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NOVEL APPROACH TO DECENTRALIZED CONTROLLER DESIGN FOR LARGE SCALE UNCERTAIN LINEAR SYSTEMS

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Abstract. *Such characteristics as large order multi-input multi-output (MIMO) system, system uncertainties, and information structure constraints. make large scale systems too complex to be effectively controlled. For the purposes of analysis and controller synthesis such systems are divided into independent sub-problems and controlled under specific information constraints. This is known as decentralized control. Analyzing stability of each of the resulting, smaller systems separately, i.e., by neglecting interconnections, is a tractable but highly conservative approach. In the previous literature on decentralized control, complex systems are divided into two classes of systems, i.e., those with strong interaction links and those with weak ones. The present paper proves that the criterion of whether a complex system is stable or not is a better basis of the breakdown of systems into classes. In the case of a stable complex system, the corresponding decentralized controller design is significantly easier than that in the unstable case. In this paper an original method of decentralized controller design is obtained for linear time-invariant large scale systems with small conservatives. The design procedure consists of two basic steps as follows. In the first step, basic properties of closed-loop controlled subsystems without interaction are determined, in such way that stability and performance of the closed-loop large scale system are guaranteed. In the second step, a decentralized control algorithm needs to be designed, which ensures the demanded subsystem closed-loop properties. If in the second step conditions are satisfied the stability and performance of subsystems and the complex plant is guaranteed, the decentralized controller design procedure performs on the subsystem level. Examples show the effectiveness of the propose method.*

Keywords: Large scale system, Equivalent subsystem method, Decentralized controller, State/output decentralized feedback

1. **Introduction.** The notion of a large scale system (LSS) indicates the fundamental characteristic of multidimensional complex system, as well as the uncertainty with decentralized control information structure [1]. The complex large scale systems to be controlled are too large and control problems are too complex. This is the reason why problems of complex systems for analysis and controller synthesis are divided into independent/almost independent sub-problems. LSS is controlled by an algorithm with information constraints, decentralized control. The classical cases of large scale systems include, e.g., power system, microgrid control, large continuous processes control, and robotics. One of the first methods of theoretical analysis of stability of complex systems and decentralized controller design used aggregation matrix method. Therefore, in the initial part of the paper, it provides a sketch of the aggregation matrix derivation and its use for the purposes of stability analysis, as well as proposal of decentralized control

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of complex continuous linear system. The decentralized control design procedure begins since 1970s [1, 2, 3, 4, 5, 6, 7]. For detailed overview of decentralized controls and their evolution, see the excellent review paper of [5]. We give an LSS which is represented by the following linear differential equation:

$$\dot{x} = Ax \tag{1}$$

The plant of (1) is composed of the interconnected subsystems as follows

$$\dot{x}_j = A_j x_j + \sum_{i=1}^{m} A_{sj} x_i, \quad j, s = 1, 2, \ldots, m, \; j \neq i \tag{2}$$

where m is the number of subsystems, $x_j \in R^{n_j}$, A_j, A_{sj} are the jth subsystem state, constant jth subsystem and interactions matrices of corresponding dimension. The stability of isolated subsystems

$$\dot{x}_j = A_j x_j \quad j = 1, 2, \ldots, m \tag{3}$$

should be determined by Lyapunov function $v_j = \left(x_j^T P_j x_j\right)^{\frac{1}{2}}$. Following from the results of [12], the jth subsystem is stable if the following inequalities hold:

$$\alpha_{j1}||x_j|| \leq v_j \leq \alpha_{j2}||x_j|| \tag{4}$$

$$\dot{v}_j \leq -\alpha_{j3}||x_j|| \quad ||grad v_j|| \leq \alpha_{j4} \tag{5}$$

where

$$\alpha_{j1} = \lambda_m^{\frac{1}{2}}(P_j) \quad \alpha_{j2} = \lambda_M^{\frac{1}{2}}(P_j) \tag{6}$$

$$\alpha_{j3} = 0.5\frac{\lambda_m(G_j)}{\lambda_M^{0.5}(P_j)} \quad \alpha_{j4} = 0.5\frac{\lambda_M(P_j)}{\lambda_m^{0.5}(P_j)} \tag{7}$$

where λ_m, λ_M are the minimum and maximum eigenvalues of the corresponding matrices. G_j is positive definite matrix satisfying Lyapunov equality $A_j^T P_j + P_j A_j = -G_j$. It is clear that if Inequalities (4), and (5) hold, the jth subsystem is asymptotically stable. Following [1, 12] stability condition for LSS can be obtained, in the form of the following aggregation matrix

$$\dot{v} \leq Wv \quad W = \{w_{ij}\} \tag{8}$$

where $v^T = [v_1 \ldots v_m]^T$ is the vector Lyapunov function [11],

$$w_{ij} = -\alpha_{j2}^{-1}\alpha_{j3} \quad j = i, \tag{9}$$

$$w_{ij} = \eta_{ij}\alpha_{j1}^{-1}\alpha_{j4} \quad j \neq i \tag{10}$$

where $\eta_{ij} = \lambda_M^{\frac{1}{2}}\left(A_{ij}^T A_{ij}\right)$.

LSS is stable if the aggregation matrix (8) is stable. From (9) it is clear that parameters α_{j2}, α_{j3} are the functions of the jth decentralized controller. Designer choosing the controller parameters needs to ensure that the Metzler matrix W will be stable. Stability of each subsystem (4), and (5) and stability of the aggregate model (8) imply connective stability [1] to the overall system.

For linear time-invariant large scale systems the decentralized controller design procedure has been obtained in the frequency and time domain. Mainly the following methods have been developed in the frequency domain: independent design method [8], sequential design [9] and method of equivalent subsystem approach [10]. The first two methods are rather conservative with complexity of their respective solution. The method of equivalent subsystem solves the proposal of the decentralized controller with necessary and sufficient stability conditions. In the time domain the three groups of method have been developed: stability analysis and decentralized control design using the aggregation matrix approach

[1], vector Lyapunov function approach [11], and much progress has been made in the control of LSS through the use of LMI-BMI; see the review article [5]. Unfortunately, when above approaches are used for stability analysis and decentralized controller design, a complete complex model of LSS needs to be applied.

In the previous literature on decentralized control, complex systems are divided into two classes of systems, i.e., with strong interaction links and with weak ones. The present work proves the criterion of whether a complex system is stable or not is a better basis of the system division into classes. This is the first new result of the paper. In the case of a stable complex system, the corresponding decentralized controller design is significantly easier than that in the unstable case. This paper is devoted to obtaining originally new analytical method to analyze the stability of LSS and to design a decentralized controller which is performed on the subsystem level. The method of decentralized controller design consists of two steps. In the first step, the designer calculates such dynamic properties of subsystems that the complex LSS should be stable. In the second step, decentralized controllers are designed in such a way that decentralized controllers of subsystems ensure the required properties. In the first step of the present method, for defined LSS structure, the demanded properties of the subsystems are determined with the necessary and sufficient conditions at which the stability of the complex system is ensured. The conservatism of the proposed method is given in the second step and depends on the chosen design procedure of the decentralized controller.

This paper is organized as follows. Section 2 provides the preliminary results needed to define an equivalent subsystem. Finally, the formulation of the problem is given. Section 3 defines an equivalent subsystem and its use for the designing of the decentralized controller to large scale systems. This paper uses two examples to compare two control methods as H_2 and regional pole placement approach in Section 4. The above mentioned examples show the effectiveness of the proposed method. In the Conclusion, Section 5 presents the advantages of the proposed method.

Hereafter, the following notation conditions will be adopted. Given a symmetric matrix $P = P^T \in R^{n\times n}$, the inequality $P > 0$, ($P < 0$) denotes matrix positive (negative) definiteness. I_n, 0_n denote the identity, (zero) matrix of corresponding dimensions.

2. Mathematical Description of Uncertain LSS and Preliminaries.

We give the uncertain polytopic LSS continuous time-invariant where the input and output matrices are in the decentralized structure

$$\dot{x} = A(\xi)x + B(\xi)u; \quad y = Cx \tag{11}$$

where $x \in R^n$, $u \in R^m$, $y \in R_i^l$ are the state, control input, controlled output, system matrices $(A(\xi), B(\xi)) = \sum_{i=1}^{N}(A_i, B_i)\xi_i$ belong to a polytopic uncertainty domain with N vertices, while for uncertainty $\xi \in \Omega_\xi$ it holds

$$\Omega_\xi = \left\{ \xi_i \geq 0,\ i = 1, 2, \ldots, N,\ \sum_{i=1}^{N} \xi_i = 1,\ \sum_{i=1}^{N} \dot{\xi}_i = 0 \right\} \tag{12}$$

Matrices A_i, B_i, C are constants and last two matrices are to be in the decentralized structure.

$$A_i = \begin{bmatrix} A_{i11} & \ldots & A_{i1m} \\ \vdots & \vdots & \vdots \\ A_{im1} & \ldots & A_{imm} \end{bmatrix} \in R^{n\times n} \tag{13}$$

$$B_i = blockdiag[B_{i1} \ldots B_{im}] \in R^{n\times m}$$

$$C = blockdiag[C_1 \ldots C_m] \in R^{l\times n} \quad i = 1, 2, \ldots, N$$

Assume that complex system (11) is centralized controllable, observable and there are no unstable fixed modes [4, 17, 21]. The system (11) can be formally decomposed to subsystems in different ways. In this paper, division of the above matrices into their sub-matrices follows from inherent properties of complex large scale systems.

Lemma 2.1. [14] *If the equality*

$$G = cH + \varrho I_n \tag{14}$$

holds for scalars $c, \varrho \in R$ *and the identity matrix* $I_n \in R^{n\times n}$*, then the eigenvalues of the matrix* G *are as follows:* $\lambda_k = c\alpha_k + \varrho$ *where* α_k *is the eigenvalue of matrix* H*,* $k = 1, 2, \ldots, n$*.*

From Lyapunov stability theory it can be obtained.

Lemma 2.2. *The sum of two matrices* $G + H \in R^{n\times n}$ *is stable if and only if positive definite Lyapunov matrix* $P > 0$ *exists such that the Lyapunov inequality*

$$(G + H)^T P + P(G + H) \leq 0 \tag{15}$$

holds.

Lemma 2.3. *Consider that uncertain system is described by (11) and gain's controller block diagonal matrix is* K*. Then the necessary and sufficient condition for the existence of a decentralized controller with gain* K *such that closed-loop system is asymptotically stable, is that*

$$\lambda(A(\xi), B(\xi), C, K) \in C^- \tag{16}$$

where C^- *is the left hand side of the complex plane.*

Definition 2.1. *Let matrix* $E = e_{ij_{n\times n}}$ *be a structured perturbation matrix of the system (11) where* $e_{ij} = 1$ *if there is an interaction connection between the subsystems* i *and* j*,* $e_{ij} = 0$ *there is no interaction connection between the subsystems* i *and* j*.*

Definition 2.2. [1] *A complex system (11) is connective stable iff it is stable for all possible entries of the matrix* $E(e_{ij})$*.*

3. **Main Results.** In this section, the original results to design robust PI, (PID) decentralized controller using the equivalent subsystem approach in time domain are obtained. Equivalent subsystem is an auxiliary subsystem to serve for design of decentralized controller which guarantees the closed-loop stability of the subsystem and performance to diagonal matrix $A_d(\xi)$ and complex uncertain LSS. Designed decentralized controller guarantees the quadratic/parameter dependent quadratic stability and performance defined by the designer for the closed-loop uncertain subsystem. The controller design procedure performs on the subsystem level. Let us split uncertain complex system (11) to the form

$$\dot{x} = (Ad(\xi) + Am(\xi))x + B(\xi)u; \quad y = Cx \tag{17}$$

or for the ith vertex of polytope

$$\begin{aligned} &\dot{x} = (Ad_i + Am_i)x + B_i u; \quad y = Cx \\ &u = blockdiag\{u_1, u_2, \ldots, u_m\}, \quad i = 1, 2, \ldots, N \end{aligned} \tag{18}$$

where $Ad(\xi)$ is the block diagonal part of matrix $A(\xi)$ and $Am(\xi) = A(\xi) - Ad(\xi)$ is diagonal off part of system (11).

Theorem 3.1. *Let us have two matrices* Ad_i*,* Am_i *with constant entries. The sum* $D_i = Ad_i + I\alpha + Am_i$ *is stable for some* α *if positive definite matrix* $P_i \in R^{n\times n}$ *exists such that the following inequality holds:*

$$(Ad_i + Am_i)^T P_i + P_i(Ad_i + Am_i) + 2\alpha P_i \leq 0 \tag{19}$$

Proof: Due to Lemma 2.2, the following inequality is obtained:

$$(Ad_i + I\alpha + Am_i)^T P_i + P_i(Ad_i + I\alpha + Am_i) \leq 0 \tag{20}$$

Small manipulation leads to (19), proving the sufficient stability condition. Let us assume that matrix D_i is stable, then using Lyapunov stability theory, a positive definite matrix P_i exists such that Inequality (19) holds, which proves the theorem.

Consider an uncertain polytopic system in the form $\dot{x} = A(\xi)x = \sum_{i=0}^{N}((Ad_i + I\alpha + Am_i)\xi_i)x$. After small modification of [15], the stability of such complex uncertain system is given in the following lemma.

Lemma 3.1. *The linear uncertain polytopic system $A(\xi)$ is stable if matrices $G, H \in R^{n\times n}$, parameter α and symmetric positive definite matrix P_i, $i = 1, 2, \ldots, N$ exist, such that the following inequality holds*

$$\begin{bmatrix} D_i^T H^T + H(Ad_i + I\alpha + Am_i) & * \\ P_i - H^T + G^T(Ad_i + I\alpha + Am_i) & -G - G^T \end{bmatrix} \leq 0 \quad i = 1, 2, \ldots, N \tag{21}$$

Inequality (21) is of the bilinear matrix inequality (BMI), and if the LSS is large order, the elimination lemma could be used [16]. To obtain the LMI formulation of (21) the linearization of non-convex diagonal part is a useful approach. Parameter α can be obtained from (19) or (21). If $\alpha \geq 0$, the complex system is stable. In this case decentralized controller should be designed in such a way that the following inequality holds for dominant eigenvalues of all closed-loop subsystems:

$$\lambda_k(Ad_i + B_i * controller) \leq \lambda_k(Ad_i), \quad k = 1, 2, \ldots, d_s;\ i = 1, 2, \ldots, N$$

where d_s is the number of the dominant eigenvalues for which the closed-loop complex system becomes stable. The decentralized controller should not lead to worse dynamic of the corresponding subsystem than that of the open-loop subsystem dynamic. In this case let us put $\alpha = 0$ for the next calculation.

If the obtained value $\alpha < 0$, then the complex system is unstable. To cope with this problem with the decentralized controller design, we have introduced the auxiliary equivalent subsystem matrix as follows: let us choose $\beta = |\alpha| + \delta$ where $\delta \geq 0$ is a small tuning parameter (for the first step $\delta = 0$).

Lemma 3.2. *Assuming that the "ideal" decentralized controller is obtained, where closed-loop complex system in the ith vertex is given as $D_i = Ad_i + I\alpha + Am_i$, $i = 1, 2, \ldots, N$, then the stability boundary of closed-loop system is as follows*

$$b_s = \alpha + \max_j (real(eig(Ad_{ij})),\ i = 1, 2, \ldots, N), \quad j = 1, 2, \ldots, m$$

Lemma 3.3. *For the case of $\alpha < 0$, large scale uncertain linear system will be stable if dominant real parts of eigenvalues of the closed-loop subsystems with decentralized controllers λ_k, where k approaches to d_s, and moves to the left on the complex plain, by the value of α.*

Proof: Lemma's argument directly follows from Equation (21) or (19).

Definition 3.1. *Equivalent subsystem is defined as*

$$Ae_i = Ad_i + I\beta \quad i = 1, 2, \ldots, N \tag{22}$$

On the basis of summary of the results the following decentralized controller design procedure has been obtained.

Algorithm for the decentralized controller design

Let us assume a decentralized controllable and observable complex system [17, 21], and there is no unstable fixed mode.

- 1) The first step calculates the value of α using for classical MIMO systems (19) or for uncertain polytopic system (21). If $\alpha \geq 0$ (complex system is stable), then decentralized controllers are designed for all subsystems using any controller design procedure such that the following inequalities for the eigenvalues hold

$$EIGCLOSEDLOOP(Ad_{ij} + B_{ij} * controller * C_j)_k \leq EIG(Ad_{ij})_k,$$
$$i = 1, 2, \dots, N,\ j = 1, 2, \dots, m,\ k = 1, 2, \dots, n_j.$$

  Stability of complex linear and nonlinear systems can be determined by another way using an aggregation matrix approach [1]. If the system is stable and the diagonal elements of the aggregation matrix are not increased by decentralized controllers, the stability of the system will not be disturbed. The results achieved in this paper support and specify the result achieved using the aggregation matrix.
- 2) Complex system is unstable, $\alpha < 0$. Lemma 3.3 gives the idea how to obtain stable LSS. While one α covers all subsystems and method of α calculation, Lemma 3.3 may be conservative. Let us put for the first step $\delta \in (0, 0.1\alpha)$ in the equivalent subsystem defined by (22). Then, let us design a robust decentralized controller to the equivalent subsystem using any design method, in such a way that all closed-loop subsystems (equivalent subsystems+decentralized controllers) are stable with performance. The complex system should be stable if both maximal values of the closed-loop subsystems eigenvalues are less than boundary of LSS stability b_s and the following inequality holds for the first d_s closed-loop dominant subsystem eigenvalues where d_s is the number of the first dominant eigenvalues for which the stability of closed-loop complex system is guaranteed.

$$EIGCLOSEDLOOP(Ad_{ij} + B_{ij} * controller * C_j)_k \leq EIG(Ad_{ij} + \alpha * I)_k,\quad k = 1, 2, \dots, d_s \tag{23}$$

  If the above conditions do not hold for ($d_s = n_j$) and the complex system is not stable, increase δ [1, 18] and repeat the calculation from second step. Check the closed-loop stability of the complex system with stable subsystems by calculation of new α. If $\alpha \geq 0$, then the complex system is stable. Note that if I-part of decentralized controller is used for all subsystems, then m new states of subsystems are introduced [13]. In this case the interaction matrix Am needs to be modified by zero elements.
- 3) If the new calculated value of $\alpha \leq 0$ and the complex system cannot be stabilized by increasing δ, then either the complex system is not decentralized controllable and observable or there is fixed mode. See [17] for characterization of fixed modes and criteria for their testing.

4. **Examples.** The following two examples aim to show in detail the procedure of decentralized controller design specifically showing the way how the first and second steps of the design procedure should be used. For the sake of success of the second step, such subsystem controller design procedure should be chosen, which guarantees that condition (23) is met. The following two methods will be used in this paper: H_2 and the regional pole placement approach. Nice examples to decentralized control can be found in literature, i.e., microgrid control [24], power system control [25], and robust control [26]. For this two examples as a mechanical system have been borrowed from [12] where the authors have used the method of aggregation matrix approach for determining the

boundary of stability. Obtained results in this paper show that the boundary of complex system stability is determined by the calculated coefficient α. Our problem is to design two robust PI decentralized controllers which will minimize, on the subsystem level, the given quadratic cost function and ensure stability and performance of the complex LSS. The mathematical model of the investigated system is as follows:

$$\dot{x}(t) = A(\xi)x(t) + B(\xi)u(t) \quad y(t) = Cx(t) \tag{24}$$

where

$$A(\xi) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/m_1 & -b_1/m_1 & 0 & b_1/m_1 \\ 0 & 0 & 0 & 1 \\ 0 & b_1/m_2 & -k_2/m_2 & -b_1/m_2 \end{bmatrix}$$

$$B(\xi) = \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Consider the following parameters of the above mechanical system

$$m_1 = 10 \text{ kg}, \quad m_2 = 30 \text{ kg}, \quad k_1 = 2.5 \text{ kN/m}, \quad k_2 = 1.5 \text{ kN/m}$$

and uncertain parameter $b_1 \in$ [25-35] Ns/m. Two subsystems will be obtained, as well as a polytopic system with two vertices for $b_1 = 25$ and $b_1 = 35$. Note that matrices $B(\xi)$, C are in the decentralized structure.

4.1. **Robust decentralized controller design using $\boldsymbol{H_2}$ approach.** To assess performance quality in the frame of H_2, the following augmented quadratic cost function is proposed

$$J_c = \int_0^\infty J(x, \dot{x}, u)\, dt \tag{25}$$

where $J(x, \dot{x}, u) = x^T Q x + \dot{x}^T S \dot{x} + u^T R u$, $Q = qI$, $q = 0.05$, $S = sI$, $s = 0.0001$, $R = rI_u$, $r = 1$.

The first step of the design procedure.

In the first step, the value of α is calculated from (21) such that LSS will be just on the boundary of stability.

$$Ad(\xi) + \alpha I_n + A_m(\xi) \tag{26}$$

The obtained value $\alpha = -0.02$ shows that LSS is unstable. Note, obtained α cover two uncertain subsystems, the result may be conservative. To relax such conservatism different α for each subsystem should be calculated (i.e., in our case, $\alpha = [\alpha_1 0; 0 \alpha_2]$). We will proceed the design procedure with one α.

The second step, robust decentralized PI controller design.

The complex system is unstable. Let us introduce an equivalent subsystem

$$Ae_i = Ad_i + I\beta \quad \beta = |-0.02| + 0.05 \tag{27}$$

The following lemma holds for the H_2 and PI controller design.

Lemma 4.1. [26] *Closed-loop jth equivalent subsystem with PI controller is stable if such auxiliary matrices $N1(j)$, $N2(j)$, positive definite Lyapunov matrix $P(i,j)$, $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, m$ and controller gain matrix $K1(j) = [kp(j)C_j \; ki(j)]$ exist that the following inequality holds*

$$W(i,j) = \{w_{kl}(i,j)\}_{\{2\times 2\}} \leq 0 \tag{28}$$

where

$$w_{11}(i,j) = N1(j)'A_v(i,j) + A_v(i,j)'N1(j) + Q + K1(j)'RK(j)$$
$$w_{12}(i,j) = P(i,j) + N2(j)A_v(i,j) - N1(j)$$
$$w_{22}(i,j) = -N2(j)' - N2(j) + S$$

where $A_v(i,j)$ is the jth equivalent subsystem in the ith vertex with integrator. On the basis of (28), the following controller gains will be obtained for the first and second equivalent subsystems:

$$R(1) = -168.9279 - \frac{115.8464}{s}$$
$$R(2) = -36.1318 - \frac{9.0135}{s}$$

Robust subsystem controllers $R(1)$, $R(2)$ ensure the stability and performance of the first and second subsystems, as well as of the complex LSS. The following eigenvalues to closed-loop decentralized controllers and LSS will be obtained for the case $i = 1$:

$$EigclosLSS = \{-0.7535 \pm 3.5897i; -.3691 \pm 1.01446i;\}$$
$$\{-.8174; -0.2713\}$$

The eigenvalues of the first and second subsystems without and with decentralized controller are as follows:

Respective eigenvalues of the subsystem $j = 1$, without and with controller:

$$Eignocotrol = \{0; -2.3956; -0.1022\}$$
$$Eigcontrol = \{-0.8845 \pm 3.8817i; -.7309\}$$

Respective eigenvalues of the subsystem $j = 2$, without and with controller:

$$Eignocotrol = \{0; -0.7688; -0.065\}$$
$$Eigcontrol = \{-0.2805 \pm 1.0107i; -.2728\}$$

The above two results indicate stability condition of LSS (23) is met, complex system being stable with the designed decentralized controllers. For the first subsystem, dominant eigenvalues are $\{0; -0.1022\}$ without control and $\{-.7309; -0.8845\}$ with control. The same holds for the second subsystem. For our example $d_s = 1$. It clearly satisfies the stability of LSS conditions. The relax approach could be used in this example. For the purposes of such a case, we have one α for each subsystem. The results of α_j calculation for our example give $\alpha_1 = -0.0107$, $\alpha_2 = -0.0123$. Because of small values of α_k the controller parameters $R(1)$, $R(2)$ virtually do not change.

4.2. **Robust controller design using regional pole placement approach.** A good controller performance should guarantee fast and well-damped behavior of the controlled closed-loop dynamics. A possible way to meet such requirement is to gather the closed-loop poles in a suitable pole region. Based on Lemma 3.3 and demanded eigenvalues of subsystems, the designer could define the region where the eigenvalues of all subsystems should remain, so that the results obtained by the regional pole placement approach in more cases will satisfy the condition of Lemma 3.3. The goal of this example is to design robust PI-D decentralized controller for mechanical system with parameters as given above. The PI part of the designed controller is output feedback and D-part is any state feedback determined by the matrix $C_d = [1\ 0]$. For the purposes of application of the regional pole placement approach to the decentralized controller design, the method described in [23] could be used. Recall the basic results for the robust controller design using the LMI region approach. An LMI region is any subset D of the complex plane that can be defined by its characteristic function

$$D = \left\{ z \in C_o : L + zM + z^T M^T < 0 \right\} \tag{29}$$

where L, M are real matrices such that $L = L^T$ is symmetric matrix and $M \in R^{d \times d}$.

$$v_1^T \begin{bmatrix} L \otimes P(\xi) + L \otimes P(\dot{\xi}) & M \otimes P(\xi) \\ M^T \otimes P(\xi) & 0 \end{bmatrix} v_1 \leq 0 \tag{30}$$

where

$$v_1^T = \left[(1_d \otimes x)^T \quad (1_d \otimes \dot{x})^T \right]$$

Inequality (30) represents the time derivative of the extended Lyapunov function for the general LMI region given by its characteristic function (29). For more detail see [23]. We wish that all poles of the closed-loop system should remain within a disk with the radius of $r = 1.6$, the center of the disk lying on the negative real axis of the complex plane, in the distance of $q = 1.8$ from the origin. Using the results of [23] leads to the PI-D controller parameters. The following controller gains will be obtained for the first and second equivalent subsystems on the basis of [23]:

$$R(1) = -36.6573 - \frac{9.9335}{s} - 7.5394s$$

$$R(2) = -35.227 - \frac{6.2104}{s} - 30.3107s$$

The following eigenvalues of closed-loop subsystems are obtained for the case of $i = 2$ $\{-3.0948; -0.6146 \pm 0.17784i\}$ for the first subsystem and $\{-1.42; -0.4085 \pm 0.193i\}$ for the second subsystem. The above example shows that eigenvalues either without controller or with it, satisfy the stability condition of LSS (23). The LSS is stable. Eigenvalues of the closed-loop complex system for both vertices are as follows:

First vertex $i = 1$

$$Eigclosedfirstvertex = \{-3.307; -0.471 \pm 1.3604i; -0.3401 \pm 0.2524i; -0.198\}$$

Second vertex $i = 2$

$$Eigclosedsecondvertex = \{-4.8775; -0.4324 \pm 1.3334i; -0.2587 \pm 0.2456i; -0.3012\}$$

Closed-loop LSS is stable with designed decentralized controller. All subsystems eigenvalues lie in the defined LMI region.

5. **Conclusion.** In the previous literature on decentralized control, complex systems are divided into two classes, i.e., with strong interaction links and with weak ones. The present work proves that division of systems into classes is more preferable to take place according to whether a complex system is stable or not. In the case of a stable complex system, the corresponding decentralized controller design is significantly easier than that in unstable case. The implementation of the proposal of the decentralized controller is performed in two steps. In the first step, the dynamic properties are calculated, of subsystems that provide stability and quality of the complex system. In the second step, at subsystem levels, decentralized regulators that meet the required dynamic properties are calculated. The examples show details of the method of calculation of a robust decentralized controller, as well as advantages of the present method. The conservative of the proposed method is significantly lower than the methods presented in the literature. The stability boundary of complex systems determines the calculated α coefficient at work. In the future, we will be devoted to control of the stability margin of the complex system by using dominant subsystem eigenvalues.

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