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DATA-DRIVEN REGULATOR PID CONTROL USING A MODEL-REFERENCE ROBUST TUNING METHOD

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Abstract. *A new data-driven model-reference robust Proportional-Integral-Derivative (PID) control is proposed for a regulator system. In the proposed method, the PID parameters are decided such that a prescribed robust stability is achieved. Furthermore, the disturbance rejection performance is optimized. Since the proposed method is designed as a model reference problem, where the reference model and the controller parameters are optimized, trade-off design between robust stability and regulator performance is achieved by selecting the prescribed robust stability. Finally, the effectiveness of the proposed method is demonstrated through numerical examples.*

Keywords: Data-driven, Model-reference robust tuning, Regulator, PID control

1. **Introduction.** Control system design methods can be divided primarily into two types: model-based methods and data-driven methods. Model-based methods require a plant model, whereas data-driven methods are model free [1, 2]. In data-driven approaches, the control performance is optimized directly from the controlled data. Even model-based approaches may have many advantages, data-driven control methods have attracted a great deal of attention because of the convenience of being model free.

As non-iterative data-driven tuning methods, the Virtual Reference Feedback Tuning (VRFT) method [3], the Fictitious Reference Iterative Tuning (FRIT) method [4], and the Non-iterative Correlation based Tuning (NCbT) method [5] have been proposed. The NCbT and VRFT methods have been proposed open-loop data-based methods, and the FRIT method is a closed-loop data-based method. Furthermore, the FRIT method has been extended to the Extended-FRIT (E-FRIT) method [6] so that reference model parameters and the control performance are simultaneously optimized.

The Proportional-Integral-Derivative (PID) [7, 8, 9, 10, 11, 12, 13] has been widely used in industry because of its usefulness. The control performance of PID control depends on the selection of the PID parameters, and there are numerous studies on the design of PID parameters. In the model-based method, the robust stability can be directly designed [14], and robust PID control methods using the sensitivity function have been designed [15, 16, 17, 18, 19]. Furthermore, data-driven model-reference robust PID control design has also been proposed [20].

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In the design method, the frequency response is estimated using the controlled data, and hence the robust stability is designed directly from the controlled data. However, in the conventional method, only the servo performance is optimized, and the regulator optimization method has not been studied. Although data-driven disturbance rejection methods have been proposed [21, 22, 23], robust stability has not been taken into account. Therefore, the present study proposes a new data-driven regulator optimization approach. In the proposed method, the controller parameters are decided such that the disturbance rejection performance is optimized subject to the prescribed robust stability.

The rest of the manuscript is organized as follows. Section 2 introduces a control law used in the proposed method, and Section 3 formulates the problem addressed in the present study. The proposed design method is given in Section 4, where the controller parameters are decided using initial input and output data. Section 5 shows numerical examples, and concluding remarks are given in Section 6.

2. **Proportional-Integral-Derivative Control System.** Consider the block diagram illustrated as Figure 1, in which $r(k)$ is a reference input at step k, $y(k)$ is a plant output, $e(k)$ is the control error, $u(k)$ is a control input, and $d(k)$ is a disturbance.

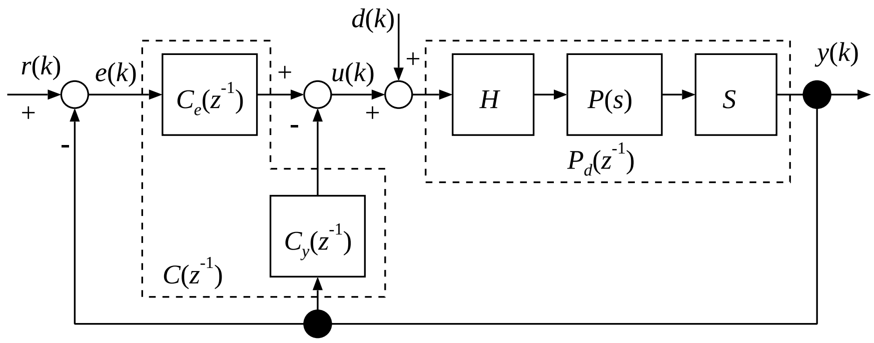

FIGURE 1. Block diagram of a discrete-time control system

In addition, $P(s)$ denotes a continuous-time plant, the dynamics of which is unknown. The continuous-time plant output is sampled by the sampler S, and the discrete-time control input is converted to a continuous-time signal by a holder H. Therefore, $P_d(z^{-1})$ denotes the unknown discrete-time plant model, and the discrete-time plant output is given as follows:

$$y(k) = P_d\left(z^{-1}\right)(u(k) + d(k)) \tag{1}$$

In the controlled system, a discrete-time PID control law is designed:

$$u(k) = C_e\left(z^{-1}\right)e(k) - C_y\left(z^{-1}\right)y(k) \tag{2}$$

$$C_e\left(z^{-1}\right) = K_p + K_i\left(\frac{T_s}{1 - z^{-1}}\right)$$

$$C_y\left(z^{-1}\right) = K_d\left(\frac{1 - z^{-1}}{T_s}\right)$$

$$e(k) = r(k) - y(k)$$

where K_p, K_i, and K_d are the proportional gain, the integral gain, and the derivative time, respectively, and T_s denotes the sampling interval. In addition, z^{-1} is the backward shift operator, and $z^{-1}y(k) = y(k-1)$.

Substituting Equation (2) into Equation (1), the plant output of the closed-loop system in discrete time is represented as follows:

$$y(k) = G_{yr}\left(z^{-1}\right) r(k) + G_{yd}\left(z^{-1}\right) d(k) \tag{3}$$

$$G_{yr}\left(z^{-1}\right) = \frac{C_e\left(z^{-1}\right) P_d\left(z^{-1}\right)}{1 + C\left(z^{-1}\right) P_d\left(z^{-1}\right)} \tag{4}$$

$$G_{yd}\left(z^{-1}\right) = \frac{P_d\left(z^{-1}\right)}{1 + C\left(z^{-1}\right) P_d\left(z^{-1}\right)} \tag{5}$$

$$C\left(z^{-1}\right) = C_e\left(z^{-1}\right) + C_y\left(z^{-1}\right)$$

3. **Problem Formulation.** In the present study, the PID control law is designed such that the regulator performance is optimized subject to a prescribed stability margin. To this end, the PID parameters are decided by solving a constrained optimization problem, which consists of a constraint condition for ensuring robustness and a performance index. In this section, the constraint condition and the performance index are defined in Subsections 3.1 and 3.2, respectively.

3.1. **Robustness constraint condition.** The stability margin is quantitatively expressed using the sensitivity function and is prescribed by the designer [15, 16]. The sensitivity function is described as follows:

$$S_f\left(z^{-1}\right) = \frac{1}{1 + C\left(z^{-1}\right) P_d\left(z^{-1}\right)} \tag{6}$$

Using the sensitivity function, the constraint that is to be satisfied by designing the controller is

$$\left|M_s - M_s^d\right| = 0 \tag{7}$$

$$M_s = \max_{\omega} \left|S_f\left(e^{-j\omega}\right)\right|$$

where M_s denotes the maximum value of the sensitivity function. In the proposed method, M_s^d is assigned by the designer, and the stability margin is adjusted arbitrarily.

The design range of M_s^d is recommended to be from 1.4 to 2.0 [10]. The stability margin with large M_s is smaller than that with small M_s, and the tracking performance with large M_s is better than that with small M_s. Therefore, the relationship between M_s and the tracking performance is a trade-off [15, 16, 20].

The objective of the present study to obtain the PID parameters that satisfy the constraint condition (Equation (7)) and minimize the objective function (Equation (8)). In the proposed method, the objective is achieved directly from input and output data without using a plant model.

3.2. **Performance objective function.** In the present study, as the model reference problem shown in Figure 2, a regulator system is designed for attenuating the deviation caused by the disturbance. In Figure 2, $M_d(z^{-1})$ denotes a reference model for the desired relationship from the disturbance to the output. The objective function is thus defined as follows:

$$J = \frac{1}{N} \sum_{k=0}^{N} \varepsilon(k)^2 \tag{8}$$

$$\varepsilon(k) = \left(G_{yd}\left(z^{-1}\right) - M_d\left(z^{-1}\right)\right) d(k)$$

where N is the data size of the evaluation, and $G_{yd}(z^{-1})$ is given by Equation (5).

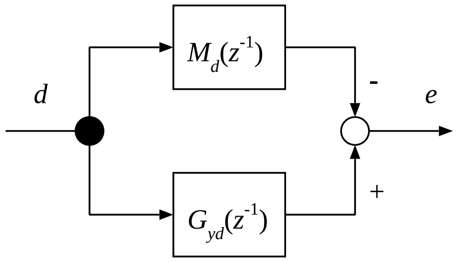

FIGURE 2. Block diagram of a model reference regulator problem

4. **Proposed Data-Driven Regulator Design.** In order to realize a data-driven robust PID control system, the constraint condition is represented using closed-loop controlled data in Subsection 4.1, and a new data-driven regulator design method is proposed in Subsection 4.2.

4.1. **Constraint condition using controlled data.** In the constraint condition, unknown $P_d(z^{-1})$ is required to calculate M_s, and hence the constraint condition is not straightforwardly realized without $P_d(z^{-1})$. In order to resolve this problem, M_s is directly obtained from closed-loop controlled data. In the present study, as the disturbance, a unit step function $d_0(k)$ is explicitly added to the control input, and the initial control input $u_0(k)$ and plant output $y_0(k)$ are obtained, where it is assumed that there is no disturbance except the explicitly added input disturbance. The frequency response is estimated from the collected closed-loop data based on [24].

In the estimation method, a bandpass filter is introduced:

$$F(s) = \frac{T_1 s}{(T_1 s + 1)(T_2 s + 1)} \tag{9}$$
$$T_1 = 100T_s$$
$$T_2 = 10T_s$$

Here, $F(s)$ is not directly used and is represented for designing a discrete-time filter. $F(s)$ is actually discretized to $F_d(z^{-1})$ with the sampling interval T_s, and the impulse response $f(k)$ of $F_d(z^{-1})$, is obtained. Using $f(k)$, the following convolution data can be obtained:

$$u_f(k) = u_0(k) * f(k) \tag{10}$$
$$y_f(k) = y_0(k) * f(k) \tag{11}$$
$$d_f(k) = d_0(k) * f(k) \tag{12}$$

Consequently, the frequency response of the controlled plant is estimated as follows:

$$\hat{P}\left(e^{-j\omega}\right) = \frac{Y_f(\omega)}{U_f(\omega) + D_f(\omega)} \tag{13}$$

where $U_f(\omega)$, $Y_f(\omega)$, and $D_f(\omega)$ are the Discrete Fourier Transforms (DFTs) of $u_f(k)$, $y_f(k)$, and $d_f(k)$, respectively.

Therefore, the constraint used in the present study is represented as follows, instead of by Equation (7):

$$\left|\hat{M}_s - M_s^d\right| = 0 \tag{14}$$

$$\hat{M}_s = \max_{\omega}\left|\hat{S}_f\left(e^{-j\omega}\right)\right|$$

$$\hat{S}_f\left(e^{-j\omega}\right) = \frac{1}{1 + C(e^{-j\omega})\hat{P}(e^{-j\omega})} \tag{15}$$

The estimated response Equation (13) used in Equation (15) is only demanded for achieving the prescribed robust stability and is not used for minimizing the performance objective function.

4.2. **Proposed data-driven regulator design.** Since $P_d(z^{-1})$ is included not only in the constraint condition (Equation (6)) but also in the objective function (Equation (8)), the controller parameters are decided such that J is minimized directly from the controlled data. In the present study, the controller parameters are optimized using the initial data u_0, y_0, and d_0. To this end, a new objective function is defined as follows:

$$J^* = \frac{1}{N}\sum_{k=0}^{N} \varepsilon^*(k)^2 \tag{16}$$

$$\varepsilon^*(k) = y_0(k) - y^*(k) \tag{17}$$

$$y^*(k) = M_r\left(z^{-1}\right) r^*(k) + M_d\left(z^{-1}\right) d_0(k)$$

where $M_r(z^{-1})$ is a reference model from $r(k)$ to $y(k)$, and $r^*(k)$ is the fictitious reference input defined as follows:

$$r^*(k) = C_e\left(z^{-1}\right)^{-1}\left\{u_0(k) + C\left(z^{-1}\right) y_0(k)\right\} \tag{18}$$

The reason for minimizing Equation (16) instead of Equation (8) is given in Appendix A.

From the relationship between Equation (4) and Equation (5), the reference model $M_r(z^{-1})$ is described as follows:

$$M_r\left(z^{-1}\right) = M_d\left(z^{-1}\right) C_e\left(z^{-1}\right)$$

Furthermore, from the definition of $r^*(k)$, $y^*(k)$ is rewritten as follows:

$$y^*(k) = M_d\left(z^{-1}\right)\left\{u_0(k) + C\left(z^{-1}\right) y_0(k) + d_0(k)\right\}$$

The reference model $M_d(z^{-1})$ is designed using the conventional regulator design method [21]:

$$M_d(s) = \frac{1}{K_m}\frac{s}{(T_0 s + 1)^{(B_0+2)}} e^{-L_0 s}$$

The designed continuous-time reference model is actually discretized with the sampling interval T_s, where the design parameters K_m, T_0, B_0, and L_0 are estimated simultaneously with the controller parameters, where B_0 is restricted to an integer.

As a result, the proposed data-driven regulator problem is formulated as a constrained optimization problem given as follows:

$$\min_{K_p, K_i, K_d, K_m, T_0, B_0, L_0} J^* \tag{19}$$

$$\text{subject to} \left|\hat{M}_s - M_s^d\right| = 0$$

In the proposed data-driven regulator design, the constrained optimization problem is solved by using only the one-shot input-output data without the controlled plant. Consequently, the PID parameters and the reference model parameters are simultaneously optimized subject to the prescribed stability margin.

In conventional data-driven methods, since the constrained optimization problem (Equation (19)) is not used, only either the regulator optimization or robust design is achieved. In the conventional method [21], assigned stability margin is not directly achieved, although it may be adjusted by selecting a design parameter.

5. **Numerical Example.** As a controlled plant, the following continuous-time model is given:

$$P(s) = \frac{1}{2s+1}e^{-0.5s} \tag{20}$$

where the dynamics is assumed to be unknown. The sampling interval is set as $T_s = 0.01$ s.

In the proposed method, since the input and output data obtained from a controlled closed-loop system are used, the closed-loop system must be stable. However, since the dynamic properties of the controlled plant are unknown, the initial PID parameters are selected by trial and error. The initial controlled data $u_0(k)$ and $y_0(k)$ are obtained using $K_p^0 = 0.03$, $K_i^0 = 0.12$, and $K_d^0 = 0.05$.

Here, $r(k) = 0$, i.e., the control input is subjected to a unit step disturbance, where the mandatory input disturbance $d_0(k)$ is assumed to be known. The controlled data disturbed by $d_0(k)$ are plotted by the dashed lines in Figure 3.

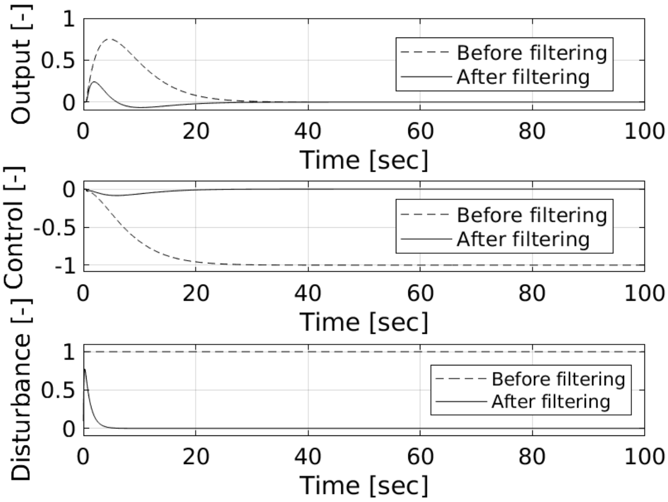

FIGURE 3. Discrete-time closed-loop experimental data using the initial PID parameters and the filtered data with sampling interval $T_s = 0.01$ s

In order to estimate the frequency response, the collected data are filtered using a bandpass filter $f(k)$, and the filtered data are plotted by the solid lines in Figure 3. The filtered data is transformed using a DFT, and the estimated frequency response is plotted in Figure 4, where the dashed lines show the true gain and phase characteristics, and the red solid lines show the estimated gain and phase characteristics. Figure 4 shows that the frequency response is sufficiently estimated.

Using the estimated frequency response, the PID parameters are decided with $M_s^d \in \{1.4, 1.6, 1.8, 2.0\}$. As an optimization calculation method, the *fmincon* function (MathWorks MATLAB R2013b) is used. The optimized PID parameters, the reference model parameters, and the M_s values are summarized in Table 1, where the estimated reference model parameters are described as a continuous-time system for readability. The average computation time for obtaining the PID parameters is 33.165 min, where CPU is Intel Core M-5Y70 Processor 1.10 GHz, Memory 8GB, and OS Windows 10 Pro 64bit. Using the obtained PID parameters, the regulator performance and robust stability are confirmed in Subsections 5.1 and 5.2, respectively.

5.1. **Regulator performance.** In Equation (20), the control results using the optimized PID parameters are shown in Figure 5, where the initial response using the initial PID

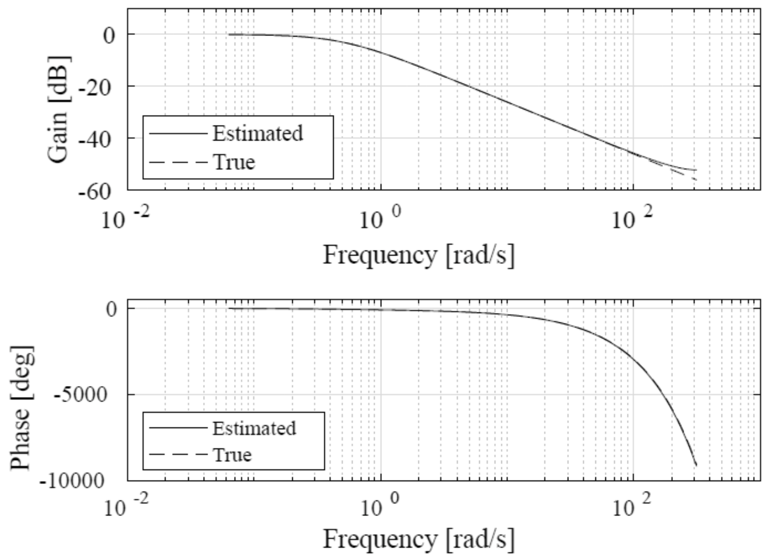

FIGURE 4. Estimated frequency response

TABLE 1. Obtained PID and reference model parameters and M_s values

M_s^d	K_p	K_i	K_d	T_0	L_0	B_0	M_s
1.4	2.005	1.085	0.3303	1.111	0.5474	0	1.400
1.6	2.589	1.505	0.5456	0.9308	0.5468	0	1.600
1.8	3.022	1.817	0.6770	0.8328	0.5578	0	1.800
2.0	3.584	2.660	0.6650	0.5640	0.6051	0	2.000

parameters is plotted by the thick-dash-dotted line, and the controlled responses using the PID parameters optimized with $M_s^d \in \{1.4, 1.6, 1.8, 2.0\}$ are plotted by the thin-dashed, thick-solid, thin-solid, and dotted lines, respectively. The proposed method is compared with the conventional method [21]. In the conventional method, the objective function is defined as follows:

$$J_{conv} = \frac{1}{N} \sum_{k=1}^{N} \left(\varepsilon^*(k)^2 + \lambda \Delta u^*(k)^2\right) \tag{21}$$

$$u^*(k) = C_e\left(z^{-1}\right) r_0(k) + C\left(z^{-1}\right) M_d\left(z^{-1}\right) r_0(k) \tag{22}$$

where λ is the weighting factor for the deviation in the control input, and $r_0(k)$ denotes the initial reference input. In Figure 5, the thick-dashed and thin-dash-dotted lines are the controlled responses using the conventional method with $\lambda \in \{10^{-2}, 10^{-1}\}$, respectively. For enhanced visibility, an enlarged view of the output responses is plotted in Figure 6. The output responses using the tuned PID parameters converge to 0 faster than the initial output response.

A quantitative evaluation of the controlled results using the following quadratic cost is shown in Table 2:

$$J_d = \frac{1}{N} \sum_{k=0}^{2.0\times 10^3} (r(k) - y(k))^2 \tag{23}$$

In the proposed method, the robust stability is directly assigned by selecting M_s^d, and the larger M_s^d is, the better the convergence response is. On the other hand, in the

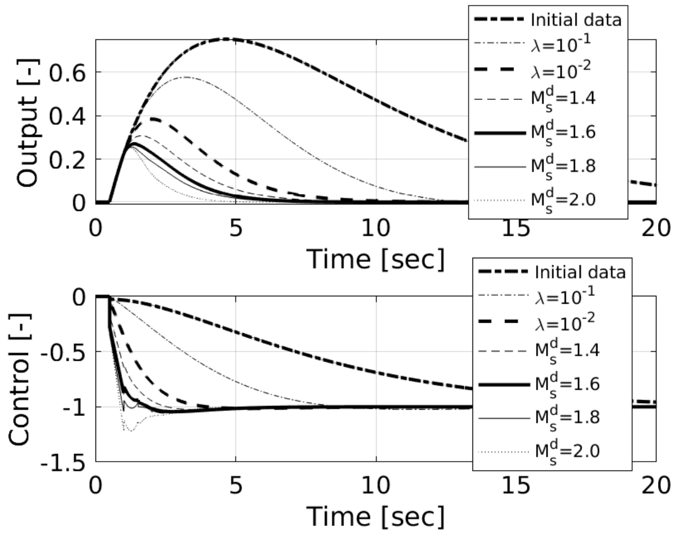

Figure 5. Output and input responses in discrete-time disturbed by the step disturbance

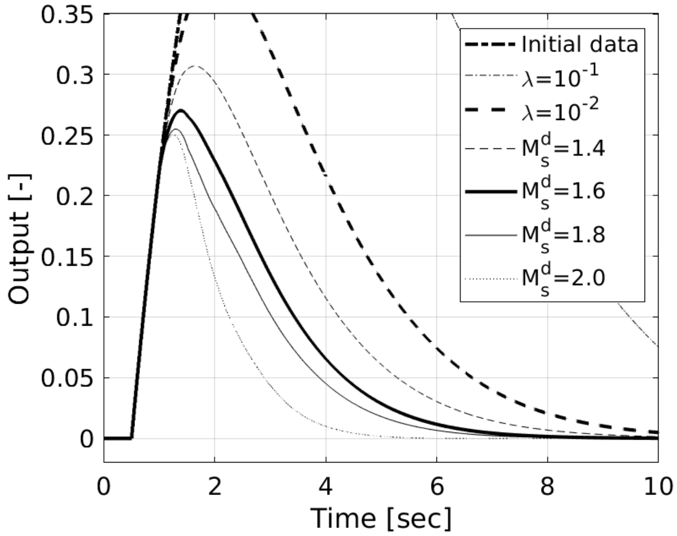

Figure 6. Enlarged view of the upper figure in Figure 5

Table 2. Regulator performance evaluation of Figure 5 using Equation (23)

Proposed method				
M_s^d	1.4	1.6	1.8	2.0
J_d	1.9×10^{-3}	1.2×10^{-3}	9.0×10^{-4}	6.2×10^{-4}

Conventional method		
λ	10^{-2}	10^{-1}
J_d	3.7×10^{-3}	1.4×10^{-2}

conventional method, the robust stability cannot be directly assigned and is adjusted by using λ. The control performance also depends on the value of λ. The smaller λ is, the better the convergence response is.

5.2. **Robust stability.** In the following scenario, the plant dynamics is changed while it is controlled, and robust stability is confirmed. The dynamics is given by Equation (20) from the start until 20 s, after which it is given by the following equations:

$$P_1(s) = \frac{1.4}{1.6s+1}e^{-0.6s} \tag{24}$$

$$P_2(s) = \frac{1.5}{1.4s+1}e^{-0.7s} \tag{25}$$

$$P_3(s) = \frac{1.5}{1.3s+1}e^{-0.7s} \tag{26}$$

$$P_4(s) = \frac{1.5}{1.2s+1}e^{-0.8s} \tag{27}$$

where the control input is disturbed by a unit step signal from the start. In this scenario, the controlled results using the optimized PID parameters in Table 1 are shown in Figure 7, Figure 8, Figure 9, and Figure 10, respectively, where the upper figure is the controlled output, and the lower figure is the control input. Figure 7 shows that all the output responses disturbed by the disturbance are well regulated after the model perturbation $P_1(s)$. On the other hand, Figure 8 shows that after the model perturbation $P_2(s)$, only the output response obtained by the proposed method with $M_s^d = 2.0$ diverges. This is because the closed-loop system with the perturbed plant is not stabilized, and the output response does not perfectly converge to 0 at 20 s. Furthermore, the controlled results for the perturbed systems with $P_3(s)$ and $P_4(s)$ are shown in Figure 9 and Figure 10, respectively. From the figures, the output responses obtained by the proposed method with $M_s^d \in \{1.8, 2.0\}$ diverge for the perturbed plant $P_3(s)$, and those of $M_s^d \in \{1.6, 1.8, 2.0\}$ also diverge for $P_4(s)$. Figure 10 shows that the closed-loop system using the proposed method with $M_s^d = 1.4$ and that of the conventional method with $\lambda \in \{10^{-2}, 10^{-1}\}$ are more robust than the other designed systems, and the output converges to 0 even when the dynamics is perturbed. Therefore, in the conventional method and the proposed method, robust stability is adjusted. However, in the conventional method, robust stability is not directly assigned. On the other hand, it is confirmed that robust stability is achieved by the proposed method, and the smaller M_s^d, the higher the robustness.

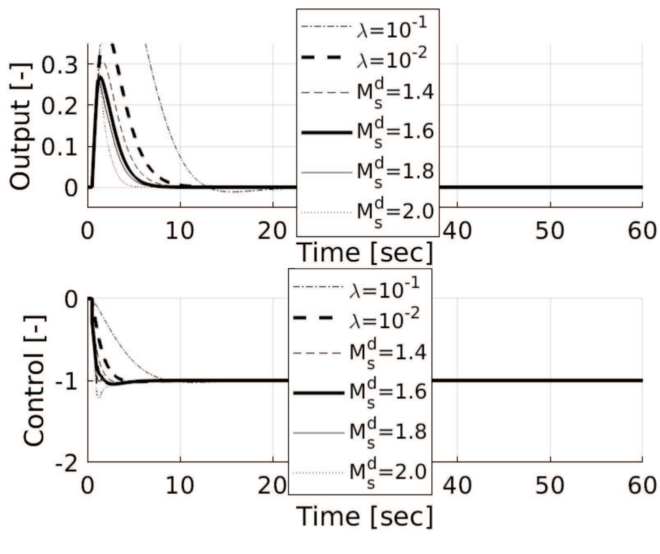

FIGURE 7. Output and input responses in discrete time, where the dynamics is changed to $P_1(s)$ after 20 s

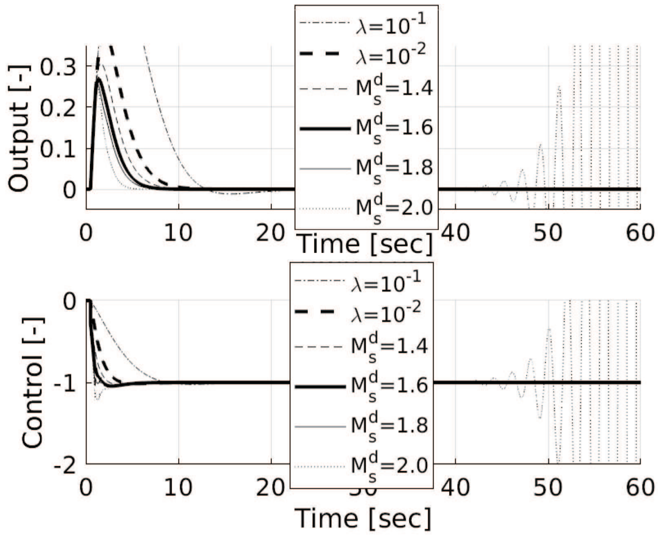

FIGURE 8. Output and input responses in discrete time, where the dynamics is changed to $P_2(s)$ after 20 s

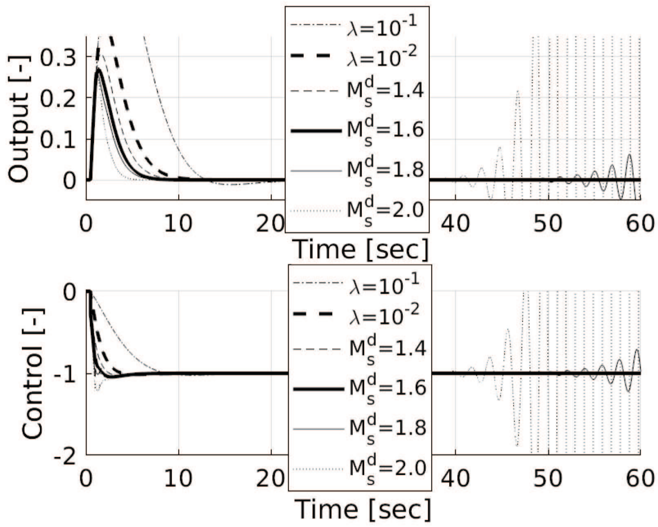

FIGURE 9. Output and input responses in discrete time, where the dynamics is changed to $P_3(s)$ after 20 s

6. **Conclusion.** The present paper proposed a new data-driven regulator design method. In the proposed method, PID controller parameters are optimized by solving a model reference problem such that a disturbance response follows a given reference model output, and the prescribed robust stability is achieved. Furthermore, a trade-off between the regulator performance and the robust stability is achieved by selecting the prescribed robust stability. From the simulation results in Subsections 5.1 and 5.2, trade-off regulator design between the tracking performance and robust stability is achieved using the proposed method.

In the proposed method, the performance is influenced by the reference model structure, and, in the future, we therefore intend to research the reference model structure design. Furthermore, the trade-off between the proposed regulator design and the conventional servo design [20] should be studied, and the noise attenuation should also be studied.

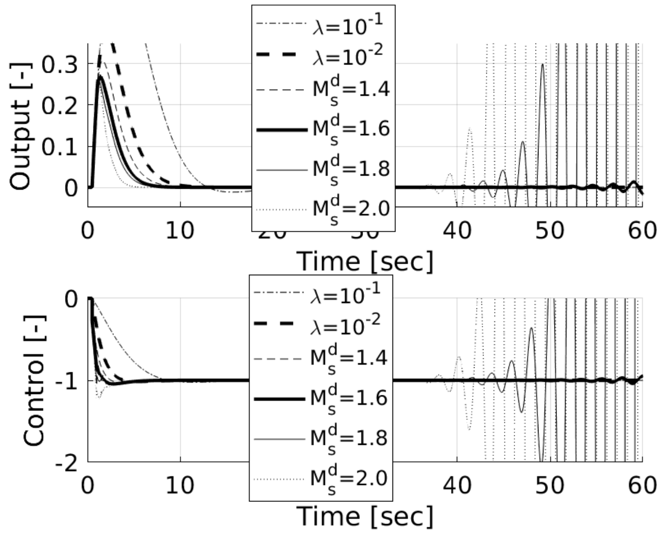

Figure 10. Output and input responses in discrete time, where the dynamics is changed to $P_4(s)$ after 20 s

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Appendix A. **Objective Function.** From Equation (1), the initial controlled data $u_0(k)$, $y_0(k)$ and $d_0(k)$ satisfy the next equation

$$y_0(k) = P_d\left(z^{-1}\right)\left(u_0(k) + d_0(k)\right) \tag{28}$$

In addition, Equation (18) gives the next equation:

$$u_0(k) = C_e\left(z^{-1}\right) r^*(k) - C\left(z^{-1}\right) y_0(k) \tag{29}$$

Substituting Equation (29) into Equation (28), the following equation is obtained:

$$\begin{aligned} y_0(k) &= P_d\left(z^{-1}\right)\left(C_e\left(z^{-1}\right) r^*(k) - C\left(z^{-1}\right) y_0(k) + d_0(k)\right) \\ &= \frac{P_d\left(z^{-1}\right) C_e\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C\left(z^{-1}\right)} r^*(k) + \frac{P_d\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C\left(z^{-1}\right)} d_0(k) \end{aligned} \tag{30}$$

From Equation (18), the initial output is rearranged as follows:

$$\begin{aligned} y_0(k) =& \frac{P_d\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C\left(z^{-1}\right)} u_0(k) + \frac{P_d\left(z^{-1}\right) C\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C\left(z^{-1}\right)} y_0(k) \\ &+ \frac{P_d\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C\left(z^{-1}\right)} d_0(k) \end{aligned} \tag{31}$$

When the optimal controller parameters that achieve the ideal disturbance response, are obtained, the next equation is satisfied:

$$M_d\left(z^{-1}\right) = \frac{P_d\left(z^{-1}\right)}{1 + P_d\left(z^{-1}\right) C^*\left(z^{-1}\right)} \tag{32}$$

where $C^*(z^{-1})$ denotes the ideal compensator of $C(z^{-1})$.

Using $C^*(z^{-1})$ instead of $C(z^{-1})$ in Equation (31), the ideal initial output data $y^*(k)$ is obtained as follows:

$$\begin{aligned} y^*(k) &= M_d\left(z^{-1}\right)u_0(k) + M_d\left(z^{-1}\right)C^*\left(z^{-1}\right)y_0(k) + M_d\left(z^{-1}\right)d_0(k) \\ &= M_d\left(z^{-1}\right)C_e^*\left(z^{-1}\right)C_e^*\left(z^{-1}\right)^{-1}\left(u_0(k) + C^*\left(z^{-1}\right)y_0(k)\right) + M_d\left(z^{-1}\right)d_0(k) \\ &= M_r\left(z^{-1}\right)r^*(k) + M_d\left(z^{-1}\right)d_0(k) \end{aligned} \tag{33}$$

where $C_e^*(z^{-1})$ denotes the ideal compensator of $C_e(z^{-1})$.

Since Equation (17) denotes the difference between the actual and ideal disturbance responses, Equation (16) is used instead of Equation (8) in the proposed method.

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