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MANAGING PORTFOLIOS WITH A FIRM-LEVEL COPULA BASED MARKET AND DEFAULT RISK INTEGRATION

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Abstract. *Corporate bonds carry market and default risk but each risk type is often analyzed separately. This study introduces an approach that synchronously incorporates market and default risk through consolidating simulated market return and default loss distributions at a firm-level. Subsequently, two sets of optimal asset-mixed portfolios are constructed. One contains corporate bonds with integrated risk while another has only market risk in concern. It is observed that the portfolios with the two risk types combined have lower holdings in corporate bonds, higher holdings in other risky assets and demonstrate a higher-risk-higher-return profile than the market-risk-only counterparts.*
Keywords: Market risk, Default risk, Risk integration, Copulas, Portfolio optimization

1. **Introduction.** Corporate bonds are one of the most important asset classes of multi-asset portfolios. Corporate debt securities make up one of the largest components of the US bond market[1], which is considered the largest securities market in the world. Like all other investments, corporate bonds carry risk. One key risk to a bondholder is that an issuer may fail to make timely payments of interest or principal. If that happens, the company will default on its bonds. This "default risk" makes the creditworthiness of the company – that is, its ability to repay debt obligations on time – an important concern to bondholders. Because of this defaultable feature, corporate bonds thus have higher-risk-higher-return profile than non-defaultable government bonds.

Investing in corporate bonds typically exposes investors to several risk factors: firstly, credit or default risk or the risk of failed timely payments; secondly, market risk or bond price fluctuation which is caused by interest rate rise or fall. Although it is known that holding corporate bonds contains primarily market and credit risk, the ways to represent all such risks are often done in separation [1]. Hence, risk of corporate bonds is measured by market metrics (e.g., volatility and value at risk) and credit metrics (e.g., credit rating) independently. However, none is available for measuring the total risk borne by corporate bond holders.

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[1]According to Securities Industry and Financial Market Association (SIFMA) in 2020, the total market value of US fixed income market is around USD 51 billion. The top-three outstanding values are US treasuries (USD 21 billion), Mortgage-backed securities (USD 11 billion) and Corporate bonds (USD 10.5 billion), respectively.

As a consequence, several decision making processes involving corporate bonds treat market and default risk of corporate bonds apart. For example, in a mean-variance portfolio optimization which is the most popular tool for portfolio construction, the typical approach often expresses risk in a form of covariance matrix but that is particularly for the market risk alone. In that case, a portfolio manager unintentionally overlooks the true risk of corporate bonds and the resulting optimal allocation is biased. Though credit risk of corporate bonds can be taken into account afterward by amending portfolio allocation, such adjustment usually neglects asset correlations and inevitably results in a sub-optimal portfolio. Accordingly, we aim to develop an approach to integrate market and credit risk in order to properly quantify the total risk of corporate bonds.

The objectives and hence the contribution of our study are twofold. Firstly, we aim to develop a novel method that is able to incorporate market risk and credit risk altogether while also contain information of returns and the combined risk in one package. Accordingly, the expected result is a joint distribution of market returns and credit losses (if there are any default events). Secondly, with the product from the first objective, we investigate impacts of integrated market and credit risk corporate bonds and market-risk-only corporate bonds when they are separately put into a portfolio optimization process. Corporate bonds with integrated risk should bear higher risk and thus have less allocation in a portfolio (to maintain the same level of risk). Risk and returns of the portfolios with different configurations are subsequently compared and analyzed.

The significance of market and default risk integration lies in the fact that corporate bonds can go default and hence the risk of such failure cannot be ignored, especially when a tradeoff between risk and return is relied on accuracy like when performing a portfolio optimization. Although the risk of default can be imposed pre- or post-optimization, information lost on market and default risk interaction is difficult to fully recompense. In consequence, we deliberately focus on capturing firm values and bond prices relationship to reflect the risk interaction in the best manner possible. The challenges we encounter are, therefore, on dealing with granularity of firm-level asset value simulations and a consequently large network of firm dependencies.

The rest of the paper is organized as follows. Section 2 reviews scholarly studies that are relevant to our work. Section 3 explains methodologies used in incorporating market and credit risk and assessing its impacts on portfolio allocation. Section 4 shows the study results with analyses and Section 5 summarizes the entire paper.

2. **Literature Review.** This section collects the published works of studies related to the integration of market and credit risk. The review of such literature summarizes the methodologies have been used and limitations known to date. Our proposed methodology establishes on the knowledge built up from these papers.

Suppose that a portfolio is composed of a defaultable asset like corporate bonds, a separate assessment of market and credit risk gives rise to a misestimation of the overall risk. The work of Jarrow and Turnbull [2] corroborates the aforementioned statement that market and credit risk are subtly related through firm's market value and repayment ability; thus, they are not easily separable. In order to structurally aggregate different types of risks, i.e., market and default risk, the "top-down" and the "bottom-up" approaches have been introduced.

2.1. **The top-down approaches.** The simplest establishment of credit and market aggregation is perhaps by the use of linear correlation[2] to account for the interactions between market and credit risk. The different risk types are then aggregated in a linear fashion with respect to implied credit-market (or expert judgment) based correlation coefficient.

The study of Fridson et al. [3] finds a positive relation between interest rates and default rates. Studies using this type of market-credit aggregation are, for example, Kuritzkes et al. [4] and Dimakos and Aas [5]. They connect market volatility to firm specific credit risk by integrating interest rate, credit spread, and foreign exchange rate risk into an overall risk of a portfolio. By using this approach, the portfolio risk measurement is improved with more accuracy.

More sophisticated methods use non-linear correlation functions in order to link marginal distributions of market and credit losses. The works of Rosenberg and Schuermann [6] and Liang et al. [7] employ copula-based approaches to model separately a joint loss distribution (between market and credit risk) and associated dependence structure.

The top-down approaches typically deal with an aggregation of market and credit loss distributions. The advantage is that they do not require much computation. Nonetheless, the downside is that it may somewhat ignore an interaction between different risk types at lower levels, e.g., individual asset level.

2.2. **The bottom-up approaches.** In bottom-up approaches, multiple risk types of individual assets are combined through a predefined model before the total risk is evaluated at a portfolio level.

2.2.1. *Integrating interest rate risk into a credit risk model.* Kijima and Muromachi [8] integrate interest rate risk into an intensity-based credit portfolio model. Then a Monte Carlo simulation is performed in order to observe the future value of the credit portfolio. Jobst et al. [9] extend this approach by introducing an independent rating migration model to express credit rating dynamics. Walder [10] introduces a slight deviation from the earlier works by assuming that interest rates and default intensities can be modeled as linear functions of state variables. In this way, an interest rate and credit risk are correlated because the short rate is part of the counterparty-specific default intensity. Grundke [11] correlates interest rate risk and a credit risk model underlying the internal ratings-based (IRB) approach proposed by the Basel Committee on Banking Supervision. A drawback of this approach is that there are only two credit states, default and no default. Moreover, credit spreads are assumed to be non-stochastic. Bo and Capponi [12] capture a stochastic process that drives default intensity and volatility of the stocks in a portfolio. They test the impact of risk integration on portfolio allocation and find that investors tend to reduce risky asset holdings during volatile markets.

2.2.2. *Integrating risk by combining market and credit loss distributions.* Barnhill and Maxwell [13] simulate a risk-free term structure, credit spreads, a foreign exchange rate, and equity market indices, which are all assumed to be correlated. Knowing the firm's credit rating at the specified time horizon, the simulated risk-adjusted term structure of interest rates can be used for discounting the future cash flows of the coupon bonds issued by that firm. Aas et al. [14] apply a copula model assuming some specific dependence structure for the aggregation of market, credit and operational risk. Zhu et al. [15] propose a copula-based hierarchical model to combine market, credit and operational risk

[2]Linear correlation is typically expressed by Pearson correlation coefficient or Pearson's r. Other variants of correlation coefficients that can capture non-linear relationship are, e.g., Spearman's ρ and Kendall's τ.

of Chinese commercial banks. The results show that portfolios created from combined risk exhibit better diversification benefits.

Based on the literature presented above, the top-down approaches are simple to implement but the relationship structure requires an assumption that all the variables are strictly linearly correlated. Besides, it is suitable for the risk integration at an aggregate (portfolio) level but not at a firm level. The bottom-up approaches try to rectify the latter limitation by formulating a model that links up market and credit risk factors at a firm level or individual asset level instead. However, this comes with an expense of a rigid model for a tractable problem which is, sometimes, inflexible to imitate real-world situations.

The gaps for improvement that we see in the bottom-up approaches are 1) a parametric model that connects market and credit losses may not able to fully express an entire interaction between market and credit risk factors – an empirical approach such as a simulation-based one has a capacity to emulate complex relationship more accurately yet requires fewer parameters; 2) market returns and default events are not necessarily related in a linear fashion – a tool that can express nonlinear dependencies for several objects like multivariate copulas can resolve such limitation with moderate computational intensity.

3. **Methodology.** Our proposed methodology is inspired by the work of Aas et al. [14] and we aim to bridge the gaps of the bottom-up approaches. Aas et al. model market, credit and operational loss distributions separately and combine them later with copulas. However, the asynchronous manner of loss distribution simulation and risk integration does not ensure that credit events will occur in a proper situation. For example, there may be an event that a firm goes default although its bond price is rising. By contrast, we resort to multivariate copula simulation to simulate credit loss and market loss distributions simultaneously. This ensures that default losses will take place in an appropriate condition, i.e., when a firm goes bankrupt, its bond price should be falling dramatically and, at the same time, the firm cannot satisfy a scheduled repayment. An investor of the firm's corporate bond thus suffers from bond price devaluation and diminished principal after a recovery process is completed.

In what follows, we elaborate the approaches and steps conducted to aggregate risk from market (price fluctuation) and credit (losses from default events) for the total risk. Later, we use the integrated risk results to study how portfolio allocation is affected when default loss is taken into consideration.

3.1. **Data.** The key data are monthly returns of the three asset classes, i.e., US government bond index, US corporate bond index and S&P500 index. Data range is from February 2003-March 2020. In addition, these data are balance sheet time series of the firms listed in S&P500 index at corresponding time periods. The corporate bond index is customized so that the constituents are the same as those in S&P500 index along the time. All data are retrieved from Bloomberg.

3.2. **Calculating firm-level default loss parameters.** We employ Merton's model to determine probability of default (PD) and loss given default (LGD) firm-by-firm by simulating the following stochastic process:

$$A_t = A_0 \cdot e^{\left(\mu_A - \frac{1}{2}\sigma_A^2\right)t + \sigma_A z_t}, \tag{1}$$

for constants $\mu_A \in \mathbb{R}$ and $\sigma_A > 0$, with a Brownian motion $(z_t)_{t \geq 0}$, where A_0 and A_t are respectively firm's asset values at the beginning and at time t. However, since asset values are balance sheet data which normally disclosed quarterly and are not usually dated back much. In our data source, Bloomberg, only a few firms have historical balance sheet data longer than 10 years. For firms with limited number of observations, we adopt an alternative introduced in Afik et al. [16] that the expected asset return can be proxied by the maximum between the historical equity return of the preceding year and the risk-free rate, i.e.,

$$\mu_A = \max\left(r_f, r_{E-1}\right), \tag{2}$$

where μ_A is an asset return, r_f is a risk-free rate and r_{E-1} is an equity return over the most recent year. For firms with abundant observations of asset values, we take an average of historical monthly asset growths as a representative of asset returns.

For asset return volatility (σ_A), we calculate the standard deviation on historical asset returns for firms with balance sheet data longer than 10 years. Otherwise, for firms with limited number of observations, the volatility is proxied by volatility of monthly returns in the most recent year.

To simulate a firm default, we also need to know the debt level a firm is obliged to pay in the future, for example, one year. Similar to asset values, the obligation can only be observed from balance sheet, so a projection of future obligation is required. Bharath and Shumway [17] propose an approximation of future obligation, or a default barrier, in 1-year ahead as:

$$D = STD + 0.5 \cdot LTD, \tag{3}$$

where D is an obligation 1-year ahead, STD is the short-term debt and LTD is the long-term debt. With the proxies of asset return and future obligation in place, PD and LGD of each firm can be estimated independently using the diffusion process (1).

Figure 1 portrays a simulation example of asset values A_t over a 12-month period after March 2020 of a selected firm. Each firm takes a simulation of 50,000 possible paths of asset values where the paths ending below the obligation line makes up default events. The frequency of asset value falling under the obligation (dotted line) over the total number

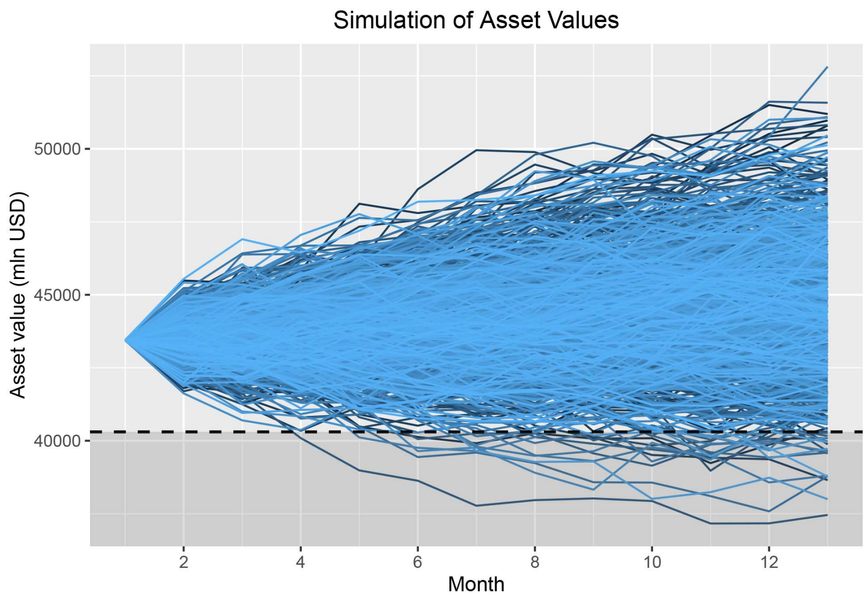


FIGURE 1. A simulation example of asset values of a selected firm. The figure shows a total of 50,000 possible paths over a horizon of 12-month (the time begins at month 1). The dotted line shows the level of obligation due in the next 12 months. The paths ending below the obligation (at month 13) is counted as default events. The probability of default is therefore the proportion of such paths over all possible paths.

of simulations thus indicates PD while the shortfall values denote LGDs of the respective firm in each scenario.

3.3. **Simulating credit losses for all firms simultaneously.** Previous subsection demonstrates a guideline for simulating PD and LGD firm-by-firm. In order to simulate PD and LGD of all firms at the same time, linkages between the firms must be formulated. We employ multivariate Gaussian copula as a tool to model dependency among corporate bond issuers through their stock returns relationship.

Firm-level PDs and LGDs extracted from Subsection 3.2 are used to simulate losses incurring due to credit events. Such default incidents are more likely to occur when firm's debt securities are in distress. In a situation where firm credit condition is sound, there appears only gain or loss from bond prices while credit loss is absent. A simulation of firm stock returns, corporate bond returns and default losses is hence done simultaneously to synchronize interactions among market and credit risk factors across firms.

A Gaussian copula probability density function (PDF) can be expressed as:

$$c(u_1,\ldots,u_n)=\frac{1}{\sqrt{\det(R)}}\exp\left(-\frac{1}{2}\begin{pmatrix}\Phi^{-1}(u_1)\\ \vdots\\ \Phi^{-1}(u_d)\end{pmatrix}^{\mathrm{T}}\cdot\left(R^{-1}-\mathbf{I}\right)\cdot\begin{pmatrix}\Phi^{-1}(u_1)\\ \vdots\\ \Phi^{-1}(u_d)\end{pmatrix}\right) \tag{4}$$

where $\mathbf{I}$ is an identity matrix, $u_i \in [0,1]$ are cumulative probabilities from an asset return distribution of firm i and $\det(R)$ is a determinant of the correlation matrix R. When performing a simulation with a multivariate Gaussian copula [18], a cumulative distribution function (CDF) and its inverse can be derived from Equation (4).

In terms of technical contribution, the use of copula helps capture nonlinear relationships of multiple firm asset returns. A copula PDF portrays asset return relationship as a probability function; hence, there are unequal chances that asset returns could move in the same direction, go away in opposite directions or be independent. Such dependencies are different from firm to firm and cannot be described straightforwardly with typical correlation coefficients. As a result, the use of copula provides more accurate result on joint default events and hence an enhancement of default loss estimation.

Figure 2 exhibits return distributions of the three asset classes in our study on diagonal panels. The upper triangular panels indicate correlations between asset classes and the lower triangular panels display their joint densities. The US Equity return distribution is a plot of S&P500 index returns and the US Bond 1-10 return distribution displays returns of US treasury with maturities between 1-10 years. The return distribution of US corporate bonds, in contrast, is a result of a multivariate copula simulation to incorporate bond returns with default losses. The subsequent return distribution therefore encapsulates both market and default risk into one package.

The US corporate bond return distribution constitutes integrated gains and losses of firms in S&P500 index intertwined with their copula dependencies. At a firm-level, PD and LGD of each company are thus affected by its own financial condition and comovement with others. Figure 3 illustrates simulated LGD distributions of two randomly selected companies in the pool. From a simulation, a shortfall of asset value from the obligation in each scenario is regarded as LGD. Therefore, when the simulation is run over multiple scenarios, those LGDs build up the distributions.

3.4. **Mean-CVaR portfolio optimization.** We set up a portfolio optimization problem on two sets of asset universes to compare the cases of market-risk-only (MKT) corporate bonds and integrated risk (INT) corporate bonds. Expected returns are taken from the historical averages and risk is measured by conditional value at risk (CVaR).

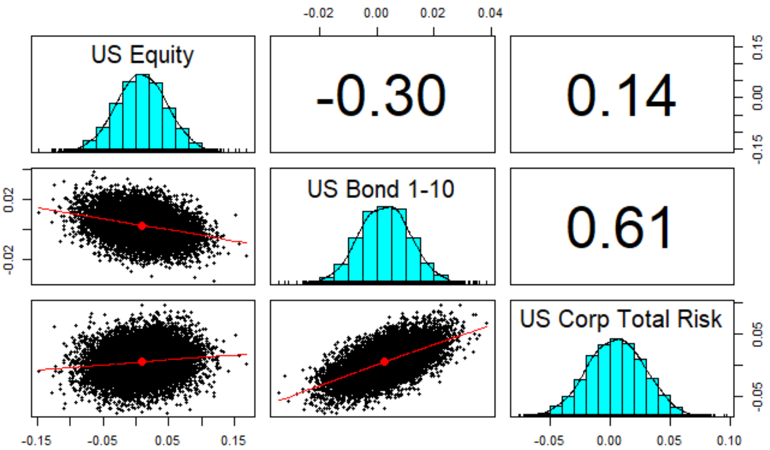


FIGURE 2. Return distributions of three asset classes. Simulated asset returns using multivariate Gaussian copula as dependency structure. "US Corp Total Risk" represents solely a distribution of the US corporate bond index returns where, in some extreme occasions, losses from default incur along with losses from falling bond prices. The two components of market and default losses constitute the total losses of corporate bonds depicted in "US Corp Total Risk".

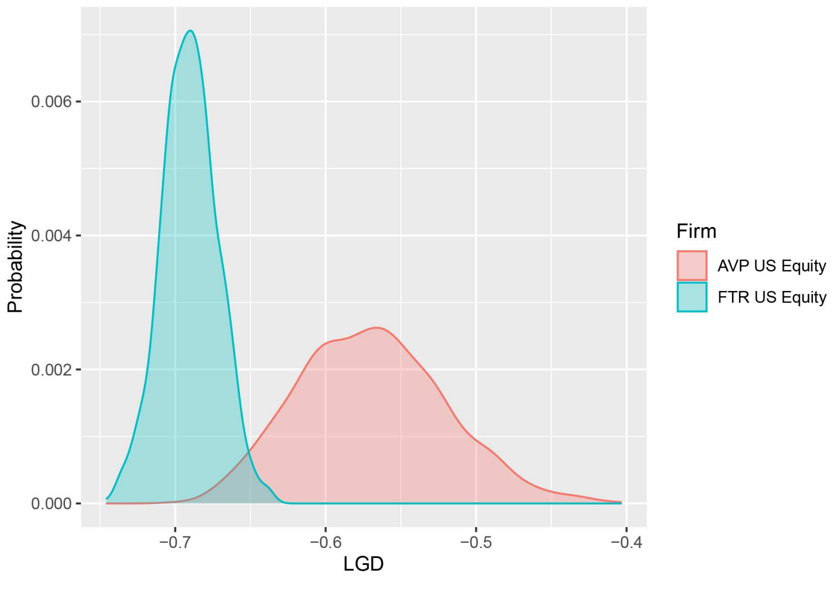


FIGURE 3. (color online) Distributions of LGDs of selected firms. From a simulation, the shortfall of asset value from the obligation in each scenario is regarded as LGD. Therefore, when the simulation is run over multiple scenarios, those LGDs build up the distributions.

Suppose there are a total of N securities with an initial price vector, $\mathbf{p}_0$, at time $t = 0$. Let $\mathbf{p}_1$ denote a random security price vector at date $t = 1$, f denote a probability density function (PDF) of $\mathbf{p}_1$ and $\mathbf{w}$ denote a vector of portfolio holdings that are chosen at date $t = 0$. The loss function at time $t = 1$ is then given by

$$l(\mathbf{w}, \mathbf{p}_1) = \mathbf{w}^{\mathrm{T}}(\mathbf{p}_0/\mathbf{p}_1 - 1).$$

Note that the loss function differs from a typical portfolio return calculation function in the sense that it interprets positive values as losses and negative values as gains. We set up a mean-CVaR portfolio optimization with the same formulation as Rockafellar and Uryasev [19] as follows:

$$\begin{aligned} \min_{\alpha,\mathbf{w},\mathbf{z}} \ & \alpha + \frac{1}{J(1-\beta)} \sum_{j=1}^{J} z_j \\ \text{subject to } & l\left(\mathbf{w}, \mathbf{p}_1^{(j)}\right) - \alpha \leq z_j, \\ & z_j \geq 0, \\ & \mathbf{w} \in \mathcal{W}, \end{aligned} \tag{5}$$

where β is a confidence level of VaR (typically set as 0.95), J is the set of contiguous intervals (bins) of discretized return distributions, z_j indicate losses beyond the VaR level (α) in which the average of them signifies CVaR and $\mathcal{W}$ is a set of portfolio constraints.

What makes the optimization problem (5) different from the original setting in [19] is that z_j are obtained from a copula-simulated distribution rather than a standard-normal-simulated one. So, z_j incorporate both market and default losses which enhance the quality of optimization input. In consequence, CVaR estimation and risk return trade-off yield more accurate optimization results and hence more informative messages for decision making.

4. Results and Discussions.

This subsection takes the resulting integrated risk of corporate bonds into further studies to better see an impact of default risk inclusion. Since incorporating default losses into total risk of corporate bonds means higher risk for them, the impact of increasing risk profile can be seen more clearly through a tradeoff between risk and return when executing asset allocation.

4.1. Effects of market and credit risk integration on risk-return profile of corporate bonds.

The first objective of our experiments is to exhibit impacts on risk-return profile of corporate bonds after incorporating default risk and market risk into the aggregate risk. Then we extend the findings to a portfolio level by studying portfolio allocations and risk-return characteristics of two sets of portfolios. The first set takes only market risk of corporate bonds into an optimization while the second one includes both market risk and credit risk of corporate bonds.

To observe the difference of corporate bond returns with integrated risk and market risk alone, Figure 4 is plotted to show return distributions of the two cases. The resulting distribution of integrated risk corporate bond returns is straightforward, when market returns and default losses are merged into a single total loss distribution, the ensuing curve shifts leftward of the market-risk-only return distribution of corporate bonds. This reflects the fact that in times of default, apart from losses of falling bond prices, there are also losses from partially redeemed principal of the bond purchased.

The statistics in Table 1 reveal that including default losses in addition to bond price returns, the resulting total return (integrated risk) distribution bears lower return. The integrated risk shows lower mean and lower minimum return due to larger losses incurred.

It is noted that the results shown in Table 1 are generated from underlying Gaussian distribution, i.e., Gaussian marginals and Gaussian copulas. This restricts that return distributions are symmetric. In addition, it is interesting that the maximum return of the integrated-risk return distribution is lower. This reflects the fact that in the scenario that one firm achieves the highest return, when a default event is included, the downfall impairs overall return of a corporate bond basket.

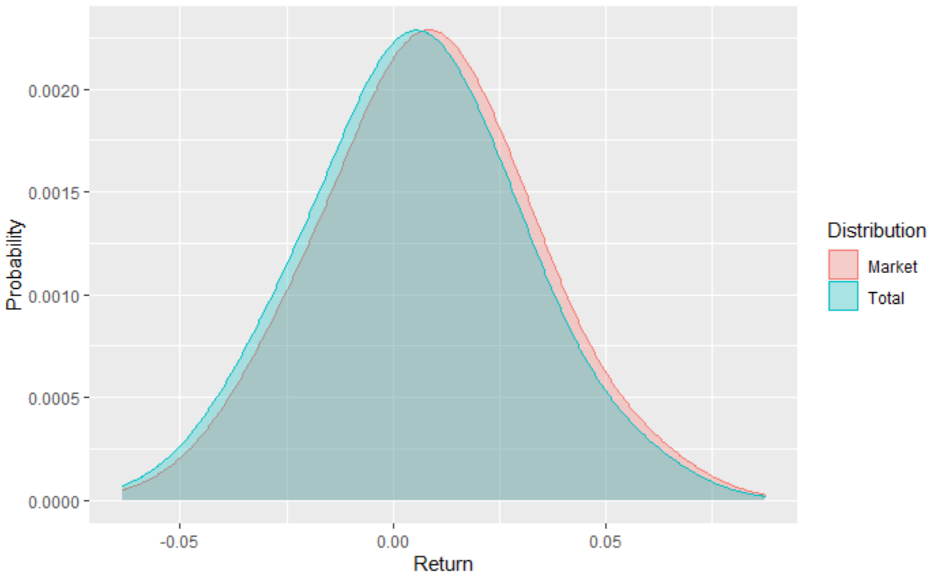


FIGURE 4. (color online) Market risk and integrated market and credit risk of corporate bonds depicted by return distribution. Return distributions of corporate bond returns when market risk is only considered (pink) versus when the two risk types are taken into account (blue). It is observed that when the risk from market and default is appraised in unison, the distribution of the total risk shifts to the left of the market-risk-only distribution. This reveals that the true risk is underestimated if market risk is only considered when investing in corporate bonds.

TABLE 1. Summary statistics of corporate bond monthly returns when default risk is and is not integrated

Statistics	Market risk only	Integrated risk
mean	0.78%	0.49%
standard deviation	2.46%	2.46%
min	−7.38%	−7.67%
max	9.93%	9.64%
5^{th}-percentile	−3.27%	−3.56%

4.2. **Effects of market and credit risk integration on portfolio allocations.** We solve the optimization problem (5) separately for the cases of MKT and INT. The expected returns and covariance matrices used in the optimization problem are extracted from simulated data prepared in Subsection 3.3. They are summarized and exhibited in Table 2.

Subsequently, an efficient frontier for each case is rendered by varying 15 different levels of target returns. Figure 5 compares the two frontiers of MKT and INT cases. MKT corporate bonds possess more efficient profile than INT counterpart – having lower risk and higher return. It can be observed that such difference in risk-return profiles of MKT and INT corporate bonds is the key that influences the shapes of efficient frontiers. Comparing between the two frontiers, the MKT case is more efficient than INT by having lower risk (CVaR) in every level of target returns.

It is unsurprising that the INT case demonstrates higher risk than that of the MKT case. However, the reason is not solely because INT portfolios hold higher-risk corporate

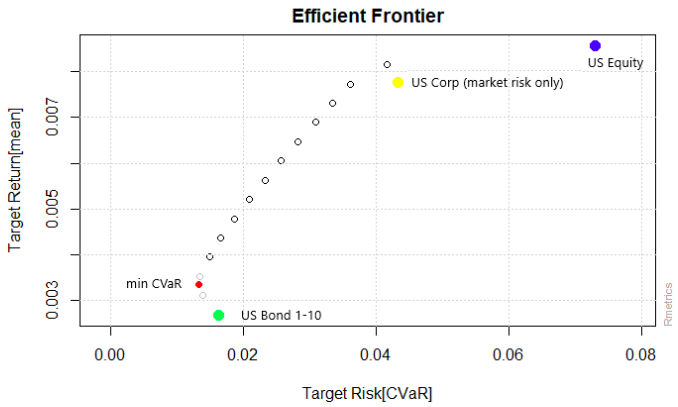
TABLE 2. Expected returns and covariance matrices for optimization problems

	US bond 1-10	US equity	US corp (MKT)
expected return	0.27%	0.86%	0.78%

covariance matrix	US bond 1-10	US equity	US corp (MKT)
US bond 1-10	0.152%	−0.011%	0.013%
US equity	−0.011%	0.008%	0.014%
US corp (MKT)	0.013%	0.014%	0.061%

	US bond 1-10	US equity	US corp (INT)
expected return	0.27%	0.86%	0.49%

covariance matrix	US bond 1-10	US equity	US corp (INT)
US bond 1-10	0.152%	−0.011%	0.013%
US equity	−0.011%	0.008%	0.014%
US corp (INT)	0.013%	0.014%	0.061%



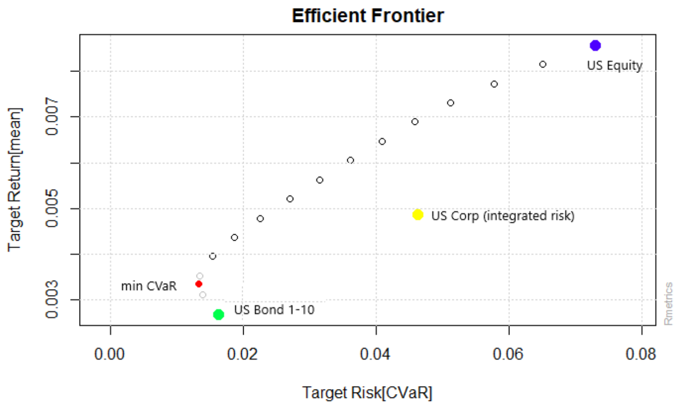


FIGURE 5. Efficient frontier comparison. The top figure (MKT) optimizes bonds (US Bond 1-10) and equities (US Equity) with corporate bonds with market risk alone (US Corp market risk only) whereas the bottom figure (INT) includes default risk and market risk of corporate bonds (US Corp integrated risk). When default risk is incorporated, corporate bonds show lower return and higher tail losses, resulting in an inferior frontier to its market-risk-only counterpart.

bonds. Rather, it is because the INT portfolios refrain from less-attractive corporate debts and hold more equities. From the lower panel of Figure 5, US corporate bonds with integrated risk significantly show inferior return per risk to that of the MKT case (upper panel). So, when MKT portfolios move along the frontier, they acquire more and more equities and expose to higher CVaR inevitably.

An evolution of asset weights when INT and MKT portfolios move along frontier is shown in Figure 6. It is clear that, for the MKT case (upper panel), MKT portfolios acquire higher returns by accumulating more corporate bonds and equities. In contrast, INT portfolios stop accumulating corporate bonds quite early and opt for holding more equities to earn higher returns. The MKT case around 50% equity at almost the end of its frontier while the INT case experiences 50% equity at the middle of the frontier.

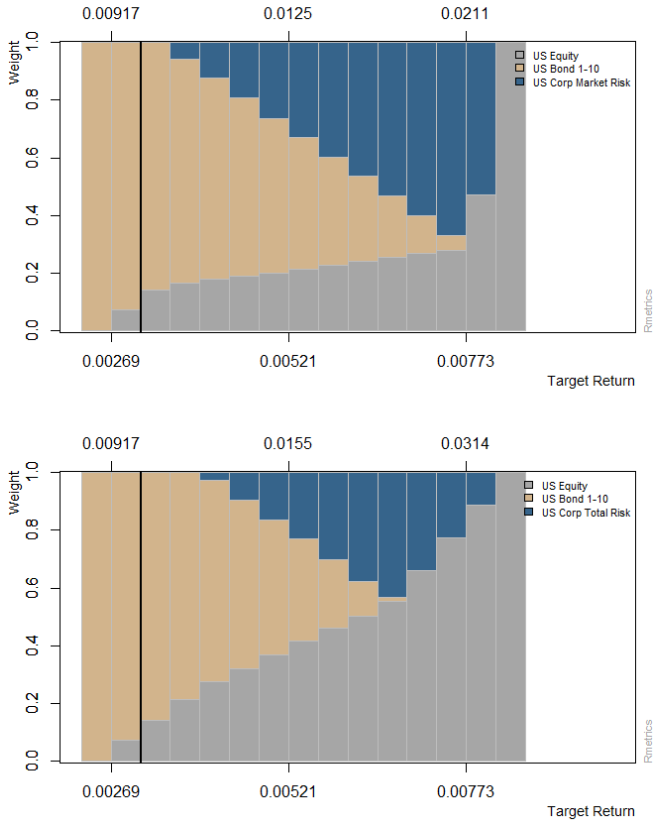


FIGURE 6. (color online) Evolution of asset allocations of portfolios optimized under different risk configurations of corporate bonds. The top horizontal axis displays portfolio volatility, the bottom horizontal axis shows portfolio target returns and the vertical axis presents levels of asset allocation. It is obvious that when the total risk of corporate bonds is taken into account, the portfolios try to evade corporate bond investment (shorten blue bars) and seek for equities to attain the target returns.

The consequence follows that the risk contribution (measured in terms of a proportion of covariance of each asset to the total variance of a portfolio) of the two cases shows a remarkable difference. Figure 7 exhibits the distinction of variance contribution of the two cases. For the MKT case, variance of low-return portfolios comes from government bonds then risk of the portfolios rises with increasing target returns. The large part of portfolio risk is primarily from corporate bonds and equities. Only at the very end of the

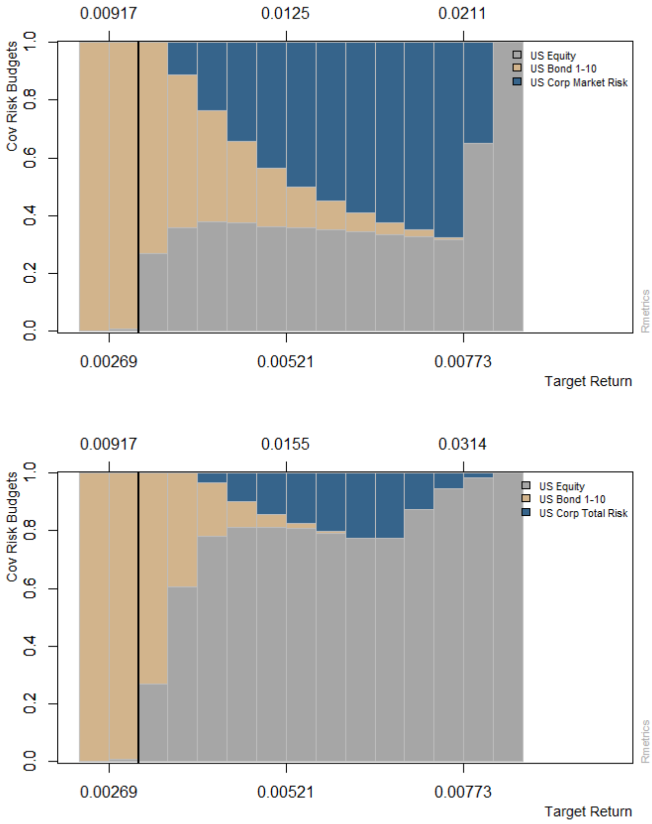


FIGURE 7. (color online) Risk contribution (from variance and covariance of each asset) of each asset in the portfolios. The top horizontal axis shows portfolio volatility, the bottom horizontal axis exhibits portfolio target returns and the vertical axis presents levels of risk contribution. As the target returns increase, the portfolios optimized with total-risk corporate bonds (the bottom figure) have fewer shares of corporate bonds and more shares of equities. This accordingly results in a larger part of risk contribution coming from equities (grey bars).

frontier that equities take all the risks. On the contrary, the INT case reveals a sharp increase of variance stemming from equities and takes a major role of risk contribution of portfolios even at the low-risk targets. For the most parts of the INT frontier, risk from equities stands at 80% or above while corporate bonds never make it over 20%.

To summarize, when default risk is taken into consideration, the resulting corporate bonds bear lower return and higher risk. Hence, portfolio optimization avoids allocating capital to corporate bonds and embraces more equities to sustain return targets. This consequently results in riskier portfolio profile for the INT case.

5. **Summary.** Our study aims to underline that risk of corporate bond investment comes from both bond prices fluctuation and losses of capital if the bond issuer goes default. In other words, corporate bonds contain both market risk and credit risk.

The potential channel to incorporate market and credit risk is by adding up losses from default events on bond returns. That is, in the event of default, holding corporate bonds incur losses from a price drop initially and, when the recovery process is gone through, the bond holders could suffer further from a debt write-off. We do this formally by simulating credit events of firms and their respective bond price movements simultaneously to create synchronous default losses and bond returns. The approach is novel and is one of the main contributions of our study.

Merging up the two distributions of default losses and market returns yields a total loss distribution of corporate bonds. As expected, the resulting integrated risk corporate bond return distribution shows inferior return and risk statistics when compared with the market-risk-only version. Two sets of portfolios are subsequently optimized to study the resulting risk-return characteristics and allocation patterns in order to observe an effect of default risk inclusion. One set takes only market risk (MKT) of corporate bonds into consideration while another integrates their market and credit risk (INT).

The optimization results show that integrating market and credit risk for corporate bonds provides riskier portfolios. This, however, is not primarily because the integrated risk corporate bonds having poorer risk-return profile, but rather the portfolios prefer equities over corporate debts. The resulting allocations thus contain more equities than the case of market-risk-only corporate bonds.

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