\sigma^2 t$. Moreover, σ^2 represents volatility and $W^C(t)$ represents the standard Brownian movement.

Second, the valuation value $V(T)$ of manufacturing equipment at the last time $t = T$ is calculated as follows [18].

$$V(T) = \max[C(T) - K, 0] \tag{40}$$

Secondly, a cash flow model is drawn up as follows [18]:

$$dC(t) = \mu C(t)dt + \sigma C(t)dW(t) \tag{41}$$

where μ, σ and $W(t)$ are the mean, volatility and Wiener process respectively. So a valuation equation is derived as follows [18]:

$$V(C(T)) = C \cdot \Phi(d_1) - K \cdot e^{-r(T-t)} \cdot \Phi(d_2) \tag{42}$$

where d_1 and d_2 are

$$d_1 = \frac{\ln(C/K) + (r + (1/2)\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{43}$$

$$d_2 = \frac{\ln(C/K) + (r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{44}$$

where r and K are a discount rate and a valuation value at the end. Additionally, $\Phi(\cdot)$ indicates a probability value of standard normal distribution and is indicated by the following equation:

$$\Phi(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} \exp\left(-\frac{1}{2}x^2\right) dx \tag{45}$$

5.2. **Numerical simulation of the trend function of an allocation of production density.** We have been working in the small and medium-sized enterprise manufacturing sector for over 20 years. At any rate, managers of small and medium-sized enterprises tend not to evaluate the management of a mathematical model. We have systematically constructed mathematical models and performed more rigorous mathematical evaluation. In this paper as well, we believe there is an advantage to conducting numerical simulations according to this baseline position.

The trend function, which denotes an expectation of a production density function, represents a lead time and depends on the tendency function coefficient, but not on volatility. The auto-correlation function also depends on the tendency function coefficient. In terms of the temporal trend of a production density function $S(t,x)$, the function is affected by the trend function and the effect of x is important, particularly in the case of non-linear terms such as a triangle function or a δ function.

- Function of auto-correlation of the distribution of the production density (Table 1). With respect to Figures 8 to 13, the influence of the stage on the auto-correlation function is identical to that of the trend function. In each Figures 8 to 13, "a" severely affects the value of the trend function. However, in terms of Figures 8 to 13, it is possible to say that "a" has less influence than "c" and "D" on the value of the trend function.
- Expected loss function (Table 2). Figures 14 to 18 show the expected loss function. These graphs correspond to the graphs of the expected high volume effective rate Figures 19 to 23 respectively. The values of the adjustment parameters in Figures 14 to 18 are given in Table 2. The graphs from Figures 14 to 18 are examples of numerical calculation of the model with respect to the uncertainty of the lead time for work. As these graphs demonstrate, the likelihood of occurrence decreases as uncertainty increases.
- Expected high volume effective rate (Table 3). Figures 19 to 23 represent the high volume effective rate of human productivity. The $py + q$ function in Figure 7 represents the expectation reduction function. Our numerical results mostly show the results, when the p and q parameters for this function are modified. The higher the average of the standard distribution, the better the productivity. As it is clear that productivity drops if σ volatility is high, the numerical results here are omitted. That is, when the lead time probability distribution follows a normal distribution, the expected high volume effective rate can be obtained. Although this is based on a relatively simple definition, it can be fully utilized as a calculation equation in the production flow system. As a method of using these results, the maximum number of production in the actual production system of the assembly line is 240 (y in Equation (33)), and the actual production number is 120(0.5) to 192(0.8). For example, the area depicted in Figure 19 is a realistic production rate. Furthermore, the value of the horizontal axis of 0.5-0.8 is also a realistic graph of the output rate with other parameters. The expected high volume effective rate on the vertical axis of the graphs is 0.8 to 0.9 such as Figure 19. The high volume effective rate of 0.5 or less is an unfavourable situation as far as management is concerned. In addition, the

fact that the trend value (mean value) of 0.73 is adopted represents the true trend value. The realistic range of G is $0.8 < G < 0.9$, and the gradient $p = 1.5$ of g is not a realistic value.

- Black-Scholes ($B \cdot S$) premium value. For Figure 24, we carried out the numerical calculation using parameter values based on Equations (42), (43) and (44). The parameters are $r = 0.1$, $C(0) = 0.1$, $\mu = 0.73$, $\sigma = 0.26$ and $K = 0.6$. Table 4 shows the premium value when the σ is changed based on Equation (42). Volatility $\sigma = 0.26$ is the lowest risk and $\sigma = 0.40$ is the highest risk in Table 4.

TABLE 1. Setting parameters for the production density distribution trend function

Figure number	a	r	c	D
Figure 8	0.3	0.5	0.5	1
Figure 9	1	0.5	0.5	1
Figure 10	0.5	0.5	0.5	1
Figure 11	0.1	0.5	0.5	1
Figure 12	1	0.5	0.2	0.7
Figure 13	1	0.5	0.1	0.6

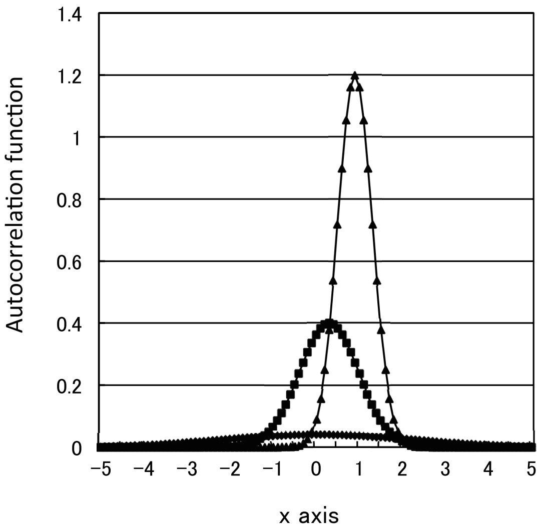

FIGURE 8. Function of autocorrelation of the distribution of the production density (Table 1)

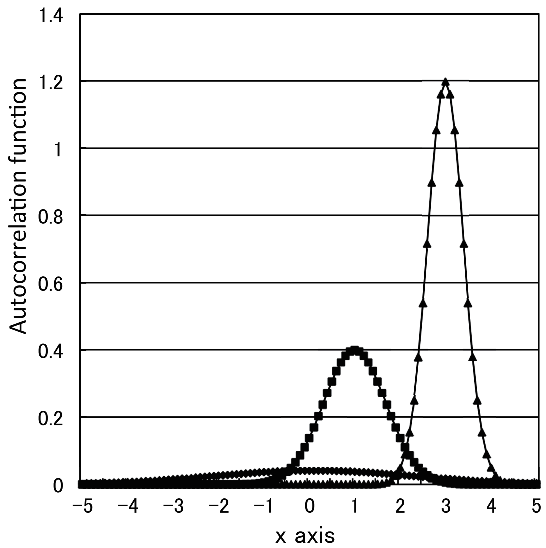

FIGURE 9. Function of autocorrelation of the distribution of the production density (Table 1)

TABLE 2. Setting parameters for Figures 14 to 18

Figure number	Average (μ)	Volatility (σ)	Gradient (p)	Constant (q)
Figure 14	0.6	0.26	1.0	0.5
Figure 15	0.73	0.26	1.0	0.5
Figure 16	0.73	0.26	0.5	0.1
Figure 17	0.73	0.26	1.5	0.1
Figure 18	0.8	0.1	1.0	0.5

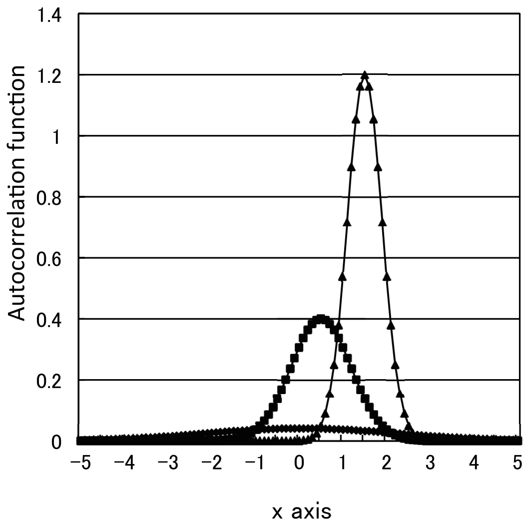

FIGURE 10. Function of autocorrelation of the distribution of the production density (Table 1)

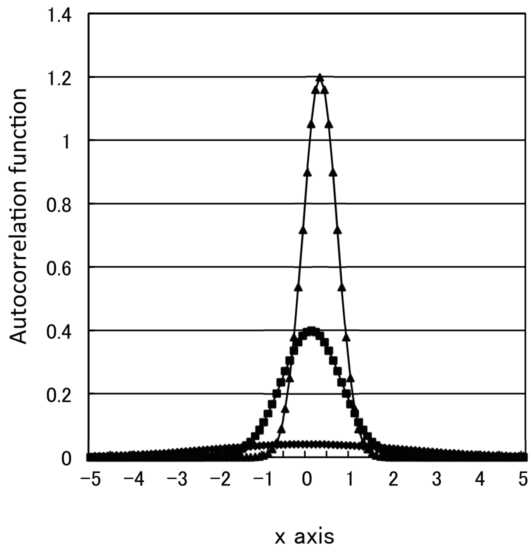

FIGURE 11. Function of autocorrelation of the distribution of the production density (Table 1)

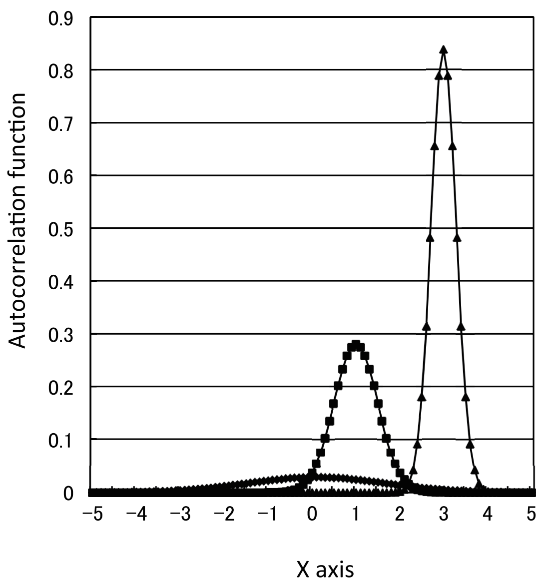

FIGURE 12. Function of autocorrelation of the distribution of the production density (Table 1)

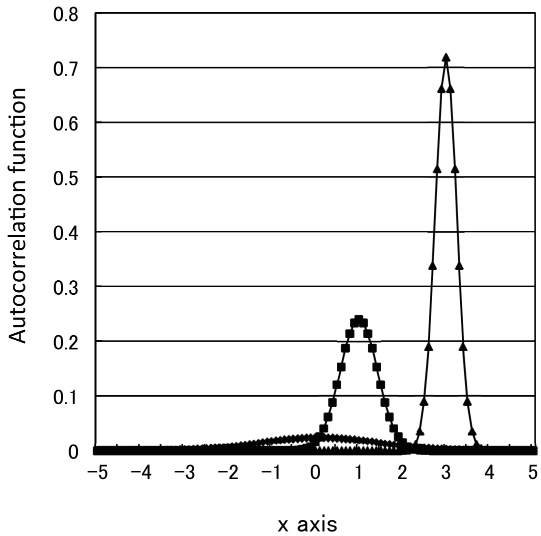

FIGURE 13. Function of autocorrelation of the distribution of the production density (Table 1)

TABLE 3. Setting parameters of expected high volume effective rate

Figure number	Average (μ)	Volatility (σ)	Gradient (p)	Constant (q)
Figure 19	0.6	0.26	1.5	0.1
Figure 20	0.73	0.26	1.0	0.5
Figure 21	0.73	0.26	0.5	0.1
Figure 22	0.73	0.26	1.5	0.1
Figure 23	0.8	0.1	1.0	0.5

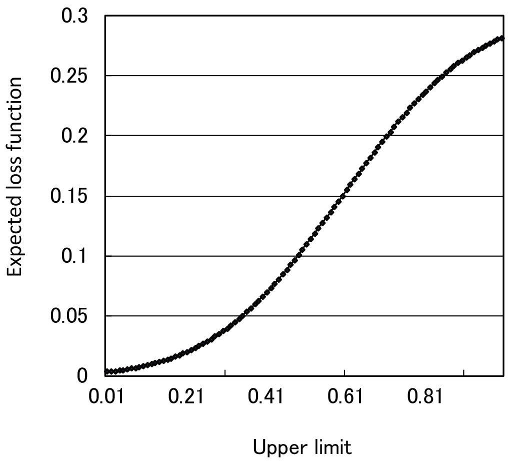

FIGURE 14. Value of loss function expected for lead time distribution

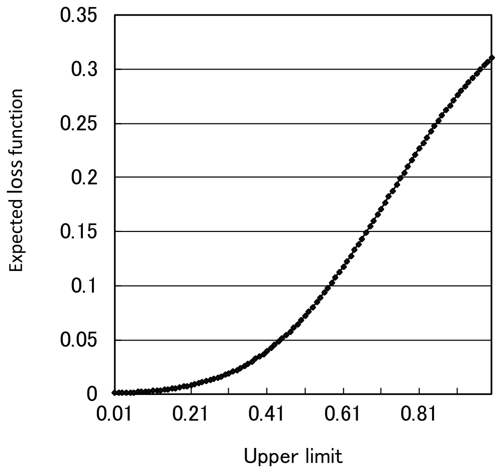

FIGURE 15. Value of loss function expected for lead time distribution

FIGURE 16. Value of loss function expected for lead time distribution

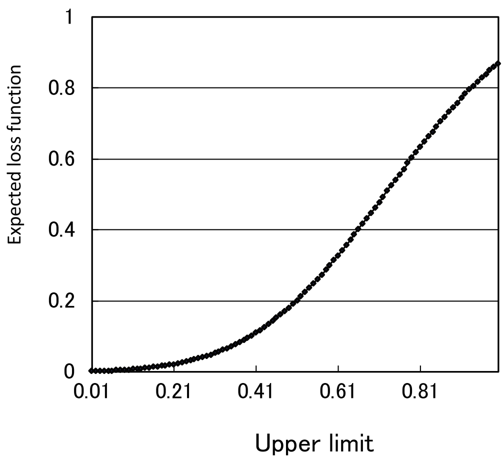

FIGURE 17. Value of loss function expected for lead time distribution

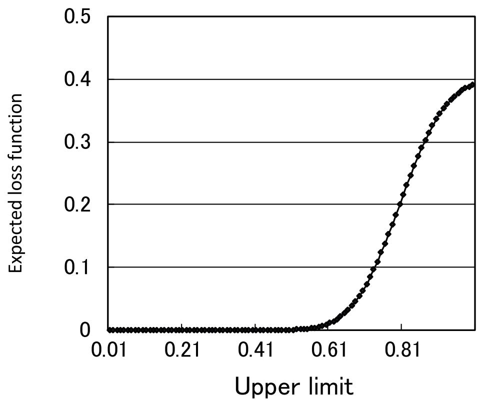

FIGURE 18. Value of loss function expected for lead time distribution

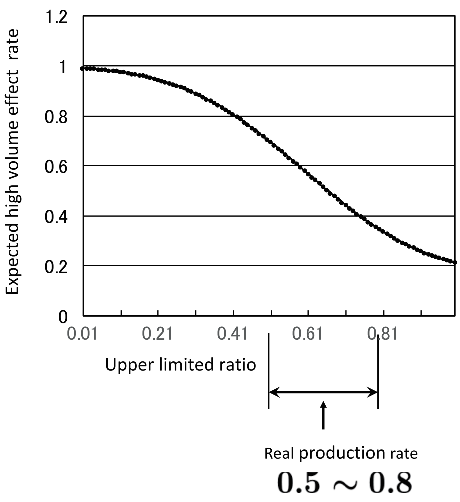

Figure 19. Expected high volume effective rate

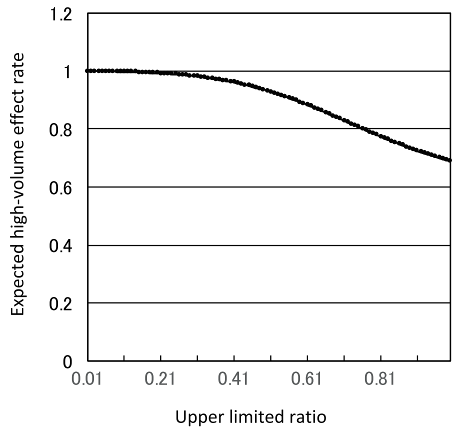

Figure 20. Expected high volume effective rate

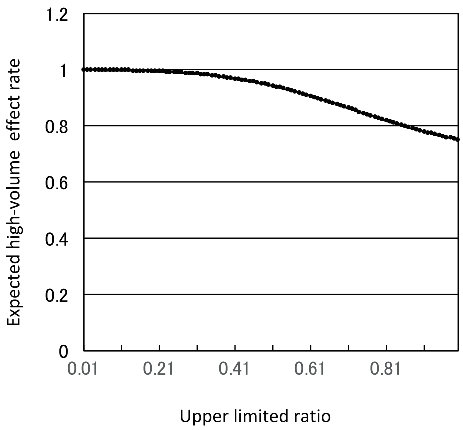

Figure 21. Expected high volume effective rate

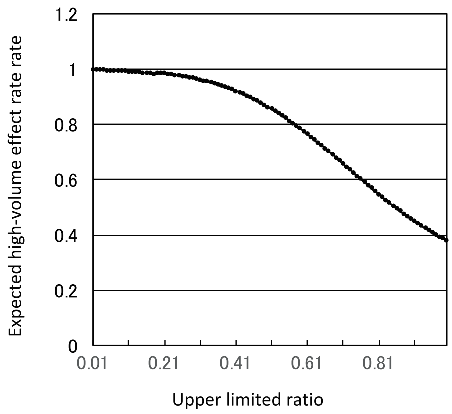

Figure 22. Expected high volume effective rate

Table 4. Setting parameters of $B \cdot S$ premium value

Volatility (σ)	$B \cdot S$ premium value
0.26	0.270
0.30	0.311
0.33	0.337
0.35	0.353
0.38	0.375
0.40	0.389

6. **Conclusion.** The distribution of production density is strongly dependent on the trend and volatility of the lead time (throughput) as an average. The mathematical stochastic distribution model of production density presented in this paper has a significant meaning, and its validity has been recognized under the terms mentioned in this paper. Next, if the timeout function follows a normal steady-state distribution, the expected high volume effective rate is presented as the inverse of the expected loss. We propose that this function is based on a relatively simple definition and can be fully utilized in

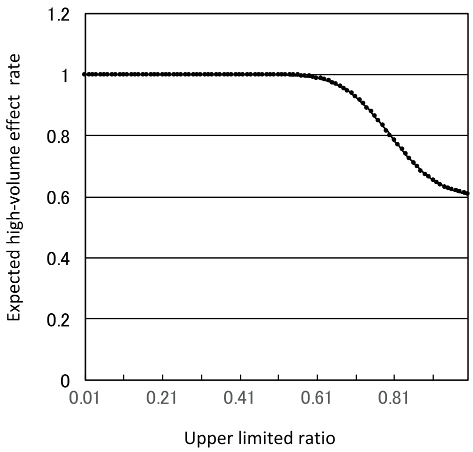

FIGURE 23. Expected high volume effective rate

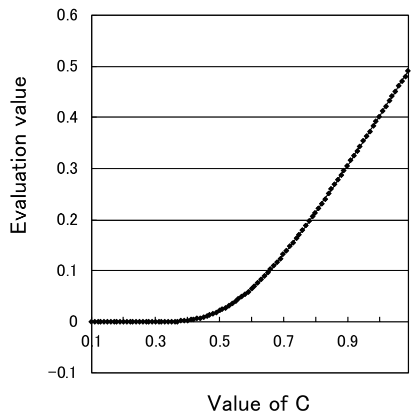

FIGURE 24. Evaluation value

evaluating a real production system. Finally, the profits valuation at the end time was evaluated using the $B \cdot S$ premium value.

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Appendix A. **Analysis of Actual Data in the Production Flow System.** Based on the control equipment, the product can be manufactured in one cycle. The rate of return required to maintain 6 pieces of equipment/day is as follows.

- (Testrun1): Because the throughput of each process (S1-S6) is asynchronous, the overall process throughput is asynchronous. In Table 6, we list the manufacturing time (min) of each process. In Table 7, we list the volatility in each process performed by the workers. Finally, Table 6 lists the target times. The theoretical throughput is obtained as $3\times 199+2\times 15 = 627$ (min). In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. In Figure 25, we plot the measurement data listed in Table 6, which represents the total working time of each worker (K1-K9). In Figure 26, we plot the data contained in Table 6, which represents the volatility of the working times.
- (Testrun2): Set to synchronously process the throughput. The target time listed in Table 8 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 9 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Testrun3-1): Introducing a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 10, the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 11 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Testrun3-2): Same as Testrun3-1.

On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput (T_s') can be obtained using the "Synchronization with preprocess" method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20\times 6 \text{ (First cycle)} + 17\times 6 \text{ (Second cycle)} \\ &\quad + 20\times 6 \text{ (Third cycle)} + 20 \text{ (Previous process)} + 8 \text{ (Idle-time)} \\ &\sim 370 \text{ (min)} \end{aligned} \tag{46}$$

The full synchronous throughput in one stage (20 min) is

$$T_s' = 3\times 120 + 40 = 400 \text{ (min)} \tag{47}$$

Using the "Synchronization with preprocess" method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed "Synchronization with preprocess" method is realistic and can be applied in flow production systems. Below, we represent for a description of the "Synchronization with preprocess".

In Table 12, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time. Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{48}$$

where the throughput of the preprocess is set to 20 (min). In Equation (48), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) $\times$ 3 (cycles).

In Table 10 and Table 12, Testrun3-1/Testrun3-2 indicate a best value for the throughput in the three types of theoretical working time. Testrun2 is an ideal production method. However, because it is difficult for talented worker, Testrun3-1/Testrun3-2 are a realistic method.

The results are as follows. This is the trend factor, which is the actual number of pieces of equipment/the target number of equipment and it is a factor that indicates the amount of manufacturing equipment parts.

Testrun1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73,
Testrun2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92,
Testrun3-1 and Testrun3-2: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95.
Volatility data represent the average value of each Testrun.

Table 5. Correspondence between the table labels and the Testrun number

	Table number	Production process	Working time	Volatility
Testrun1	Table 6	Asynchronous process	627 (min)	0.29
Testrun2	Table 8	Synchronous process	500 (min)	0.06
Testrun3-1	Table 10	Synchronous process	470 (min)	0.03
Testrun3-2	Table 12	"Synchronization with preprocess" method	470 (min)	0.03

Table 6. Testrun1

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 7. Volatility of Table 6

	S1	S2	S3	S4	S5	S6
K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

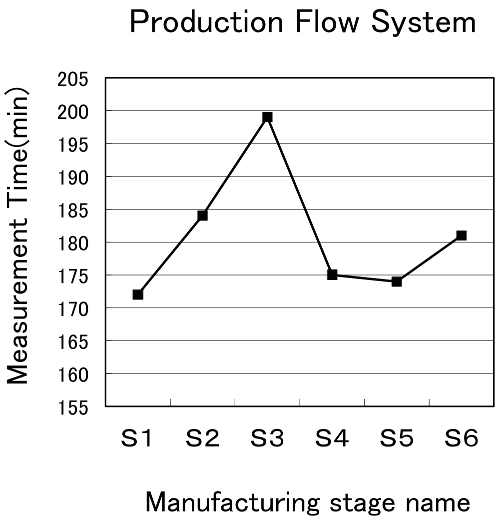

FIGURE 25. Total work time for each stage (S1-S6) in Table 6

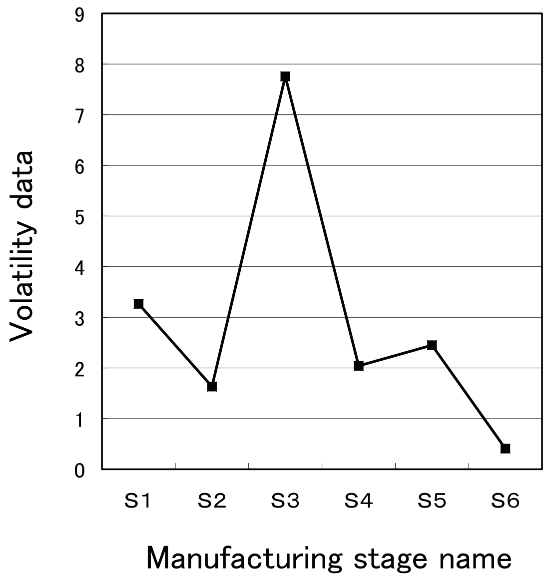

FIGURE 26. Volatility data for each stage (S1-S6) in Table 6

TABLE 8. Testrun2

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

TABLE 9. Volatility of Table 8

	S1	S2	S3	S4	S5	S6
K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 10. Testrun3-1

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180

TABLE 11. Variance of Table 10

	S1	S2	S3	S4	S5	S6
K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.33	3	3	0	0	0
K5	1.33	1.33	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.33	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0

TABLE 12. Testrun3-2. K5 (*): Preprocess.

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	*16	*16	*16	*17	*17	*16	*16
K6	16	18	18	18	18	18	18
K7	20	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	25	25	25	25	25	25
Total	168	165	164	163	166	165	164

TABLE 13. Volatility of Table 12. K5 (*): Preprocess.

	S1	S2	S3	S4	S5	S6
K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*0	*0	*0.33	*0.33	*0	*0
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	2	2	2.33	2	2	2.33
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	1.67	1.67	1.67	1.67	1.67	1.67