International Journal of Innovative
Computing, Information and Control
Volume **17**, Number **6**, December **2021**

ICIC International ©2021 ISSN 1349-4198
pp. 2057–2070

VARIABLE ACCEPTANCE SAMPLING PLAN BASED ON THE LIFETIME PERFORMANCE INDEX C_L FOR WEIBULL PRODUCTS

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Received May 2021; revised September 2021

Abstract. *Acceptance sampling plan is mainly based on producer risks $\tilde{\alpha}$ or consumer risks $\tilde{\beta}$. Therefore, we will discuss variable acceptance sampling plan in testing products meet process requirements and then protect producers and consumers to reduce cost and time for checking failure. Besides, we also assume that the lifetime of the testing product has a Weibull distribution with the given (or known) shape parameter β and unknown scale parameter λ. And we use the lifetime performance index C_L to assess the performance of products. Finally, we propose the maximum likelihood estimator of C_L, where L is the known lower specification limit, and then develop a variable acceptance sampling plan based on the lifetime performance index C_L.*

Keywords: Variable acceptance sampling plan, Weibull distribution, Lifetime performance index, Maximum likelihood estimator, First failure-censored sampling plan

1. **Introduction.** Acceptance sampling is an important field of statistical quality control that was popularized by Dodge and Romig and originally applied by the US military to the testing of bullets during World War II. If every bullet was tested in advance, no bullets would be left to ship. If, on the other hand, none were tested, malfunctions might occur in the field of battle, with potentially disastrous results [1]. Acceptance sampling is a "middle ground" approach between the extremes of 100% inspection and no inspection. It often provides a methodology for moving between these extremes as sufficient information is obtained on the control of the manufacturing process that produces the product. Although there is no direct control of quality in the application of an acceptance sampling plan to an isolated lot, when that plan is applied to a stream of lots from a supplier, it becomes a means of providing protection for both the producer of the lot and the consumer. It also provides for an accumulation of quality history regarding the process that produces the lot, and it may provide feedback that is useful in process control, such as determining when process controls at the supplier's plan plant are not adequate. Finally, it may place economic or psychological pressure on the supplier to improve the production process. Two kinds of errors may be committed in acceptance sampling. A good lot will be rejected called a Type I error. The risk of making a Type I error is called producer's risk

DOI: 10.24507/ijicic.17.06.2057

or $\tilde{\alpha}$-risk. Accept a lot of poor quality called a Type II error. The risk of making a Type II error is called consumer's risk or $\widetilde{\beta}$-risk [2].

There are a number of different ways to classify acceptance sampling plans. One major classification is by attributes and variables. Attributes are quality characteristics that are expressed on a "go, no-go" basis. The attribute sampling plans include single sampling plan, double sampling plan and multiple sampling plan. Variables, of course, are quality characteristics that are measured on a numerical scale. There are many advantages in variable sampling plans. The primary advantage of variable sampling plans is that the same operating characteristic (OC) curve can be obtained with a smaller sample size than would be required by an attribute sampling plan. The second advantage is that measurement data usually provide more information about the manufacturing process or the lot than do attributes data. Generally, numerical measurements of quality characteristics are more useful than simple classification of the item as defective or non-defective. A final point to be emphasized is that when acceptable quality levels are very small, the sample sizes required by attribute sampling plans are very large. Under these circumstances, there may be significant advantages in switching to variable measurement. Thus, as many manufacturers begin to emphasize allowable numbers of defective parts per million, variable sampling becomes very attractive [2]. Recently, several variable single sampling plans have been developed by considering numerous process capability indices (PCIs) [1,3-13].

In life testing experiments, the experimenter may not always be in a position to observe the lifetimes of all the products (or items) put on test. This may be because of time limitation and/or other restrictions (such as money, material resources, mechanical or experimental difficulties) on data collection. Therefore, censored samples may arise in practice. In this study, we consider the case of first-failure censored sampling plan. Due to the fact that sometimes the lifetime of a product is quite long, a right type II censored sample plan for such a product can be too long. Johnson [14] proposed the first-failure censored sampling plan in which the experimenter can decide to group the test units into several sets (each set is an assembly of test units), and then run all the test units simultaneously until the first failure in each group. Such plans are usually feasible when test facilities are scarce but test material is relatively cheap. Balasooriya [15] examined the failure-censored sampling plan for the two-parameter exponential distribution based on testing m random samples, each of size n, one after the other. That procedure is based on exact results, and only the first failure time of each sample is needed. The Balasooriya sampling plan is compared with traditional sampling plans using a sample of size $m \cdot n$ [16]. The first-failure censored sampling plan has an advantage in terms of shorter test-time and a saving of resources. Recently, Lee et al. [17] constructed a maximum likelihood estimator (MLE) of C_L with the first failure-censored sampling plan under the Gompertz distribution. The MLE of C_L is then utilized to develop a new hypothesis testing procedure in the condition of known L. The new hypothesis testing procedure is a quality performance assessment system in enterprise resource planning (ERP). The managers can then employ the new testing procedure to determine whether the lifetime performance of products adheres to the required level. The managers can also utilize this procedure to enhance product process capability.

The Weibull distribution is useful in a great variety of applications, particularly as a model for product life. It has also been used as the distribution of strength of certain materials. It is named after Weibull [18], who popularized its use among engineers. One reason for its popularity is that it has a great variety of shapes. This makes it extremely flexible in fitting data, and it empirically fits many kinds of data. The ability of Weibull failure rate function to describe increasing or decreasing failure rates contributed to making the Weibull distribution popular for lifetime data analysis [19]. The Weibull distribution has been used extensively in reliability engineering as a model of time to failure for electrical and mechanical components and systems. Examples of situations in which the Weibull distribution has been used include electronic devices such as memory elements, mechanical components such as bearings, and structural elements in aircraft and automobiles [2]. The Weibull distribution includes the exponential and the Rayleigh distributions as special cases. The exponential and the Rayleigh distributions have been recognized as a useful model for the analysis of lifetime data. The Weibull distribution family has played an important role in the analysis of lifetime data. Suppose that the lifetime of products may be modeled by a Weibull distribution. Let X denote the lifetime of such a product and X has the Weibull distribution with the probability density function (p.d.f.) is

$$f_X(x) = \frac{\beta}{\lambda^\beta} x^{\beta-1} \exp\left[-\left(\frac{x}{\lambda}\right)^\beta\right], \quad x > 0,\ \lambda > 0,\ \beta > 0. \tag{1}$$

The parameter β is called the shape parameter, and the parameter λ is called the scale parameter. For the special case $\beta = 1$, the Weibull distribution is the simple exponential distribution. For the special case $\beta = 2$, the Weibull distribution is the Rayleigh distribution. In addition, for $3 \le \beta \le 4$, the shape of the Weibull distribution is close to that of the normal distribution [19].

Under the assumption of Weibull distribution, the main aim of this research is to construct a maximum likelihood estimator (*MLE*) of C_L with the first failure-censored sampling plan. The *MLE* of C_L has a good property, that is, we can derive the exact probability distribution of *MLE* of C_L. The *MLE* of C_L is then utilized to develop a new variable sampling in the condition of known L. The variable acceptance sampling plan procedure is used to deal with the product acceptance decision making problem in the condition of known L.

The rest of this paper is organized as follows. Section 2 introduces some properties of the lifetime performance index C_L for lifetime of product with the Weibull distribution and discusses the relationship between the lifetime performance index C_L and conforming rate. Section 3 then presents the *MLE* of the lifetime performance index C_L and its statistical properties based on the first failure-censored sampling plan. Section 4 develops a new variable acceptance sampling plan based on the lifetime performance index C_L. Two numerical examples and concluding remarks are made in Section 5 and Section 6, respectively.

2. **The Lifetime Performance Index and the Conforming Rate.** The process capability index is a measure of the ability of the process to manufacture product that meets the specifications. The index C_L was developed to provide a dimensionless quantity for measuring the performance of a process. Clearly, a longer lifetime implies a better product quality. Hence, the lifetime is a larger-the-better type quality characteristic. The lifetime is generally required to exceed L years to both be economically profitable and investors. Montgomery [20] developed a capability index C_L for properly measuring the larger-the-better quality characteristic. C_L is defined as follows:

$$C_L = \frac{\mu - L}{\sigma}, \tag{2}$$

where μ, σ, and L are the process mean, the process standard deviation and the lower specification limit, respectively.

To assess the lifetime performance of products, C_L can be defined as the lifetime performance index. Let X have the Weibull distribution with p.d.f. as (1). Moreover, there are several important properties, as follows.

• The lifetime performance index C_L can be rewritten as:

$$C_L = \frac{\mu - L}{\sigma} = \frac{\lambda\Gamma\left(\frac{\beta+1}{\beta}\right) - L}{\lambda\sqrt{\Gamma\left(\frac{\beta+2}{\beta}\right) - \Gamma^2\left(\frac{\beta+1}{\beta}\right)}} = \frac{1}{\Delta}\left[\Gamma\left(\frac{\beta+1}{\beta}\right) - \frac{L}{\lambda}\right], \tag{3}$$

where the process mean $\mu = E(X) = \lambda\Gamma\left(\frac{\beta+1}{\beta}\right)$, the process standard deviation $\sigma = \sqrt{VAR(X)} = \lambda\Delta$, $\Delta = \sqrt{\Gamma\left(\frac{\beta+2}{\beta}\right) - \Gamma^2\left(\frac{\beta+1}{\beta}\right)}$ and L is the lower specification limit.

• The cumulative distribution function (c.d.f.) of X is given by

$$F_X(x) = 1 - \exp\left[-\left(\frac{x}{\lambda}\right)^{\beta}\right], \quad x > 0,\ \beta > 0,\ \lambda > 0. \tag{4}$$

• The failure rate function $r_X(x)$ is defined by

$$r_X(x) = \frac{f_X(x)}{1 - F_X(x)} = \frac{\beta x^{\beta-1}}{\lambda^{\beta}}, \quad x > 0,\ \beta > 0,\ \lambda > 0. \tag{5}$$

When the process mean $\mu = \lambda\Gamma\left(\frac{\beta+1}{\beta}\right)$ $(> L)$, then the lifetime performance index $C_L > 0$. From (5) and the shape parameter β is given or known, we can see that the larger the process mean μ, the smaller the failure rate $r_X(x)$ and the larger the lifetime performance index C_L. Therefore, the lifetime performance index C_L reasonably and accurately represents the lifetime performance of new product.

To determine whether the lifetime of products X is consistently achieved by manufacturers and delivered within their required specifications L preset by customers, we define the conforming rate of product. If the lifetime of a product X exceeds the lower specification limit L, then the product is defined as a conforming product. The ratio of conforming products is known as the conforming rate, and can be defined as:

$$P_r = P(X \geq L) = \exp\left[-\left(\frac{L}{\lambda}\right)^{\beta}\right] = \exp\left\{-\left[\Gamma\left(\frac{\beta+1}{\beta}\right) - C_L\Delta\right]^{\beta}\right\},$$

$$-\infty \leq C_L \leq \frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}. \tag{6}$$

Obviously, a strictly increasing relationship exists between conforming rate P_r and the lifetime performance index C_L. Because $0 \leq P_r \leq 1$, $-\infty \leq C_L \leq \frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}$. Since a one-to-one mathematical relationship exists between the conforming rate P_r and the lifetime performance index C_L. Therefore, utilizing the one-to-one relationship between P_r and C_L, lifetime performance index can be a flexible and effective tool, not only evaluating product quality, but also for estimating the conforming rate P_r.

3. **The Maximum Likelihood Estimator of Lifetime Performance Index.**

In lifetime testing experiments of products, the experimenter may not always be in a position to observe the lifetimes of all the items on test due to time limitation and/or other restrictions (such as money, material resources, mechanical or experimental difficulties) on data collection. Therefore, censored samples may arise in practice. In this paper, we consider the case of the first-failure censored sampling plan. And in order to evaluate the lifetime performance of product it requires providing an estimator for C_L with the exact probability distribution.

Let X denote the lifetime of such a product and X has the Weibull distribution with the p.d.f. given as (1). Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the order statistics of a random sample of size n from the Weibull distribution with p.d.f. as (1). The p.d.f. of the first order statistic $X_{(1)}$ can be obtained

$$f_{X_{(1)}}(x) = \frac{\beta x^{\beta-1}}{\lambda_*^{\beta}} \exp\left[-\left(\frac{x}{\lambda_*}\right)^{\beta}\right], \quad x > 0, \tag{7}$$

where $\lambda_* = n^{-1/\beta}\lambda > 0$, $\beta > 0$.

Let $X_{(1)1} < X_{(1)2} < \cdots < X_{(1)m}$ be the first-failure censored set of first order statistics of m samples of size n from the Weibull distribution with p.d.f. as (1). By using (7), the joint p.d.f. of $X_{(1)1}, X_{(1)2}, \ldots, X_{(1)m}$ is given as follows:

$$\begin{aligned} f_{X_{(1)1},\ldots,X_{(1)m}}(x_{(1)1}, x_{(1)2}, \ldots, x_{(1)m}) &= m! \prod_{i=1}^{m} f_{X_{(1)}}(x_{(1)i}) \\ &= m! \prod_{i=1}^{m} \left\{ \frac{\beta x_{(1)i}^{\beta-1}}{\lambda_*^{\beta}} \exp\left[-\left(\frac{x_{(1)i}}{\lambda_*}\right)^{\beta}\right]\right\} \\ &= \left(\frac{m!\beta^m}{\lambda_*^{m\beta}} \prod_{i=1}^{m} x_{(1)i}^{\beta-1}\right) \cdot \exp\left(-\frac{\sum_{i=1}^{m} x_{(1)i}^{\beta}}{\lambda_*^{\beta}}\right), \end{aligned} \tag{8}$$

where $\lambda_* = n^{-1/\beta}\lambda > 0$, $\beta > 0$. Under the fact that β is known or given, so the likelihood function can be written as

$$L(\lambda_*) = \left(\frac{m!\beta^m}{\lambda_*^{m\beta}} \prod_{i=1}^{m} x_{(1)i}^{\beta-1}\right) \cdot \exp\left(-\frac{\sum_{i=1}^{m} x_{(1)i}^{\beta}}{\lambda_*^{\beta}}\right), \tag{9}$$

The log-likelihood function is given by

$$\ln L(\lambda_*) = \ln m! + m\ln\beta - m\beta\ln\lambda_* + (\beta-1)\sum_{i=1}^{m} \ln\left(x_{(1)i}^{\beta}\right) - \frac{\sum_{i=1}^{m} x_{(1)i}^{\beta}}{\lambda_*^{\beta}}. \tag{10}$$

The differentiation of (10) with respect to λ_* yields

$$\frac{d}{d\lambda_*}\ln L(\lambda_*) = -\frac{m\beta}{\lambda_*} + \frac{\beta\sum_{i=1}^{m} x_{(1)i}^{\beta}}{\lambda_*^{\beta+1}}, \tag{11}$$

Hence, the *MLE* $\hat{\lambda}_*$ of λ_* is $\hat{\lambda}_* = \left(\frac{1}{m}\sum_{i=1}^{m} X_{(1)i}^{\beta}\right)^{\frac{1}{\beta}}$ by solving the equation $\frac{d}{d\lambda_*}\ln L(\lambda_*) = 0$. Under $\lambda_* = n^{-1/\beta}\lambda$, we obtain that the *MLE* $\hat{\lambda}$ of λ is given by $\hat{\lambda} = \left(\frac{n}{m}\sum_{i=1}^{m} X_{(1)i}^{\beta}\right)^{\frac{1}{\beta}}$ based on the invariance of *MLE* [21] and $\frac{2m\hat{\lambda}^{\beta}}{\lambda^{\beta}} \sim \chi^2_{(2m)}$. Further, by using (3) and the invariance of *MLE*, the *MLE* of C_L can be written as given by

$$\hat{C}_L = \frac{1}{\Delta}\left[\Gamma\left(\frac{\beta+1}{\beta}\right) - \frac{L}{\hat{\lambda}}\right]. \tag{12}$$

4. **Variable Acceptance Sampling Plan Based on the Lifetime Performance Index.** Suppose that the quality characteristic of interest has the lower specification limit (L) and follows a Weibull distribution. A common approach to the design of an

acceptance sampling plan is to require that the OC curve passes through two designated points. In a purchasing contract, both consumer and producer will set their requirements with the risks they could suffer. The consumer requests that not too many "bad" lots shall be accepted, and the producer requests that not too many "good" lots shall be rejected. Thus, a designed acceptance sampling plan should be made to satisfy these somewhat opposing requirements. It is usual to use the AQL (acceptable quality level or acceptable quality limit) and $LTPD$ (lot tolerance percent defective) points on OC curve for this purpose. That is, if the quality level of the submitted lot is at $C_L = C_{AQL}$ (in high quality), the probability of acceptance must be greater than $1 - \tilde{\alpha}$ (where $\tilde{\alpha}$ is usually called the producer's risk). And if the quality level of the submitted lot is only at $C_L = C_{LTPD}$ (in low quality), the probability of acceptance would be no more than $\tilde{\beta}$ (where $\tilde{\beta}$ is usually called the consumer's risk). We will discuss variable acceptance sampling plan in testing products meet process requirements and then protect producers and consumers to reduce cost and time for checking failure. The variable acceptance sampling plan based on the lifetime performance index C_L is stated as follows.

Step 1. Given n, lower specification limit L, producer's risk $\tilde{\alpha}$ and consumer's risk $\tilde{\beta}$, process level (C_{AQL}, C_{LTPD}), we can calculate the parameters (m, C_0) of acceptance sampling plan.

Step 2. Given the first-failure censored set of first order statistics of m samples of size n, $X_{(1)1} < X_{(1)2} < \cdots < X_{(1)m}$, and to calculate the value of the MLE $\hat{C}_L$ by using (12).

Step 3. Make a decision with rule as follows:

If $\hat{C}_L \geq C_0$, then lots shall be accepted, otherwise lots shall be rejected, where C_0 is decision critical value.

The parameters (m, C_0) of variable acceptance sampling plan based on the lifetime performance index C_L should be satisfied with the following two conditions:

$$P(C_{AQL}) = P\left(\hat{C}_L \geq C_0 \,|C_L = C_{AQL}\right) \geq 1 - \tilde{\alpha}, \tag{13}$$

and

$$P(C_{LTPD}) = P\left(\hat{C}_L \geq C_0 \,|C_L = C_{LTPD}\right) \leq \tilde{\beta}. \tag{14}$$

By $\frac{2m\hat{\lambda}^\beta}{\lambda^\beta} \sim \chi^2_{(2m)}$, (13) and (14) can be rewritten as follows:

$$P(C_{AQL}) = P\left\{\chi^2_{(2m)} \geq 2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_{AQL}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_0}\right]^\beta\right\} \geq 1 - \tilde{\alpha}, \tag{15}$$

and

$$P(C_{LTPD}) = P\left\{\chi^2_{(2m)} \geq 2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_{LTPD}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_0}\right]^\beta\right\} \leq \tilde{\beta}. \tag{16}$$

By using (15) and (16), we can attain

$$2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_{AQL}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_0}\right]^\beta \leq \chi^2_{(2m),\tilde{\alpha}}, \text{ and } 2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_{LTPD}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta} - C_0}\right]^\beta \geq \chi^2_{(2m),1-\tilde{\beta}}, \tag{17}$$

where $C_{AQL} > C_{LTPD}$ and $\chi^2_{(2m),p}$ represents the lower p percentile of $\chi^2_{(2m)}$.

When the $2m$ degrees of freedom is rather large, one may obtain Wilson and Hilferty approximations to the $\chi^2_{(2m),p}$, as

$$\chi^2_{(2m),p} \cong 2m\left(\frac{z_p}{3\sqrt{m}}+1-\frac{1}{9m}\right)^3, \tag{18}$$

where z_p is the lower p percentage point of the standard normal distribution [22,23].

By using Wilson and Hilferty approximations, (17) can be rewritten as follows:

$$2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{AQL}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_0}\right]^\beta \le 2m\left(\frac{z_{\tilde{\alpha}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^3, \tag{19}$$

and

$$2m\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{LTPD}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_0}\right]^\beta \ge 2m\left(\frac{z_{1-\tilde{\beta}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^3. \tag{20}$$

By using (19) and (20), we can attain

$$\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{AQL}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{LTPD}}\right]^\beta \le \left(\frac{3\sqrt{m}z_{\tilde{\alpha}}+9m-1}{3\sqrt{m}z_{1-\tilde{\beta}}+9m-1}\right)^3. \tag{21}$$

Let $K=\left[\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{AQL}}{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{LTPD}}\right]^{\beta/3}$, (21) can be rewritten as follows:

$$9(K-1)m+3\left(Kz_{1-\tilde{\beta}}-z_{\tilde{\alpha}}\right)\sqrt{m}-(K-1)\le 0. \tag{22}$$

m must satisfy (22) and $m>0$, we can attain the solution of m as follows:

$$m=\left\lceil\left[\frac{\left(Kz_{1-\tilde{\beta}}-z_{\tilde{\alpha}}\right)+\sqrt{\left(Kz_{1-\tilde{\beta}}-z_{\tilde{\alpha}}\right)^2+4(K-1)^2}}{6(K-1)}\right]^2\right\rceil \tag{23}$$

where $\lceil u\rceil = \text{Min}\{y|u\le y \text{ and } y\in Z\}$ [24].

By using (19) and (20), we can attain the decision critical value C_0 as follows:

$$\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{LTPD}}{\left(\frac{z_{1-\tilde{\beta}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^{3/\beta}} \le C_0 \le \frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{AQL}}{\left(\frac{z_{\tilde{\alpha}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^{3/\beta}}. \tag{24}$$

Let $C_1=\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{LTPD}}{\left(\frac{z_{1-\tilde{\beta}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^{3/\beta}}$ and $C_2=\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-\frac{\frac{\Gamma\left(\frac{\beta+1}{\beta}\right)}{\Delta}-C_{AQL}}{\left(\frac{z_{\tilde{\alpha}}}{3\sqrt{m}}+1-\frac{1}{9m}\right)^{3/\beta}}$, so $\forall C_0\in[C_1,C_2]$, and C_0 is linear combination of C_1 and C_2. C_0 can be expressed as follows:

$$C_0=tC_1+(1-t)C_2,\quad 0\le t\le 1. \tag{25}$$

Under $\beta=0.93, 2.8, 1, 2$; $t=0.5$, different values of $\tilde{\alpha}$, $\tilde{\beta}$, C_{AQL} and C_{LTPD} are given, we can calculate the acceptance sampling plan of parameters (m, C_0) in Tables 1-4, respectively.

TABLE 1. The acceptance sampling plan of parameters (m, C_0) under $\beta = 0.93$, $t = 0.5$

$\beta = 0.93$		$C_{AQL} = 0.88$		$C_{AQL} = 0.86$		$C_{AQL} = 0.84$	
$\tilde{\alpha}$	$\tilde{\beta}$	$C_{LTPD} = 0.70$		$C_{LTPD} = 0.65$		$C_{LTPD} = 0.60$	
		m	C_0	m	C_0	m	C_0
0.010	0.010	12	0.81005	14	0.77590	16	0.74257
0.010	0.025	10	0.80080	12	0.76572	14	0.73162
0.010	0.050	9	0.79204	11	0.75626	12	0.71902
0.010	0.075	9	0.78751	10	0.74790	12	0.71297
0.010	0.100	8	0.77984	10	0.74347	11	0.70463
0.025	0.010	10	0.81672	12	0.78416	13	0.75143
0.025	0.025	9	0.80926	10	0.77431	11	0.74019
0.025	0.050	8	0.80110	9	0.76471	10	0.72937
0.025	0.075	7	0.79346	8	0.75604	9	0.71992
0.025	0.100	7	0.78946	8	0.75133	9	0.71453
0.050	0.010	8	0.82271	10	0.79156	11	0.76028
0.050	0.025	7	0.81503	8	0.78178	9	0.74927
0.050	0.050	6	0.80621	7	0.77170	8	0.73814
0.050	0.075	6	0.81027	7	0.76588	8	0.73146
0.050	0.100	5	0.79275	6	0.75698	7	0.72231

TABLE 2. The acceptance sampling plan of parameters (m, C_0) under $\beta = 2.8$, $t = 0.5$

$\beta = 2.8$		$C_{AQL} = 1.88$		$C_{AQL} = 1.86$		$C_{AQL} = 1.84$	
$\tilde{\alpha}$	$\tilde{\beta}$	$C_{LTPD} = 1.65$		$C_{LTPD} = 1.60$		$C_{LTPD} = 1.55$	
		m	C_0	m	C_0	m	C_0
0.010	0.010	36	1.76295	31	1.72763	27	1.69216
0.010	0.025	31	1.75261	27	1.71615	23	1.67877
0.010	0.050	28	1.74285	24	1.70478	21	1.66662
0.010	0.075	25	1.73463	22	1.69621	19	1.65653
0.010	0.100	23	1.72774	20	1.68804	18	1.64891
0.025	0.010	30	1.77219	25	1.73773	22	1.70346
0.025	0.025	26	1.76200	22	1.72624	19	1.69035
0.025	0.050	22	1.75107	19	1.71413	17	1.67752
0.025	0.075	20	1.74311	17	1.70470	15	1.66662
0.025	0.100	19	1.73695	16	1.69740	14	1.65816
0.050	0.010	25	1.78140	21	1.74820	19	1.71513
0.050	0.025	21	1.77098	18	1.73645	16	1.70205
0.050	0.050	18	1.76016	16	1.72474	14	1.68877
0.050	0.075	16	1.75171	14	1.71498	12	1.67726
0.050	0.100	15	1.74500	13	1.70720	11	1.66800

5. **Numerical Examples.** In this section, we propose the acceptance sampling plan to one practical dataset and one simulated dataset. Example 5.1 considered a practical dataset consisting of $nm = 60$ ($n = 6$, $m = 10$) observations from a lifetime test of insulating fluid [19]. Example 5.2 considered a simulated dataset consisting of $nm = 190$

TABLE 3. The acceptance sampling plan of parameters (m, C_0) under $\beta = 1$, $t = 0.5$

$\beta = 1$		$C_{AQL} = 1.56$		$C_{AQL} = 1.54$		$C_{AQL} = 1.52$	
$\tilde{\alpha}$	$\tilde{\beta}$	$C_{LTPD} = 1.40$		$C_{LTPD} = 1.35$		$C_{LTPD} = 1.30$	
		m	C_0	m	C_0	m	C_0
0.010	0.010	192	1.50356	116	1.48133	73	1.46370
0.010	0.025	162	1.51130	98	1.49106	61	1.47645
0.010	0.050	138	1.51938	83	1.50155	52	1.48952
0.010	0.075	123	1.52562	74	1.50958	46	1.50035
0.010	0.100	112	1.53106	67	1.51690	42	1.50932
0.025	0.010	165	1.49623	100	1.47216	63	1.45233
0.025	0.025	137	1.50383	83	1.48170	52	1.46447
0.025	0.050	115	1.51184	69	1.49215	43	1.47803
0.025	0.075	101	1.51820	61	1.50009	38	1.48823
0.025	0.100	92	1.52345	55	1.50722	34	1.49794
0.050	0.010	143	1.48913	87	1.46334	55	1.44144
0.050	0.025	117	1.49646	71	1.47254	45	1.45279
0.050	0.050	97	1.50426	58	1.48276	37	1.46537
0.050	0.075	84	1.51061	51	1.49043	32	1.47575
0.050	0.100	75	1.51616	46	1.49714	29	1.48403

TABLE 4. The acceptance sampling plan of parameters (m, C_0) under $\beta = 2$, $t = 0.5$

$\beta = 2$		$C_{AQL} = 1.30$		$C_{AQL} = 1.28$		$C_{AQL} = 1.26$	
$\tilde{\alpha}$	$\tilde{\beta}$	$C_{LTPD} = 1.10$		$C_{LTPD} = 1.05$		$C_{LTPD} = 1.00$	
		m	C_0	m	C_0	m	C_0
0.010	0.010	69	1.20066	57	1.16578	49	1.13099
0.010	0.025	60	1.19191	50	1.15575	43	1.11963
0.010	0.050	52	1.18288	44	1.14559	37	1.10758
0.010	0.075	47	1.17606	40	1.13788	34	1.09899
0.010	0.100	44	1.17058	37	1.13126	32	1.09195
0.025	0.010	58	1.20886	48	1.17522	41	1.14159
0.025	0.025	49	1.19995	41	1.16500	35	1.12996
0.025	0.050	42	1.19075	35	1.15430	30	1.11783
0.025	0.075	38	1.18387	32	1.14654	28	1.10945
0.025	0.100	35	1.17795	29	1.13934	25	1.10091
0.050	0.010	49	1.21703	41	1.18457	35	1.15213
0.050	0.025	41	1.20821	34	1.17439	29	1.14054
0.050	0.050	35	1.19904	29	1.16375	25	1.12854
0.050	0.075	31	1.19191	26	1.15563	22	1.11900
0.050	0.100	28	1.18562	24	1.14876	20	1.11076

($n = 10$, $m = 19$) observations from a failure-censored sample of a computer-generated Weibull distribution with p.d.f. as (1) and $\lambda = 1$ and $\beta = 3$ [25].

Example 5.1. (Insulating fluid example). *Table 5 shows 60 times to breakdown in minutes of an insulating fluid subjected to high voltage stress (see [19, pp.462]). Let*

TABLE 5. Times to insulating fluid breakdown

1.89	2.75	2.15	0.70	0.20	1.70	0.18	0.82	0.06	0.78
4.03	0.00	1.08	3.82	2.12	2.17	10.6	2.06	3.57	8.71
1.54	2.17	2.57	8.11	3.97	1.82	1.63	0.49	1.13	2.1
0.31	0.66	1.17	3.17	1.56	9.99	0.71	0.02	6.63	7.21
0.66	1.99	3.87	5.55	1.34	2.24	0.93	0.50	1.08	3.83
1.30	0.64	2.80	0.80	1.49	0.55	4.75	3.72	2.44	5.13

X denote the lifetime of insulating fluid data and X have the Weibull distribution with p.d.f. and c.d.f. as (1) and (4), respectively. By using the "shape-first" approach and the least square estimation method to estimate β [26]. The least square estimation of β is $\hat{\beta} = 0.93$. We can apply the Gini statistics [27] to testing the hypothesis that the lifetime of insulating fluid data from the Weibull distribution with the p.d.f. is $f_X(x) = \frac{0.93}{\lambda^{0.93}} x^{-0.07} \exp\left[-\left(\frac{x}{\lambda}\right)^{0.93}\right]$, $x > 0$, $\lambda > 0$. Because p-value of the hypothesis testing is 0.38698 ($> \alpha^* = 0.05$), we can conclude the lifetime of insulating fluid data from the Weibull distribution with the p.d.f. is $f_X(x) = \frac{0.93}{\lambda^{0.93}} x^{-0.07} \exp\left[-\left(\frac{x}{\lambda}\right)^{0.93}\right]$, $x > 0$, $\lambda > 0$, at level $\alpha^* = 0.05$.

The times are divided into ten sets randomly. Table 6 shows the ten sets as follows.

TABLE 6. The times of ten groups

Set 1	**Set 2**	**Set 3**	**Set 4**	**Set 5**	**Set 6**	**Set 7**	**Set 8**	**Set 9**	**Set 10**
1.82	0.71	1.34	**0.7**	**0.20**	1.89	**0.18**	0.66	**0.06**	**0.78**
9.99	**0.00**	**1.08**	3.57	8.71	**0.66**	0.82	1.99	2.75	1.70
2.24	10.6	2.17	1.13	2.10	1.30	2.06	0.64	0.50	2.17
0.31	1.63	4.03	6.63	7.21	2.15	0.49	**0.02**	3.72	2.12
3.87	8.11	1.54	1.08	3.83	3.82	3.55	2.57	1.49	3.97
2.80	3.17	4.75	2.44	5.13	5.55	0.80	1.17	0.93	1.56

Based on Table 6, the first-failure censored order statistics data of $m = 10$ samples of size $n = 6$ is $\{x_{(1)1}, x_{(1)2}, \ldots, x_{(1)10}\} = \{0.00, 0.02, 0.06, 0.18, 0.20, 0.31, 0.66, 0.70, 0.78, 1.08\}$.

The acceptance sampling plan based on the lifetime performance index C_L is stated as follows.

Step 1. Given $\beta = 0.93$, $n = 6$, lower specification limit $L = 0.14$, producer's risk $\tilde{\alpha} = 0.025$ and consumer's risk $\tilde{\beta} = 0.01$, process level $(C_{AQL}, C_{LTPD}) = (0.88, 0.70)$, we can calculate that the parameters (m, C_0) of acceptance sampling plan is $(10, 0.81672)$ by Table 1.

Step 2. Given the first-failure censored order statistics of size $m = 10$ of size $n = 6$ is $\left\{x_{(1)1}, x_{(1)2}, \ldots, x_{(1)10}\right\} = \{0.00, 0.02, 0.06, 0.18, 0.20, 0.31, 0.66, 0.70, 0.78, 1.08\}$, and to calculate the value of the maximum likelihood estimator $\hat{C}_L$ by using (12) as follows:

$$\hat{C}_L = \frac{1}{1.11293}\left[\Gamma\left(\frac{0.93+1}{0.93}\right) - \frac{0.14}{2.65159}\right] = 0.88181, \quad (26)$$

where $\Delta = \sqrt{\Gamma\left(\frac{0.93+2}{0.93}\right) - \Gamma^2\left(\frac{0.93+1}{0.93}\right)} = 1.11293$ and $\hat{\lambda} = \left(\frac{6}{10}\sum_{i=1}^{10} x_{(1)i}^{0.93}\right)^{\frac{1}{0.93}} = 2.65159$.

Step 3. By $\hat{C}_L = 0.88181$ and $C_0 = 0.81672$, because $\hat{C}_L \geq C_0$, we can make a decision: The lots of insulating fluid shall be accepted.

Table 7. Order rounded observations in each set

Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
0.6794	0.4642	0.4335	0.3223	0.3980	0.4928	0.3895	0.4269	0.1556	0.2717
0.7093	0.6493	0.6131	0.5008	0.4252	0.6424	0.4324	0.5876	0.6445	0.8063
0.7574	0.8981	0.6575	0.8426	0.4723	0.6628	0.4343	0.6549	0.7609	0.8575
0.7945	0.9466	0.7156	0.8600	0.6354	0.6768	0.4673	0.6617	1.0386	0.9329
1.0529	0.9973	0.8505	0.9251	0.7495	0.9836	0.5392	0.8231	1.0645	0.9484
1.2536	1.0478	0.8574	1.0521	0.7662	1.0730	0.5462	0.9014	1.1652	1.0534
1.2892	1.0539	0.9652	1.1163	0.8472	1.0854	0.7553	0.9868	1.3060	1.0656
1.3056	1.1102	0.9682	1.2449	0.8677	1.0988	0.9615	0.9919	1.3070	1.1648
1.3110	1.5865	1.1811	1.3049	1.1278	1.4663	1.1084	1.0906	1.4282	1.1959
1.3788	1.6852	1.3116	1.3343	1.2715	1.6829	1.5057	1.4353	1.4581	1.2632
Set 11	**Set 12**	**Set 13**	**Set 14**	**Set 15**	**Set 16**	**Set 17**	**Set 18**	**Set 19**	
0.3123	0.3185	0.5083	0.6819	0.3799	0.3136	0.6077	0.2029	0.2033	
0.4303	0.4365	0.5407	0.7367	0.3882	0.4710	0.7082	0.2845	0.6991	
0.4961	0.4500	0.6074	0.8570	0.5044	0.5439	0.7779	0.5205	0.7438	
0.5151	0.5411	0.6709	0.8732	0.5385	0.6005	0.7916	0.5342	0.8027	
0.6091	0.5477	0.7886	0.9149	0.6464	0.9792	0.8685	0.7280	0.8249	
0.6201	0.8759	0.9344	0.9359	0.7970	0.9981	0.8924	0.7569	0.8677	
0.6621	0.9395	0.9947	0.9369	0.8990	1.0233	0.9838	0.8279	1.0339	
0.6898	0.9464	1.0969	0.9831	0.9807	1.1519	0.9989	0.9227	1.0882	
1.1301	1.1407	1.1323	1.1722	1.0535	1.1629	1.0789	1.1873	1.2657	
1.2171	1.2280	1.2548	1.6736	1.6670	1.4977	1.1765	1.5337	1.4722	

Example 5.2. (Simulation example). *A generated sample of 190 observations from the Weibull distribution with parameters $\lambda = 1$ and $\beta = 3$ is randomly grouped into 19 sets. The ordered observations in each set are shown in Table 7 [25]. Let X denote the lifetime of product and X have the Weibull distribution with p.d.f. and c.d.f. as (1) and (4), respectively. By using the "shape-first" approach and the least square estimation method to estimate β [26]. The least square estimation of β is $\hat{\beta} = 2.8$. We can apply the Gini statistics [27] to testing the hypothesis that the lifetime of generated sample from the Weibull distribution with the p.d.f. is $f_X(x) = \frac{2.8}{\lambda^{2.8}} x^{1.8} \exp\left[-\left(\frac{x}{\lambda}\right)^{2.8}\right]$, $x > 0$, $\lambda > 0$. Because p-value of the hypothesis testing is 0.7366 ($> \alpha^* = 0.05$), we can conclude the lifetime of generated sample from the Weibull distribution with the p.d.f. is $f_X(x) = \frac{2.8}{\lambda^{2.8}} x^{1.8} \exp\left[-\left(\frac{x}{\lambda}\right)^{2.8}\right]$, $x > 0$, $\lambda > 0$, at level $\alpha^* = 0.05$.*

Based on Table 7, the first-failure censored order statistics data of $m = 19$ samples of size $n = 10$ is $\left\{x_{(1)1}, x_{(1)2}, \ldots, x_{(1)19}\right\} = \{0.1556, 0.2029, 0.2033, 0.2717, 0.3123, 0.3136, 0.3185, 0.3223, 0.3799, 0.3895, 0.3980, 0.4269, 0.4335, 0.4642, 0.4928, 0.5083, 0.6077, 0.6794, 0.6819\}$.

The acceptance sampling plan based on the lifetime performance index C_L is stated as follows.

Step 1. Given $\beta = 2.8$, $n = 10$, lower specification limit $L = 0.25$, producer's risk $\tilde{\alpha} = 0.025$ and consumer's risk $\tilde{\beta} = 0.1$, process level $(C_{AQL}, C_{LTPD}) = (1.88, 1.65)$, we can calculate that the parameters (m, C_0) of acceptance sampling plan are $(19, 1.73695)$ by Table 2.

Step 2. Given the first-failure censored order statistics of size $m = 19$ of size $n = 10$ is $\left\{x_{(1)1}, x_{(1)2}, \ldots, x_{(1)19}\right\} = \{0.1556, 0.2029, 0.2033, 0.2717, 0.3123, 0.3136, 0.3185, 0.3223,$

$0.3799, 0.3895, 0.3980, 0.4269, 0.4335, 0.4642, 0.4928, 0.5083, 0.6077, 0.6794, 0.6819\}$, and to calculate the value of the *MLE* $\hat{C}_L$ by using (12) as follows:

$$\hat{C}_L = \frac{1}{0.34427}\left[\Gamma\left(\frac{2.8+1}{2.8}\right) - \frac{0.25}{1.00943}\right] = 1.86711, \tag{27}$$

where $\Delta = \sqrt{\Gamma\left(\frac{2.8+2}{2.8}\right) - \Gamma^2\left(\frac{2.8+1}{2.8}\right)} = 0.34427$ and $\hat{\lambda} = \left(\frac{10}{19}\sum_{i=1}^{19} x_{(1)i}^{2.8}\right)^{\frac{1}{2.8}} = 1.00943$.

Step 3. By $\hat{C}_L = 1.86711$ and $C_0 = 1.73695$, because $\hat{C}_L \geq C_0$, we can make a decision: The lots of product shall be accepted.

6. **Conclusions.** Acceptance sampling is a statistical procedure used for determining whether to accept or reject a production lot of material. The primary advantage of the variable sampling plan is that it can achieve the same operating characteristic (OC) curve with a smaller sample size compared with the attribute sampling plan. In life testing experiments, censored samples may arise in practice. The first-failure censored sampling plan has an advantage in terms of shorter test-time and a saving of resources, and Weibull distribution is useful in a great variety of applications, particularly as a model for product life. So, we consider the case of the first failure-censored sampling plan and Weibull distribution, and our study develops a variable acceptance sampling plan based on the lifetime performance index C_L. The variable acceptance sampling plan procedure is used to deal with the product acceptance decision making problem. Finally, an example and tables of the required sample size and the corresponding critical value are presented to demonstrate the applicability of the proposed variable acceptance sampling plan. The existing studies of variables sampling plan included statistical hypothesis testing with complete sample, statistical hypothesis testing with censored sample and planning of life test with cost constraint. The result of this study can be utilized to deal with non-normal quality data, determine the correct sample size with first-failure censored sampling plan rapidly, and the package of the product acceptance decision making system is finished by computer programming language easily. In future research on this problem, it would be interesting to deal with the Weibull products with series systems [19] under the progressively type II right censoring schemes.

Acknowledgements. The authors are very much grateful to the associate editor and reviewers for their suggestions and helpful comments which led to the improvement of this paper. In addition, the authors also thank the Ministry of Science and Technology, Taiwan (Plan No.: MOST 108-2221-E-415-007 and MOST 109-2221-E-415-012) and Shih Chien University, Taiwan (Plan No.: USC 107-08-01004) for the financial support.

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