International Journal of Innovative Computing, Information and Control
Volume **18**, Number **1**, February **2022**

ICIC International ©2022 ISSN 1349-4198
pp. **289–304**

DISTRIBUTED ADAPTIVE NEURAL NETWORK CONTROL FOR A CLASS OF UNCERTAIN HETEROGENEOUS MULTI-AGENT SYSTEMS

Yongqing Fan[1,2], Xiaoyan Ren[1,2] and Zhen Li[1,2]

[1]School of Automation
[2]Xi'an Key Laboratory of Advanced Control and Intelligent Process
Xi'an University of Posts and Telecommunications
No. 618, Chang'an West Street, Chang'an District, Xi'an 710121, P. R. China
{ fanyongqing; lizhen }@xupt.edu.cn; RXYrenxiaoyan@163.com

Received July 2021; revised November 2021

Abstract. *In this study, a distributed adaptive robust control scheme of the cooperative tracking stabilization based on radial basis function neural network (RBFNN) is developed for a class of uncertain multi-agent systems with different subsystems. Multi-agent dynamical systems in followers are supposed to be different dynamical behaviors due to their different equations, and each follower has the leader system with a series of similar parameters. By using the properties of similarity among each agent, the feedback control with robust terms, coupling weights adaptive laws and the neural network weights are designed for the consensus of heterogeneous multi-agent systems, which break the limitation of existing works for heterogeneous multi-agent systems with the same structure. The states of each follower synchronize to the dynamical behavior of the leader reference model, and all signals in the closed-loop systems can be guaranteed to be uniformly ultimately bounded (UUB). Finally, by employing the relationship of undirected connected communication graphs for every multi-agent system, three simulation examples are verified by good tracking performances.*

Keywords: Radial basis function neural network (RBFNN), Distributed control, Adaptive control, Heterogeneous multi-agent systems, Uniformly ultimately bounded (UUB)

1. **Introduction.** In recent decades, multi-agent systems have become an interesting research subject in more sophisticated and intelligent demand, and have been widely applied to various engineering fields, such as flocking [1, 2], communication online planning [3], sensor networks [4], control of multi-robot systems [5], and air vehicles [6]. Because there exist lots of different complex relationships and couplings among each sub-system (e.g., high-cost of agent and impractical applications), how to design some series satisfactory cooperative controllers is very significant and tough work in the whole control process of the multi-agent systems. Recently, distributed control schemes based on each local information have been studied in [7, 8, 9, 10]. In [11, 12], a cooperative global tracking controller based on the observer was presented for multi-agent systems with unknown external interference, but the drawback of this method only can be suitable for linear multi-agent systems. Leader-following consensus is also proposed for multi-agent systems in [13, 14]. Finite-time consensus tracking control for multi-agent systems was researched in [15, 16]. In these aforementioned works, the main contribution is that the leaderless consensus and the synchronization of leader-follower systems are designed for accurate model or dynamical with some certain conditions being satisfied, but it is necessary to point out that the unstructured uncertainties of multi-agent systems are not considered

DOI: 10.24507/ijicic.18.01.289

[7, 8, 9, 10, 11, 12, 13], or some conditions need to be satisfied such as Lipschitz constant of the nonlinear functions in [14], continuous homogeneous of nonlinear functions in [15], upper right Dini derivative of nonlinear functions in [16]. Accordingly, these designed control algorithm schemes can be only utilized to some special structure of multi-agent systems.

Neural network (NN) has the effective ability of training and learning [17], so the result is that NN can be utilized to many application fields. Especially, RBFNN (radial basis function neural network) as a generalized approximation to counteract the unstructured uncertainties has been received growing concern by researchers in many different fields [18, 19, 20, 21, 22, 23]. Consequently, NNs are employed to deal with the unknown model and further realize the tracking stabilization or synchronization of multi-agent systems with unstructured uncertainties. For example, a distributed synchronization adaptive NN control based on observer was proposed for a class of multi-agent systems with uncertainties in [24]. For the tracking of stochastic multi-agent systems, a novel RBFNN adaptive control was designed to resolve the unknown terms of interactions and coupling among each agent system in [25]. Output feedback tracking controller with reduced order filters and NN technique was proposed for second-order multi-agent systems with unknown uncertainties in [26]. Unfortunately, these existing research works just only focus on the multi-agent dynamical systems with the same agent dynamics in [24, 25, 26], and the differential equation of each agent system was identical in the literature. Actually, in many real applications, the structure of every agent system is totally different in a network; in this case, the non-identical multi-agent systems commonly are called as heterogeneous multi-agent systems with different dynamical behaviors. Therefore, the control schemes in the mentioned literature will be invalid to control the stabilization or synchronization of heterogeneous multi-agent systems. According to this point, it is badly in need of some novel controls that can be exploited to not only suit for identical multi-agent systems but also be valid to heterogeneous multi-agent systems.

Recently, many researchers are very keen on the research of consensus tracking or synchronization for some class of heterogeneous multi-agent systems, for instance, multiple uncertain nonlinear strict-feedback form [27], the first-order and second-order multi-agent unknown nonlinear systems [28]. However, the control approaches only suit to the class of multi-agent systems with Lipschitz conditions [27, 28]. Besides, the specific structure of these heterogeneous multi-agent systems is another limitation for design control scheme. From the view of mathematics, there exist more similar characters of many multi-agent dynamical systems in many real networks (e.g., many identical plants of structure in power network systems [29], the electric power system with the connection of synchronous machines in [30], and other similar composite systems in [31, 32, 33]).

In this paper, motivated by the definition of similar considerations in [29, 30, 31, 32, 33], the nodes in large-scale systems are called as similar agents (identical agent is the special case), and then the control for stabilization or synchronization needs to be exploited urgently for this kind of heterogeneous multi-agent systems. It is well known that the decentralized control has been successfully used to control the stabilization or synchronization of large-scale interconnected systems, but this control approach could not be applied to the multi-agent systems owing to some couplings among each agent dynamical system [29, 30, 31, 32]. Cooperative tracking control is a popular design method for multi-agent systems, and many outstanding studied results were raised. For example, the authors studied NN adaptive control for multi-agent with unknown high-order nonlinear dynamics in [34], and the control of each follower only depended on its own states, rather than the coupling strength among every agent system. For a kind of multi-agent systems with higher-order nonlinear in [35], the synchronization control was designed, and the prescribed performance was guaranteed with the connection of weighted directed graph for every agent dynamical system. Nevertheless, it is worth to emphasize that the particular common problem is that each agent is assumed to be identical with the same dynamic behavior. The controls in these works may lose efficacy when the agent of multi-agent systems was nonidentical with difference.

Up to this point, a worthy research question arises that how to design a consensus protocol to satisfy a class of heterogeneous multi-agent systems with different structures and nonidentical dynamical behaviors, where the design control can be utilized to homogenous multi-agents but also heterogeneous multi-agents with different structures and nonidentical dynamic behaviors. Inspired by the qualities in [29, 30, 31, 32], this paper attempts to design a novel stabilization controller by using the information of each agent for heterogeneous multi-agent dynamical systems. Compared with the identical multi-agent systems, the main contributions of the proposed control are described in two aspects. Firstly, different from the control for heterogeneous multi-agent systems with the same composition in [1, 2, 5, 34, 35], a novel robust NN adaptive feedback control architecture is devised for the cooperative tracking of heterogeneous multi-agent systems with the same or different composition, which can be utilized to solve the cooperative stabilization of multi-agent systems whether each agent system is identical or nonidentical; in this case, the controller in this paper is more generalized than other existing works. Secondly, the feedback control gain can be easily obtained by solving the given Hurwitz matrix rather than complex algebraic Riccati equation of other works in the procedure of designing.

The rest of this paper is organized as follows. Section 2 describes the multi-agent systems with similar qualities of each agent system, and some definitions and preliminaries are proposed. In Section 3, the detailed procedure design of the distributed NN adaptive control with reference tracking stability analysis is presented. Three examples are simulated to demonstrate the effectiveness of the control plan in Section 4. The conclusions are summarized in Section 5.

2. **Preliminaries and System Descriptions.** In this section, it is necessary to be supplemented that the knowledge of graph theory, which will be used to demonstrate the interconnection relationship of each agent system in follower (2) and leader system (1).

Define each agent system as one node, a graph $\mathbb{G} = \{\nu, \varepsilon\}$ with the node set $\nu = \{n_1, n_2, \ldots, n_N\}$ and the edge set $\varepsilon = \{(n_i, n_j) \in \nu \times \nu\}$, where (n_i, n_j) indicates the relationship of communication from the ith node to the jth node. $\mathbb{N}_i = \{j | (n_j, n_i) \in \varepsilon\}$ denotes the ith neighbour set. Let a matrix $\mathbb{A} = [a_{ij}]_{N\times N}$, where $a_{ij} = 1$ if $(n_j, n_i) \in \varepsilon$, if otherwise, $a_{ij} = 0$. Degree matrix is defined as $\mathbb{D} = diag\{d_1, d_2, \ldots, d_N\}$, in which $d_i = \sum_{j\in\mathbb{N}_i} a_{ij}$. Therefore, the Laplacian matrix $\mathbb{L} = \mathbb{D} - \mathbb{A} \triangleq [l_{ij}]$ is associated with the undirected graph $\mathbb{G}$. For leader system, the leader adjacency matrix $\mathbb{A}_0 = diag\{a_{10}, a_{20}, \ldots, a_{N0}\}$ is denoted, where $a_{i0} > 0$ if there exists the relationship of the ith agent to the leader system directly, $a_{i0} = 0$ otherwise. Then, apparently matrix $\mathbb{H} = \mathbb{L} + \mathbb{A}_0$ is a nonnegative matrix for an appropriate matrix $\mathbb{A}_0$.

The considered heterogeneous multi-agent systems consist of one leader and N followers, whose communication topology graph is represented by $\mathbb{G}$, the dynamical model of leader is described by the following differential equation

$$\dot{x}_0 = A_0 x_0 + B_0 r(x_0, t) \tag{1}$$

where $x_0 \in R^{n\times 1}$ represents the state vector; $A_0 \in R^{n\times n}$ and $B_0 \in R^{n\times m}$ are two known constant matrices; $r(x_0, t) = K_0 x_0 + s(t)$ with $K_0 \in R^{m\times n}$, in which K_0 is chosen such that $A_0 + B_0 K_0$ satisfies Hurwitz that will be a known matrix given by designer, the remainder $s(t) \in R^{m\times 1}$ is proposed a known input signal vector with bounded.

Now, the ith follower multi-agent system is defined as

$$\dot{x}_i = A_i x_i + B_i[u_i + f_i(x_i) + \omega_i(t)], \quad i = 1, 2, \dots, N \tag{2}$$

where $x_i = [x_{i1}, \dots, x_{in}]^T$ denotes the state vector of the ith follower; control input is $u_i \in R^{m\times 1}$; $f_i(x_i) \in R^{m\times 1}$ denotes the unknown nonlinear function that will be approximated by using RBFNN; $\omega_i(t) = [\omega_{i1}, \omega_{i2}, \dots, \omega_{im}]^T$ is the unknown external disturbance vector with bounded, which means that $|\omega_{ik}| \le \bar{\omega}_{ik}$ ($k = 1, 2, \dots, m$) is satisfied, and $\bar{\omega}_{ik}$ is a known constant. In this paper, we denote maximum vector as $\bar{\omega}_i = [\bar{\omega}_{i1}, \dots, \bar{\omega}_{im}]^T$; $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are two known appropriate dimension matrices which are presented by two different from the A_0 and B_0 in (1), respectively.

Remark 2.1. *Each agent system in (2) is heterogeneous in this paper, which admits a characteristic of different form for the multi-agent systems with identical dynamical behavior in* [11, 12, 24, 25, 26, 34, 35]*. Especially, the systems followers are the same in the literature, if $A_i = A_0$ and $B_i = B_0$.*

Definition 2.1. [32]*. Suppose there exist matrices $K_i \in R^{m\times n}$ and other invertible matrices $J_i \in R^{n\times n}$, and then system (2) is called as a similar composite system with the following conditions for $i, s = 1, 2, \dots, N$*

$$\begin{cases} J_i^{-1}(A_i + B_iK_i)J_i = J_s^{-1}(A_s + B_sK_s)J_s \\ J_i^{-1}B_i = J_s^{-1}B_s \end{cases} \tag{3}$$

Based on Definition 2.1, Definition 2.2 is expounded as follows.

Definition 2.2. *The ith agent system in follower system (2) and the leader system (1) are said to be a similar composite system (1), if there exist matrices $K_i \in R^{m\times n}$ and $K_0 \in R^{m\times n}$ for invertible matrices $J_i \in R^{n\times n}$ and $J_0 \in R^{n\times n}$ such that*

$$\begin{cases} J_i^{-1}(A_i + B_iK_i)J_i = J_0^{-1}(A_0 + B_0K_0)J_0 \\ J_i^{-1}B_i = J_0^{-1}B_0 \end{cases} \tag{4}$$

where matrices pairs (J_i, K_i) and (J_0, K_0) are corresponding to the ith subsystem (2) and leader model system (1), which are defined as the transformation. All the nominal subsystems (2) are said to be similar to the leader model dynamics (1).

Remark 2.2. *By analyzing Definition 2.2, it is easily known that the ith follower is similarity equivalent to the leader system through feedback control $u_i = K_ix_i$, which means that the ith follower and the leader have some common eigenvalues. Therefore, the follower systems (2) and the leader (1) are called as similar composite systems* [32]*, where some similar dynamics behaviors exist in this class of multi-agent systems. In fact, such kind of similar composite systems can be found in real applications, for instance, two coupled inverted pendulum systems* [36, 37]*, and electric power systems* [38]*. In these applications, similar structure of each system exists because of the composing among each subsystem. Consequently, it is known that the multi-agent system in follower (2) is composed of N multi-agent systems via different linear couplings among them.*

In this paper, the uncertain model of $f_i(x_i)$ is supposed to exist in each subsystem, so the approximation of RBFNN is employed to compensate the uncertainties.

Assumption 2.1. *The uncertainty function $Y_i(x_i)$ can be approximated by the RBF neural network as the following universal approximation property:*

$$Y_i(x_i) = W_i^T \phi_i(x_i), \quad \forall x_i \in \Omega_{x_i} \tag{5}$$

where $W_i \in R^{l\times m}$ denotes the ideal weight matrix that can be updated online automatically. $\Omega_{x_i} \in R^{n\times 1}$ is the approximation domain that should be chosen large sufficiently. $\phi_i(x_i) = [\phi_{i1}(x_i), \ldots, \phi_{im}(x_i)]^T$ is the activation function vector with m denoting the number of basis functions. In general, the Gaussian function is selected as

$$\phi_i(x_i) = \exp\left[-\frac{(x_i - c_{im_i})^T(x_i - c_{im_i})}{2b_{im_i}^2}\right], \quad i = 1, 2, \ldots, l \tag{6}$$

where $\| * \|$ denotes 2-norm, $c_{im_i} = [c_{im_{i1}}, c_{im_{i2}}, \ldots, c_{im_{il}}]^T$ denotes the center of the receptive field, and b_{im_i} represents the width of Gaussian function. On a compact set Ω_{x_i}, any continuous function can be approximated by RBFNN with arbitrary accuracy ϵ_i which is written as follows:

$$Y_i(x_i) = W_i^{*T}\phi_i(x_i) + \epsilon_i \tag{7}$$

ϵ_i represents the approximation accuracy and satisfies $|\epsilon_i| \le \bar{\epsilon}_i$, with $\bar{\epsilon}_i$ as a known positive constant, and W_i^{*T} is an ideal weight vector and is defined as $W_i^{*T} = \arg\min_{W\in\Omega_W} \sup_{W\in\Omega_{x_i}} \left|Y_i(x_i) - W_i^T\phi_i(x_i)\right|$, Ω_W and Ω_{x_i} are corresponding to the compact regions of W and x_i.

If the similarity properties of Definition 2.2 and Assumption 2.1 are satisfied, the following control objective in this paper will be designed.

Control objective: For any given input signal s by designer, in order to make each agent in follower (2) track the dynamical behavior of the leader system (1) perfectly, a distributed RBFNN adaptive controller scheme is proposed such that all signals in the closed-loop follower multi-agent system (2) can be guaranteed to be uniformly ultimately bounded (UUB), and the tracking error can be satisfied with bounded small value.

3. **Distributed Adaptive RBFNN Control Design.** By using the similar character of Definition 2.2, we define the tracking error as $\sigma_i = J_i^{-1}x_i - J_0^{-1}x_0 = z_i - z_0$, and let $e_i \in R^n$ denote the local cooperative tracking error. Then, the following equation is formed

$$e_i = \sum_{k\in N_i} a_{ik}(z_i - z_k) + a_{i0}(z_i - z_0) \tag{8}$$

where a_{ik} and a_{i0} are defined as the elements of graph $\mathbb{G}$ that are shown in Section 2.

Remark 3.1. *As shown in (8), the tracking error of followers and leader has its characteristic, where every agent in systems is transformed by the similar matrices J_i and J_0. The advantage of this mapping transformation is that the consensus of each agent with nonidentical dynamical behaviors or different structures can be guaranteed. Obviously, if $J_i = I_{n_i\times n_i}$ and $J_0 = I_{n_0\times n_0}$ are satisfied, multi-agent systems (1) and (2) are equal to the systems in* [11, 12, 24, 25, 26, 34, 35].

The distributed adaptive RBFNN control is sketched as follows

$$u_i = u_{i1} + u_{i2} \tag{9}$$

where $u_{i1} = K_ix_i + F\sigma_i + c_iFe_i - \hat{W}_i^T\phi_i(x_i)$ is proposed, in which $K_ix_i + F\sigma_i + c_iFe_i$ is a linear feedback that deals with the stability of follower systems with coupling strengths tracking the leader system, and $\hat{W}_i^T\phi_i(x_i)$ ($\hat{W}_i$ is the estimation value of W_i^*) is utilized to approximate the unknown nonlinear terms in each agent system in follower (2). Another robust control $u_{i2} = \begin{cases} -\frac{\bar{B}_0^TPe_i\sum_{k=1}^m\bar{\omega}_{ik}}{\left\|\sigma^T\left[H\otimes\left(P\bar{B}_0\right)\right]\right\|}, & \bar{B}_0^TPe_i \neq 0 \\ 0, & \bar{B}_0^TPe_i = 0 \end{cases}$ is designed to eliminate the robust from the unknown external disturbance with bounded. We denote $\tilde{W}_i$ as the estimation error of W_i^*, and define $\tilde{W}_i = \hat{W}_i - W_i^*$. The adaptive law about estimation $\hat{W}_i$ is constructed as the following expression form

$$\dot{\hat{W}}_i = -\eta_{wi}\hat{W}_i + \alpha_{wi}\phi_i(x_i)e_i^T P\bar{B}_0 \tag{10}$$

$c_i \in R$ denotes the coupling weight between the subsystems which is projected as

$$\dot{c}_i = -\eta_{ci}c_i + \alpha_{ci}e_i^T P\bar{B}_0 F e_i \tag{11}$$

where the parameters η_{wi} and α_{wi} are two matrices with some positive constants, η_{ci} and α_{ci} are also positive constants which will be given by designer, and $\bar{B}_0$ is defined as shown in (12).

The control gain matrix F and positive symmetric matrix P in (9) can be obtained by solving the following Lyapunov function for any given positive matrix Q

$$P\left(\bar{A}_0 + \bar{B}_0F\right) + \left(\bar{A}_0 + \bar{B}_0F\right)^T P = -Q \tag{12}$$

where $\bar{A}_0 = J_0^{-1}(A_0 + B_0K_0)J_0$, $\bar{B}_0 = J_0^{-1}B_0$.

Substituting (9) to (2), then the multi-agent system in follower (2) becomes

$$\dot{x}_i = (A_i + B_iK_i)x_i + B_iF\sigma_i + c_iB_iFe_i + B_i\left[-\tilde{W}_i^T\phi_i(x_i) + \epsilon_i + \omega_i(t) + u_{i2}\right] \tag{13}$$

owing to the similar elements K_i, J_i, K_0, J_0 between the leader model (1) and follower (2), if we define the state transformation as $z_i = J_i^{-1}x_i$ and $z_0 = J_0^{-1}x_0$, so the leader model reference is obtained as

$$\dot{z}_0 = J_0^{-1}\dot{x}_0 = J_0^{-1}(A_0 + B_0K_0)J_0z_0 + J_0^{-1}B_0s(t) \tag{14}$$

the ith follower dynamical subsystem can be written as

$$\begin{aligned}\dot{z}_i =\ & J_i^{-1}(A_i + B_iK_i)J_iz_i + J_i^{-1}B_iF\sigma_i + c_iJ_i^{-1}B_iFe_i \\ & + J_i^{-1}B_i\left[-\tilde{W}_i^T\phi_i(x_i) + \epsilon_i + \omega_i(t) + u_{i2}\right]\end{aligned} \tag{15}$$

Now, we consider the similar property of Definition 2.2 between the leader system (1) and each multi-agent system (2), and then (15) is equal to the following result

$$\begin{aligned}\dot{z}_i =\ & J_0^{-1}(A_0 + B_0K_0)J_0z_i + J_0^{-1}B_0F\sigma_i + c_iJ_0^{-1}B_0Fe_i \\ & + J_0^{-1}B_0\left[-\tilde{W}_i^T\phi_i(x_i) + \epsilon_i + \omega_i(t) + u_{i2}\right]\end{aligned} \tag{16}$$

According to the tracking error $\sigma_i = z_i - z_0$, the time derivative respect to (14) and (16) is resulted as

$$\dot{\sigma}_i = \left(\bar{A}_0 + \bar{B}_0F\right)\sigma_i + c_i\bar{B}_0Fe_i + \bar{B}_0\left[-\tilde{W}_i^T\phi_i(x_i) + \epsilon_i + \omega_i(t) - s(t) + u_{i2}\right] \tag{17}$$

If we denote $\sigma = \left[\sigma_1^T, \ldots, \sigma_N^T\right]^T$, $\epsilon = diag\{\epsilon_1, \ldots, \epsilon_N\}$, $\tilde{W}^T = diag\left\{\tilde{W}_1^T, \ldots, \tilde{W}_N^T\right\}$, $\phi(x) = diag\{\phi_1(x_1), \ldots, \phi_N(x_N)\}$, $c = diag\{c_1, \ldots, c_N\}$, $\omega = diag\{\omega_1(t), \ldots, \omega_m(t)\}$, $\bar{S} = diag\{\underbrace{s(t), \ldots, s(t)}_{m}\}$, $u_2 = diag\{u_{12}, u_{22}, \ldots, u_{m2}\}$, then (17) can be rewritten as

$$\begin{aligned}\dot{\sigma} =\ & \left[I_N \otimes \left(\bar{A}_0 + \bar{B}_0F\right) + (cH) \otimes \left(\bar{B}_0F\right)\right]\sigma \\ & + \left(I_N \otimes \bar{B}_0\right)\left[-\tilde{W}^T\phi(x) + \epsilon + \omega - \bar{S} + u_2\right]\end{aligned} \tag{18}$$

where I_N is a unit matrix with N dimension, and $\otimes$ denotes Kronecter produce.

In terms of the control task, the following statements of Theorem 3.1 are obtained.

Theorem 3.1. *Consider the multi-agent subsystem (2) and the leader reference model (1) with similar characters K_i, J_i, K_0, J_0, if Assumption 2.1 is satisfied, the control (9) with adaptive laws (10) and (11) can promote all signals in closed-loop (18) to satisfy UUB.*

Proof: Selecting the candidate Lyapunov function as follows

$$V = \frac{1}{2}\sigma^T(H \otimes P)\sigma + \frac{1}{2}tr\left(\tilde{W}^T\alpha_w^{-1}\tilde{W}\right) + \frac{1}{2}\alpha_c^{-1}c^T c \tag{19}$$

along with the closed-loop system (18), the time derivative is obtained as

$$\begin{aligned}\dot{V} =& \frac{1}{2}\sigma^T\left\{H \otimes \left[P\left(\bar{A}_0 + \bar{B}_0F\right) + \left(\bar{A}_0 + \bar{B}_0F\right)^T P\right]\right\}\sigma + c\sigma^T H^2 \otimes \left(P\bar{B}_0F\right)\sigma \\ &+ \alpha_c^{-1}c^T\dot{c} + \sigma^T\left(H \otimes P\bar{B}_0\right)\left[-\tilde{W}^T\phi(x) + \epsilon\right] + tr\left(\tilde{W}^T\alpha_w^{-1}\dot{\hat{W}}\right) \\ &+ \sigma^T\left(H \otimes P\bar{B}_0\right)\left(\omega - \bar{S} + u_2\right)\end{aligned} \tag{20}$$

Since the following inequality holds

$$\begin{aligned}&\sigma^T\left[H \otimes \left(P\bar{B}_0\right)\right]\left(\omega - \bar{S} + u_2\right) \\ &= \sum_{i=1}^N e_i^T P\bar{B}_0\begin{bmatrix}\omega_{i1}(t) \\ \omega_{i2}(t) \\ \vdots \\ \omega_{im}(t)\end{bmatrix} - \sum_{i=1}^N\begin{cases} e_i^T P\bar{B}_0\frac{\bar{B}_0^T Pe_i\sum_{k=1}^m\bar{\omega}_{ik}}{\left\|\sigma^T\left[H\otimes\left(P\bar{B}_0\right)\right]\right\|}, & \bar{B}_0^T Pe_i \neq 0 \\ 0, & \bar{B}_0^T Pe_i = 0\end{cases} \\ &\leq \sum_{i=1}^N\left\|\sigma^T\left[H \otimes \left(P\bar{B}_0\right)\right]\right\|\left(\|\omega_i\| - \bar{\omega}_i\right) \leq 0\end{aligned} \tag{21}$$

with (21), (20) becomes that

$$\begin{aligned}\dot{V} \leq& -\frac{1}{2}\sigma^T(H \otimes Q)\sigma + \sigma^T\left[H \otimes \left(P\bar{B}_0\right)\right]\epsilon + \sum_{i=1}^N\Big[-e_i^T P\bar{B}_0\tilde{W}_i^T\phi_i(x_i) + \tilde{W}_i^T\alpha_{w_i}^{-1}\dot{\hat{W}}_i \\ &+ \alpha_{c_i}^{-1}c_i\dot{c}_i + c_ie_i^T P\bar{B}_0Fe_i\Big]\end{aligned} \tag{22}$$

Adaptive laws (10) and (11) are applied to (22), let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum and maximum eigenvalue of matrix $(\cdot)$, respectively, then it yields

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(H \otimes Q)\sigma^T\sigma + \|\sigma\| \cdot \left\|H \otimes \left(P\bar{B}_0\right)\right\|\bar{\epsilon} - \sum_{i=1}^N\frac{\eta_{ci}}{\alpha_{ci}}c_i^2 - \sum_{i=1}^N\frac{\eta_{wi}}{\alpha_{wi}}\hat{W}_i^T\tilde{W}_i \tag{23}$$

Since the following inequality holds

$$-\frac{\eta_{wi}}{\alpha_{wi}}\hat{W}_i^T\tilde{W}_i = -\frac{\eta_{wi}}{\alpha_{wi}}\tilde{W}_i^T\tilde{W}_i - \frac{\eta_{wi}}{\alpha_{wi}}W_i^T\tilde{W}_i \leq -\frac{1}{2}\frac{\eta_{wi}}{\alpha_{wi}}\tilde{W}_i^T\tilde{W}_i + \frac{1}{2}\frac{\eta_{wi}}{\alpha_{wi}}W_i^T W_i \tag{24}$$

(23) follows that

$$\begin{aligned}\dot{V} \leq& -\frac{1}{2}\lambda_{\min}(H \otimes Q)\sigma^T\sigma - \frac{\eta_c}{2\alpha_c}c^T c - \frac{1}{2}\frac{\eta_w}{\alpha_w}tr\left(\tilde{W}^T\tilde{W}\right) \\ &+ \|\sigma\| \cdot \left\|H \otimes \left(P\bar{B}_0\right)\right\|\bar{\epsilon} + \frac{1}{2}\frac{\eta_w}{\alpha_w}tr\left(W^T W\right)\end{aligned} \tag{25}$$

Denote $\chi = \min\left\{\frac{\lambda_{\min}(H\otimes Q)}{\lambda_{\max}(H\otimes P)}, \frac{\eta_c}{\alpha_c}, \frac{\eta_w}{\alpha_w}\right\}$, and $\varrho = \|\sigma\| \cdot \left\|H \otimes \left(P\bar{B}_0\right)\right\|\bar{\epsilon} + \frac{1}{2}\frac{\eta_w}{\alpha_w}tr\left(W^T W\right)$, then inequality (25) is equal to the following form

$$\dot{V} \leq -\chi V(t) + \varrho \tag{26}$$

Now, multiplying by $e^{\chi t}$ on the both sides of (26), and then integrating over $[0, t]$, it follows as

$$0 \leq V(t) \leq \left[V(0) - \frac{\varrho}{\chi}\right] e^{-\chi t} + \frac{\varrho}{\chi} \tag{27}$$

With the result of (27), it is known that every signal in the closed-loop systems can be satisfied the condition of semi-global UUB, which completes the proof for Theorem 3.1.

Remark 3.2. *From the stabilization analysis of Theorem 3.1, it is easily concluded that similar condition (4) in Definition 2.2 can be satisfied, so the followers can follow the tracks of the reference leader dynamical system with better performance.*

4. **Simulation Examples.** In this section, three different classes of multi-agent systems with their similar elements are provided to demonstrate the effectiveness of the proposed control approach.

Example 4.1. *The network topology diagram of multi-agents is shown as Figure 1.*

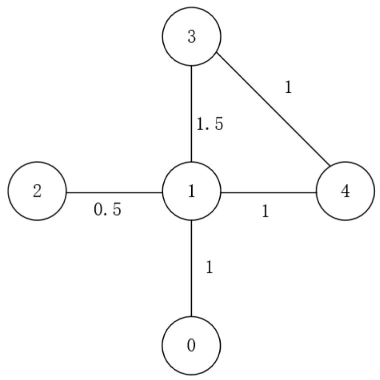


FIGURE 1. Communication graph of one leader and four followers

In order to show some advantages of the proposed control by comparing with other controllers, the dynamics of agents are all same, which are described as the following form:

$$\begin{cases} \dot{x}_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_0 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [K_0 x_0 + s(t)] \\ \dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u_i + f_i(x_i) + \omega_i(t)], \quad i = 1, 2, 3, 4 \end{cases} \tag{28}$$

where $K_0 = \begin{bmatrix} -1 & -2 \end{bmatrix}$, $s(t) = 0$. In followers $f_i(x_i) = \sin(x_i)$, $\omega_i(t)$ is defined the same as in [39], which denotes the external disturbance. Because of $A_i = A_0$ and $B_i = B_0$, the similar parameters are selected as $K_i = K_0$, $J_i = J_0$. For a positive matrix $Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, positive matrix $P = \begin{bmatrix} 2.2113 & 0.0476 \\ 0.0476 & 0.0327 \end{bmatrix}$ and control gain $F = \begin{bmatrix} -20 & -30 \end{bmatrix}$ are obtained.

The initial states of the agents are selected as $x_0(0) = [0; 0]$, $x_{i1}(0) = [5; -3; 6; 7; 2]$, $x_{i2}(0) = [2; -1; 2; 1; 3]$, and the initial values of coupling weight in (11) are chosen as $c_i(0) = 1$. For the four unknown nonlinear function $f_i(x_i)$, RBFNNs are employed to approximate these terms, 20 nodes are selected with center space $[-1; 1] \times [-1; 1]$, the width is chosen as 0.2, the parameters are selected as $\eta_{w1} = 0.03 I_{20\times20}$, $\eta_{w2} = 0.25 I_{20\times20}$, $\eta_{w3} = 0.14 I_{20\times20}$, $\eta_{w4} = 0.12 I_{20\times20}$, $\alpha_{w1} = 0.03 I_{20\times20}$, $\alpha_{w2} = 0.01 I_{20\times20}$, $\alpha_{w3} = 0.04 I_{20\times20}$, $\alpha_{w4} = 0.05 I_{20\times20}$, $\eta_{c_1} = 0.2$, $\eta_{c_2} = 0.35$, $\eta_{c_3} = 0.3$, $\eta_{c_4} = 0.4$, $\alpha_{c_1} = 0.03$, $\alpha_{c_2} = 0.05$, $\alpha_{c_3} = 0.08$, $\alpha_{c_4} = 0.06$, the simulation results are shown in Figures 2 and 3.

Figure 2 shows the comparison of the consensus by applying different control schemes. 2(a) shows the time response of consensus among the states x_{01} and x_{i1} by using the control method in [39], and 2(b) shows the result by applying the proposed control (9) in this paper. Similarly, the consensus of x_{i2} tracking with x_{02} is illustrated as 2(c) by the control design in [39], while 2(d) demonstrates the well consensus with fast speed by

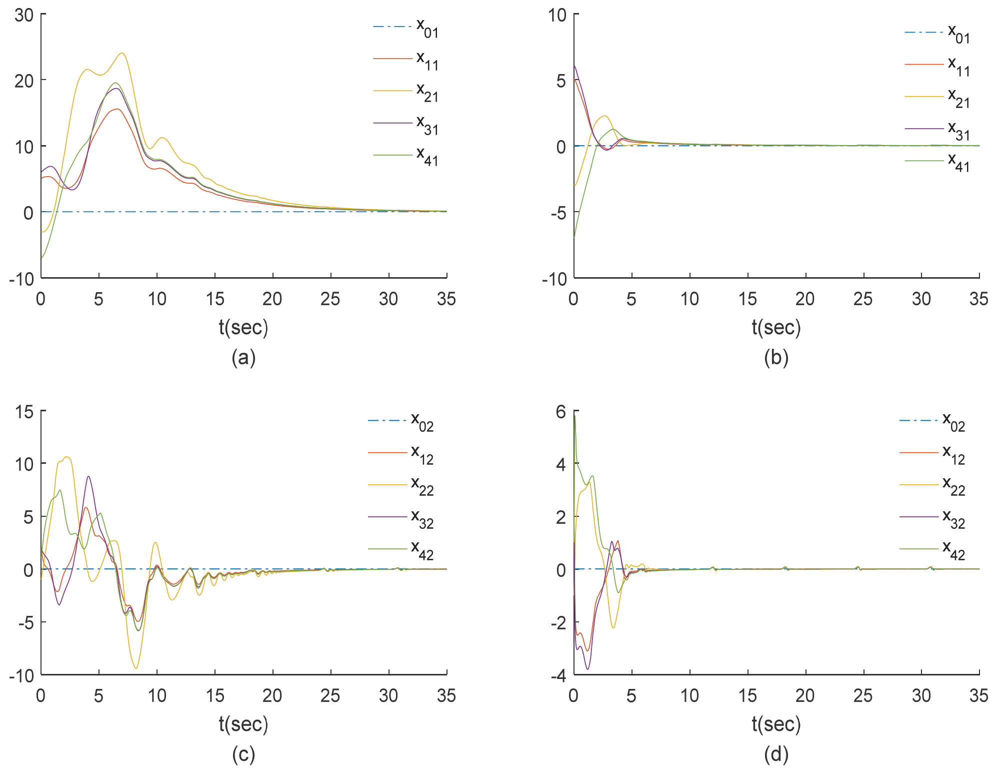


FIGURE 2. (color online) (a) Time response of x_{01} and x_{i1} in [39]; (b) time response of x_{01} and x_{i1} by controller (9); (c) time response of x_{02} and x_{i2} in [39]; (d) time response of x_{02} and x_{i2} by controller (9)

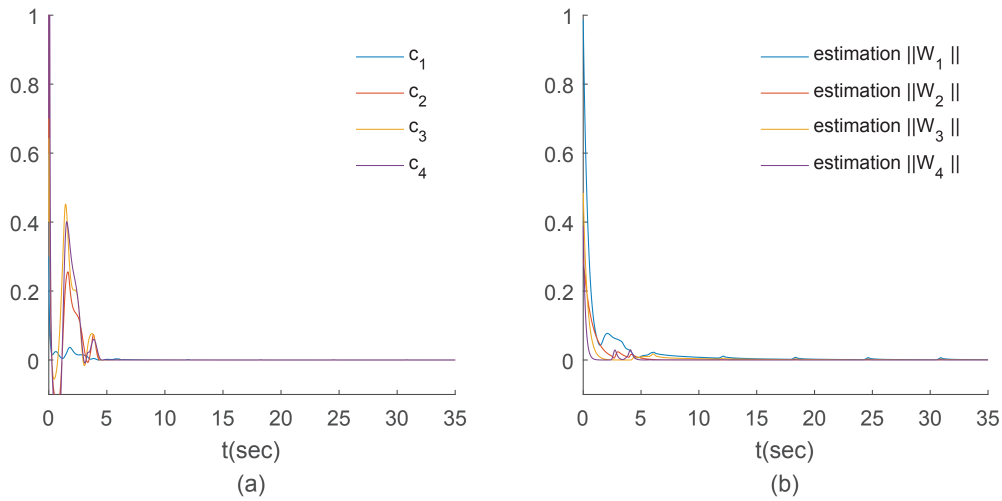


FIGURE 3. (color online) (a) Time response of couping c_i in the RBFNNs control (9); (b) the norm of estimation weights $\hat{W}_i$ in the RBFNNs control (9)

adopting the control approach (9) in this paper. These comparisons show that the states x_{i1} and x_{i2} in followers can track to the states x_{01} and x_{02} with fast velocity by employing the presented control (9). Time responses of coupling strength in control (9) are shown in Figure 3(a), and the corresponding estimation of weights is described in Figure 3(b).

If we consider six agents with one leader and five followers system, their relationship is explained by the communication graph in Figure 4. The dotted line indicates that the actual relationship between the two agent systems is virtually non-existent, which means the follower systems are controlled to track the dynamical behavior of leader system. The full line stands for the coupling strength among each agent system in follower system.

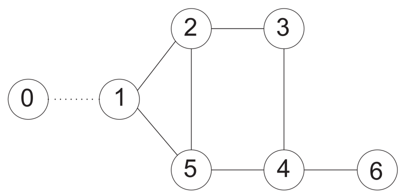


FIGURE 4. Communication graph including one leader and six followers

Example 4.2. *Different from Example 4.1, real application such as the following multiple marine surface vehicles with identical dynamical behavior is proposed*

$$\dot{x} = Ax_i + B[f_i(x_i) + u_i + \omega_i] \tag{29}$$

where $A = \begin{bmatrix} O_{3\times3} & I_{3\times3} \\ O_{3\times3} & O_{3\times3} \end{bmatrix}$, $B = \begin{bmatrix} O_{3\times3} \\ \mathcal{M}_i^{-1} \end{bmatrix}$, $\mathcal{M}_i = \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix}$, $f_i(x_i) = -\mathcal{D}_i\theta_i - \mathcal{G}_i(\theta_i)$, and $\omega_i = \mathcal{R}(\psi_i)b_i$, $\mathcal{D}_i = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0.1 \\ 0 & 0.1 & 0.5 \end{bmatrix}$. The unknown nonlinear function vector and the bounded disturbance in follower systems are chosen as $\mathcal{G}_i(x_i) = \left[-0.13x_{i4}x_{i5}^2, -0.24x_{i5}x_{i4}, -0.1x_{i6}x_{i4}\right]^T$, $\omega_i = [0.2\sin(0.6t)\cos(t), 1.9\sin(t)\cos(0.8t), -2.2\cos(t) + 0.5\sin(t)]^T$. The input reference signal of the leader system is selected as

$$s(t) = \begin{cases} [0;0;0], & 0 < t \leq 20 \\ \left[30;30;\dfrac{\pi}{5}\right], & 20 < t \leq 40 \\ [0;0;0], & 40 < t \leq 60 \end{cases} \tag{30}$$

In the light of Definition 2.2, the similar matrices between the leader and each subsystem in followers can be calculated as $J_0 = I_{6\times6}$ and $J_i = J_0$, the other similar matrix K_i is picked such that the matrix $\bar{A} = (A+BK_i)$ satisfies the condition of Hurwitz matrix as

$$K_i = \begin{bmatrix} -33.8625 & 0 & 0 \\ 0 & -44.4109 & -1.4595 \\ 0 & 0.2884 & -3.5741 \\ -24.1875 & 0 & 0 \\ 0 & -31.8326 & -1.3441 \\ 0 & 3.8996 & -2.4424 \end{bmatrix}^T$$

On the basis of the positive matrix $\bar{Q} = diag\{20, \ldots, 20\}_{6\times6}$, the positive matrix P is obtained by solving the following Lyapounov function

$$P\bar{A} + \bar{A}^T P = -\bar{Q} \tag{31}$$

$$P = \begin{bmatrix} 31.8095 & 0 & 0 & 7.6190 & 0 & 0 \\ 0 & 31.8215 & 0.0001 & 0 & 7.6115 & 0.2066 \\ 0 & 0.0001 & 31.8217 & 0 & -0.2065 & 7.6116 \\ 7.6190 & 0 & 0 & 18.7937 & 0 & 0 \\ 0 & 7.6115 & -0.2065 & 0 & 18.7856 & 0.0002 \\ 0 & 0.2066 & 7.6116 & 0 & 0.0002 & 18.7859 \end{bmatrix}$$

Initial states are given as $x_0 = [0.8, 0.2, 1, 0, 0, 0]^T$, $x_1 = [1.2, 1.4, 1.5, 0, 0, 0]^T$, $x_2 = [0.6, 1.7, 0.3, 0, 0, 0]^T$, $x_3 = [1.3, 0.4, 0.7, 0, 0, 0]^T$, $x_4 = [1.1, 0.4, 0.5, 0, 0, 0]^T$, $x_5 = [0.9, 0.8, 0.2, 0, 0, 0]^T$, $x_6 = [0.8, 0.6, 0.7, 0, 0, 0]^T$. According to the dimension of multiple marine surface vehicles (29), the uncertain nonlinearities of six sub-systems in followers are necessary to be approximated by taking advantage of some RBFNNs with 20 nodes. The center space is defined on $[-1, -1, -1, -1, -1, -1]^T \times [1, 1, 1, 1, 1, 1]^T$, and the width is defined as 0.2. The parameters in controls are selected as $\eta_{w1} = 0.05 I_{20\times20}$, $\eta_{w2} = 0.1 I_{20\times20}$, $\eta_{w3} = 0.125 I_{20\times20}$, $\eta_{w4} = 0.15 I_{20\times20}$, $\eta_{w5} = 0.175 I_{20\times20}$, $\eta_{w6} = 0.2 I_{20\times20}$, $\alpha_{w1} = 0.01 I_{20\times20}$, $\alpha_{w2} = 0.02 I_{20\times20}$, $\alpha_{w3} = 0.03 I_{20\times20}$, $\alpha_{w4} = 0.04 I_{20\times20}$, $\alpha_{w5} = 0.05 I_{20\times20}$, $\alpha_{w6} = 0.06 I_{20\times20}$. The initial values of coupling among follower sub-systems are selected as $c_1(0) = 0.7$, $c_2(0) = 0.2$, $c_3(0) = 0.6$, $c_4(0) = 0.3$, $c_5(0) = 0.5$, $c_6(0) = 0.6$, with $\eta_{c_1} = 0.1$, $\eta_{c_2} = 0.2$, $\eta_{c_3} = 0.3$, $\eta_{c_4} = 0.4$, $\eta_{c_5} = 0.5$, $\eta_{c_6} = 0.6$, $\alpha_{c_1} = 0.03$, $\alpha_{c_2} = 0.09$, $\alpha_{c_3} = 0.07$, $\alpha_{c_4} = 0.08$, $\alpha_{c_5} = 0.04$, $\alpha_{c_6} = 0.01$. Under the condition of diverse input references for three different periods of time (31), the simulation results of tracking reference are depicted in Figures 5 and 6.

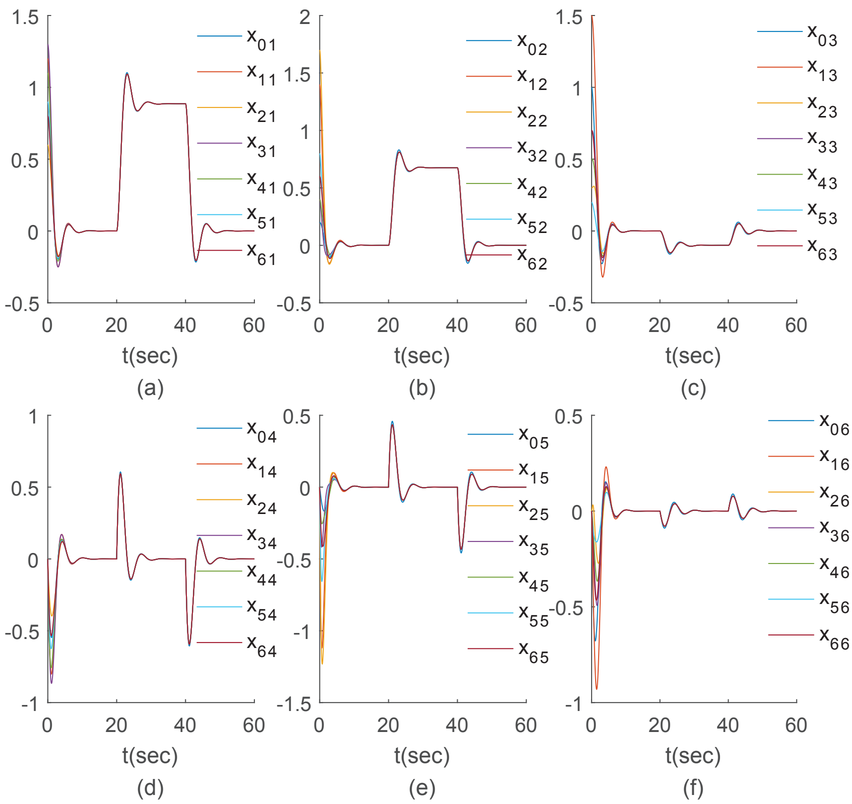


FIGURE 5. (color online) (a) Time response of follower states x_{i1} and leader x_{01}; (b) time response of follower states x_{i2} and leader x_{02}; (c) time response of follower states x_{i3} and leader x_{03}; (d) time response of follower states x_{i4} and leader x_{04}; (e) time response of follower states x_{i5} and leader x_{05}; (f) time response of follower states x_{i6} and leader x_{06}

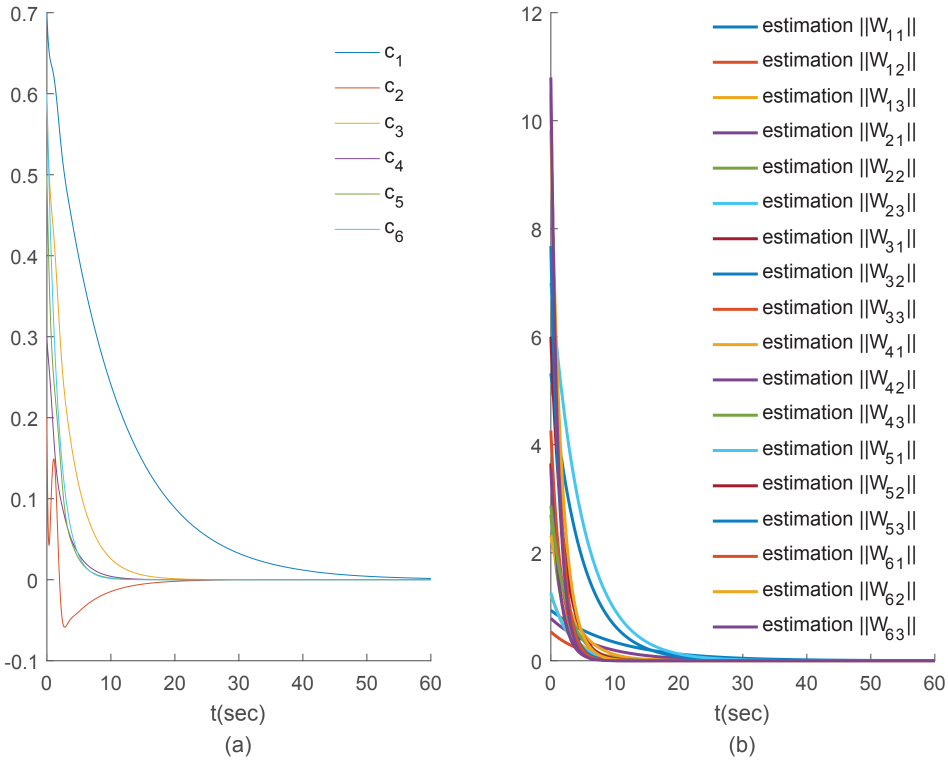


FIGURE 6. (color online) (a) The coupling strength among follower multi-agent systems; (b) the norm of estimation weight in the RBFNNs control

From Figure 5, although the input reference of leader system is changed with different time periods, it is known that good tracking performances still can be kept, the corresponding of position in each sub-system is described in Figures 5(a)-5(c), and the velocity tracking of the each state also can be maintained in Figures 5(d)-5(f). The coupling strengths adaptive tend to be stabilized at zero domain that are shown in Figure 6(a), and the norms of estimation weight vector are also bounded as the descriptions in Figure 6(b), with which all signals in the closed-loop system satisfy the condition of UUB.

Example 4.3. *The following harmonic oscillator is considered as leader system (1):*

$$A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad K_0 = \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad s(t) = 0$$

The initial state is chosen as $x_0(0) = [1, 2]^T$, the state trajectories of dynamical leader system are shown in Figure 7(a). The trajectories of the two states in leader system are exhibited as the phase plane in Figure 7(b).

In this example, the different matrices in each heterogeneous agent of the followers are listed as $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$, $A_4 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$, $A_5 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$, $A_6 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$, $B_i = B_0$ $(i = 1, 2, 3, 4, 5, 6)$, $f_i = x_{i1}x_{i2}^2 + x_{i2}\sin(x_{i1})$, $\omega_1(t) = 0.3\sin(t)\cos(2t)$, $\omega_2(t) = 0.9\sin(2t)\cos(t)$, $\omega_3(t) = 1.2\cos(t) + \sin(t)$, $\omega_4(t) = 0.5\sin(3t)\cos(5t)$, $\omega_5(t) = \cos(2t) + \sin(4t)$, $\omega_6(t) = 0.6\cos(t) + \sin(3t)$. We know that $\bar{\omega}_1 = 0.3$, $\bar{\omega}_2 = 0.9$, $\bar{\omega}_3 = 2.2$, $\bar{\omega}_4 = 0.5$, $\bar{\omega}_5 = 2$, $\bar{\omega}_6 = 1.6$. With these known matrices in this example, the distributed adaptive control in [24] will be invalid because it only can be used to the identical matrix in every agent, but this example is proposed with different matrices. Obviously, compared with [24], the proposed control method in this paper has

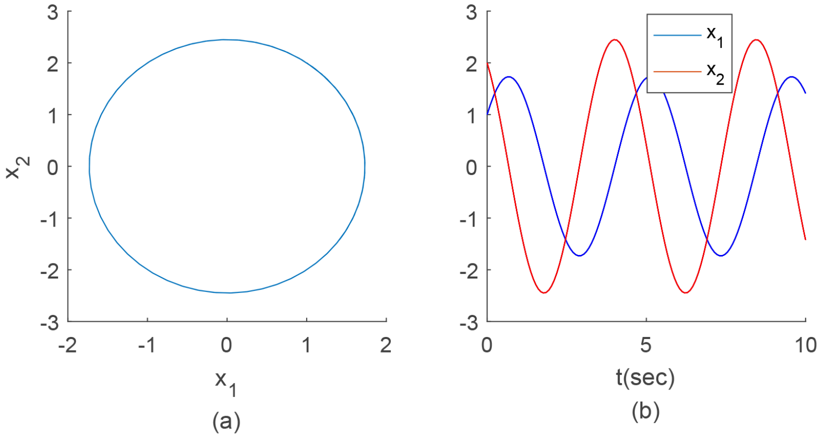


FIGURE 7. (a) Phase-plane diagram of the harmonic oscillator; (b) states time response of the harmonic oscillator

some novel advantages. Besides that, it is noted the similar information between each multi-agent system and the leader system as shown in Definition 2.2, and then these similar elements about matrices or vectors can be obtained as $J_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J_i = J_0$, $F_0 = r$, $F_1 = \begin{bmatrix} -3 & 0 \end{bmatrix}$, $F_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $F_3 = \begin{bmatrix} -1 & -1 \end{bmatrix}$, $F_4 = \begin{bmatrix} -1 & -2 \end{bmatrix}$, $F_5 = \begin{bmatrix} 0 & -1 \end{bmatrix}$, $F_6 = \begin{bmatrix} -1 & 1 \end{bmatrix}$. The six initial state vectors in follower sub-systems are provided as $x_1 = [1.1, 1.8]^T$, $x_2 = [1.4, 2.2]^T$, $x_3 = [0.5, 1.6]^T$, $x_4 = [0.6, 1.5]^T$, $x_5 = [0.8, 1.7]^T$, $x_6 = [1.2, 0.7]^T$. The original values of the coupling strength are chosen as $c_1(0) = 0.2$, $c_2(0) = 0.6$; $c_3(0) = 0.9$; $c_4(0) = 0.3$; $c_5(0) = 0.8$; $c_6(0) = 0.5$. Six RBFNNs are utilized to compensate for the six unknown nonlinear functions of the each sub-system in followers, for each RBFNN, 20 nodes are chosen with center space $[-1, 1]^T \times [-1, 1]^T$, and the width and other parameters in adaptive laws are chosen as Example 4.2. Applying the designed control scheme, the simulation results are depicted in Figure 8.

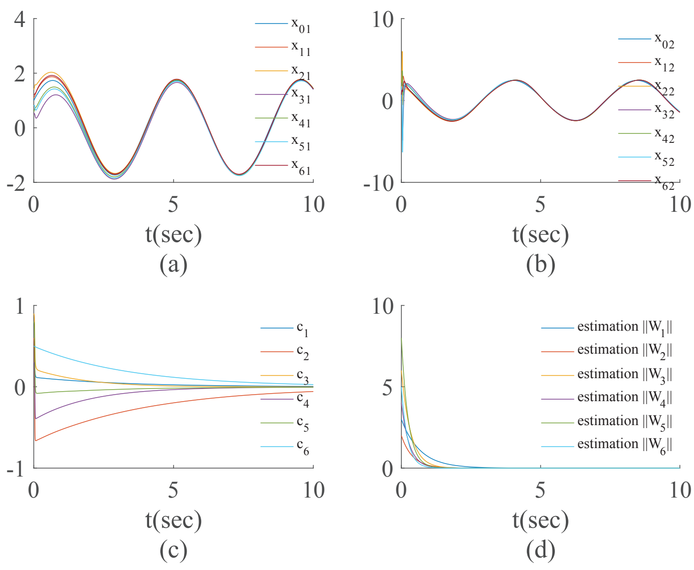


FIGURE 8. (color online) (a) Time response of follower states x_{i1} and leader x_{01}; (b) time response of follower states x_{i2} and leader x_{02}; (c) time response of coupling strength; (d) the estimation weight norm in the RBFNNs control

Although the multi-agents in followers are heterogeneous, the two figures (a) and (b) in Figure 8 show that the two states of each multi-agent system in follower can track the dynamical behavior of the given leader system with good tracking. Figure 8(c) describes the time response of the couplings strength that all can be ensured to be bounded, and the estimation weight vectors in RBFNNs can be guaranteed to be UUB as shown in Figure 8(d).

5. **Conclusions.** A distributed RBFNN control scheme is designed to track the dynamical behavior of leader system for follower multi-agent system with similar characteristics. The proposed distributed controls are constructed by the series of similar matrices or vectors, and the stabilization criteria are derived by the design control gain of linear feedback and robust terms of the unknown external disturbance with bounded. The effectiveness of the control scheme has been verified by three kinds of examples. In future, the control design for multi-agent systems with different dimensions is our further work.

Acknowledgment. This work is supported by the National Natural Science Foundation of China under Grant (62003262, 51875457, 61903298), Shaanxi Provincial Department of Science and Technology key project in the field of industry (2018ZDXM-GY-039), National Natural Science Foundation of Shaanxi under Grant 2019JQ-341, Shaanxi Provincial Scientific and Technological Activities for Overseas Staff Preferential Projects under Grant 35. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] Z. Chen, H.-T. Zhang, M.-C. Fan, D. Wang and D. Li, Algorithms and experiments on flocking of multiagents in a bounded space, *IEEE Transactions on Control Systems Technology*, vol.22, no.4, pp.1544-1549, 2014.

[2] R. Olfati-Saber, Flocking for multi-agent dynamic systems: Algorithms and theory, *IEEE Transactions on Automatic Control*, vol.51, no.3, pp.401-420, 2006.

[3] F. Wu, S. Zilberstein and X. Chen, Online planning for multi-agent systems with bounded communication, *Artificial Intelligence*, vol.175, no.2, pp.487-511, 2011.

[4] C.-K. Tham and J.-C. Renaud, Multi-agent systems on sensor networks: A distributed reinforcement learning approach, *2005 International Conference on Intelligent Sensors, Sensor Networks and Information Processing*, pp.423-429, 2005.

[5] T. L. Huntsberger, A. Trebi-Ollennu, H. Aghazarian, P. S. Schenker, P. Pirjanian and H. D. Nayar, Distributed control of multi-robot systems engaged in tightly coupled tasks, *Autonomous Robots*, vol.17, no.1, pp.79-92, 2004.

[6] M. G. Earl and R. D'Andrea, A decomposition approach to multi-vehicle cooperative control, *Robotics and Autonomous Systems*, vol.55, no.4, pp.276-291, 2007.

[7] J. A. Fax and R. M. Murray, Information flow and cooperative control of vehicle formations, *IEEE Transactions on Automatic Control*, vol.49, no.9, pp.1465-1476, 2004.

[8] W. Yu, W. Ren, W. X. Zheng, G. Chen and J. Lü, Distributed control gains design for consensus in multi-agent systems with second-order nonlinear dynamics, *Automatica*, vol.49, no.7, pp.2107-2115, 2013.

[9] X. Xu, Z. Li and L. Gao, Distributed adaptive tracking control for multi-agent systems with uncertain dynamics, *Nonlinear Dynamics*, vol.90, no.4, pp.2729-2744, 2017.

[10] Q. Wang, J. Fu and J. Wang, Fully distributed containment control of high-order multi-agent systems with nonlinear dynamics, *Systems & Control Letters*, vol.99, pp.33-39, 2017.

[11] J.-W. Zhu, G.-H. Yang, W.-A. Zhang and L. Yu, Cooperative tracking control for linear multi-agent systems with external disturbances under a directed graph, *International Journal of Systems Science*, vol.48, no.13, pp.2683-2691, 2017.

[12] J.-W. Zhu, G.-H. Yang, W.-A. Zhang and L. Yu, Cooperative fault tolerant tracking control for multiagent systems: An intermediate estimator-based approach, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2017.2753383, 2017.

[13] M. Zhao, C. Peng, W. He and Y. Song, Event-triggered communication for leader-following consensus of second-order multiagent systems, *IEEE Transactions on Cybernetics*, vol.48, no.6, pp.1888-1897, 2018.

[14] W. Zou, Z. Xiang and C. K. Ahn, Mean square leader-following consensus of second-order nonlinear multiagent systems with noises and unmodeled dynamics, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.49, no.12, pp.2478-2486, 2019.

[15] X. Li, M. Z. Chen and H. Su, Finite-time consensus of second-order multi-agent systems via a structural approach, *Journal of the Franklin Institute*, vol.353, no.15, pp.3876-3896, 2016.

[16] F. Sun, W. Zhu, Y. Li and F. Liu, Finite-time consensus problem of multi-agent systems with disturbance, *Journal of the Franklin Institute*, vol.353, no.12, pp.2576-2587, 2016.

[17] D. Kurniadi, E. Abdurachman, H. L. H. S. Warnars and W. Suparta, Predicting student performance with multi-level representation in an intelligent academic recommender system using backpropagation neural network, *ICIC Express Letters, Part B: Applications*, vol.12, no.10, pp.883-890, 2021.

[18] C. Yang, Y. Jiang, Z. Li, W. He and C.-Y. Su, Neural control of bimanual robots with guaranteed global stability and motion precision, *IEEE Transactions on Industrial Informatics*, vol.13, no.3, pp.1162-1171, 2017.

[19] Y.-J. Liu, S. Lu and S. Tong, Neural network controller design for an uncertain robot with time-varying output constraint, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.47, no.8, pp.2060-2068, 2017.

[20] C. Yang, X. Wang, Z. Li, Y. Li and C.-Y. Su, Teleoperation control based on combination of wave variable and neural networks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.47, no.8, pp.2125-2136, 2017.

[21] F. Wang, B. Chen, C. Lin, J. Zhang and X. Meng, Adaptive neural network finite-time output feedback control of quantized nonlinear systems, *IEEE Transactions on Cybernetics*, vol.48, no.6, pp.1839-1848, 2018.

[22] C. Yang, X. Wang, L. Cheng and H. Ma, Neural-learning-based telerobot control with guaranteed performance, *IEEE Transactions on Cybernetics*, vol.47, no.10, pp.3148-3159, 2017.

[23] K. Lu, Z. Liu, C. L. P. Chen and Y. Zhang, Event-triggered neural control of nonlinear systems with rate-dependent hysteresis input based on a new filter, *IEEE Transactions on Neural Networks and Learning Systems*, vol.31, no.4, pp.1270-1284, 2020.

[24] Z. Peng, D. Wang, H. Zhang and G. Sun, Distributed neural network control for adaptive synchronization of uncertain dynamical multiagent systems, *IEEE Transactions on Neural Networks and Learning Systems*, vol.25, no.8, pp.1508-1519, 2014.

[25] F. Wang, B. Chen, C. Lin and X. Li, Distributed adaptive neural control for stochastic nonlinear multiagent systems, *IEEE Transactions on Cybernetics*, vol.47, no.7, pp.1795-1803, 2017.

[26] L. Zhao and Y. Jia, Neural network-based adaptive consensus tracking control for multi-agent systems under actuator faults, *International Journal of Systems Science*, vol.47, no.8, pp.1931-1942, 2016.

[27] J. Chen, J. Li and X. Yuan, Global fuzzy adaptive consensus control of unknown nonlinear multiagent systems, *IEEE Transactions on Fuzzy Systems*, vol.28, no.3, pp.510-522, 2020.

[28] W. Wang and S. Tong, Observer-based adaptive fuzzy containment control for multiple uncertain nonlinear systems, *IEEE Transactions on Fuzzy Systems*, vol.27, no.11, pp.2079-2089, 2019.

[29] C. De Araujo and J. De Castro, Application of power system stabilisers in a plant with identical units, *Generation Transmission & Distribution IEE Proceedings C*, vol.138, no.1, pp.11-18, 1991.

[30] L. Bakule and J. Lunze, Decentralized design of feedback control for large-scale systems, *Kybernetika*, vol.24, no.8, pp.1-3, 1988.

[31] X.-G. Yan and G.-Z. Dai, Technical communique: Decentralized output feedback robust control for nonlinear large-scale systems, *Automatica*, vol.34, no.11, pp.1469-1472, 1998.

[32] X.-G. Yan, J. Lam and G.-Z. Dai, Decentralized stabilization for nonlinear similar composite systems with uncertainty, *IFAC World Congress*, vol.32, no.2, pp.3445-3449, 1999.

[33] Y.-Q. Fan, Y.-H. Wang, Y. Zhang and Q.-R. Wang, The synchronization of complex dynamical networks with similar nodes and coupling time-delay, *Applied Mathematics and Computation*, vol.219, no.12, pp.6719-6728, 2013.

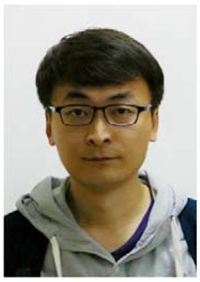
[34] P. Shi and Q. Shen, Cooperative control of multi-agent systems with unknown state-dependent controlling effects, *IEEE Transactions on Automation Science and Engineering*, vol.12, no.3, pp.827-834, 2015.

[35] H. A. Hashim, S. El-Ferik and F. L. Lewis, Neuro-adaptive cooperative tracking control with prescribed performance of unknown higher-order nonlinear multi-agent systems, *International Journal of Control*, DOI: https://doi.org/10.1080/00207179.2017.1359422, 2017.
[36] L. Shi and S. K. Singh, Decentralized control for interconnected uncertain systems: Extensions to higher-order uncertainties, *International Journal of Control*, vol.57, no.6, pp.1453-1468, 1993.
[37] P. Koluda, P. Perlikowski, K. Czolczynski and T. Kapitaniak, Synchronization configurations of two coupled double pendula, *Communications in Nonlinear Science and Numerical Simulation*, vol.19, no.4, pp.977-990, 2014.
[38] M. K. Sundareshan and R. M. Elbanna, Qualitative analysis and decentralized controller synthesis for a class of large-scale systems with symmetrically interconnected subsystems, *Automatica*, vol.27, no.2, pp.383-388, 1991.
[39] Y. Zhou, X. Yu, C. Sun and W. Yu, Robust synchronisation of second-order multi-agent system via pinning control, *IET Control Theory & Applications*, vol.9, no.5, pp.775-783, 2015.

Author Biography

Yongqing Fan received her M.S. degree in School of Applied Mathematics from Guangdong University of Technology, Guangzhou, China, in 2009, and the Ph.D. degree in School of Automation, Guangdong University of Technology, Guangzhou, China, in 2012. From 2017 to 2018, she was an academic visitor with Swansea University, Swansea, U.K. She is currently an associate professor with the School of Automation, Xi'an University of Posts and Telecommunications, Xi'an, China. Her research interests include robot systems, fuzzy adaptive control, neural network adaptive control, nonlinear multi-agent systems, nonlinear systems and dynamical networks control.

Xiaoyan Ren received her B.S. degree from Henan University of Engineering in 2019. Currently she is pursuing the M.S. degree in control theory and control engineering from, School of Automation, Xi'an University of Posts and Telecommunications, China. Her research interests include fuzzy control, neural network control, intelligent control and their applications.



Zhen Li received the B.S. and M.S. degrees in fundamental mathematics from Yangzhou University, Jiangsu, China, in 2009 and 2013, and the Ph.D. degree in control science and engineering from Donghua University, Shanghai, China, in 2016, respectively. He is currently a lecturer with the School of Automation, Xi'an University of Posts and Telecommunications, China. His current research interests include hybrid dynamical systems, multiagent systems, and distributed control systems.