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NFTSMC-BASED ROBUST CONTROL APPROACH FOR AUTONOMOUS SPACECRAFT RENDEZVOUS

Xingbo Wang[1,2], Aiwen Han[1,2], Haikuo He[3,*] and Zhifeng Gao[1,2]

[1]College of Automation
[2]College of Artificial Intelligence
Nanjing University of Posts and Telecommunications
No. 9, Wenyuan Road, Qixia District, Nanjing 210023, P. R. China
wangxbo@njupt.edu.cn; leisurewen@163.com; gaozhifeng80@126.com

[3]Department of Computer and Information Engineering
Hebei Petroleum University of Technology
No. 2, Xueyuan Road, High-Tech Development Zone, Chengde 067000, P. R. China
[*]Corresponding author: hehaikuo@cdpc.edu.cn

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Abstract. *In this paper, the problem of uncertain autonomous spacecraft rendezvous is studied using disturbance observer and nonsingular fast terminal sliding mode control (NFTSMC) techniques. Taking the accuracy of spacecraft rendezvous process into account, a disturbance observer is designed to identify the external disturbance and parameter uncertainty, which affect the spacecraft attitude stability, and the obtained estimation will be used to the compensation part design of the rendezvous controller. And then, a continuous NFTSMC-based robust control scheme is developed with the aid of Lyapunov stability criteria, which could achieve the better rendezvous performance than some existing results; the finite time stability of the closed loop system is analyzed using Lyapunov approach. Finally, a simulation example is given to illustrate the superiority of the designed scheme.*

Keywords: Spacecraft rendezvous, Disturbance observer, Finite time stability, Nonsingular fast terminal sliding mode control (NFTSMC)

1. **Introduction.** In recent years, spacecraft rendezvous has gradually become an important research content in the field of aerospace engineering, and it could be seen as the chaser spacecraft through a series of orbital maneuvers close to a target spacecraft [1,2]. The Clohessey and Wiltshire (C-W) equations are often used to describe the relative motion between two spacecraft in the close range rendezvous phase [3]. Many rendezvous control schemes have been proposed to achieve the pre-designated rendezvous mission by using the C-W equations. For instance, the problem of spacecraft rendezvous with disturbance and input saturation is investigated in [4] by using Lyapunov method. In [5], a delay-dependent feedback controller is proposed by solving the solution of the parameterized Lyapunov equation to solve the circular orbit rendezvous problem under actuator saturation case. The non-fragile robust $\mathcal{H}_\infty$ controller is designed in [6] for an uncertain spacecraft rendezvous system with poles and input constraints. Using linear matrix inequality (LMI) technique, a reliable control scheme is proposed in [7] for the spacecraft rendezvous problem with actuator partial loss of effectiveness fault. In [8], a robust $\mathcal{H}_\infty$ controller is designed for the spacecraft rendezvous system with multi-source interferences by combining Lyapunov theory and LMI technique. On this basis, an observer-based output feedback control strategy is presented in [9] for the spacecraft rendezvous system by

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combining Riccati equation and gain scheduling technique. For the spacecraft rendezvous problem in unknown actuator faulty case, an adaptive fault-tolerant control scheme is proposed in [10]. In [11], a robust control strategy is proposed for spacecraft rendezvous systems with unknown inertial parameter using both sliding mode control and neural network technology. It should be noted that most spacecraft rendezvous control schemes described above only guarantee the asymptotic stability result, and the autonomous spacecraft rendezvous control approach with finite time convergence is very limited.

On the other hand, the closed loop control system under finite time control scheme has better robustness and higher control accuracy [12]. Therefore, the finite time control scheme is more desirable for autonomous spacecraft rendezvous in actual engineering application. To achieve the finite time stability, some control strategies have been proposed recently. For example, the chattering-free full-order recursive sliding mode control is proposed in [13] for rigid spacecraft to achieve the finite time attitude synchronization. On this basis, an adaptive sliding mode disturbance observer is built in [14] for spacecraft rendezvous systems, and a finite time sliding mode control in terms of dual quaternion is developed, but the sliding mode control approaches developed in [13,14] have the annoying singularity problem. In order to eliminate this problem, some nonsingular TSM control (NTSMC) approaches have been proposed and applied to trajectory tracking control of robotic manipulator, consensus tracking of multi-agent systems and attitude control of rigid spacecraft [15-18]. Moreover, some composite approaches are developed, such as disturbance observer (DOB) based TSM control scheme for nonlinear systems subject to disturbances [19,20], extended state observer (ESO) based NTSM control scheme for a maneuvering target [21]. To the best of our knowledge, these novel composite control approaches have not been used to solve the problem of autonomous spacecraft rendezvous control.

According to the above discussion, a novel autonomous spacecraft rendezvous control scheme is proposed during the final closing phase by utilizing both DOB and NFTSMC techniques in this paper. Disturbance compensation control is critical to spacecraft rendezvous; otherwise, the rendezvous task will fail. To handle this problem, a novel DOB is designed to estimate the lumped disturbance. Then, a DOB-NFTSMC based robust controller is designed for the spacecraft rendezvous system to achieve the better disturbance rejection performance, which guarantees that the relative position and velocity could converge to a small region in finite time based on Lyapunov theory. Finally, simulation results are given to illustrate the effectiveness of the proposed DOB-NFTSMC based robust control scheme.

The rest of the paper is organized as follows. In Section 2, the spacecraft rendezvous problem formulation is provided. The disturbance observer design approach is derived in Section 3, followed by the BOD-NFTSMC based robust control scheme presented in Section 4. Section 5 shows the simulation results and Section 6 gives the conclusions.

2. **Problem Statement and Preliminaries.**

The schematic diagram of spacecraft rendezvous system in circular orbit is depicted in Figure 1; it could demonstrate that a chasing spacecraft gradually catches up to a target spacecraft in a circular orbit after the continuous maneuvering. The C-W equations are used to describe the proximity relative motion dynamic model between two spacecraft [8]

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = \frac{1}{m}\left(u_x + \omega_x\right) \\ \ddot{y} + 2n\dot{x} = \frac{1}{m}\left(u_y + \omega_y\right) \\ \ddot{z} + n^2 z = \frac{1}{m}\left(u_z + \omega_z\right) \end{cases} \tag{1}$$

where n is the constant angular velocity of the target spacecraft, m is the mass of the chaser spacecraft. x, y and z are the components of the relative position along each axis. u_x, u_y and u_z are the control input forces of the chaser spacecraft. ω_x, ω_y and ω_z denote the space environment disturbance.

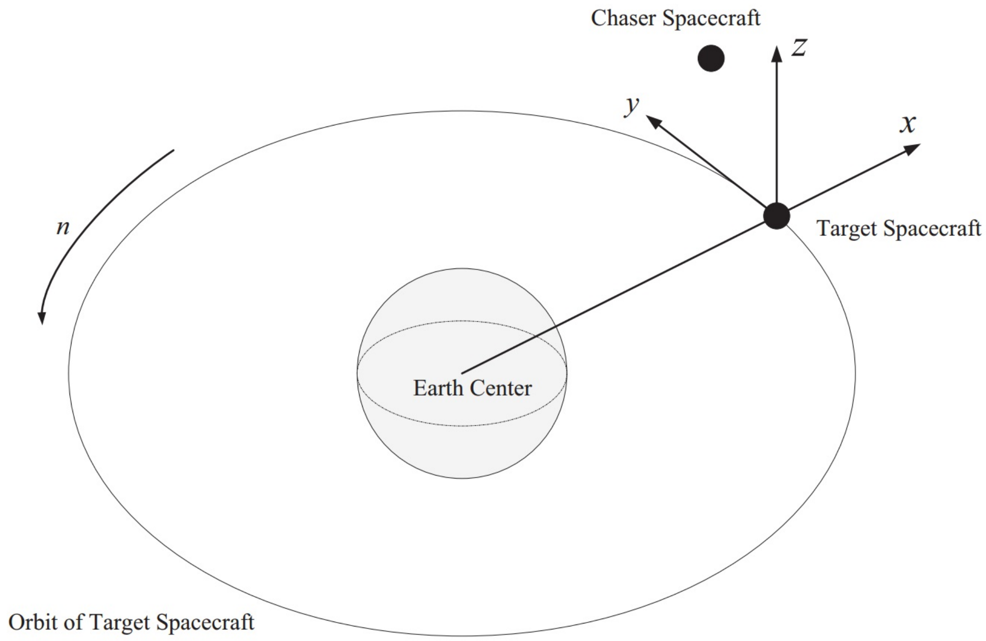

FIGURE 1. The schematic diagram of spacecraft rendezvous system

Consider sensor measurement error and space environment disturbance, the target angular velocity n is not measured precisely and it is usually modeled as [8]

$$n = n_0(1+\xi) \tag{2}$$

where n_0 is the ideal target constant angular velocity, and ξ is the uncertainty.

Define $q_1 = [x, y, z]^T$, $q_2 = [\dot{x}, \dot{y}, \dot{z}]^T$, $u = [u_x, u_y, u_z]^T$, and $\omega = [\omega_x, \omega_y, \omega_z]^T$. After some manipulations, the rendezvous system (1) could be transformed into the following form

$$\dot{q}_1 = q_2 \tag{3}$$

$$\dot{q}_2 = (A_1 + \Delta A_1)q_1 + (A_2 + \Delta A_2)q_2 + B(u+\omega) \tag{4}$$

where

$$A_1 = \begin{bmatrix} 3n_0^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n_0^2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2n_0 & 0 \\ -2n_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \frac{1}{m}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta A_1 = \begin{bmatrix} 3n_0^2(2\xi+\xi^2) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n_0^2(2\xi+\xi^2) \end{bmatrix}, \quad \Delta A_2 = \begin{bmatrix} 0 & 2\xi n_0 & 0 \\ -2\xi n_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $d \triangleq m\Delta A_1 q_1 + m\Delta A_2 q_2 + \omega$ denote the lumped disturbance; system (3)-(4) could be further rewritten as

$$\dot{q}_1 = q_2 \tag{5}$$

$$\dot{q}_2 = A_1 q_1 + A_2 q_2 + Bu + Bd \tag{6}$$

The objective of this study is to design a DOB-NFTSMC based robust control scheme to drive any arbitrary q_1 and q_2 to a desired residual set in finite time.

In the following, the assumption and lemma are introduced to derive the results of this study.

Assumption 2.1. *The lumped disturbance d is norm bounded and satisfies the inequality $\|d\| \le l_\delta$, where l_δ is a positive constant. Meanwhile, the derivative of the lumped disturbance d is assumed to be unknown but bounded.*

Lemma 2.1. [1] *There exist some positive real contants $a_1, a_2, \dots, a_n$ and $0 < p < 2$; then the following inequality holds*

$$\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^p \le \left(a_1^p + a_2^p + \cdots + a_n^p\right)^2$$

3. **The Design of Disturbance Observer.** As we all know, the uncertain perturbation and unknown external disturbance are the important factors affecting the success of spacecraft's autonomous rendezvous, which need to be real-time online identified and then are used to the compensation control design to ensure the completion of the rendezvous mission. In this section, a sliding mode disturbance observer will be designed to obtain the estimated value of the lumped disturbance d in (5)-(6).

Define a sliding mode variable

$$s_1 = \dot{q}_1 - \bar{z}$$

z has the following dynamics equation

$$\dot{\bar{z}} = A_1 q_1 + A_2 q_2 + Bu + B\hat{d} + k_1 s_1 + v \tag{7}$$

where $\hat{d}$ is the estimated value of the lumped disturbance d, $v = k_2 \mathrm{sgn}(s_1)$ is the sliding mode term, k_1 and k_2 are the gain parameters of the auxiliary system (7).

From (6) and (7), it can be derived that

$$\dot{s}_1 = B\tilde{d} - k_1 s_1 - k_2 \mathrm{sgn}(s_1) \tag{8}$$

where $\tilde{d} = d - \hat{d}$ is the error of disturbance estimation.

According to Assumption 2.1, it is assumed that the upper bound of $\dot{d}$ is the unknown positive constant scalar γ. To obtain the estimated value of the uncertainty d, the following adaptive sliding mode DOB is designed as

$$\hat{d} = w + k_3 q_2 \tag{9}$$

$$\dot{w} = k_3\left(-A_1 q_1 - A_2 q_2 - Bu - B\hat{d}\right) + (k_4 + \hat{\gamma})\,\mathrm{sgn}(v) \tag{10}$$

where w is an auxiliary variable, and $\hat{\gamma}$ is the estimated value of unknown constant γ. k_3 and k_4 are the gain parameters to be determined later.

After some manipulations, the following dynamics could be obtained as

$$\dot{\tilde{d}} = \dot{d} - k_3 B\tilde{d} - (k_4 + \hat{\gamma})\,\mathrm{sgn}\left(\tilde{d}\right) \tag{11}$$

Now, the first main result of this study is given for the proposed adaptive sliding mode DOB.

Theorem 3.1. *For the dynamics equation (7) and the disturbance observer (9)-(10), if the gain parameters are selected as*

$$k_1 > 0,\ k_2 = \|B\|\left(l_\delta + \left\|\hat{d}\right\| + \overline{\gamma}\right),\ k_3 > 0,\ k_4 > 0 \tag{12}$$

and the adaptive update algorithm is designed as

$$\dot{\hat{\gamma}} = -\sigma_1\hat{\gamma} + \sigma_0\left\|B^{-1}v\right\| \tag{13}$$

where $\bar{\gamma}$, σ_0 and σ_1 are three positive scalars, then, the disturbance error $\tilde{d}$ in (11) will converge into a small set near the origin in finite time.

Proof: The detailed proof is given in the following.

Step 1: Firstly, it is proved that $s_1 = 0$ will be reached in finite time. Choose the following Lyapunov function

$$V_1 = \frac{1}{2}s_1^T s_1 \tag{14}$$

Taking the derivative of V_1 along the trajectory (8) yields

$$\begin{aligned}\dot{V}_1 &= s_1^T \dot{s}_1 = -k_1 s_1^T s_1 - k_2\|s_1\| + s_1^T B\tilde{d} \le -k_1 s_1^T s_1 - k_2\|s_1\| + \|B\|\|s_1\| \left\|\tilde{d}\right\| \\ &= -k_1 s_1^T s_1 - \left(k_2 - \|B\| \left\|\tilde{d}\right\|\right)\|s_1\| \end{aligned} \tag{15}$$

According to (12), Inequality (15) can be rewritten as

$$\dot{V}_1 < -2k_1 V_1 - \left(k_2 - \|B\| \left\|\tilde{d}\right\|\right)\|s_1\| < -2k_1 V_1 - \sqrt{2}\|B\|\bar{\gamma}V^{\frac{1}{2}} \triangleq -\varrho_0 V_1 - \varrho_1 V^{\frac{1}{2}} \tag{16}$$

where $\varrho_0 = 2k_0$ and $\varrho_1 = \sqrt{2}\|B\|\bar{\gamma}$.

From Lemma 2.1, it can be seen that the sliding variable s_1 converges to zero in finite time $t_1 \le \frac{2}{\varrho_0}\ln\frac{\varrho_0 V_1^{\frac{1}{2}}(0)+\varrho_1}{\varrho_1}$, where $V_1(0)$ is the initial value of V_1. After that, $s = \dot{s} = 0$ holds all the time. According to the equivalent output injection principle, $B\tilde{d}$ is equivalent to the term v in (8). That is, $\left(\tilde{d}\right)_{eq} = B^{-1}v$.

Step 2: When $s_1 = 0$ has been reached after finite time t_1, the finite time stability of $\tilde{d}$ will be proved. Choose the following Lyapunov function

$$V_2 = \frac{1}{2}\tilde{d}^T\tilde{d} + \frac{1}{2\sigma_0}\tilde{\gamma}^2 \tag{17}$$

where $\tilde{\gamma} = \gamma - \hat{\gamma}$.

Taking the derivative of V_2 yields

$$\dot{V}_2 = \tilde{d}^T\dot{\tilde{d}} + \frac{1}{\sigma_0}\tilde{\gamma}\dot{\tilde{\gamma}} \le -k_3\|B\|\tilde{d}^T\tilde{d} - k_4\left\|\tilde{d}\right\| - \hat{\gamma}\left\|\tilde{d}\right\| + \tilde{d}^T\dot{d} - \frac{1}{\sigma_0}\tilde{\gamma}\dot{\hat{\gamma}} \tag{18}$$

Substituting the adaptive update algorithm (13) into (18), it can be seen that

$$\begin{aligned}\dot{V}_2 &\le -k_3\|B\|\tilde{d}^T\tilde{d} - k_4\left\|\tilde{d}\right\| - \hat{\gamma}\left\|\tilde{d}\right\| + \gamma\left\|\tilde{d}\right\| - \frac{1}{\sigma_0}\tilde{\gamma}\left(-\sigma_1\hat{\gamma} + \sigma_0\left\|\tilde{d}\right\|\right) \\ &\le -k_3\|B\|\tilde{d}^T\tilde{d} - k_4\left\|\tilde{d}\right\| + \tilde{\gamma}\left\|\tilde{d}\right\| + \frac{\sigma_1}{\sigma_0}\tilde{\gamma}\hat{\gamma} - \tilde{\gamma}\left\|\tilde{d}\right\| \\ &= -k_3\|B\|\tilde{d}^T\tilde{d} - k_4\left\|\tilde{d}\right\| + \frac{\sigma_1}{\sigma_0}\tilde{\gamma}\hat{\gamma}\end{aligned} \tag{19}$$

Note that the following inequality holds

$$\frac{\sigma_1}{\sigma_0}\tilde{\gamma}\hat{\gamma} = \frac{\sigma_1}{\sigma_0}\tilde{\gamma}(\gamma - \tilde{\gamma}) \le \frac{\sigma_1}{\sigma_0}\left(\frac{\gamma^2}{2} - \frac{\tilde{\gamma}^2}{2}\right) \tag{20}$$

Substituting Inequality (20) into (19) yields

$$\begin{aligned}\dot{V}_2 &\le -k_3\|B\|\tilde{d}^T\tilde{d} - k_4\left\|\tilde{d}\right\| + \frac{\sigma_1}{\sigma_0}\left(\frac{\gamma^2}{2} - \frac{\tilde{\gamma}^2}{2}\right) \le -k_4\left\|\tilde{d}\right\| + \frac{\sigma_1}{\sigma_0}\left(\frac{\gamma^2}{2} - \frac{\tilde{\gamma}^2}{2}\right) \\ &= -k_4\left\|\tilde{d}\right\| - \frac{\sigma_1}{2\sigma_0}\left(\tilde{\gamma}^2\right)^{\frac{1}{2}} + \frac{\sigma_1}{2\sigma_0}\left(\tilde{\gamma}^2\right)^{\frac{1}{2}} - \frac{\sigma_1}{2\sigma_0}\tilde{\gamma}^2 + \frac{\sigma_1}{2\sigma_0}\gamma^2\end{aligned} \tag{21}$$

Note that the term $\frac{\sigma_1}{2\sigma_0}\left(\tilde{\gamma}^2\right)^{\frac{1}{2}} - \frac{\sigma_1}{2\sigma_0}\tilde{\gamma}^2 + \frac{\sigma_1}{2\sigma_0}\gamma^2$ has an upper bound $\tau = \frac{\sigma_1+4\sigma_1\gamma^2}{8\sigma_0}$, and then (21) can be transformed into

$$\dot{V}_2 \le -\min\left\{k_4, \frac{\sigma_1}{\sqrt{2}\sigma_0}\right\}\left(\left\|\tilde{d}\right\| + \frac{1}{\sqrt{2}}\left(\tilde{\gamma}^2\right)^{\frac{1}{2}}\right) + \tau \tag{22}$$

In terms of Lemma 2.1, Inequality (22) could be rewritten as

$$\dot{V}_2 \le -\min\left\{k_4, \frac{\sigma_1}{\sqrt{2}\sigma_0}\right\} V^{\frac{1}{2}} + \tau \tag{23}$$

According to [23], it could be seen that the estimation error $\tilde{d}$ will be driven into a bounded set $\Omega_0 = \left\{\tilde{d} \in R^n \middle| \left\|\tilde{d}\right\| \le \frac{\tau}{(1-\varepsilon_0)\min\left\{k_4, \frac{\sigma_1}{\sqrt{2}\sigma_0}\right\}} \triangleq \bar{\tau}\right\}$ with $0 < \varepsilon_0 < 1$ in finite time $t_2 = t_1 + \frac{2V_2^{\frac{1}{2}}(t_1)}{\varepsilon_0 \min\left\{k_4, \frac{\sigma_1}{\sqrt{2}\sigma_0}\right\}}$. The proof is completed.

Remark 3.1. *Different from the existing disturbance estimation schemes developed in [14,19,20], the proposed adaptive sliding mode DOB has one obvious advantage, namely, it could achieve the finite time convergence of the disturbance error* $\tilde{d}$.

Remark 3.2. *Note that, the designed sliding mode disturbance observer (9)-(10) is a prerequisite for the composite autonomous rendezvous controller in the next section, the conclusion of Theorem 3.1 is a sufficient condition for the existence of the designed sliding mode disturbance observer (9)-(10), which plays the very important role in the design process of the composite autonomous rendezvous controller developed in this study.*

4. **NFTSMC-Based Robust Control Scheme.** The objective of this section is to design an NFTSMC-based robust controller for the considered spacecraft rendezvous system (1), such that the relative position signal q_1 could track the desired final relative position q_{1d}.

Define the following error variables

$$e_1 = q_{1d} - q_1, \quad e_2 = \dot{q}_{1d} - q_2 \tag{24}$$

Then, the following tracking error dynamic equations are derived as

$$\dot{e}_1 = e_2 \tag{25}$$

$$\dot{e}_2 = A_1 e_1 + A_2 e_2 - B(u + d) + \ddot{q}_{1d} - A_2 \dot{q}_{1d} - A_1 q_{1d} \tag{26}$$

In terms of control accuracy and convergence speed, the NFTSMC-based robust control scheme has the obvious advantage compared with the existing linear SMC, where only asymptotical stability is achieved. In this following, a novel NFTSMC surface is designed to achieve autonomous spacecraft rendezvous with high control performance.

$$s_2 = e_1 + h_1 \mathrm{sig}^{r_1}(e_1) + h_2 \mathrm{sig}^{r_2}(e_2) \tag{27}$$

where $s_2 = [s_{21}, s_{22}, s_{23}]^T$, $h_1 > 0$, $h_2 > 0$ and $1 < r_2 < 2$, $r_2 < r_1$.

The first time derivative of s_2 can be calculated as

$$\dot{s}_2 = \dot{e}_1 + h_1 r_1 \mathrm{diag}(|e_1|)^{r_1 - 1}\dot{e}_1 + h_2 r_2 \mathrm{diag}(|e_2|)^{r_2 - 1}\dot{e}_2 \tag{28}$$

Choosing a Lyapunov function $W = \frac{1}{2}e_1^T e_1$, it can be seen that

$$\dot{W} = e_1^T \dot{e}_1 = e_1^T e_2 \tag{29}$$

Meanwhile, it follows from $s_2 = 0$ that

$$\begin{aligned} e_2 &= -h_2^{-\frac{1}{r_2}} \mathrm{sig}^{\frac{1}{r_2}}(e_1 + h_1 \mathrm{sig}^{r_1}(e_1)) \\ &= -h_2^{-\frac{1}{r_2}} \mathrm{sgn}(e_1 + h_1 \mathrm{sig}^{r_1}(e_1))|e_1 + h_1 \mathrm{sig}^{r_1}(e_1)|^{\frac{1}{r_2}} \end{aligned}$$

$$= -h_2^{-\frac{1}{r_2}}\mathrm{sgn}(e_1)(|e_1| + h_1|e_1|^{r_1})^{\frac{1}{r_2}} \tag{30}$$

Substituting (30) into (29) yields

$$\begin{aligned}\dot{W} &\le -h_2^{-\frac{1}{r_2}}\sum_{i=1}^{3}|e_{1,i}|(|e_{1,i}| + h_1|e_{1,i}|^{r_1})^{\frac{1}{r_2}} \\ &\le -2^{\frac{1-r_2}{r_2}}h_2^{-\frac{1}{r_2}}\sum_{i=1}^{3}|e_{1,i}|^{\frac{1}{r_2}+1} - 2^{\frac{1-r_2}{r_2}}\left(\frac{h_1}{h_2}\right)^{\frac{1}{r_2}}\sum_{i=1}^{3}|e_{1,i}|^{\frac{r_1}{r_2}+1}\end{aligned} \tag{31}$$

According to the definition of W, it is obtained that

$$\dot{W} \le -2^{\frac{1-r_2}{2r_2}}3^{-\frac{1}{r_2}}h_2^{-\frac{1}{r_2}}W^{\frac{r_2+1}{2r_2}} - 2^{\frac{1-r_2}{2r_2}}3^{-\frac{r_1}{r_2}}\left(\frac{h_1}{h_2}\right)^{\frac{1}{r_2}}W^{\frac{r_1+r_2}{2r_2}}$$

According to the range of gains (27), it is known that $\frac{2}{4} \le \frac{r_2+1}{2r_1} \le 1$, $\frac{r_1+r_2}{2r_2} > 1$. Then, it is concluded that the error e_1 will reach zero in a finite time.

Theorem 4.1. *Consider the uncertain rendezvous system (1) and the designed NFTSMC surface (27), if the NFTSMC-based robust control scheme is designed as*

$$\begin{aligned}u = B^{-1}\bigg(&-A_1q_1 - A_2q_2 + \ddot{q}_{1d} + h_2^{-1}r_2^{-1}\mathrm{sig}^{2-r_2}(e_2) + \frac{h_1r_1}{h_2r_2}\mathrm{sig}^{2-r_2}(e_2)\,\mathrm{diag}(|e_1|)^{r_1-1} \\ &+ l_1s_2 + l_2\mathrm{sig}^{\rho}(s_2) - \hat{d}\bigg)\end{aligned} \tag{32}$$

where $l_1 > 0$ and $l_2 > 0$ are two positive scalars, $0 < \rho < 1$. Then, the sliding mode variable s_2 will converge to the following region in finite time

$$|s_2| \le \chi = \min(\chi_1, \chi_2), \quad \chi_1 = \frac{\bar{\tau}}{l_1m}, \ \chi_2 = \left(\frac{\bar{\tau}}{l_2m}\right)^{\frac{1}{\rho}}$$

$\bar{\tau}$ has been defined in Ω_0 on page 6.

$$|e_1| \le \left(\frac{\chi}{h_1}\right)^{\frac{1}{r_1}}, \quad |e_2| \le \left(\frac{\chi}{h_2}\right)^{\frac{1}{r_2}}$$

Proof: Consider the following Lyapunov function

$$V_3 = \frac{1}{2}s_2^Ts_2 \tag{33}$$

Substituting (28) into the derivative of (33) yields

$$\begin{aligned}\dot{V}_3 &= s_2^T\dot{s}_2 = s_2^T\left[\dot{e}_1 + h_1r_1\,\mathrm{diag}\left(|e_1|\right)^{r_1-1}\dot{e}_1\right] + s_2^T\left[h_2r_2\,\mathrm{diag}\left(|e_2|\right)^{r_2-1}\dot{e}_2\right] \\ &= s_2^T\left[\dot{e}_1 + h_1r_1\,\mathrm{diag}\left(|e_1|\right)^{r_1-1}\dot{e}_1\right] \\ &\quad + s_2^T\left[h_2r_2\,\mathrm{diag}\left(|e_2|\right)^{r_2-1}\left(\ddot{q}_{1d} - (A_1q_1 + A_2q_2 + Bu + Bd)\right)\right]\end{aligned}$$

Substituting (32) into the above $\dot{V}_3$ yields

$$\begin{aligned}\dot{V}_3 &= s_2^T\left(\dot{e}_1 + h_1r_1\,\mathrm{diag}\left(|e_1|\right)^{r_1-1}\dot{e}_1\right) - s_2^Th_2r_2\,\mathrm{diag}\left(|e_2|\right)^{r_2-1}\bigg(h_2^{-1}r_2^{-1}\mathrm{sig}^{2-r_2}(e_2) \\ &\quad + l_1s_2 + l_2\mathrm{sig}^{\rho}(s_2) + \frac{h_1r_1}{h_2r_2}\mathrm{sig}^{2-r_2}(e_2)\mathrm{diag}(|e_1|)^{r_1-1} + B\tilde{d}\bigg) \\ &= -s_2^Th_2r_2\,\mathrm{diag}(|e_2|)^{r_2-1}\left(l_1s_2 + l_2\mathrm{sig}^{\rho}(s_2) + B\tilde{d}\right)\end{aligned}$$

$$= -h_2 r_2 l_1 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} s_{2i}^2 - h_2 r_2 l_2 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}|^{\rho+1} + \frac{h_2 r_2}{m} \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}| \left|\tilde{d}_i\right| \tag{34}$$

In terms of the designed finite time disturbance observer, the disturbance error $\tilde{d}$ after the time t_2 converges to the range τ, it follows from (34) that

$$\dot{V}_3 \le -h_2 r_2 l_1 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} s_{2i}^2 - h_2 r_2 l_2 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}|^{\rho+1} + \frac{h_2 r_2 \tau}{m} \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}| \tag{35}$$

In the following, (35) will be divided into two cases for discussion.

Case 1: Inequality (35) can be rewritten as the following form

$$\dot{V}_3 \le -h_2 r_2 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}| \left(l_1 - \frac{\bar{\tau}}{m|s_{2i}|}\right) |s_{2i}| - h_2 r_2 l_2 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} |s_{2i}|^{\rho+1} \tag{36}$$

If $|s_i| > \frac{\bar{\tau}}{k_1 m}$ and $e_2 \ne 0$, then there exist two positive scalars c_1 and c_2, such that the following inequality holds

$$\begin{aligned} \dot{V}_3 &\le -c_1 \sum_{i=1}^{3} |s_{2i}|^2 - c_2 \sum_{i=1}^{3} |s_{2i}|^{1+\rho} \le -c_1 \sum_{i=1}^{3} |s_{2i}|^2 - c_2 \sum_{i=1}^{3} \sqrt{\left(|s_{2i}|^2\right)^{1+\rho}} \\ &= -c_1 V_3 - c_2 V_3^{\frac{1+\rho}{2}} \end{aligned} \tag{37}$$

According to Lemma 2.1, it can be seen from (37) that the finite time stability is guaranteed. Namely, the tracking error system will converge to the following region in finite time

$$|s_{2i}| \le \frac{\bar{\tau}}{l_1 m} \tag{38}$$

Case 2: Inequality (35) can also be rewritten as the following form

$$\dot{V} \le -h_2 r_2 l_1 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} s_{2i}^2 - h_2 r_2 \sum_{i=1}^{3} |e_{2i}|^{r_2-1} \left(l_2 - \frac{\bar{\tau}}{m|s_{2i}|^{\rho}}\right) |s_{2i}|^{1+\rho} \tag{39}$$

If $|s_i| > \left(\frac{\bar{\tau}}{l_2 m}\right)^{1/\rho}$ and $e_2 \ne 0$, then there exist two positive scalars c_3 and c_4, such that the following inequality holds

$$\begin{aligned} \dot{V} &\le -c_3 \sum_{i=1}^{3} |s_{2i}|^2 - c_4 \sum_{i=1}^{3} |s_{2i}|^{1+\rho} \le -c_3 \sum_{i=1}^{3} |s_{2i}|^2 - c_4 \sum_{i=1}^{3} \sqrt{(|s_{2i}|^2)^{1+\rho}} \\ &= -c_3 V - c_4 V^{\frac{1+\rho}{2}} \end{aligned} \tag{40}$$

Similar to Case 1, the tracking error system will converge to the following range in finite time

$$|s_{2i}| \le \left(\frac{\bar{\tau}}{k_2 m}\right)^{\frac{1}{\rho}} \tag{41}$$

The last step is to show that $e_2 = 0$ is not an attractor in the reaching stage.

Substituting (32) into (26) for $e_2 = 0$, it could be obtained as

$$\dot{e}_2 = -l_1 s_2 - l_2 \text{sig}^{\rho}(s_2) + B\left(d - \hat{d}\right) = -l_1 s_2 - l_2 \text{sig}^{\rho}(s_2) + B\tilde{d} \tag{42}$$

namely

$$\dot{e}_{2i} = \begin{cases} -\left(l_1 - \dfrac{\tilde{d}_i}{ms_{2i}}\right) s_{2i} - l_2 \mathrm{sig}^\rho(s_{2i}) \neq 0 & \text{for } |s_{2i}| > \dfrac{\tau}{k_1 m} \\ -l_1 s_{2i} - \left(l_2 - \dfrac{\tilde{d}_i}{m \cdot \mathrm{sig}^\rho(s_{2i})}\right) \mathrm{sig}^\rho(s_{2i}) \neq 0 & \text{for } |s_{2i}| > \left(\dfrac{\tau}{l_2 m}\right)^{\frac{1}{\rho}} \end{cases} \tag{43}$$

where $\tilde{d}_i$ is the ith component of vector $\tilde{d}$.

It means that the finite time reachability of s_2 could also be guaranteed if $e_2 = 0$.

In the following, the finite time reachability of the tracking error e_1 and its derivative e_2 will be further analyzed. When the sliding mode variable s_2 enters the region $|s_2| < \chi$, we have

$$e_{1i} + h_1 \mathrm{sig}^{r_1}(e_{1i}) + h_2 \mathrm{sig}^{r_2}(e_{2i}) = \varphi_i, \ |\varphi_i| \leq \chi \tag{44}$$

According to $e_{2i} \neq 0$, (44) could be rewritten as

$$e_{1i} + h_1 \mathrm{sig}^{r_1}(e_{1i}) + \left(h_2 - \frac{\varphi_i}{\mathrm{sig}^{r_2}(e_{2i})}\right) \mathrm{sig}^{r_2}(e_{2i}) = 0 \tag{45}$$

If $\left(h_2 - \frac{\varphi_i}{\mathrm{sig}^{r_2}(e_{2i})}\right) > 0$, (45) is the same as (27), it could be seen that the velocity error will converge to the following region in finite time

$$|e_{2i}| \leq \left(\frac{\varphi_i}{h_2}\right)^{\frac{1}{r_2}} \leq \left(\frac{\chi}{h_2}\right)^{\frac{1}{r_2}} \tag{46}$$

Similarly, according to $e_{2i} \neq 0$, (44) could be rewritten as

$$e_{1i} + \left(h_1 - \frac{\varphi_i}{\mathrm{sig}^{r_1}(e_{1i})}\right) \mathrm{sig}^{r_1}(e_{1i}) + h_2 \mathrm{sig}^{r_2}(e_{2i}) = 0 \tag{47}$$

Using the same analysis as that in (45), if $\left(h_1 - \frac{\varphi_i}{\mathrm{sig}^{r_1}(e_{1i})}\right) > 0$, it follows from (34) that the relative position tracking error will converge to the following region in finite time

$$|e_{1i}| \leq \left(\frac{\varphi_i}{h_1}\right)^{\frac{1}{r_1}} \leq \left(\frac{\chi}{h_1}\right)^{\frac{1}{r_1}} \tag{48}$$

This completes the proof.

Remark 4.1. *To reduce the chattering in the designed NFTSMC based robust control scheme (32), as discussed in [20-22], the discontinuous* $\mathrm{sign}(s_i)$ *could be replaced with the continuous saturation functions* $\mathrm{sat}(s_i/\omega_i)$ $(i = 1, 2, 3)$*, where* $\mathrm{sat}(x) = x$ *if* $|x| < 1$ *and* $\mathrm{sat}(x) = \mathrm{sign}(x)$ *otherwise.*

Remark 4.2. *Compared with the traditional linear SMC used in [24-26], an NFTSMC-based robust control scheme has been proposed in this paper to study the position tracking problem of autonomous spacecraft rendezvous system, it could improve the control performance of spacecraft rendezvous process, which is mainly due to the finite time disturbance observer design and the application of NFTSMC technology.*

5. **Numerical Example.** In this section, the final approach phase of a spacecraft rendezvous mission is numerically simulated to demonstrate the effectiveness of the designed control scheme in this study. In a rendezvous mission, it is assumed that the target spacecraft is moving along a geosynchronous orbit in constant angular velocity, the chaser spacecraft is in the final approaching phase, and the two spacecraft are relatively static in initial zero relative velocity. Based on the above descriptions, the simulation will be

Table 1. The simulation parameters

Variable name	Value
The mass of the chaser	$m = 300$ kg
The target constant angular velocity	$n_0 = 7.2722 \times 10^{-5}$ rad/s
The initial relative position	$q_1(0) = [60m, 55m, 50m]^T$
The initial relative velocity	$q_2(0) = [0, 0, 0]^T$
The uncertainty parameter	$\xi = 13.751 \cos(0.01t)$
The desired final relative position	$q_{1d} = [5m, 0, 0]^T$
The external disturbance	$\omega_x = \omega_y = \omega_z = 0.01 \sin(0.01t)$

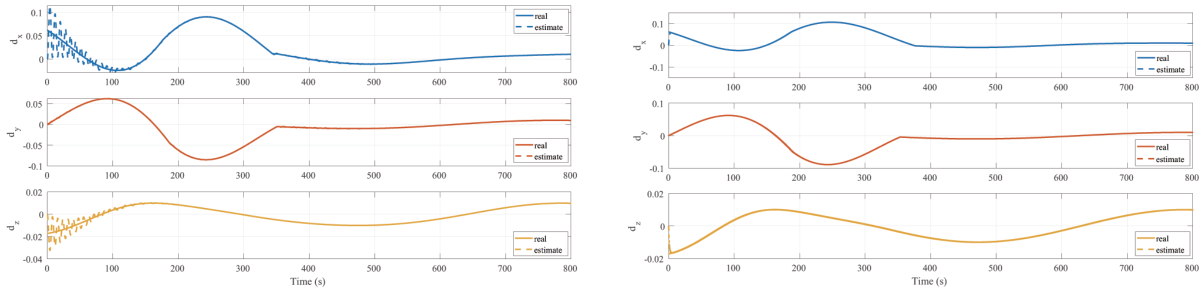

Figure 2. The disturbance estimation curves using ESO approach (left) and DOB approach (right)

performed in Matlab simulink platform and the simulation conditions are summarized as in Table 1.

In order to make the simulation more realistic and fair, the control input torque is limited to $0.5N$. The gain parameters of the designed DOB (9)-(10) and NFTSMC-based robust controller (32) are selected as $k_1 = 35$, $k_2 = 1$, $k_3 = 1$, $k_4 = 0.05$, $\sigma_0 = 0.1$, $\sigma_1 = 4$, $h_1 = 0.7$, $h_2 = 300$, $r_1 = 1.4$, $r_2 = 1.2$, $l_1 = 0.02$, $l_2 = 0.001$ and $\rho = 0.5$.

For the aim of simulation comparisons, the ESO-NTSMC based robust controller developed in [1] and the DOB-NFTSMC based robust controller developed in this paper are used to control the considered spacecraft rendezvous system respectively. The disturbance estimation curves are shown in Figure 2. From Figure 2, it is not difficult to find that the DOB designed in this paper has better disturbance estimation performance than the ESO designed in [1], and it is very important to enhance the anti-disturbance ability of the closed loop rendezvous systems. The relative position output curves are depicted in Figure 3. It is easily seen from Figure 3 that the accuracy of the x, y, and z positional accuracy using DOB-NFTSMC based robust control scheme is 0.05, 0.2, and 0.2, which are better than that using the ESO-NTSMC based robust control scheme. The relative velocity output curves are depicted in Figure 4, and it is not difficult to find from Figure 4 that the speed accuracy using DOB-NFTSMC based robust control scheme is 5×10^{-4}, which is better than the speed accuracy using the ESO-NTSMC based robust control scheme. The control input curves along each axis are shown in Figure 5, it could be seen that the control input curves depicted in Figure 5 using the DOB-NFTSMC based robust control scheme do not exceed the limit and there is no chattering. By comparisons, it is known that the disturbance observer and the finite time NFTSMC based robust controller designed in this study could achieve the satisfactory anti-disturbance ability and improve the control precision.

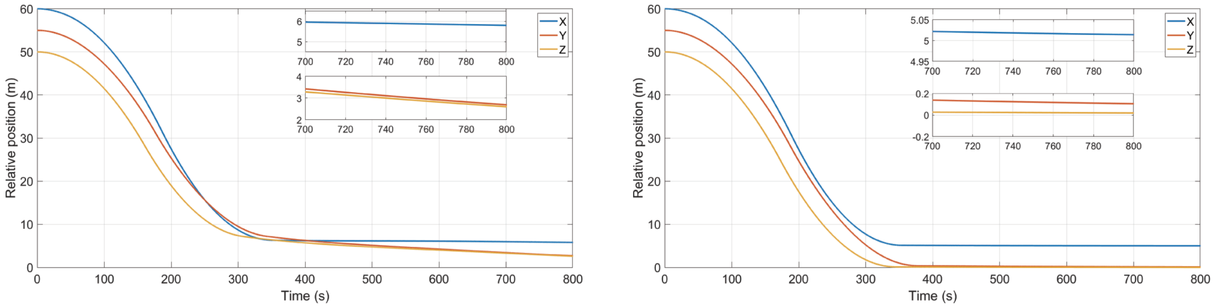

FIGURE 3. (color online) The relative position output curves using ESO-NTSMC based robust controller (left) and DOB-NFTSMC based robust controller (right)

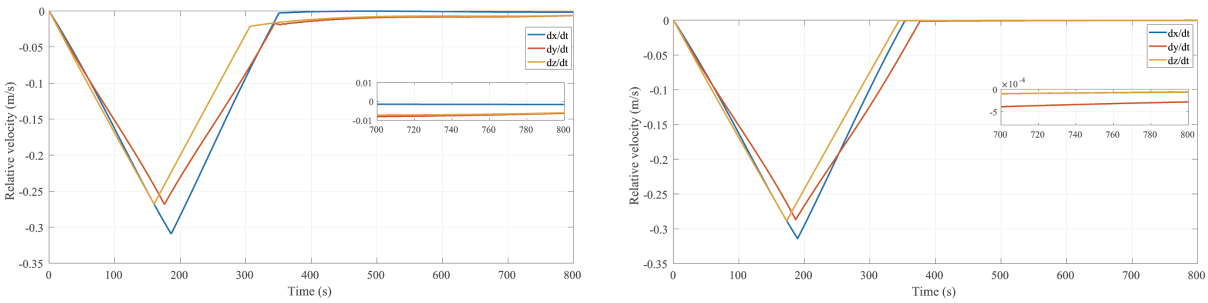

FIGURE 4. (color online) The relative velocity output curves using ESO-NTSMC based robust controller (left) and DOB-NFTSMC based robust controller (right)

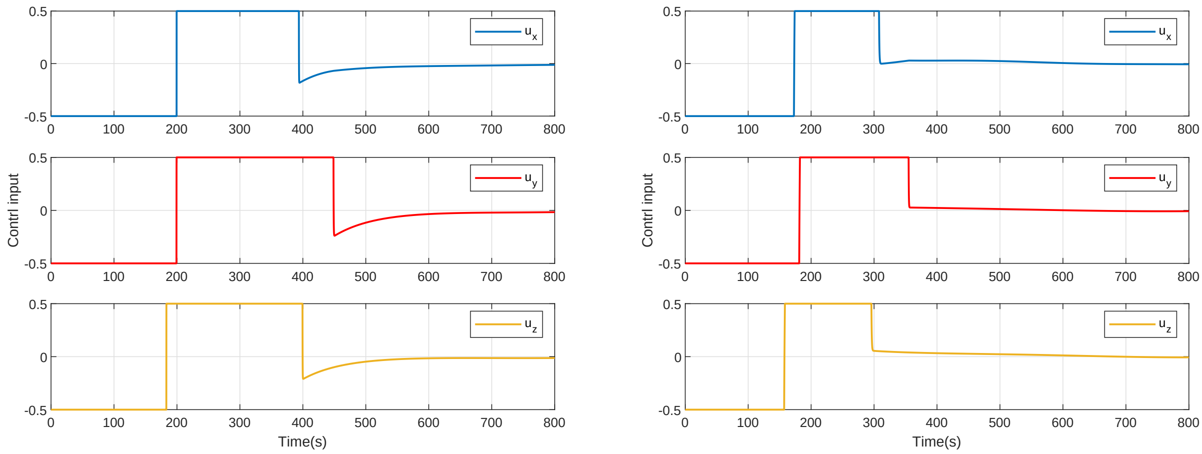

FIGURE 5. The control input curves using ESO-NTSMC based robust controller (left) and DOB-NFTSMC based robust controller (right)

6. **Conclusions.** In this paper, a novel DOB-NFTSMC based robust control scheme is proposed for the circular orbital autonomous spacecraft rendezvous system subject to model uncertainty and external disturbance. The finite time disturbance observer is designed to identify the lumped disturbance, and the NFTSMC technique is used to design the robust controller for the considered uncertain spacecraft rendezvous system. Numerical simulations are given to illustrate the feasibility of the proposed control approach.

In future work, fault tolerant control problem of the autonomous spacecraft rendezvous system in actuator or sensor faulty case will be our research interests.

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Author Biography

Xingbo Wang received the Ph.D. degree from Shandong University, Jinan, China, in 2011. Since 2012, he has been a Lecturer with Nanjing University of Posts and Telecommunications. His main research interests include optimal estimation and intelligent control of robots.

Aiwen Han received the B. Eng. degree from Wuhan Textile University, China in 2016. She is now pursuing a Master degree in Control Engineering from Nanjing University of Posts and Telecommunications. Her research interests include fault tolerant control with its application in flight control system.

Haikuo He received the master degree from Hebei University of Science and Technology, China in 2010. She is currently an associate professor with Hebei Petroleum University and Technology, China. Her research interests include fault tolerant control with its application in nonlinear systems.

Zhifeng Gao received the Ph.D. degree from Nanjing University of Aeronautics and Astronautics, China, in 2011. He is currently an Associate Professor with Nanjing University of Posts and Telecommunications, China. His research interests include fault tolerant control with its application in nonlinear systems.