

## ADAPTIVE H-INFINITY VIBRATION CONTROL OF FLEXIBLE STRUCTURE USING FUZZY APPROXIMATOR

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**ABSTRACT.** *An adaptive fuzzy H-Infinity control scheme is presented for a flexible cantilever beam system using fuzzy approximator. At first, a novel adaptive H-Infinity controller is designed for flexible cantilever beam, while ensuring the control system to converge to equilibrium point in a short period of time. In the presence of unknown model uncertainties and external disturbances, an adaptive fuzzy H-Infinity controller is employed to compensate such system nonlinearities, which can achieve H-Infinity performance indicators and improve the tracking performance. Numerical simulations for a flexible cantilever beam are investigated to verify the effectiveness of the proposed adaptive fuzzy controller and demonstrate the satisfactory tracking performance and robustness.*

**Keywords:** Flexible beam, Adaptive control, H-Infinity control, Fuzzy control, Piezoceramic sensor and actuator

**1. Introduction.** During the past few years, vibration control using piezoceramic material has been widely studied and applied. The advantages of piezoceramic material include low-power consumption, rapid response, compactness, and easy implementation. Usually there are vibrations because flexible structures are lightly damped owing to small material damping and lack of air damping in space, having a negative influence on the control accuracy and leading to fatigue damage to components. Therefore, active vibration control for space flexible structures is an important concern.

Some advanced control algorithms such as feed back controller [11], sliding mode controller [2], finite volume formulation [3], and adaptive controller [4,9]. Adaptive boundary controller [5], PID controller [6], finite element model method [7], hybrid adaptive sliding mode/Lyapunov controller [8], point-to-point motion controller [10] have been widely investigated and applied to controlling the vibration of flexible structures. Halim and Moheimani [1] designed a feedback controller that suppresses vibration of flexible structures to minimize the spatial  $H_\infty$  norm of the closed-loop system. Wang et al. [2] presented an experimental study of an adaptive robust sliding-mode control scheme for active vibration control of a flexible beam using PZT sensor and actuator. Fallah and Ebrahimnejad [3] proposed a finite volume formulation for the free vibration analysis of the smart beams with piezoelectric sensors and actuators. Oveisi and Gudarzi [4] used two different adaptive control algorithms for the vibration control of geometrically nonlinear beam. Ge et al. [5] used the adaptive boundary control technique to suppress the vibration of a

Euler-Bernoulli beam system. Jovanovic et al. [6] presented an active vibration control of a smart cantilever composite beam using a PID controller. Zhang and Li [7] developed a finite element model of a piezoelectric actuator and cantilever to suppress vibration effectively. Azadi et al. [8] presented a hybrid adaptive sliding mode/Lyapunov controller for both the rotational maneuver and the vibration control of smart flexible. Koofgar and Amelian [9] presented a novel adaptive-based vibration control algorithm for smart structures. Kim et al. [10] developed a point-to-point motion control method for accurate positioning and vibration suppression of a vertical XY positioning system. Liu et al. [11] proposed an adaptive  $H_\infty$  sliding mode controller for uncertain Markovian jump systems with time-delay.

In nonlinear systems, there are often unknown nonlinearities. Recently, many researchers are working on neural networks and fuzzy system, which do not need the precise models and could approach the nonlinear smooth function with arbitrary precision [16,17]. Lee [12] presented active vibration control of a cantilever beam structural system by combining the adaptive input estimation method with the fuzzy robust controller. Xu and Chen [13] presented an LMI (linear matrix inequality)-based fuzzy approach of modeling and active vibration control of geometrically nonlinear flexible plates. Nasser et al. [14] presented a comprehensive methodology for the structural active vibration damping using a fuzzy logic control. Li et al. [15] presented an adaptive fuzzy sliding mode based active vibration control. Qiu et al. [16] designed the design and implementation of a fuzzy sliding mode control algorithm and a composite controller to dampen the vibration of a flexible manipulator.

Motivated by the above researchers, an adaptive fuzzy H-Infinity control approach is designed to realize the vibration control for cantilever beam. The fuzzy control and the  $H_\infty$  robust control are successfully integrated with cantilever beam system so that satisfactory tracking performance and strong robustness can be achieved. The advantages of the proposed controller are summarized as the following.

- 1) The fuzzy control method combined with the H-Infinity control for cantilever beam makes the algorithm obtain the self-learning ability and adjust the fuzzy parameters. Hence, the approach not only removes some of the fundamental limitations of the traditional approach but also provides improved tracking accuracy.
- 2) The proposed fuzzy H-Infinity control adds additional compensators for achieving and improving the system stability, hence obtaining desired system behavior and performance. Thus, the entire closed-loop system meets the expectations indicators of dynamic and static performance and achieves accurate tracking performance.

The paper is organized as follows. In Section 2, the dynamic equation of piezoceramic cantilever beam is established. In Section 3, an adaptive H-Infinity controller method is derived to guarantee the stability of the closed-loop system and good tracking performance. In Section 4, an adaptive fuzzy H-Infinity controller method is proposed to guarantee the stability of the closed-loop system and good tracking performance. Simulation examples are presented in Section 5 to illustrate the excellent performance of the proposed control schemes. Conclusions are given in Section 6.

**2. Dynamic Model of Piezoceramic Cantilever Beam.** As shown in Figure 1, the piezoceramic actuator and sensor are symmetrically pasted on the top and bottom of cantilever beam, the length, width and thickness of the piezoceramic element are respectively  $L_p$ ,  $b$  and  $t_p$ .  $X_1$  and  $X_2$  are the distances between both ends of the piezoceramic and fixed ends of cantilever beam respectively.

We assume that the piezoceramic patches are pasted well on the cantilever beam and the stickup layer of piezoceramic patches has no effect on the dynamic characteristics of

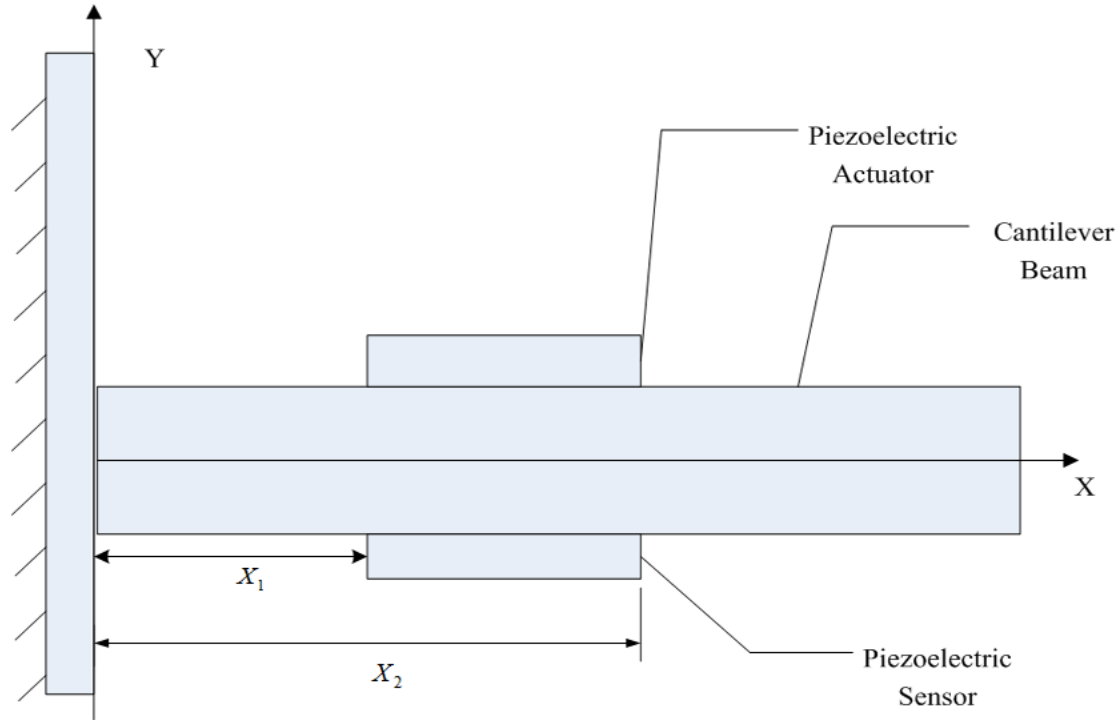


FIGURE 1. Model of piezoceramic cantilever beam

the cantilever beam. According to the direct piezoceramic effect, the output charge of piezoceramic sensor which is caused by the beam deflection is

$$Q(t) = \frac{1}{2}d_{31}t_bE_p \int_{x_1}^{x_2} \frac{\partial^2 w(x,t)}{\partial x^2} dx \quad (1)$$

where  $d_{31}$  is a piezoceramic constant,  $E_p$  is spring modulus of piezoceramic patches, and  $w$  is beam deflection.

Under the action of the input voltage, the piezoceramic actuator torque for the cantilever beam is

$$M(x,t) = K_a U_a(t) [h(x-x_1) - h(x-x_2)] \quad (2)$$

where  $h(x)$  is Heaviside step function,  $U_a(t)$  is input voltage, and  $K_a = \frac{1}{2}bd_{31}E_p(t_b + t_p)$  is coupling coefficient.

The dynamics of cantilever beam vibration is as follows

$$\rho_1 \frac{\partial^2 w(x,t)}{\partial t^2} + c \frac{\partial w(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 w(x,t)}{\partial t^2} \right) = \frac{\partial^2}{\partial x^2} M(x,t) \quad (3)$$

where  $\rho_1$ ,  $c$ ,  $EI$  are linear density, damping coefficient and flexural rigidity of the beam respectively.

When the cantilever beam does flexural vibration, by means of the preceding  $N$  modals of the cantilever beam, the deflection of the beam  $w$  can be expressed as

$$w(x,t) = \sum_{i=1}^n \phi_i(x) q_i(t) = \phi q \quad (4)$$

where  $\phi = [\phi_1(x), \phi_2(x), \dots, \phi_n(x)]$  is mass-normalized orthogonal modal matrix,  $q = [q_1(x), q_2(x), \dots, q_n(x)]$  is modal coordinates vector, and thus the displacement of the free

end of the cantilever beam is

$$y = w(L_b, t) = \sum_{i=1}^n \phi_i(L_b)q_i(t) = \phi_i q \tag{5}$$

The output voltage of piezoceramic sensors can be expressed as

$$U_s(t) = \frac{Q(t)}{C_p} = \frac{d_{31}t_b E_p}{2C_p} \sum_{i=1}^n [\phi'_i(x_2) - \phi'_i(x_1)] q_i(t) = \sum_{i=1}^n G_i q_i(t) = Gq \tag{6}$$

where  $C_p$  is capacitance of piezoceramic patch,  $G = [G_1, G_2, \dots, G_n]$ ,  $K_s = \frac{d_{31}t_b E_p}{2C_p}$ ,  $G_i = K_s [\phi'_i(x_2) - \phi'_i(x_1)]$ .

Using Equations (3) and (4), modal equation of motion of piezoceramic smart cantilever beam under the action of a piezoceramic actuator can be expressed as

$$\ddot{q}_i(t) + 2\xi_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = B_i U_a \tag{7}$$

where  $\xi_i$  is the  $i$ th order structure damping,  $\omega_i$  is the  $i$ th natural frequency, and  $B_i = K_a [\phi'_i(x_2) - \phi'_i(x_1)]$ .

**3. Design of Adaptive Robust H-Infinity Controller.** After the model of piezoceramic cantilever beam is analyzed, in this section, an adaptive H-Infinity controller method is derived for the piezoceramic cantilever beam to guarantee the stability of the closed-loop system and good tracking performance. The adaptive H-Infinity control approach is designed for the trajectory tracking of cantilever beam as shown in Figure 2. The control target is to achieve real-time compensation for the model uncertainties and external disturbance.

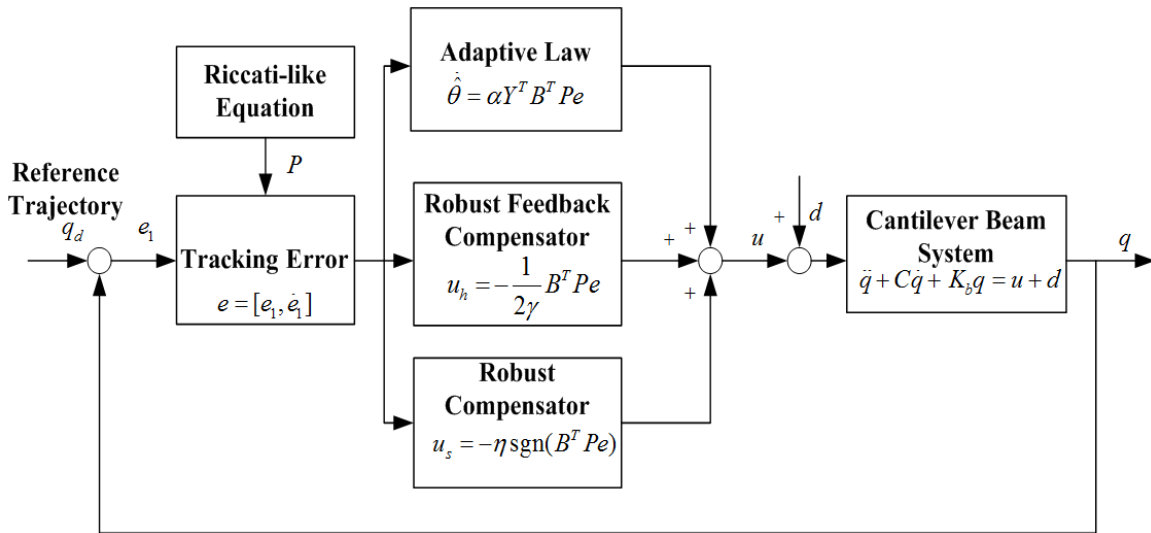


FIGURE 2. Block diagram of an adaptive  $H_\infty$  vibration control for a cantilever beam

The vector form of piezoceramic cantilever beam model can be written as

$$\ddot{q} + C\dot{q} + Kq = u + d \tag{8}$$

where  $C, K \in R^{i \times i}$ ,  $C$  is damping term,  $K$  is frequency term,  $u$  is input term,  $u = B_i U_a$ , and  $d$  is external disturbance.

Equation (8) can be rewritten as:

$$\ddot{q} = F(x) + u + d \tag{9}$$

where

$$F(x) = -C\dot{q} - K_bq \tag{10}$$

Define the tracking error as follows

$$e_1 = q - q_d \tag{11}$$

where  $q_d(t)$  is reference trajectory, and  $q(t)$  is actual trajectory.

Defining  $e = [e_1, \dot{e}_1]^T$ , the tracking error dynamic equation can be written as:

$$\dot{e} = A_0e + B[F(x) + u + d - \ddot{q}_d] \tag{12}$$

where  $A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ . Select a matrix  $K = [k_1 \ k_2]$ , which can make the matrix  $A = A_0 - BK = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$  as Hurwitz matrix. Then Equation (12) can be rewritten as

$$\dot{e} = Ae + B[Ke + F(x) + u + d - \ddot{q}_d] \tag{13}$$

Considering the uncertainty of the cantilever beam system, the adaptive  $H_\infty$  method is proposed to approximate  $F(x)$ , which can make tracking error of the cantilever beam system to the minimum.

**Theorem 3.1.** *If the cantilever beam system adopts adaptive control law*

$$u = -Y\hat{\theta} + \ddot{q}_d - Ke + u_h + u_s \tag{14}$$

with

$$u_h = -\frac{1}{2\gamma}B^TPe \tag{15}$$

$$u_s = -\eta \text{sgn}(B^TPe) \tag{16}$$

$$\dot{\hat{\theta}} = \alpha Y^T B^T Pe \tag{17}$$

where  $u_h$  is used for achieving tracking performance,  $u_s$  is used to reduce the effects of interference,  $\eta > |\Delta F|$ , and the symmetric matrix  $P = P^T > 0$  satisfies the following Riccati-like equation:

$$PA + A^T P + Q + PB \left( \frac{1}{\rho^2} I - \frac{1}{\gamma} I \right) B^T P = 0 \tag{18}$$

where attenuation factor  $\rho > 0$ , H-Infinity control gain  $\gamma > 0$ , weight matrix  $Q = Q^T > 0$ . The following conclusion can be drawn: if  $d \in L_2[0, \infty)$ , the cantilever beam system can implement the following H-Infinity tracking performance:

$$\frac{1}{2} \int_0^T e^T Q e dt \leq e^T(0) P e(0) + \frac{1}{2\alpha} \tilde{\theta}^T(0) \tilde{\theta}(0) + \frac{1}{2} \rho^2 \int_0^T \|d\|^2 dt \tag{19}$$

**Proof:** Substituting Equation (14) into Equation (13) yields

$$\dot{e} = Ae + B \left[ -Y \left( \hat{\theta} - \theta^* \right) + \Delta F + u_h + u_s + d \right] \tag{20}$$

Define the adaptive parameter estimation error as

$$\tilde{\theta} = \hat{\theta} - \theta^* \tag{21}$$

Then Equation (20) becomes

$$\dot{e} = Ae + B \left[ -Y \tilde{\theta} + \Delta F + u_h + u_s + d \right] \tag{22}$$

Define the Lyapunov function as

$$V = \frac{1}{2}e^T P e + \frac{1}{2\alpha}\tilde{\theta}^T \tilde{\theta} \tag{23}$$

The time derivative of  $V$  is

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} + \frac{1}{\alpha}\tilde{\theta}^T \dot{\tilde{\theta}} \\ &= \frac{1}{2}e^T (A^T P + PA) e - \tilde{\theta}^T Y^T B^T P e + u_h^T B^T P e + u_s^T B^T P e + \Delta F^T B^T P e \\ &\quad + d^T B^T P e + \frac{1}{\alpha}\tilde{\theta}^T \dot{\tilde{\theta}} \\ &= -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 d^T d - \frac{1}{2}\left(\frac{1}{\rho}B^T P e - \rho d\right)^T \left(\frac{1}{\rho}B^T P e - \rho d\right) - \tilde{\theta}^T Y^T B^T P e \\ &\quad + u_s^T B^T P e + \Delta F^T B^T P e + \frac{1}{\alpha}\tilde{\theta}^T \dot{\tilde{\theta}} \end{aligned} \tag{24}$$

Since

$$\begin{aligned} u_s^T B^T P e + \Delta F^T B^T P e &= -\eta \operatorname{sgn}(B^T P e) + \Delta F^T B^T P e \\ &\leq -\eta |B^T P e| + |\Delta F| |B^T P e| \\ &= -(\eta - |\Delta F|) |B^T P e| \leq 0 \end{aligned} \tag{25}$$

we can obtain

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 d^T d \tag{26}$$

Integrating Inequality (26) from  $t = 0$  to  $t = T$  generates

$$V(T) - V(0) \leq -\frac{1}{2}\int_0^T e^T Q e dt + \frac{1}{2}\rho^2 \int_0^T \|d\|^2 dt \tag{27}$$

Since  $V(T) \geq 0$ , we can obtain the H-Infinity tracking performance represented in Equation (19)

$$\begin{aligned} \frac{1}{2}\int_0^T e^T Q e dt &\leq V(0) + \frac{1}{2}\rho^2 \int_0^T \|d\|^2 dt \\ &= e^T(0) P e(0) + \frac{1}{2\alpha}\tilde{\theta}^T(0)\tilde{\theta}(0) + \frac{1}{2}\rho^2 \int_0^T \|d\|^2 dt \end{aligned} \tag{28}$$

Let  $d \in L_\infty[0, \infty)$ ,  $\|d\| \leq \varepsilon_d$  for some  $\varepsilon_d > 0$ . Inequality (27) can be rewritten as

$$\dot{V} \leq -\frac{1}{2}\lambda_q \|e\|^2 + \frac{1}{2}\rho^2 \varepsilon_d^2 \tag{29}$$

where  $\lambda_q$  denotes the minimum eigenvalue of  $Q$ .

For any small  $\delta > 0$ , if we choose  $\lambda_q > \frac{\rho^2 \varepsilon_d^2}{\delta^2}$ , we can obtain

$$\dot{V} \leq -\tau \|e\|^2 < 0 \tag{30}$$

where  $\tau > 0$ . This implies that  $\dot{V}$  is a negative definite, and all variables are bounded. From Barbalat Lemma,  $e$  asymptotically converges to zero.

Regardless of model uncertainties and external disturbances, the ideal control law can be designed as

$$u^* = C\dot{q} + K_b q + \ddot{q}_m + k^T e \tag{31}$$

where control parameter  $k = (k_2, k_1)^T$ , and trajectory tracking error  $e = [e_1, \dot{e}_1]^T$ .

Substituting Equation (29) into Equation (8), we can get the closed loop system as

$$\ddot{e}_1 + k_1 \dot{e}_1 + k_2 e_1 = 0 \tag{32}$$

It can be obviously found that the tracking error  $e_1$  will tend to zero sympathetically and the stability of the control system can be ensured if proportional gain  $k_1$  and derivative gain  $k_2$  are chosen in the favorable situation.

**4. Design of Adaptive Fuzzy H-Infinity Controller.** In this section, the control law (31) is difficult to implement in practical engineering. Then an adaptive fuzzy controller is derived to approximate the ideal control law (31). The block diagram of the adaptive fuzzy H-Infinity control is shown in Figure 3.

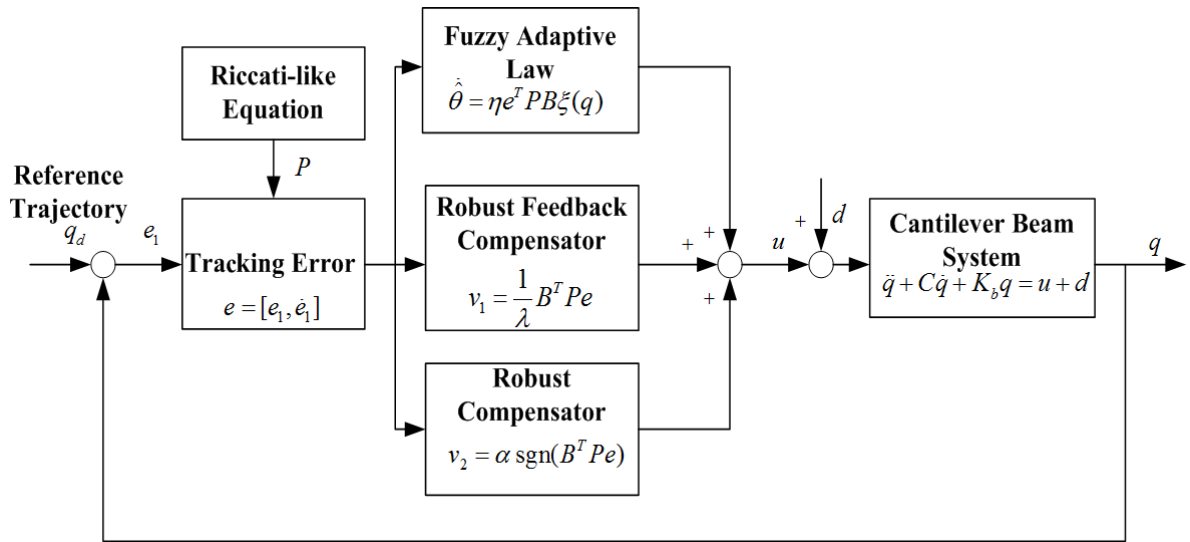


FIGURE 3. Block diagram of an adaptive fuzzy  $H_\infty$  vibration control for a cantilever beam

The fuzzy logic system which is used to approximate the ideal controller is represented as

$$u_c(X|\theta) = \sum_{i=1}^N \theta_i \xi_i(X) = \theta^T \xi(X) \tag{33}$$

where  $X$  is a fuzzy system input,  $u_c$  is a fuzzy system, and  $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_N]^T$  is the adaptive fuzzy parameter, the membership function  $\xi(X) = [\xi_1(x) \ \xi_2(x) \ \dots \ \xi_N(x)]^T$ ,  $\xi_i(x) = \frac{\prod_{j=1}^n \mu_j^i(x_j)}{\sum_{i=1}^N \prod_{j=1}^n \mu_j^i(x_j)}$ .

The adaptive fuzzy controller is designed as

$$u = u_c(X|\theta) + u_r \tag{34}$$

where  $u_c$  is the fuzzy controller and  $u_r$  is the robust controller. The fuzzy controller is designed to approximate the ideal control law (31) to relax the requirement of strict mathematical model. The robust controller is developed to achieve a specified H-Infinity robust tracking performance.

Substituting Equation (34) into Equation (8) yields

$$\ddot{q} = -C\dot{q} - K_b q + u_c(X|\theta) + u_r + d \tag{35}$$

Transforming Equation (31) yields

$$\ddot{q}_m = u^* - C\dot{q} - K_b q - k^T e \tag{36}$$

Equation (36) subtracting Equation (35) becomes

$$\ddot{e}_1 = u^* - k^T e - u_c(X|\theta) - u_r - d \tag{37}$$

Suppose that state vector is designed as  $X = e = (e_1, \dot{e}_1)^T$ , the state space-equation has form as

$$\dot{e} = Ae + B [u^* - u_c(e|\theta) - u_r - d] \tag{38}$$

where  $A = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

We define the optimal approximation constants  $\theta^*$ ,

$$\theta^* = \arg \min_{\theta \in \Omega} \left[ \sup_{x \in U_c} |u_c(e|\theta) - u^*(e)| \right] \tag{39}$$

where  $\Omega$  is the bound set of  $\theta$ .

Define the fuzzy minimum approximation error as

$$w = u^*(e) - u_c(e|\theta^*) \tag{40}$$

**Assumption 4.1.** *There exists a positive constant  $\alpha$  such that  $\|(w(e))_i\| \leq \alpha$ , where  $1 \leq i \leq 2$ .*

Using Equation (34) and Equation (40), Equation (38) can be rewritten as

$$\dot{e} = Ae - B\tilde{\theta}^T \xi(e) + Bw - Bu_r - Bd \tag{41}$$

where parameter estimation error  $\tilde{\theta} = \hat{\theta} - \theta^*$ .

**Theorem 4.1.** *Consider the nonlinear cantilever beam system defined in Equation (8). The control law is designed as follows:*

$$u = u_c(X|\theta) + u_r \tag{42}$$

where the fuzzy controller is given as  $u_c(X|\theta) = \theta^T \xi(X)$ , and the robust controller  $u_r = v_1 + v_2$  with

$$v_1 = \frac{1}{\lambda} B^T P e \tag{43}$$

$$v_2 = \alpha \text{sgn} (B^T P e) \tag{44}$$

$$\dot{\hat{\theta}} = \dot{\tilde{\theta}} = \eta e^T P B \xi(e) \tag{45}$$

where positive constant  $\eta > 0$  and  $\alpha > 0$ , the symmetric positive definite matrix  $P = P^T$  satisfies the following Riccati-like equation:

$$PA + A^T P + Q - \frac{2}{\lambda} P B B^T P + \frac{1}{\rho^2} P B B^T P = 0 \tag{46}$$

where the condition for the existence of solution is  $2\rho^2 \geq \lambda$ , attenuation factor  $\rho > 0$ , weight matrix  $Q = Q^T > 0$  and gain matrix  $\lambda > 0$ . Then, using the proposed control law, the variables of the closed-loop system (8) are bounded, the stability and the robustness of the closed-loop cantilever beam system can be obtained and the following H-Infinity tracking performance can be achieved:

$$\int_0^T e^T Q e \leq e^T(0) P e(0) + \frac{1}{\eta} \tilde{\theta}^T(0) \tilde{\theta}(0) + \rho^2 \int_0^T \|d\|^2 dt \tag{47}$$

where  $e(0)$  and  $\tilde{\theta}(0)$  are the initial values of  $e$  and  $\tilde{\theta}$  respectively,  $Q = Q^T > 0$ ,  $P = P^T > 0$ ,  $\rho$  and  $\eta$  are positive constant.

**Proof:** Choose a Lyapunov function candidate as follows

$$V = \frac{1}{2} e^T P e + \frac{1}{2\eta} \text{tr} \left( \tilde{\theta}^T \tilde{\theta} \right) \tag{48}$$

where  $\text{tr}(\bullet)$  is the trace operator, and  $\eta$  is the positive constant.

The time derivative of  $V$  is

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} + \frac{1}{\eta} \text{tr} \left( \tilde{\theta}^T \dot{\tilde{\theta}} \right) \tag{49}$$

Substituting Equation (41) into Equation (49) yields

$$\begin{aligned} \dot{V} = & \frac{1}{2} \left[ e^T A^T - \xi^T \tilde{\theta} B^T + \omega^T B^T - v_1^T B^T - v_2^T B^T - d^T B^T \right] P e \\ & + \frac{1}{2} e^T P \left[ A e - B \tilde{\theta}^T \xi + B \omega - B v_1 - B v_2 - B d \right] + \frac{1}{\eta} \text{tr} \left( \tilde{\theta}^T \dot{\tilde{\theta}} \right) \end{aligned} \tag{50}$$

Substituting Equation (43), Equation (44) into Equation (50) yields

$$\begin{aligned} \dot{V} = & \frac{1}{2} e^T \left( A^T P + P A - \frac{2}{\lambda} P B B^T P \right) e + e^T P B \omega - e^T P B d \\ & - \alpha e^T P B \text{sgn} (B^T P e) + \text{tr} \left[ \tilde{\theta}^T \left( \frac{1}{\eta} \dot{\tilde{\theta}} - \xi e^T P B \right) \right] \end{aligned} \tag{51}$$

Substituting Equation (45) into Equation (51) yields

$$\dot{V} = \frac{1}{2} e^T \left( A^T P + P A - \frac{2}{\lambda} P B B^T P \right) e + e^T P B \omega - e^T P B d - \alpha e^T P B \text{sgn} (B^T P e) \tag{52}$$

According to Riccati-like Equation (46) and Assumption 4.1, the derivative of  $V$  can be bounded as

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e - \frac{1}{2\rho^2} e^T P B B^T P e + e^T P B \omega - \alpha e^T P B \text{sgn} (B^T P e) - e^T P B d \\ \leq & -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d^T d - \alpha \sum_{i=1}^2 |(B^T P e)_i| + \sum_{i=1}^2 |\omega_i| \cdot |(B^T P e)_i| \\ \leq & -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 \|d\|^2 \end{aligned} \tag{53}$$

Integrating Inequality (53) from  $t = 0$  to  $t = T$  generates

$$V(T) - V(0) \leq -\frac{1}{2} \int_0^T e^T Q e dt + \frac{1}{2} \rho^2 \int_0^T \|d\|^2 dt \tag{54}$$

Since  $V(T) \geq 0$ , the  $H_\infty$  tracking performance is obtained in Equation (47)

$$\begin{aligned} \frac{1}{2} \int_0^T e^T Q e dt & \leq V(0) + \frac{1}{2} \rho^2 \int_0^T \|d\|^2 dt \\ & = \frac{1}{2} e^T(0) P e(0) + \frac{1}{2\eta} \tilde{\theta}^T(0) \tilde{\theta}(0) + \frac{1}{2} \rho^2 \int_0^T \|d\|^2 dt \end{aligned} \tag{55}$$

Let  $d \in L_\infty[0, \infty)$ ,  $\|d\| \leq \varepsilon_d$  for some  $\varepsilon_d > 0$ . From Inequality (51), we can get

$$\dot{V} \leq -\frac{1}{2} \lambda_q \|e\|^2 + \frac{1}{2} \rho^2 \varepsilon_d^2 \tag{56}$$

where  $\lambda_q$  denotes the minimum eigenvalue of  $Q$ .

For any small  $\delta > 0$ , if we choose  $\lambda_q > \frac{\rho^2 \varepsilon_d^2}{\delta^2}$ , we can obtain  $\dot{V} \leq -\tau \|e\|^2 < 0$ , where  $\tau > 0$ . This implies that  $\dot{V}$  is a negative definite, and all variables are bounded. From Barbalat Lemma, it can be proved that  $e$  can asymptotically converge to zero.

5. **Simulation.** According to the dynamics characteristics, the main vibration is determined by the structure's first several modes. To illustrate the simulation of this flexible structure vibration suppression, here we only choose the first mode system. In this section, we will evaluate the proposed adaptive  $H_\infty$  control scheme. The parameters of the cantilever beam are chosen as:  $C = 0.18$ ,  $K = 56.4$ . The parameter  $C$  of the sliding mode surface is not obtained by a quantitative calculation formula. Generally speaking, the larger  $C$  is selected, the faster the system will converge. Therefore, this paper selects  $C$  to be appropriately large. The gain coefficient  $K$  of the sliding mode switching term ensures the robustness of the system. The larger gain coefficient will cause larger chattering of the system, so it can only be selected appropriately. The desired trajectories are described by  $q = 0$ . Control parameters are selected as  $k_1 = 5$ ,  $k_2 = 16$ ,  $\rho = 2$ ,  $\gamma = 1$ ,  $\alpha = 15$ ,  $\eta = 1000$ ,  $Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ .

In order to testify the vibration control performance and robustness of the vibration control system in the presence of large disturbance, now white noise disturbance  $d = 100\text{randn}(1,1)$  is added all the time, and the control action is implemented after 10s. Figure 4 depicts cantilever beam tracking trajectories using the adaptive fuzzy H-Infinity control law under large steady disturbance. The actual motion trajectory of the cantilever beam is consistent with the desired reference trajectory in a short finite time. Figure 5 depicts position tracking comparisons of adaptive fuzzy H-Infinity control under steady disturbance with adaptive fuzzy H-Infinity control under large transient disturbance.

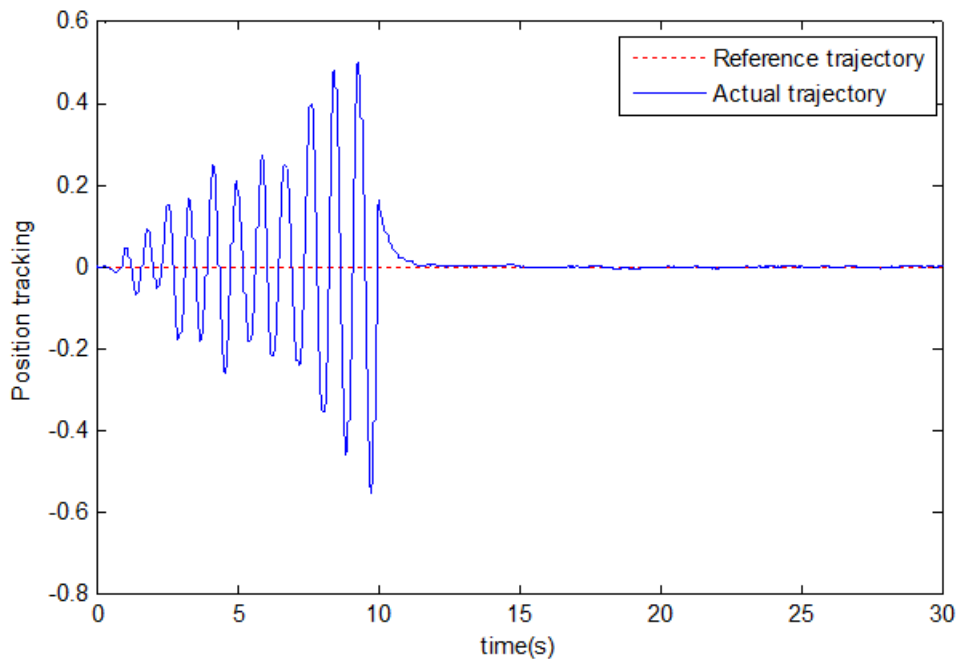


FIGURE 4. Position tracking under steady disturbance

The white noise disturbance  $d = 100\text{randn}(1,1)$  is added during 0-10 second, and the control action is implemented after 10s. Figure 6 draws the trajectories of cantilever beam using the adaptive fuzzy  $H_\infty$  control law under large transient disturbance. It can be observed intuitively that the actual motion trajectory of the cantilever beam is consistent with the desired reference trajectory in a short finite time, demonstrating that the adaptive fuzzy H-Infinity tracking performance is satisfactory as we expected. Figure 7 depicts position tracking comparisons of adaptive fuzzy H-Infinity control under steady

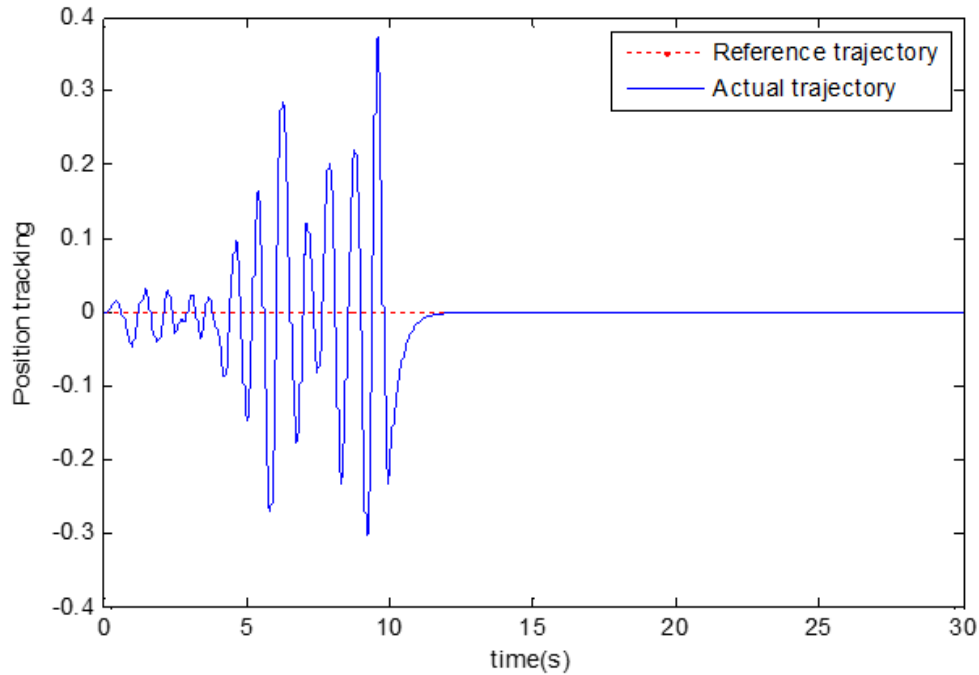


FIGURE 5. Position tracking under transient disturbance

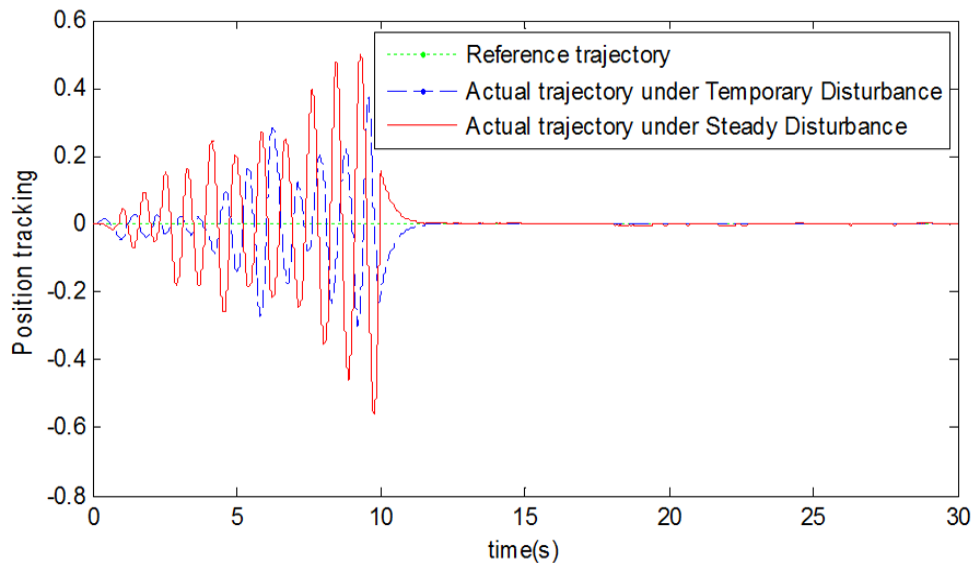


FIGURE 6. Position tracking comparisons of adaptive fuzzy H-Infinity control under steady disturbance with adaptive fuzzy H-Infinity control under transient disturbance

disturbance with adaptive fuzzy H-Infinity control under large transient disturbance. It can be observed that the trajectory under large transient disturbance is much smoother.

In order to demonstrate the advantages of the proposed controller, at first, the adaptive H-Infinity control is compared to PD control. From Figure 7 and Figure 8, the tracking position of adaptive H-Infinity control converges to zero in about 4 second and the trajectory is much smoother which improves the dynamic behavior of the cantilever beam and verifies that the adaptive H-Infinity control law can ensure the stability of the system. Then a comparable investigation is accomplished between the proposed adaptive fuzzy H-Infinity control and adaptive H-Infinity control. From Figure 9 and Figure 10, the

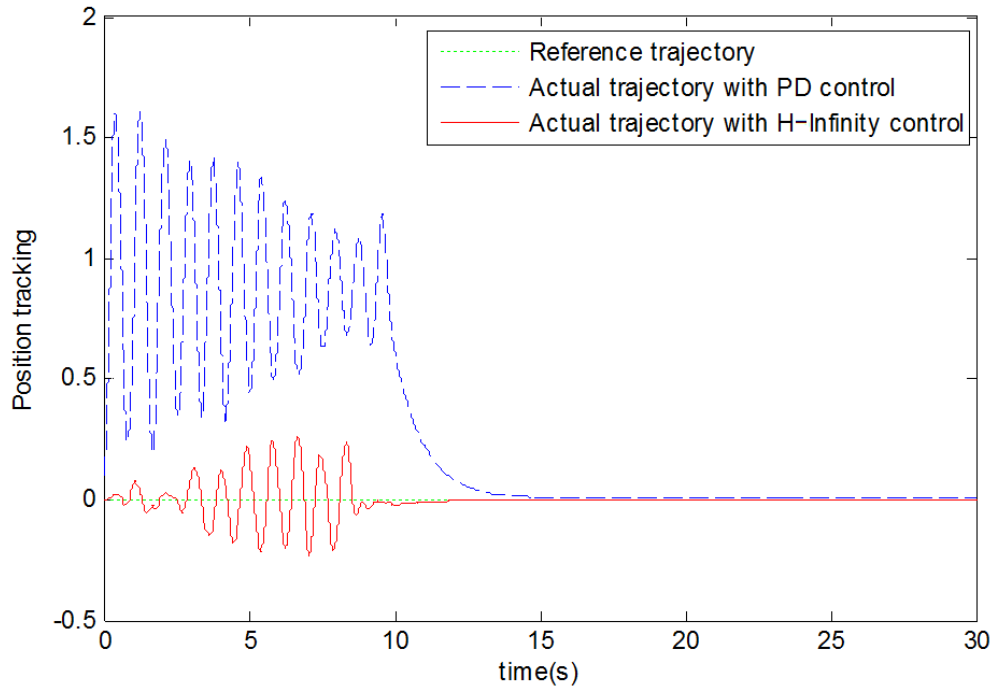


FIGURE 7. Position tracking comparisons of PD control with adaptive H-Infinity control under steady disturbance

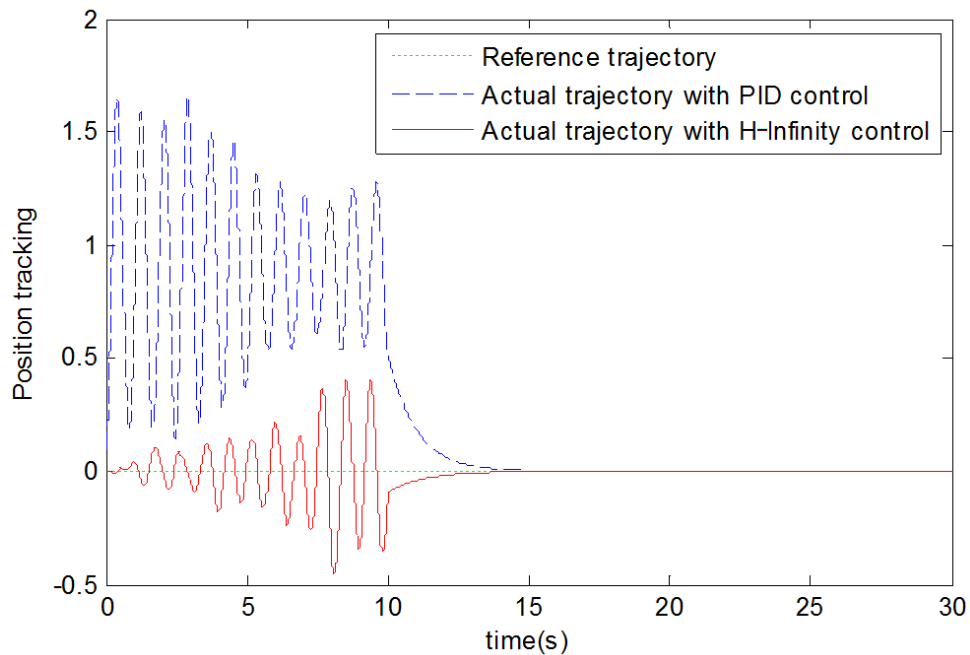


FIGURE 8. Position tracking comparisons of PD control with adaptive H-Infinity control under transient disturbance

tracking position of adaptive fuzzy H-Infinity control converges to zero in about 2 second and the trajectory is smoother.

**6. Conclusions.** An adaptive H-Infinity controller is proposed which makes the system can accurately track the reference trajectory and implement H-Infinity tracking performance. Then fuzzy control and robust control technology is combined, a novel adaptive

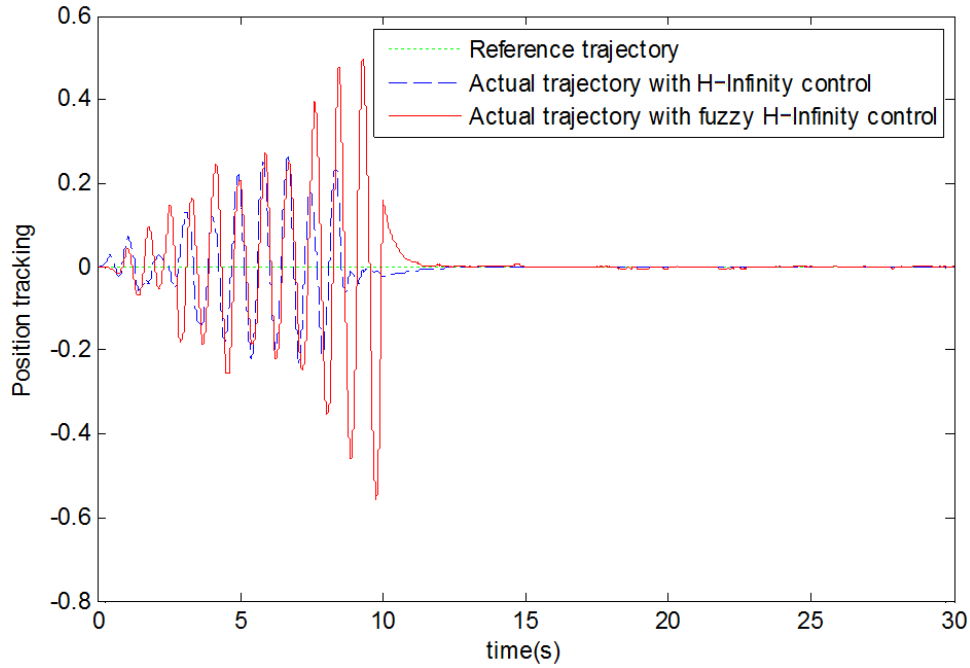


FIGURE 9. Position tracking comparisons of adaptive H-Infinity control with adaptive fuzzy H-Infinity control under steady disturbance

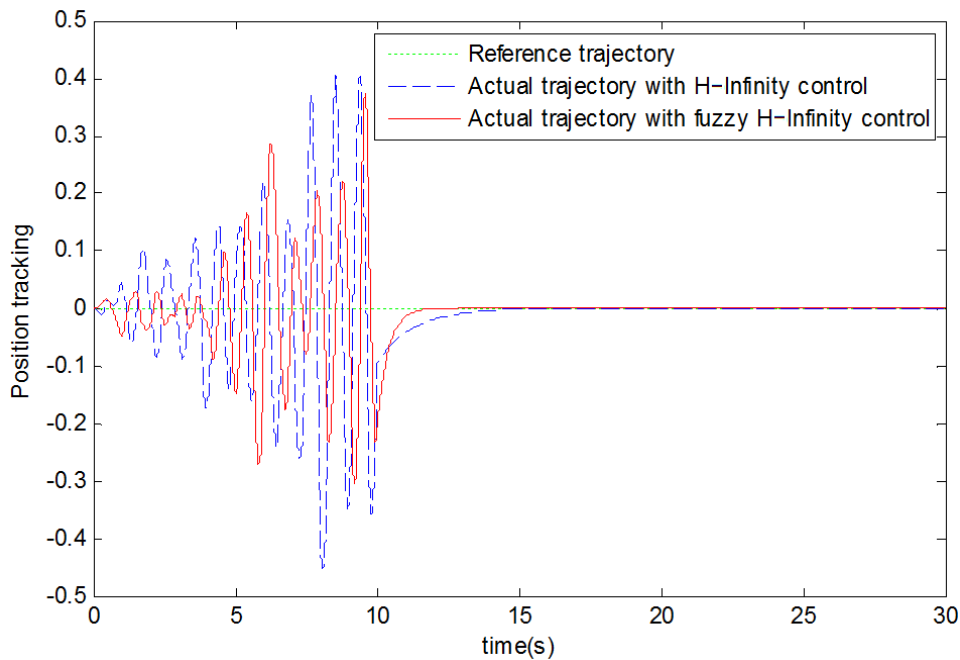


FIGURE 10. Position tracking comparisons of adaptive H-Infinity control with adaptive fuzzy H-Infinity control under transient disturbance

fuzzy H-Infinity controller is developed for vibration control of flexible cantilever beam. Fuzzy controller is used to approximate the ideal control input of beam system, an H-Infinity controller is used to improve the robustness of the system, so that the system can accurately track the reference trajectory and achieve H-Infinity performance indicators. Compared with the adaptive H-Infinity control scheme, the proposed controller has better dynamic performance and stronger robustness against modeling error and inevitable

disturbances. Some other intelligent controllers such as fuzzy neural network control and real-time experiment verification will be the future directions.

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