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IDENTIFICATION OF MODEL FREE NONLINEAR SYSTEM VIA PARAMETRIC DYNAMIC NEURAL NETWORKS WITH IMPROVED LEARNING

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Abstract. *In this paper, a parametric dynamic neural networks based identification algorithm with the improved learning law is developed for model free nonlinear system. Unlike the commonly used neural network based identification method, a parameter error driven updating law is synthesized to ensure improved performance in terms of steady-state error and robust convergence. The stability of the proposed identification method with and without external disturbance is proved using the Lyapunov approach. Simulation results show that the proposed identification method can yield accurate online identification of the model free nonlinear system, even when encountering noise disturbance compared with the commonly used neural network based identification method.*

Keywords: Parametric dynamic neural networks, Identification, Nonlinearity, Online learning

1. **Introduction.** The identification of nonlinear dynamic systems with unknown models is often a prerequisite for successful analysis and control. Neural network, owing to their good generalization and nonlinear approximation ability, is widely used to identify model free nonlinear systems and exhibit higher performance compared to other identification methods. The reported neural network identifiers may be classified into two categories on the basis of the neural network structure used, namely, static neural network [1-3] and dynamic neural network [4,5]. The main drawback of the static neural network is that the function approximation treatment makes it easy to fall into local optimum. Dynamic neural network (DNN) method combines feedback information to provide an effective means to solve a wide range of identification problems. However, the structure of dynamic neural network lacks a unified form.

The Hopfield network is a typical dynamic neural network in which every processing unit is connected to all other units [6]. A large number of neural network structures have been developed from Hopfield neural network, falling into two main categories, namely, high order neural networks [7,8] and multi-layer dynamic neural network [9-12]. Multiple nonlinear functions in high order neural networks are used to approximate nonlinear dynamics, which brings the curse of dimensionality problem with the increase of order.

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Multilayer dynamic neural networks which contain additional hidden layers combined with a dynamic operator is not easy to design the online updating law. In general, popular learning rule such as back propagation algorithm is used to design the online weight update laws of dynamic neural networks, and then suitable candidates of Lyapunov function are proposed to ensure the stability of the system [13,14]. In order to solve the locally minimal convergence problem caused by backpropagation algorithms, a novel updating laws of multilayer dynamic neural networks is proposed in [15,16], where the global asymptotic error stability is guaranteed by defining a Lyapunov function candidate based on quadratic functions of the weights and the estimation errors. A robust noise for human activity recognition using convolutional neural network is proposed in [17], which can realize the identification of dynamic system directly from the data information.

In [18], it was claimed that dynamic neural networks can be described by a more parsimonious form called parametric dynamic neural networks, where the compact parametric form is derived by extracting the parameter matrix of correlation weight multiplied by the correlation input and output state. Therefore, the online update law used in general adaptive identification methods, such as the least square method [19], and gradient descent method [20], can be used to design the weight update law of the parametric neural network. However, both of the least square method and gradient descent method are based on the identification errors between the outputs of the estimation model and the system response itself to generate the adaptive law, so the presence of noise or inaccuracies in the identified data could thus strongly affect the accuracy of the identification. As suggested in [21-23], the use of adaptive laws based on parameter estimation error could greatly improve the estimation accuracy. The present study is primarily motivated by the use of such an adaptive law for design of a novel identification method based on parametric dynamic neural networks. The stability and robustness of the proposed estimation method are demonstrated using the Lyapunov approach. The effectiveness of the proposed identification method is illustrated via a typical nonlinear system.

The notable contributions of the study include (i) design of a more parsimonious dynamic neural network model that relies on parameter matrices of linear matrix and weights which simplifies the training problem and leads to more efficient models; (ii) the synthesis of an adaptive updating law based on the state identification error plus the filtered parameter error to achieve high accuracy and greater robustness; and (iii) the Lyapunov approach proves that the identification error is exponential convergence without disturbance and UUB stability in the presence of a bounded disturbance.

The paper is organized in three sequential sections. The identification algorithm is presented in Section 2. Simulation results illustrating the effectiveness of the proposed scheme are presented and discussed in Section 3. Major conclusions of the study are summarized in Section 4.

2. Identification Algorithm.

The nonlinearity and model uncertainty of most real systems pose great challenges to controller design. Therefore, it is necessary to determine the system model before considering system control. In the past decades, several traditional nonlinear identification methods have been proposed. However, most of these studies rely on the priori information of the system model. This assumption is difficult to satisfy for most real systems when getting accurate models is difficult or impossible. Therefore, the general nonlinear system with unknown model is considered in this paper, such that

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathfrak{R}^n$ is the state variable, $u \in \mathfrak{R}^p$ is the input vector, and $f(\cdot)$ is unknown continuous nonlinear smooth function.

2.1. **Identifier design using the general dynamic neural network.** The following single layer DNN is used to identify the nonlinear system (1)

$$\dot{\hat{x}} = A\hat{x} + W_1\sigma(\hat{x}) + W_2\phi(\hat{x})\delta(u) \tag{2}$$

where $\hat{x} \in \mathfrak{R}^n$, $A \in \mathfrak{R}^{n\times n}$, $W_1, W_2 \in \mathfrak{R}^{n\times n}$ are the identification state, the linear matrix and the weights of the DNN, respectively. $\sigma(\hat{x}) = [\sigma(\hat{x}_1), \ldots, \sigma(\hat{x}_n)]^T \in \mathfrak{R}^{n\times 1}$, $\phi(\hat{x}) = diag[\phi(\hat{x}_1), \ldots, \phi(\hat{x}_n)]^T \in \mathfrak{R}^{n\times n}$. The differentiable input-output function $\delta(\cdot) : \mathfrak{R}^p \to \mathfrak{R}^n$ is assumed to be bounded $\|\delta(u)\|^2 \le \bar{u}$. The activation functions $\sigma(\cdot)$, $\phi(\cdot)$ are generally selected as sigmoid function, i.e., $\sigma(\cdot) = \frac{a_1}{\left(1+e^{-b_1(\hat{x})}\right)-c_1}$, $\phi(\cdot) = \frac{a_2}{\left(1+e^{-b_2(\hat{x})}\right)-c_2}$.

There definitely exist nominal constant values of the weights W_1^*, W_2^* and nominal constant Hurwitz matrix A^* such that the nonlinear system (1) can be described by the following DNN model

$$\dot{x} = A^*x + W_1^*\sigma(x) + W_2^*\phi(x)\delta(u) + \xi \tag{3}$$

where W_1^*, W_2^* are assumed to be the bounded unknown idea matrices, i.e., $W_1^*\Lambda_1^{-1}W_1^{*T} \le \overline{W}_1$, $W_2^*\Lambda_2^{-1}W_2^{*T} \le \overline{W}_2$, where Λ_1^{-1}, Λ_2^{-1} are the positive definite symmetric matrices, $\overline{W}_1$, $\overline{W}_2$ are prior known matrices, and ξ is the modeling error.

The identification error is defined as

$$e = x - \hat{x} \tag{4}$$

Then from (2) and (3), one can obtain the error dynamics equation

$$\dot{e} = A^*e + \widetilde{A}\hat{x} + \widetilde{W}_1\sigma(\hat{x}) + \widetilde{W}_2\phi(\hat{x})\delta(u) + W_1^*\widetilde{\sigma} + W_2^*\widetilde{\phi}\delta(u) + \xi \tag{5}$$

where $\widetilde{W}_1 = W_1^* - W_1$, $\widetilde{W}_2 = W_2^* - W_2$, $\widetilde{A} = A^* - A$.

By designing the following Lyapunov function candidate

$$L = e^TPe + \eta_1^{-1}tr\left\{\widetilde{A}^TP\widetilde{A}\right\} + \eta_2^{-1}tr\left\{\widetilde{W}_1^TP\widetilde{W}_1\right\} + \eta_3^{-1}tr\left\{\widetilde{W}_2^TP\widetilde{W}_2\right\} \tag{6}$$

then we can get the online updating laws of the proposed DNN identifier as described in the following Theorem 2.1.

Theorem 2.1. *By properly designing the Lyapunov function as shown in (6), we obtain the updating laws as*

$$\begin{cases} \dot{A} = \eta_1 e\hat{x}^T \\ \dot{W}_1 = \eta_2 e\sigma^T(\hat{x}) \\ \dot{W}_2 = \eta_3\phi(\hat{x})\delta(u)e^T \end{cases} \tag{7}$$

where η_1, η_2, η_3 represent the learning rates of the DNN identifier. The identifier (2) with the updating laws (7) can guarantee the following stable properties $e \in L_\infty \in L_2$, $W_{1,2} \in L_\infty$, $A \in L_\infty$ and $\lim_{t\to\infty} e = 0$, $\lim_{t\to\infty} \dot{W}_{1,2} = 0$.

The nonlinear system identification with DNN was originally proposed in our previous study [9]. The detailed proof of Theorem 2.1 is referred to [9]. It can be seen from (7) that the updating law is only based on the identification error e without considering the parameters error, which may cause parameter overflow problems. In this paper, the identifier structure is improved by more effective parametric DNN, which makes it easier to design the improved weight update law.

2.2. **Identifier design with the parametric dynamic neural network with improved learning.** It is well known that dynamic neural network can approximate the general nonlinear system (1) to any degree with the following form [24]

$$\dot{x}_i = -\alpha_i x_i + \sum_j w_{ij} S_j(x_j, u_j) \tag{8}$$

where x_i is the state of the ith neuron, α_i is the constant which is usually assumed to be known in advance, w_{ij} is the synaptic weight connecting the jth input to the ith neuron, and the nonlinear mapping S_j constituting of the jth state x_j and input u_j to the relational neuron.

A more efficient parameterization of dynamic neural network model with the simplest architecture has been introduced in [18], such that

$$\dot{x}_i = -\alpha_i x_i + \sum_{j=1}^{n} w_{ij}\sigma(x_j) + \sum_{j=1}^{p} \lambda_{ij} u_j \tag{9}$$

where w_{ij} and λ_{ij} are updated weights, $\sigma(\cdot)$ is sigmoid function which is defined as $\sigma(\cdot) = a/\left(1+e^{-bx}\right) - c$, where a, b, c are designed constants. Figure 1 shows the block diagram of the dynamic neural network model (9). $u_1, \ldots, u_p$ form the input vector, $\lambda_{i1}, \ldots, \lambda_{ip}$ denote the input weights, $x_1, \ldots, x_n$ represent the output of the parametric DNN, $\omega_{i1}, \ldots, \omega_{in}$ represent the weights related to $x_1, \ldots, x_n$, the nonlinear active function is selected as $\sigma(x_i) = \frac{a}{1+\exp(-bx_i)} - c$, and the differential in Formula (9) can be equivalently rewritten as $\frac{1}{s+\alpha_i}$.

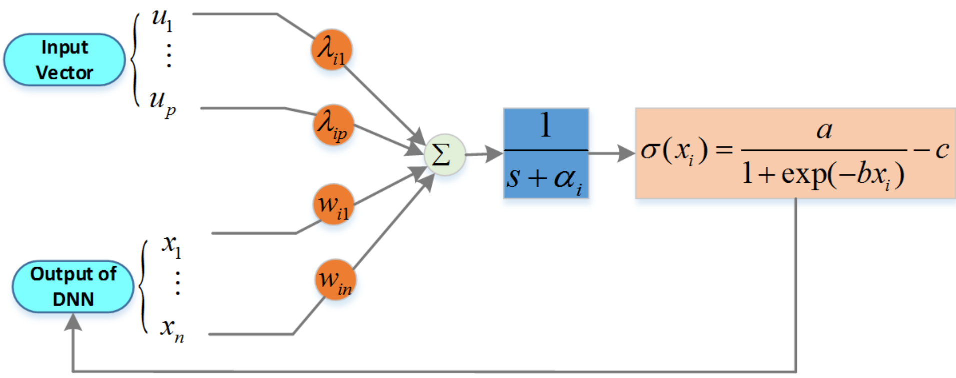

FIGURE 1. Block structure of the parametric dynamic neural network

Remark 2.1. *The use of input affine dynamic neural network architecture (9) to approximate the nonautonomous systems (1) is advantageous, since many important nonlinear control schemes require input affine nonlinear models.*

The parametric dynamic neural network is formed by a single layer of n units as in Equation (9). For the convenience of analysis, the vectorized expression of (9) is obtained with the following form

$$\dot{x} = -ax + w\sigma(x) + \lambda u + \xi \tag{10}$$

where $x \in \Re^n$ is the state vector, $a \in \Re^{n\times n}$ is the unknown matrix for the linear part of parametric dynamic neural network model, $w \in \Re^{n\times n}$, $\sigma(x) = [\sigma(x_1), \ldots, \sigma(x_n)]^T \in \Re^n$, $\lambda \in \Re^{n\times m}$, $u \in \Re^p$ is the input vector, and ξ denotes modeling error and disturbances.

Furthermore, we define the vector notations composed of unknown parameters of parametric dynamic neural network as $\theta = [a, w, \lambda]^T$ and the regressor vector as $\psi = [x, \sigma(x), u]^T$, then the compact form of (10) becomes

$$\dot{x} = \theta^T \psi \tag{11}$$

Remark 2.2. *Several adaptive identifiers have been proposed for system (11), where the adaptive laws are all designed by minimizing the residual identifier error (i.e., error between system state x and the identifier output $\hat{x}$) based on least square method or gradient method. However, the identifier weight convergence was not guaranteed. As indicated in [25], the convergence of the identifier weights is essential for the convergence of the control. This paper will present a novel adaptive law to 'direct' identify the unknown parameters of compact parametric dynamic neural network in (11).*

Next, we will design an improved weight update law to ensure the convergence of state identification error and parameters error. Thus, define the filtered variables x_f and ψ_f of x and ψ, as:

$$\begin{cases} l\dot{x}_f + x_f = x, \quad x_f(0) = 0 \\ l\dot{\psi}_f + \psi_f = \psi, \quad \psi_f(0) = 0 \end{cases} \tag{12}$$

where l is the designed filter constant.

Then from (11) and (12) we can get

$$x_f = \frac{x - x_f}{l} = \theta^T \psi_f \tag{13}$$

Further we define the filtered regression matrix $E(t)$ and $F(t)$ vector as

$$\begin{cases} \dot{E}(t) = -\eta E(t) + \psi_f(t)\psi_f(t)^T, \quad E_1(0) = 0 \\ \dot{F}(t) = -\eta F(t) + F_f^T(t)[(x(t) - x_f(t))/l]^T, \quad F_1(0) = 0 \end{cases} \tag{14}$$

where η is the designed filter constant.

From (14) one can get

$$\begin{cases} E(t) = \displaystyle\int e^{-\eta(t-r)} \psi_f(r)\psi_f^T(r)dr \\ F(t) = \displaystyle\int e^{-\eta(t-r)} \psi_f(r)[(x(r) - x_f(r))/l]^T dr \end{cases} \tag{15}$$

Definition 2.1. [25] *A vector or matrix function Φ is persistently excited (PE) if there exist $\tau > 0$, $\varepsilon > 0$, such that $\int_t^{t+\tau} \Phi(r)\Phi(r)^T dr > \varepsilon I$, $\forall t \geq 0$. Since $\Phi(r)\Phi(r)^T$ is always positive semi definite, the PE condition requires that its integral over any interval of time of length is a positive definite matrix.*

Remark 2.3. *If the repressor vector Φ is PE, then Φ_f defined in (12) is PE, because Φ_f is the filtered version of Φ in terms of a minimum strictly proper transfer function $1/(ks+1)$ in (12) as proved in [25]. Moreover, based on Definition 2.1, if Φ_f is PE, the inequality $\int_t^{t+\tau} \Phi_f^T(r)\Phi_f(r)dr > \varepsilon I$ is true for all $t > 0$, $\varepsilon > 0$. Then $\int_t^{t+\tau} e^{-l(t-r)}\Phi_f^T(r)\Phi_f(r)dr > \varepsilon I$ holds for all $t > 0$, $\varepsilon > 0$.*

Consider the following identifier

$$\dot{\hat{x}} = \hat{\theta}^T \psi + Ke \tag{16}$$

where $e = x - \hat{x}$, $\hat{\theta} = \left[\hat{\alpha}, \hat{w}, \hat{\lambda}\right]^T$, $K > 0$ is a designed parameter.

From (11) and (17) we can get

$$\dot{e} = \dot{x} - \dot{\hat{x}} = \theta^T \psi - \hat{\theta}^T \psi - Ke + \xi = -Ke + \widetilde{\theta}^T \psi + \xi \tag{17}$$

where $\widetilde{\Theta} = \Theta - \hat{\Theta}$ is the parameter identification error.

Finally, we denote another auxiliary vector as

$$M(t) = E(t)\hat{\Theta} - F(t) \tag{18}$$

where $\hat{\Theta}$ is the estimation of Θ. It is clear that $M(t)$ can be calculated based on Equation (9).

Remark 2.4. *From (14)-(16), we have* $M(t) = E(t)\hat{\Theta} - F(t) = E(t)\hat{\Theta} - E(t)\Theta = -E(t)\widetilde{\Theta}$. *As can be seen,* $M(t)$ *is composed of weights error* $\widetilde{\Theta}$, *which is used to design the improved updating law in the next analysis.*

Then by using the auxiliary vector $M(t)$, *one can have the following improved updating law*

$$\dot{\hat{\theta}} = \Gamma[\psi e - \rho M] \tag{19}$$

where $\Gamma = \Gamma^T > 0$, $\rho > 0$ *is positive constant.*

Theorem 2.2. *Considering system (1) with the identifier (17) and parameters adaptive law (19), then the convergent properties of identification error as well as parameters error can be obtained as the following*

(i) With the assumption that $\xi = 0$, *we have* $e, \theta \in L_\infty$ *and* $\lim_{t\to\infty} e = 0$;

(ii) With the assumption that ξ *is bounded, then we have* $e, \theta \in L_\infty$.

Proof: Choose a Lyapunov function as

$$V = e^T Pe + \frac{1}{\Gamma} tr\left\{\widetilde{\theta}^T P\widetilde{\theta}\right\} \tag{20}$$

Case i. If $\xi = 0$, then from (17)-(19) and $\dot{\widetilde{\theta}} = -\dot{\hat{\theta}}$, one can get the differential of (20) as

$$\begin{aligned}
\dot{V} &= \left(\dot{e}^T Pe + e^T P\dot{e}\right) + \frac{2}{\Gamma} tr\left\{\dot{\widetilde{\theta}}^T P\widetilde{\theta}\right\} \\
&= \left[\widetilde{\theta}^T\psi - Ke\right]^T Pe + e^T P\left(\widetilde{\theta}^T\psi - Ke\right) + \frac{2}{\Gamma} tr\left\{\dot{\widetilde{\theta}}^T P\widetilde{\theta}\right\} \\
&= 2e^T P\widetilde{\theta}^T\psi - 2e^T PKe - 2(\psi e - \rho M)P\widetilde{\theta} \\
&= -2e^T PKe + 2\rho\widetilde{\theta}^T PM \\
&= -2e^T PKe + 2\rho\widetilde{\theta}^T P\left(E(t)\hat{\Theta} - F(t)\right) \\
&= -2e^T PKe + 2\rho\widetilde{\theta}^T P\left(E(t)\hat{\Theta} - E(t)\Theta\right) \\
&= -2e^T PKe - 2\rho\widetilde{\theta}^T PE(t)\widetilde{\theta} \le 0
\end{aligned} \tag{21}$$

From (21) we know that $e, \theta \in L_\infty$. Furthermore, one can infer from (17) that $\dot{e} \in L_\infty$. Based on the non-increasing property of the function V, the integral of V on both sides from 0 to ∞ can be obtained

$$\int_0^\infty \left(-2e^T PKe - 2\rho\widetilde{\theta}^T PE(t)\widetilde{\theta}\right) = [V_x(0) - V_x(\infty)] < \infty \tag{22}$$

Therefore, $e \in L_2$ can be obtained from (22). It can be concluded that $e \in L_2 \cap L_\infty$ and $\dot{\Delta}x, \dot{\Delta}y \in L_\infty$. It is thus obtained from Barbalat's Lemma [26] that $\lim_{t\to\infty} e = 0$.

Case ii. For bounded ξ, by designing the same Lyapunov function as Formula (20), one obtains

$$\begin{aligned}
\dot{V} &= \left(\dot{e}^T Pe + e^T P\dot{e}\right) + \frac{2}{\Gamma} tr\left\{\dot{\widetilde{\theta}}^T P\widetilde{\theta}\right\} \\
&= \left[\widetilde{\theta}^T\psi - Ke + \xi\right]^T Pe + e^T P\left(\widetilde{\theta}^T\psi - Ke + \xi\right) + \frac{2}{\Gamma} tr\left\{\dot{\widetilde{\theta}}^T P\widetilde{\theta}\right\}
\end{aligned}$$

$$
\begin{aligned}
&= 2e^TP\widetilde{\theta}^T\psi - 2e^TPKe - 2(\psi e - \rho M)P\widetilde{\theta} + 2e^TP\xi \\
&= -2e^TPKe + 2\rho\widetilde{\theta}^TPM + 2e^TP\xi \\
&= -2e^TPKe + 2\rho\widetilde{\theta}^TP\left(E(t)\hat{\Theta} - F(t)\right) + 2e^TP\xi \\
&= -2e^TPKe + 2\rho\widetilde{\theta}^TP\left(E(t)\hat{\Theta} - E(t)\Theta\right) + 2e^TP\xi \\
&= -2e^TPKe - 2\rho\widetilde{\theta}^TPE(t)\widetilde{\theta} + 2e^TP\xi \\
&\le -2e^TPKe - 2\rho\widetilde{\theta}^TPE(t)\widetilde{\theta} + e^TP\Lambda_2Pe + \xi^T\Lambda_2^{-1}\xi \\
&\le -\mu_1(\|e\|) - \mu_2\left(\left\|\widetilde{\theta}\right\|\right) + \mu_3\left(\|\xi\|\right)
\end{aligned} \tag{23}
$$

where μ_1, μ_2, μ_3 are positive constants, and Λ_1, Λ_2 positive definite matrixes.

It can be seen from (23) that V is input-to-state stability (ISS) Lyapunov function, so by Theorem 1 in [16] we can get the stability of the system such that if the model error ξ is bounded, then the updating law (19) can make the identification procedure stable, i.e., $e, \theta \in L_\infty$. The overall structure of the proposed parametric DNN identifier is shown in Figure 2.

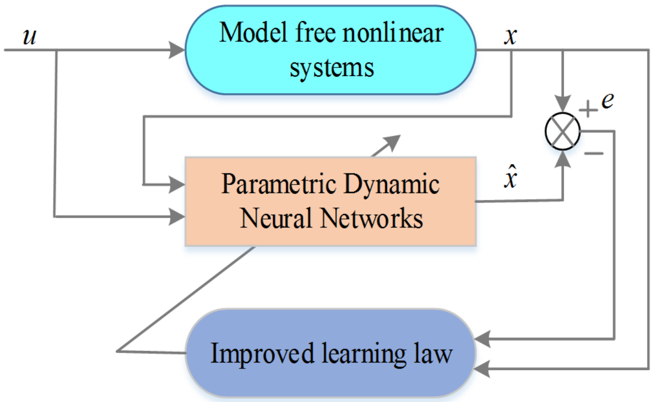

FIGURE 2. Block structure of the identification scheme

Remark 2.5. *It is well known that the commonly used backpropagation tuning law of the neural network based identification algorithm generally leads to unbounded weights which is also known as parameter overflow problem. In this paper, we proposed a modified weights tuning laws considering the parameter identification errors as shown in (19), where the first term is the standard backpropagation approach and the second term consisting of parameter filtering error information is used to guarantee bounded weights. Moreover, the NN identifier structure is improved by the more compact parametric dynamic neural networks, which makes the relevant learning algorithms in adaptive control theory can be used to further design the adaptive control law in the future research. As a final note, input-to-state stability property of the closed loop identification process makes the system more stable.*

3. **A Case Research: Simulation.** The effectiveness of the proposed identification method is demonstrated by comparing our previous results with those obtained using the identification error driven method as shown in [9], where the updating law is obtained from Equation (7) in Section 2.1.

Two examples are presented to illustrate the theoretical results.

Example 3.1. *We consider the following nonlinear system*

$$\begin{aligned}\dot{x}_1 &= \alpha_1 x_1 + \beta_1 sign(x_2) + u_1 \\ \varepsilon \dot{x}_2 &= \alpha_2 x_2 + \beta_2 sign(x_1) + u_2\end{aligned} \tag{24}$$

where we use the same parameter $\alpha_1 = -5$, $\alpha_2 = -10$, $\beta_1 = 3$, $\beta_2 = 2$, $x_1(0) = -5$, $x_2(0) = -5$ *as in [9]. The given nonlinear system, even simple, is interesting enough, since it has multiple isolated equilibriums [9]. Using the parameter embedding technique, the model used here is singularly perturbed and the small parameter* ε *is positive and smaller than 1. The input signals are selected as:* u_1 *is a sinusoidal wave* ($u_1 = 8\sin(0.05t)$) *and* u_2 *is a saw-tooth function with the amplitude 8 and frequency 0.02 Hertz.*

a) Without modeling error and disturbances ($\xi = 0$)*. We want to compare our result with that in [9]. For the fair comparison, we choose exactly the same model and input signal. Only one-time scale* ($\varepsilon = 1$) *is considered. For the activation function here we select hyperbolic tangent. This left the only difference from [9] is the neural network itself. Under the online adaptive updating algorithm (19), the identification process is conducted. The results are shown in the following Figures 3-6. Figure 3 and Figure 5 illustrate identification results of nonlinear system (24) in the absence of modeling error and disturbances using the proposed method and the method shown in Reference [9]. Both the methods demonstrate good convergence. However, the method in Reference [9] yields higher estimation error compared with the proposed method, especially at the turning point,*

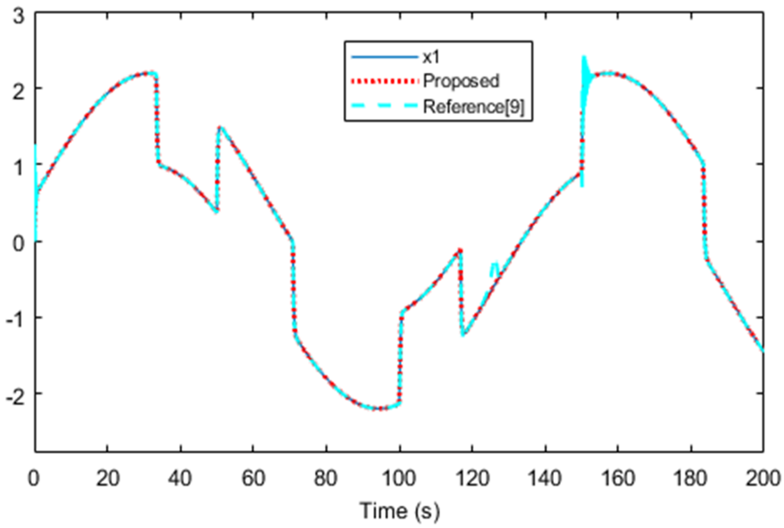

Figure 3. Identification result for x_1

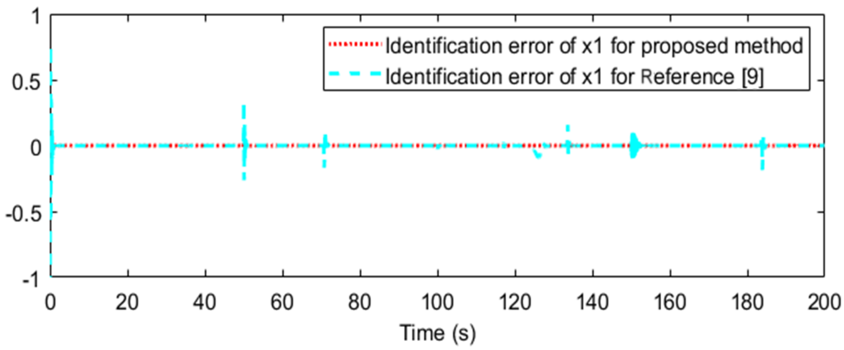

Figure 4. Identification error for x_1

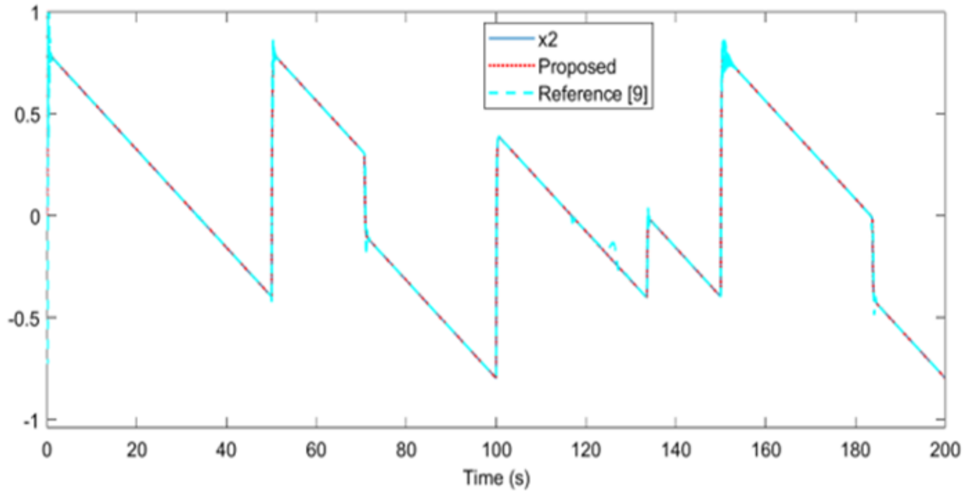

FIGURE 5. Identification result for x_2

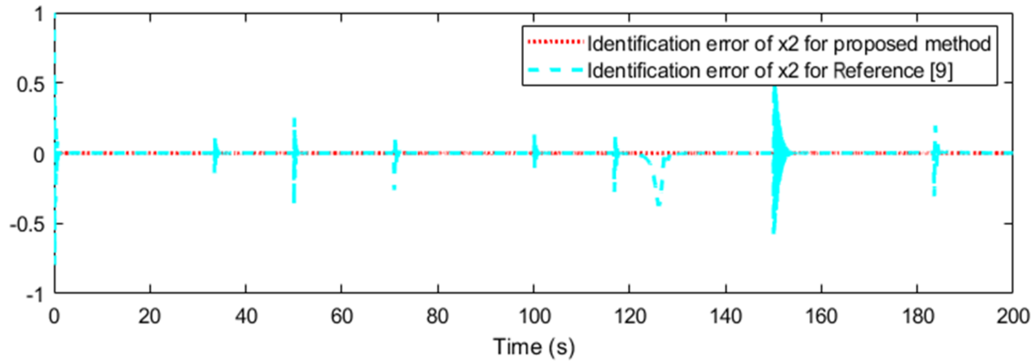

FIGURE 6. Identification error for x_2

which is illustrated in Figure 4 and Figure 6. This is attributable to contributions of the filtered regression matrix $E(t)$ and $F(t)$ vector as shown in (14) and the corresponding auxiliary vector $M(t)$ expressed in Equation (18) that make up the updating law for the proposed identification method. The adaptive law in Reference [9] as shown in Equation (7), is based on the identification error and the regression vector, which may degrade the identification performance in the presence of modeling error and disturbances. The adaptive law in the proposed estimation method, on the other hand, is based on an explicit expression of the parameter identification errors, which is realized by introducing appropriate filter operations as shown in Equation (19). The proposed method thus exhibits relatively better performance in terms of modeling error and disturbances.

b) With modeling error and disturbances ($\xi \neq 0$). The superior robustness of the proposed method compared with the RLS method was more evident in the presence of disturbances. Simulations were performed considering noise signal as shown in Figure 7, while the parameters for implementation of the proposed and Reference [9] estimation methods were taken as those stated above. It is observed from Figure 8 and Figure 10 that the proposed method can identify the nonlinear system (24) even under noise disturbance, which proves the convergence stability as depicted in Theorem 2.2. The method in Reference [9], however, exhibits relatively larger deviation and fluctuation in the identified nonlinear system (24) compared with the proposed identification method, which is illustrated via the identification error signals as shown in Figure 9 and Figure 11.

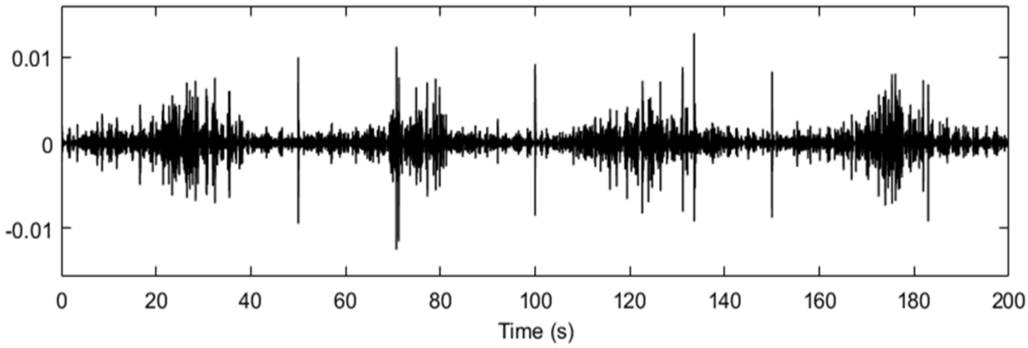

FIGURE 7. Disturbances

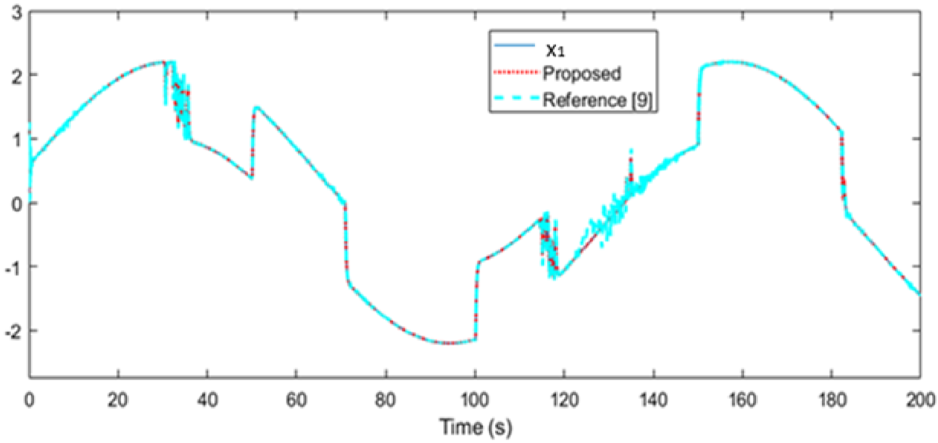

FIGURE 8. Identification result for x_1 with disturbance

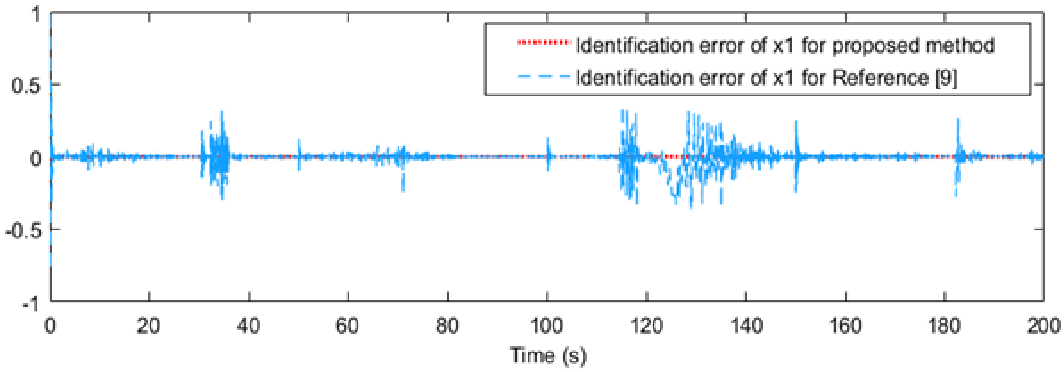

FIGURE 9. Identification error for x_1 with disturbance

To show the identification performance of the proposed algorithm, the performance index – root mean square (RMS) for the state error has been adopted for the purpose of comparison.

$$RMS = \sqrt{\sum_{i=1}^{n} e^2(i) \Big/ n} \qquad (25)$$

where n *is the number of the simulation steps, and* $e(i)$ *is the difference between the state variables in model and system at the ith step.*

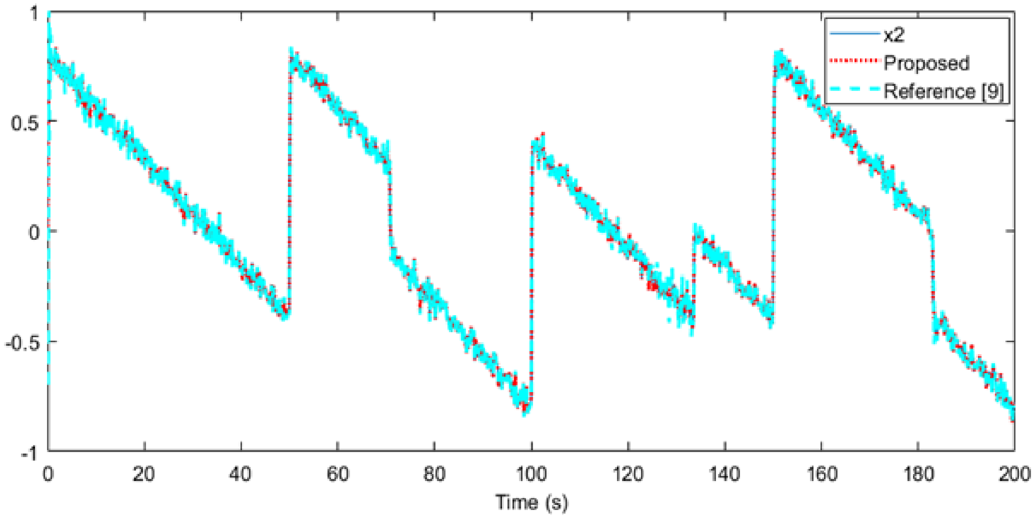

FIGURE 10. Identification result for x_2 with disturbance

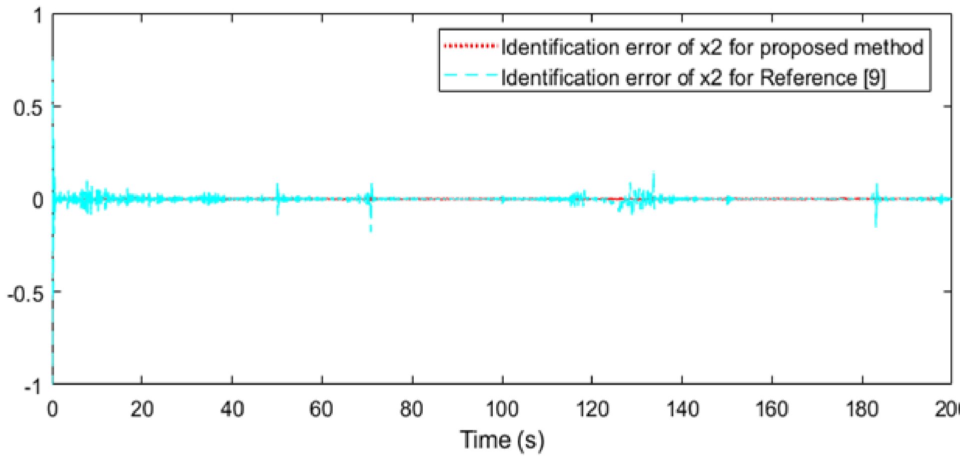

FIGURE 11. Identification error for x_2 with disturbance

The RMS values of all state variables as shown in Table 1 demonstrate that the identification performance has been improved compared to those in [9].

TABLE 1. The RMS values for state variable

	Without disturbance		With disturbance	
	X_1	X_2	X_1	X_2
Proposed	0.000131	0.000271	0.00260	0.00565
Reference [9]	0.000273	0.000595	0.00283	0.00760

Example 3.2. *DC servomotor system with uncertain load is used to illustrate the usefulness and flexibility of the proposed identifiers. DC motor modeling can be separated into electrical and mechanical two subsystems due to the difference of time scales. The model of DC motor [27] is shown as follows*

$$\begin{aligned} \dot{\omega} &= i \\ \varepsilon \dot{i} &= -\omega - i + u + L \end{aligned} \tag{26}$$

where ω is related to the speed of motor, i is related to circuit current, ε is the time scale, u is related to the input voltage to the circuit, and L denotes the uncertain load.

The DNN identifiers with the single layer structure (2) corresponding to the updating laws (7) and the proposed structure (10) corresponding to the updating laws (19) are used to identify the DC motor model (26). The parameter of time scale is $\varepsilon = 0.5$ and input signal is $u = 3\sin 0.5t$. The uncertain load L is selected as a band limited white noise with noise power 0.1. Comparative identification results are illustrated as in Figures 12-15. It can be seen that the proposed identifier using the parametric DNN with improved learning law is more accurate than the identifier using the general DNN with the backpropagation learning law as our previous research [9]. The reason is that the updating law as shown in (7) is based on the identification error, which will influence the accuracy of the identification results due to the time varying parameters in the practical system. However, the parameter errors are considered in the updating law (19) of the proposed identifiers, which greatly improves the accuracy of the identification results.

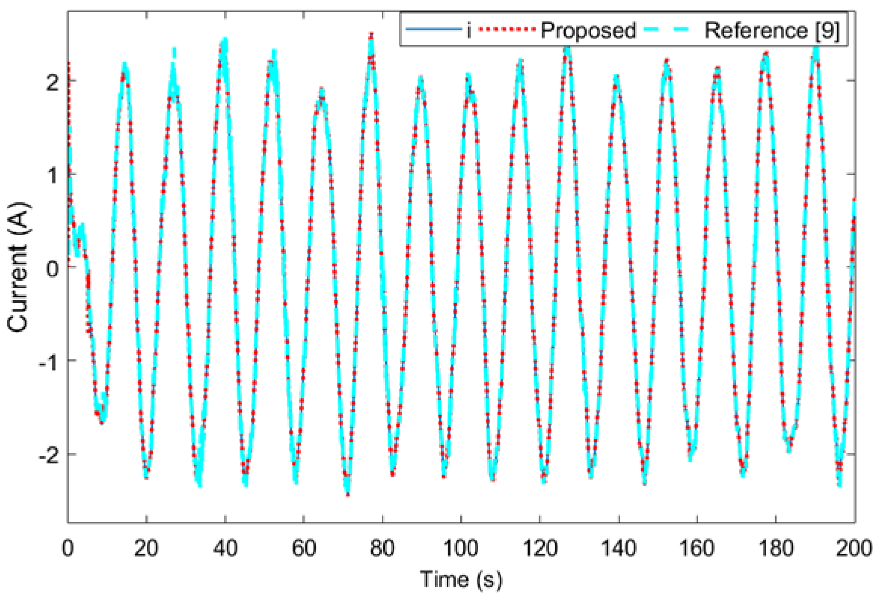

Figure 12. Comparative identification result of current

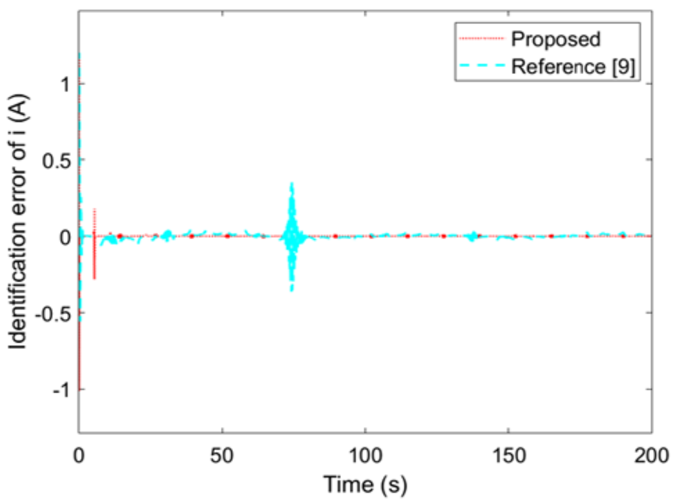

Figure 13. Comparative identification error of current

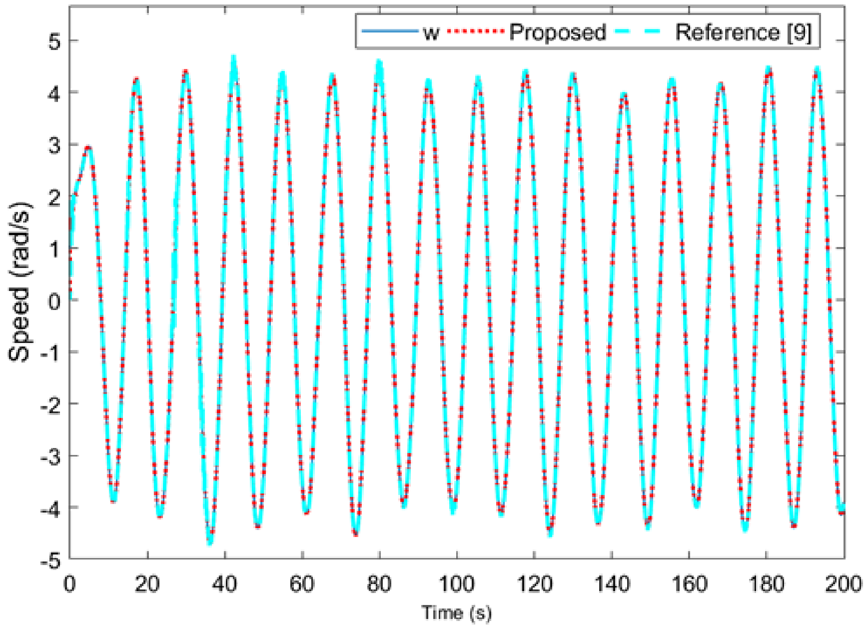

FIGURE 14. Comparative identification result of motor speed

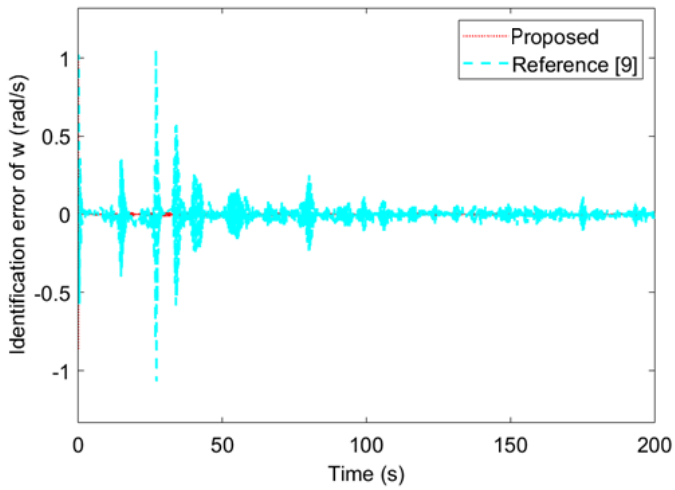

FIGURE 15. Comparative identification error of motor speed

4. **Conclusions.** This study reported a novel robust identification method via parametric dynamic neural networks with improved learning. Exponential convergence without disturbance and UUB stability with the bounded disturbance are proved via Lyapunov method. The online learning law of the proposed estimation method is based on the identification error and the filtered parameters error, which can avoid the disturbance interference problem caused by the high order signals and easily be applied in the model free nonlinear systems to improve the identification performance. The simulation results show that the proposed identification method demonstrates improved performance even in the presence of external disturbances compared with the commonly used neural network identification method.

It should be pointed out that the proposed parametric DNN identifier is based on the availability of all the systems states. This may be a rather stringent requirement for practical applications. Future research will be dedicated to developing a novel identifier with unknown state and an effective control method of model free nonlinear system based on the identification result.

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