International Journal of Innovative
Computing, Information and Control
Volume **17**, Number **5**, October **2021**

ICIC International ©2021 ISSN 1349-4198
pp. **1549–1558**

LARGE-SCALE POSITIVE SYSTEM CONTROL: DECENTRALIZED CONTROL APPROACH

Jana Paulusová and Vojtech Veselý

Institute of Robotics and Cybernetics
Slovak University of Technology
Ilkovicova 3, Bratislava 81219, Slovak Republic
{ jana.paulusova; vojtech.vesely }@stuba.sk

Received April 2021; revised July 2021

Abstract. *The paper deals with the problem of proportional-integral (PI) decentralized controller design for continuous-time invariant positive systems. The proposed method is based on the notion of the equivalent subsystem in the time domain. Using the decentralized controller design procedure for the subsystem level, the complex system's closed-loop stability and the Metzler matrix properties are guaranteed (after some iteration). Two examples, state-feedback and PI output-feedback with $\mathcal{L}_2$-gain illustrate the effectiveness of the proposed method.*

Keywords: Decentralized systems, Linear systems, Output feedback

1. **Introduction.** Designing a control system to control the positive uncertain dynamic large-scale system is one of the most significant control theory challenges. Such systems to be controlled are too large and too complex for a centralized controller. This is why a complex system is divided into many small interconnected subsystems with interconnections and controlled by a controller with information constraints-decentralized controller [1]. Overview of decentralized controller designs and their evolution is given in [2]. Two large groups of methods were developed to design decentralized controllers for linear time-invariant large-scale systems. First is controller design in the frequency domain [3, 4, 5] and references therein. The second is in the time domain using the vector Lyapunov function [6]. Much progress has been made by using linear-bilinear matrix inequalities (LMI-BMI) [2]. Decentralized controllability, observability and decentralized stabilization of complex systems are given in [7]. For all methods, instead of analyzing stability [5] and decentralized controller design of each of the resulting smaller subsystems separately, i.e., by neglecting interconnections is tractable, but in results, it gives a highly conservative approach [1].

Positive systems were first introduced in 1979 [8, 9]. A linear system is positive if its state and output are non-negative for any non-negative input and initial state. Many systems, such as temperature control, concentration control, liquid level control, bioprocesses belong to the class of positive systems [10, 11, 12]. It was reported in [13] that for positive systems, the $\mathcal{L}_1$-gain/$\mathcal{L}_\infty$-gain as performance is suitably employed. Explicit formulation of the $\mathcal{L}_\infty$-gain with distributed delays is presented in [14]. The state-feedback stabilization for positive linear systems is described in [10]. Static output-feedback stabilization for linear time-invariant (LTI) positive systems is given in [15] and state stabilization of discrete-time positive systems is introduced in [16].

DOI: 10.24507/ijicic.17.05.1549

The decentralized control of positive systems can be found in [17, 18, 19] and references therein where the authors of the proposed methods try to reduce the conservativeness of existing methods. As an example, the use of bounded real lemma [17] can be used to propose a static structure of the state feedback decentralized controller. In this contribution, the size of interaction (strong, weak) does not charge from the direction of the decentralized controller design for a positive system; therefore, the method proposed in this paper provides conservativeness reduction. This paper aims to develop a new design procedure for the LTI positive system based on the above-mentioned short survey, the PI decentralized controller design with output-feedback and $\mathcal{L}_2$-gain performance. The decentralized control design procedure is based on the equivalent subsystem approach developed in this paper.

The paper is organized as follows. Section 2 presents the preliminaries. Section 3 brings the results of obtaining equivalent subsystem and ensuring stability in the complex plane. Section 4 describes the PI controller design for positive systems. Section 5 shows the effectiveness of the proposed method using two examples. The decentralized state-feedback is proposed for the first example, and PI decentralized controller design using the $\mathcal{L}_2$ gain approach is presented for the second one. The paper is closed by concluding remarks in Section 6.

Notation: The set of real numbers is $\mathbb{R}$, $\mathbb{R}^n$ denotes the n-column real vector, $\mathbb{R}^{n\times n}$ is the set of all $n \times n$ dimensional real matrices, $\mathbb{R}^n_{++}$ represents the n dimensional vectors with positive entries, $P > 0$ denotes the positive definiteness of the matrix, A^T denotes the matrix transpose, $A \in \mathbb{M}^n$ is $n \times n$ dimensional Metzler matrix, and $P \in \mathbb{D}^n_{++}$ is the diagonal positive definite matrix.

2. **Preliminaries.**

Let us introduce the following definitions and propositions [9, 20].

Definition 2.1. *A linear system is said to be positive (internally positive) if its state and output are non-negative for all non-negative initial state and input.*

Definition 2.2. *A linear matrix $A \in \mathbb{M}^n$ is said to be Metzler if its off-diagonal entries are all non-negative, i.e., $a_{ij} \geq 0$ $(i \neq j)$.*

Proposition 2.1. *Let us consider the continuous-time LTI system as follows*

$$\begin{aligned} \dot{x} &= Ax + B_w w \\ z &= C_z x \end{aligned} \tag{1}$$

Then the system (1) *is positive if and only if $A \in \mathbb{M}^n$ (A is Metzler matrix of the size $n \times n$), $B_w \geq 0$ and $C_z \geq 0$.*

Proposition 2.2. [21] *For the given $A \in \mathbb{M}^n$, the following conditions are equivalent.*

1) The matrix A is Hurwitz stable.

2) There is the vector $g \in \mathbb{R}^n_{++}$ such that $Ag < 0$.

3) There exists $P \in \mathbb{D}^n_{++}$ such that $A^T P + PA < 0$.

4) There exists $W \in \mathbb{R}^{n\times n}$ such that $W + W^T > 0$ and $A^T W + W^T A < 0$.

Notation $\mathbb{R}^n_{++}$ defines $\mathbb{R}^n_{++} := \{x \in \mathbb{R}^n, x > 0\}$.

The other stability conditions for positive systems with the partition of the matrix A and two lower-order matrices of the model can be found in [13].

3. **Equivalent Subsystem Approach.**

Consider the following positive continuous time system described as

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t)$$

$$\begin{aligned} z(t) &= C_z x(t) \\ y(t) &= C x(t) \end{aligned} \tag{2}$$

where $x(t)$, $u(t)$, $y(t)$, $w(t)$, $z(t)$ are the state, control input, controlled output, disturbance input and measured output of system, respectively.

Let us assume that the matrix $A \in \mathbb{M}^n$ and other matrices B, B_w, C_z, C are known constant block diagonal (*blkdiag*) matrices with non-negative entries in the decentralized structure

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & \dots & A_{1m} \\ \vdots & \dots & \vdots \\ A_{m1} & \dots & A_{mm} \end{bmatrix} \in \mathbb{R}^{n \times n} \\ B &= blkdiag[B_1, \dots, B_m] \in \mathbb{R}^{n \times k} \\ B_w &= blkdiag[B_{w1}, \dots, B_{wm}] \\ C &= blkdiag[C_1, \dots, C_m] \in \mathbb{R}^{l \times n} \\ C_z &= blkdiag[C_{z1}, \dots, C_{zm}] \end{aligned} \tag{3}$$

Division of the above matrices into submatrices results from inherent properties of the Metzler system (2).

The main goal of this paper is to design a PI decentralized controller for the ith, $i = 1, 2, \dots, m$ subsystem with the following algorithm

$$u_i = k_{Pi} y_{ki} + k_{Ii} \int_{t_0}^{t} y_i(\tau) d\tau \tag{4}$$

The algorithm (4) should guarantee the closed-loop stability of a complex plant (2). Due to (3) the plant can be rewritten to this form

$$\begin{aligned} A &= A_d + A_m \\ A_d &= blkdiag[A_{11}, \dots, A_{mm}] \end{aligned} \tag{5}$$

Let us introduce the following lemmas.

Lemma 3.1. *If $A \in \mathbb{R}^{n \times n}$ with $\{\lambda_1, \dots, \lambda_n\}$ eigenvalues, then for matrix $A + \alpha I$, $\alpha \geq 0$ the eigenvalues are $\{\lambda_1 + \alpha, \dots, \lambda_n + \alpha\}$.*

Lemma 3.2. *Every eigenvalue of matrix A lies in a complex plane within at least one of the Gershgorin discs:*

$$|\lambda - a_{ii}| \leq R_i = \sum_{j \neq i}^{n} |a_{ij}|, \quad j = 1, 2, \dots, n \tag{6}$$

Now, a new equivalent subsystem will be introduced. Let us denote

$$p_0 = \max(real(eig(A_m))) \tag{7}$$

Furthermore, $p = p_0 + \delta$, $(\delta > 0)$ denotes the demanded closed-loop complex system degree of stability. Due to Lemma 3.1, all eigenvalues of matrix $\bar{A}_m$ (8) lie in the left half complex plane (matrix $\bar{A}_m$ is stable).

$$\bar{A}_m = A_m - pI \in \mathbb{R}^{n \times n} \tag{8}$$

On the basis of (8) for the equivalent subsystem, the following formula is obtained

$$A_e = A_d + pI = blkdiag[A_{e1}, \dots, A_{em}] \tag{9}$$

It follows that if complex plant $A_e + A_m$ can be stabilized by the decentralized control (4), there exists such value of δ which gives the closed-loop complex system stability with the PI decentralized controllers [22, 23]. To check the Metzler plant stability with the decentralized controller, we define the Lyapunov function (Proposition 2.2). For this case, the following inequality is obtained

$$\left(\bar{A}_m + A_{ec}\right)^T P + P\left(\bar{A}_m + A_{ec}\right) + Q < 0 \tag{10}$$

where A_{ec} is the matrix of the closed-loop equivalent diagonal subsystem and

$$\begin{aligned} &Q \in \mathbb{R}^{n\times n} > 0, \\ &A_{ec} = blkdiag[A_{ec1}, \ldots, A_{ecm}], \\ &A_{eci} = A_{ei} + B_i(PI\ controller) \end{aligned}$$

The obtained results are summarized in the next theorem.

Theorem 3.1. *Let the ith decentralized PI control* (4) *guarantees the closed-loop stability of the ith equivalent Metzler subsystem for* $i = 1, 2, \ldots, m$. *Then the decentralized PI control guarantees the closed-loop stability of the positive system* (2).

Proof: Equations (8) and (9) imply if $A_{ec} + \bar{A}_m$ is a stable Metzler matrix because $A_{ec} + \bar{A}_m = A_{dc} + A_m$, the complex plant will be stable and Metzler, $A_{dc} = A_d + B(PI\ controller)$.

4. **PI Controller Design.** Consider the equivalent subsystem that is defined (e is omitted)

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + B_i u_i + B_{wi}w_i \\ z_i &= C_{zi}x_i \\ y_i &= C_i x_i \end{aligned} \tag{11}$$

where $i = 1, 2, \ldots, m$, $\dot{x}_i \in \mathbb{R}^{n_i}$, $y_i \in \mathbb{R}^{l_i}$, $u_i \in \mathbb{R}^{m_i}$, $z_i \in \mathbb{R}^{d_i}$, $w_i \in \mathbb{R}^{m_i}$ are state, controlled output, controller output, measured output and disturbance input of the ith subsystem, respectively. We assume that $\sum_{i=1}^{m} m_i = k$, $\sum_{i=1}^{m} n_i = n$.

Let us assume that A_{ii} is equivalent to subsystem (9) Metzler matrix and entries to all matrices B_i, B_{wi}, C_i, C_{zi}, which are non-negative.

This paper aims to design a PI controller for the subsystem (11) such that close system matrix A_c is Metzler using an equivalent subsystem approach. Let us consider the control algorithm for the ith subsystem as follows

$$u_i = k_{Pi}y_i + k_{Ii}e_i \tag{12}$$

The control algorithm (12) includes the proportional part P and approximate integral part. For the sake of achieving the stable Metzler matrix, the following auxiliary system will be applied

$$\dot{e}_i = y_i - \alpha e_i = C_i x_i - \alpha e_i \tag{13}$$

where $\alpha > 0$ is the small tuning parameter. The design control procedure requires the addition of the system (13) into the real plant. Considering the above results, let us introduce the following formula

$$\dot{x}_{ni} = A_{ci}x_{ni} \tag{14}$$

where

$$\begin{aligned} x_{ni} &= \begin{bmatrix} x_i \\ e_i \end{bmatrix}, \\ A_{ci} &= \begin{bmatrix} A_{ii} + B_i k_{Pi} C_i & B_i k_{Ii} \\ C_i & -\alpha I_i \end{bmatrix} \end{aligned}$$

Note that from (14), the following condition will be obtained: if $\alpha = 0$ and A_{ci} requires having the Metzler properties, the stability of A_{ci} with $k_{Ii} > 0$ is not possible to guarantee. The proposed approach ensures the stability and Metzler properties for $\alpha > 0$.

For the guaranty of the Metzler properties for matrices A_{ci} (where $i = 1, 2, \dots, m$), the PI controller designer should ensure the following conditions:

- diagonal entries of the matrix $A_{ii} + B_i k_{Pi} C_i$ are negative;
- diagonal off part of matrix $A_{ii} + B_i k_{Pi} C_i$ is not negative;
- in the case where all entries of the matrix B_i are non-negative, the I part of the ith controller gain requires to be positive (positive feedback).

Note that the dimension of the matrix A_{ci} is not too large for LMI (BMI) controller calculation. The three above conditions are easily implemented in the corresponding script.

5. **Examples.** The following academic example has been considered to show each step of the PI controller design procedure. The Metzler matrix A, positive matrices B, $B_w = B$, C and $C_z = C$ are as follows

$$A = \begin{bmatrix} -0.35 & 0.3 & 0.28 & 0.1 \\ 0.05 & -0.71 & 0.1 & 0.25 \\ 0.12 & 0.05 & -0.65 & 0.31 \\ 0.27 & 0.13 & 0.07 & -0.7 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1.1 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Two second-order subsystems are considered. Using (9) (in the case when $\delta = 0.01$ and $p = 0.3211$), the following equivalent subsystem is obtained

$$A_e\{j\} = A_d\{j\} + pI, \quad j = 1, 2$$

The first equivalent subsystem is unstable.

Example 5.1. *The aim is to design a state-feedback decentralized controller in the form* $u_j = -K_j x_j$. *The numerical solution for each subsystem has been carried out by standard MATLAB script (with* $R\{j\} = 1$, $Q\{j\} = 1$*):*

$$[K_j, S, E] = lqr(A_e\{j\}, B\{j\}, Q\{j\}, R\{j\}).$$

The obtained results are as follows: the gain matrix for the first equivalent subsystem $K_1 = [\ 0.9845 \quad 0.2605\ ]$ *and for the second equivalent subsystem* $K_2 = [\ 0.7583 \quad 0.2234\ ]$. *The control law gain matrix* $K \in \mathbb{R}^{2\times 4}$ *is*

$$K = \begin{bmatrix} 0.9845 & 0.2605 & 0 & 0 \\ 0 & 0 & 0.7583 & 0.2234 \end{bmatrix}.$$

The closed-loop real plant with the proposed decentralized state-feedback controller is stable and Metzler:

$$A_{cl} = \begin{bmatrix} -1.33458 & 0.0395 & 0.28 & 0.1 \\ 0.05 & -0.71 & 0.1 & 0.25 \\ 0.12 & 0.05 & -1.4841 & 0.0643 \\ 0.27 & 0.13 & 0.07 & -0.7 \end{bmatrix} \in \mathbb{M}^4$$

with closed-loop eigenvalues (EV)

$$EV = \{-1.6063;\quad -1.2695; -0.4767;\quad -0.8761\}.$$

Simulation results using the state-feedback decentralized controller are shown in Figures 1-3. In the simulations x, r, y, u *are the state variable, reference signal, measured output and controller output, respectively. The state variables vector* $x(t)$ *as well as the output variables vector* $y(t)$ *are positive when the input in the closed-loop system is positive vector* $v^T = [\ 1.1326 \quad 0.5223\ ]$ *and* $u(i) = -Kx(i) + v$. *The control law gain matrix* K, *(with* $R\{j\} = 1$, $Q\{j\} = 1$, *where* $j = 1, 2$*) implies stable positive matrix of the closed-loop system matrix* A_{cl}.

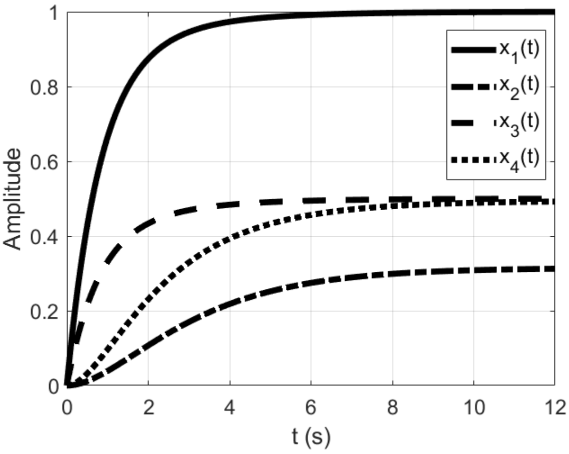

FIGURE 1. State variables response $x(t)$ using state-feedback decentralized controller (Example 5.1)

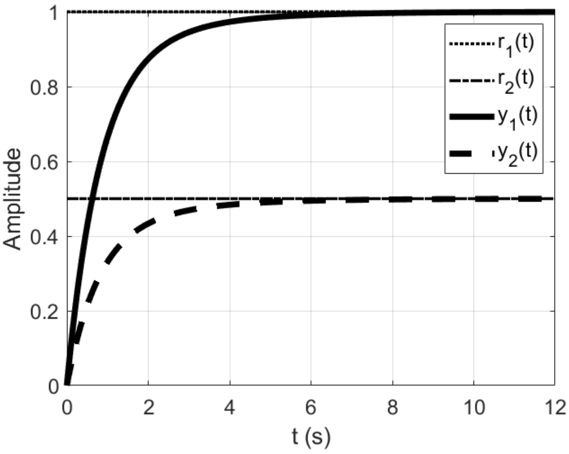

FIGURE 2. Simulation results $r(t)$, $y(t)$ using state-feedback decentralized controller (Example 5.1)

Example 5.2. *This example aims to design a decentralized PI controller for each subsystem with output-feedback by using the* $\mathcal{L}_2$*-gain (*$\mathcal{H}_\infty$*) approach. The control algorithm is in the form* (12). *The* $\mathcal{L}_2$*-gain robust PI controller design procedure for equivalent subsystem can be obtained by using the standard approach presented in* [24]. *These results are summarized in Lemma 5.1* [24].

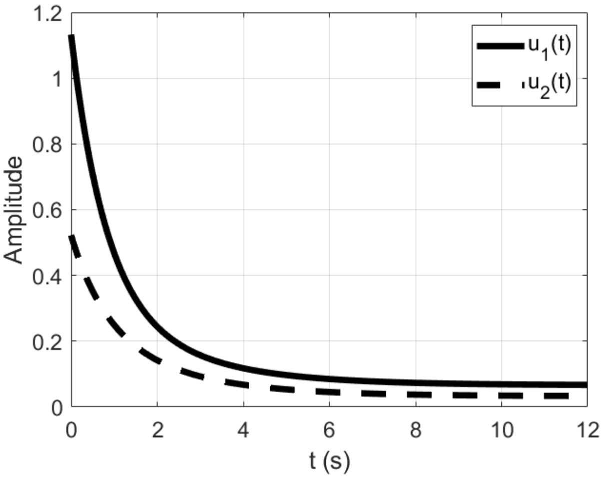

FIGURE 3. Controller output $u(t)$ using state-feedback decentralized controller (Example 5.1)

Lemma 5.1. *Let us control the* ith *equivalent subsystem* (11) *with a PI control algorithm* (12)*. The closed-loop subsystem is stable if there exists Lyapunov function with* $P_i > 0$*, matrices* N_1*,* N_2*,* N_3 *and* $\mathcal{L}_2$*-gain* $p_i = \frac{\|z_i\|}{\|w_i\|}$ *such that it holds*

$$B_{ei} = v_i(t)^T W_i v_i(t) < 0 \tag{15}$$

where $i = 1, 2, \ldots, m$, $v_i^T = \begin{bmatrix} \dot{x}_i^T & x_i^T & w_i^T \end{bmatrix}$, $W_i = \{w_{kli}\}_{3\times 3}$

$$\begin{aligned} w_{11i} &= N_1^T + N_1 \\ w_{12i} &= P_i - N_1^T A_{ci} + N_2 \\ w_{13i} &= -N_1^T B_{wi} + N_3 \\ w_{22i} &= C_{zi}^T C_{zi} - N_2^T A_{ci} - A_{ci}^T N_2 \\ w_{23i} &= -N_2^T B_{wi} - A_{ci}^T N_3 \\ w_{33i} &= -p_i I_i - N_3^T B_{wi} - B_{wi}^T N_3 \end{aligned}$$

and $A_{ci} = A_{ii} + B_i K_i$*,* $K_i = \begin{bmatrix} k_{Pi} C_i & k_{Ii} \end{bmatrix}$.

If Inequality (15) *holds, the equivalent subsystem is stable with* $\mathcal{L}_2$*-gain* $p_i < 1$*. The obtained following results for the above example are* ($\alpha = 0.37$) *as follows.*

For the first subsystem

$$R_1(s) = -72.9005 + \frac{4.9834}{s}, \quad p_1 = 0.5086$$

for the second subsystem

$$R_2(s) = -72.876 + \frac{4.9843}{s}, \quad p_2 = 0.5097$$

The closed-loop real Metzler system with the proposed decentralized controllers ($R_1(s)$ *and* $R_2(s)$) *is*

$$A_{cl} = \begin{bmatrix} -73.2505 & 0.3 & 4.9834 & 0.28 & 0.1 & 0 \\ 0.05 & -0.71 & 0 & 0.1 & 0.25 & 0 \\ 1 & 0 & -0.37 & 0 & 0 & 0 \\ 0.12 & 0.05 & 0 & -80.8136 & 0.31 & 5.4827 \\ 0.27 & 0.13 & 0 & 0.07 & -0.7 & 0 \\ 0 & 0 & 0 & 1 & 0 & -0.37 \end{bmatrix} \in \mathbb{M}^6,$$

with closed-loop eigenvalues (EV)

$$EV = \{-73.3135; \quad -80.8864; -0.8858; \quad -0.5237; -0.3012; \quad -0.3020\}.$$

Considering the above results, we can conclude that designed PI decentralized controllers 5.2 guarantee the closed-loop stability of complex plant, and the closed-loop system matrix is Metzler. The maximal value of closed-loop transfer function norm ($\mathcal{L}_2$-gain) is $p = 0.5097$*. The closed-loop system matrix satisfies the properties of the Metzler matrix.*

Simulation results using the decentralized PI controller for each subsystem with output-feedback by using the $\mathcal{L}_2$-gain ($\mathcal{H}_\infty$) approach are shown in Figures 4-6. In the simulations x, r, y, u are the state variable, reference signal, measured output and controller output, respectively. In smaller box (Figure 6), a zoom in the time range [0.1-12]s is shown to highlight the detail of controller output. The numerical solution has been carried out by MATLAB using YALMIP [25]. *The simulations have been done using Simulink (in MATLAB). Simulation results illustrate the effectiveness of the proposed method and show that PI controller design for a positive system can still be considered as partially open problem.*

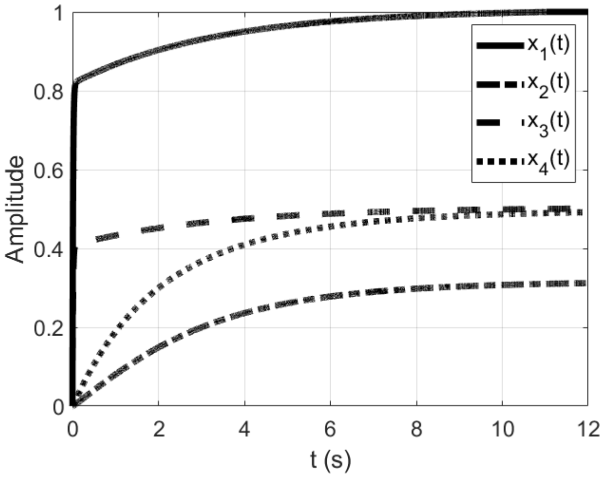

FIGURE 4. State variables response $x(t)$ using decentralized PI controller for each subsystem with output-feedback by using the $\mathcal{L}_2$-gain ($\mathcal{H}_\infty$) approach (Example 5.2)

6. **Conclusion.**

This paper deals with the novel method for the decentralized controller designs for the large-scale Metzler plants using the equivalent subsystem approach in the time-domain. The decentralized control design procedure is carried out on the subsystem level in such a way that the closed-loop real plant is stable and Metzler.

The obtained results show the decentralized control's effectiveness of feedback state and output using the $\mathcal{L}_2$-gain approach. There is no need to use the iteration process to check the stability of a closed-loop complex system, which is illustrated by the above described two examples. The obtained results for the subsystem level $\mathcal{L}_2$-gain are for $p = 0.5097$.

For the sake of achieving the required Metzler properties of the closed-loop system, only a state P controller has to be designed for positive systems. In this paper, the control algorithm includes the proportional part P and the approximate integral part. The obtained results, illustrated in this paper, show the applicability of the designed controller. The proposed approach guarantees Metzler properties of the closed-loop system.

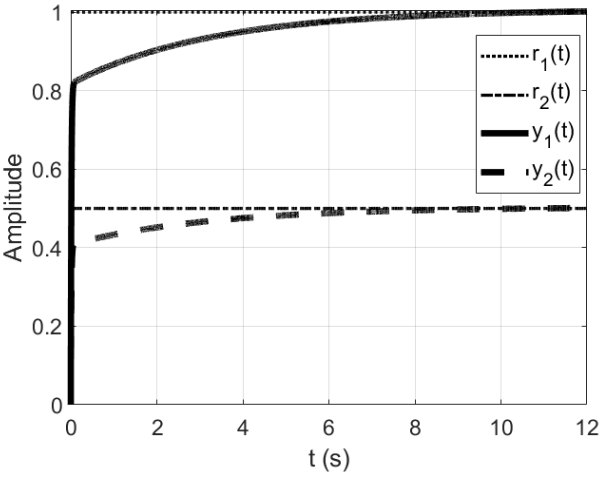

FIGURE 5. Simulation results $r(t),\ y(t)$ using decentralized PI controller for each subsystem with output-feedback by using the $\mathcal{L}_2$-gain ($\mathcal{H}_\infty$) approach (Example 5.2)

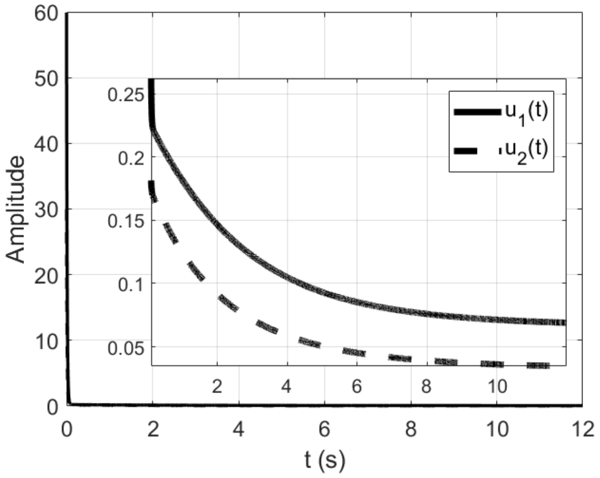

FIGURE 6. Controller output $u(t)$ (zoom in the time range [0.1-12]s) using decentralized PI controller for each subsystem with output-feedback by using the $\mathcal{L}_2$-gain ($\mathcal{H}_\infty$) approach (Example 5.2)

Future research will focus on methods, further reducing the conservativeness and look for the mathematical processing between the equivalent subsystem design of a decentralized controller and ensure the stability and quality control of the complex system.

Acknowledgment. The work has been supported by Grant 1/0049/20 of the Slovak Scientific Grant Agency. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] D. D. Siljak, *Large-Scale Dynamic Systems: Stability and Structure*, North-Holland, New York, 1978.

[2] L. Bakule, Decentralized control: An overview, *Annual Reviews in Control*, vol.32, pp.87-98, 2008.

[3] M. Hovd and S. Skogestad, Improved independent design of robust decentralized control, *Journal of Process Control*, vol.3, no.1, pp.43-51, 1993.

[4] M. Hovd and S. Skogestad, Sequential design of decentralized controllers, *Automatica*, vol.30, no.10, pp.1601-1607, 1994.

[5] A. Kozaková, V. Veselý and V. Kučera, Robust decentralized controller design based on equivalent subsystems, *Automatica*, vol.107, pp.29-35, 2019.

[6] V. M. Matrosov, On the stability of motion, *Journal of Applied Mathematics and Mechanics*, vol.26, no.6, pp.1337-1353, 1962.

[7] N. Wa, R. Li and B. Hu, Decentralized controllability and observability, and decentralized stabilization, *Proc. of the 1988 IEEE Int. Conf. on Systems, Man, and Cybernetics*, pp.598-601, DOI: 10.1109/ICSMC.1988.754370, 1988.

[8] D. G. Leunberger, *Introduction to Dynamic Systems: Models, and Applications*, Springer-Verlag, Berlin, 1979.

[9] T. Kaczorek, *Positive 1D and 2D Systems*, Springer-Verlag, London, 2002.

[10] M. A. Rami and F. Tadeo, Controller synthesis for positive linear systems with bounded controls, *IEEE Trans. Circuits Syst.*, vol.54, pp.151-156, 2007.

[11] Z. He, L. Wu and X. Su, H_∞ control of linear positive systems: Continuous- and discrete-time cases, *International Journal of Innovative Computing, Information and Control*, vol.5, no.6, pp.1747-1756, 2009.

[12] T. Jitwang and D. Puangdownreong, Application of cuckoo search to robust PIDA controller design for liquid-level system, *International Journal of Innovative Computing, Information and Control*, vol.16, no.1, pp.189-205, 2020.

[13] Y. Ebihara, D. Peaucelle and D. Arzelier, L_1 gain analysis of linear positive systems and its application, *Proc. of the 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, Orlando, FL, USA, pp.4029-4034, 2011.

[14] J. Shen and J. Lam, Static output-feedback stabilization with optimal L_1-gain for positive linear systems, *Automatica*, vol.63, pp.248-253, 2016.

[15] M. A. Rami, Solvability of static output-feedback stabilization for LTI positive systems, *Systems and Control Letters*, vol.60, pp.704-708, 2011.

[16] D. Krokavec and A. Filasová, Stabilization of discrete-time LTI positive systems, *Archives of Control Sciences*, vol.27, pp.575-594, 2017.

[17] T. Tanaka and C. Langbord, The bounded real lemma for internally positive systems and H_∞ structured static state feedback, *IEEE Trans. Automatic Control*, vol.56, no.9, pp.2218-2223, 2011.

[18] C. Marcello, *Robust and Decentralized Control of Positive Systems: A Convex Approach*, Ph.D. Thesis, Automatic Control Laboratory (IfA) of ETH Zürich, Switzerland, 2016.

[19] D. Peaucelle, D. Alzelier, O. Bachelier and J. Bernussou, A new robust D-stability condition for real convex polytopic uncertainty, *Systems and Control Letters*, vol.40, pp.21-30, 2000.

[20] R. Ma and M. Ma, Stabilization design of a class of switched positive linear systems with linear programming, *ICIC Express Letters*, vol.13, no.9, pp.859-866, 2019.

[21] L. Farina and S. Rinaldi, *Positive Linear Systems: Theory and Applications*, John Wiley & Sons, 2000.

[22] V. Veselý, Large scale dynamic system stabilization using the principle of dominant subsystems approach, *Kybernetika*, vol.24, pp.48-61, 1993.

[23] D. D. Siljak, *Large-scale Dynamic System: Stability and Structure*, Amsterdam, North-Holland, The Netherlands, 1978.

[24] V. Veselý, A. Ilka, L. Körösi and M. Ernek, Master-slave H_∞ robust controller design for synchronization of chaotic systems, *Modeling, Identification and Controls*, vol.40, pp.41-50, 2019.

[25] J. Löfberg, YALMIP: A toolbox for modeling and optimization in MATLAB, *Proc. of the CACSD Conference*, Taipei, Taiwan, 2004.